Division Of Algebraic Expressions
Ex-8.1

8. DIVISION OF ALGEBRAIC EXPRESSIONS.

1. Degree of a polynomial in one Variable:-

In a polynomial in one variable, the highest power of the variable is called its degree.

(i) $2x^3+5x^2-7$.

highest power of a polynomial $=3$.

degree $=3$.

(ii) $5x^2+3x+2$.

highest power of a polynomial $=2$.

degree $=2$.

(iii) $2x+x^2-8$.

highest power of a polynomial $=2$.

degree $=2$.

(iv) $\frac{1}{2}y^7-12y^6+48y^5-10$.

highest power of a polynomial $=7$.

degree $=7$.

(v) $3x^3+1$

highest power of a polynomial $=3$.

degree $=3$.

(vi) 5

highest power of a polynomial $=0$, degree $=0$.

(vii) $20x^3+12x^2y^2x-10y^2+20$.

highest power of a polymqal $=4$, degree $=4$.

2. Polynamials :-

An algebraic expression in which the variables involved have only non-negative integral powers, is called a polynomial.

In the given expressions

(i), (iv), (v) are not polynomials.

because. The expressions in which the variables involved have only non-negative integral powers.

3. (i) $3+6x+x^2+5x^4$ (or) $5x^4+x^2+6x+3$.

degree = 4.

(ii) $4+a^2+5a^6$ (or) $5a^6+a^2+4$.

degree = 6.

(iii) x^6-5x^3+4 (or) $4-5x^3+x^6$

degree = 6.

(iv) y^6+9y^3-22 (or) $-22+9y^3+y^6$

degree = 6.

(v) $a^6+\frac{27}{136}a^3-\frac{48}{136}$ (or), $-\frac{48}{136}+\frac{27}{136}a^3+a^6$.

degree = 6.

(vi) $a^2+\frac{25}{12}a+1$ (or), $1+\frac{25}{12}a+a^2$

degree = 2.

Division Of Algebraic Expressions

Ex 8.2

Exercise - 8.2.

Divide:

1. $6x^3y^2z^2$ by $3x^2yz$.

we have, $\Rightarrow \dfrac{6x^3y^2z^2}{3x^2yz} = 2xyz$.

$$\frac{6x^3y^2z^2}{3x^2yz} = 2xyz$$

2. $15m^2n^2$ by $5m^2n^2$

$$\frac{15m^2n^3}{5m^2n^2} = \frac{15.m.m.n.n.n}{5.m.m.n.n}$$

$$= 3n.$$

3. $24a^3b^3$ by $-8ab$.

$$\frac{24a^3b^3}{-8ab} = \frac{24.a.a.a.b.b.b}{-8.a.b}$$

$$= -3a^2b^2.$$

4. $-21abc^2$ by $7abc$.

$$\frac{-21abc^2}{7abc} = \frac{-21a.b.c.c}{7.a.b.c}$$

$$= -3c.$$

5. $72xyz^2$ by $-9xz$

$$\frac{72xyz^2}{-9xz} = \frac{8\times 9.x.y.z.z}{-9.x.z} = -8yz.$$

Solution-06:-

$$\frac{-72a^4b^5c^8}{-9a^2b^2c^3} = \frac{+72\cdot a\cdot a\cdot a\cdot a\cdot b\cdot b\cdot b\cdot b\cdot b\cdot c\cdot c\cdot c\cdot c\cdot c\cdot c\cdot c\cdot c}{+9\cdot a\cdot a\cdot b\cdot b\cdot c\cdot c\cdot c}$$

$$= 8a^2b^3c^5$$

Solution-07:-

$$\frac{16m^3y^2}{4m^2y} = \frac{16\cdot m\cdot m\cdot m\cdot y\cdot y}{4\cdot m\cdot m\cdot y}$$

$$= 4m\cdot y.$$

Solution-08:-

$$\frac{32m^2n^3p^2}{4m.n.p} = \frac{32\ m\cdot m\cdot n\cdot n\cdot n\cdot p\cdot p}{4\cdot m\cdot n\cdot p}$$

$$= 8mn^2p.$$

Division Of Algebraic Expressions
Ex 8.3

Exercise - 8.3

Divide.

1. $x+2x^2+3x^4-x^5$ by $2x$

$$\frac{x+2x^2+3x^4-x^5}{2x} = \frac{x(1+2x+3x^3-x^4)}{2x}$$

$$= \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^3.x}{2.x} - \frac{x^4.x}{2x}$$

$$= \frac{1}{2} + x + \frac{3}{2}x^3 - \frac{1}{2}x^4.$$

2. $y^4-3y^3+\frac{1}{2}y^2$ by $3y$.

$$\frac{y^4-3y^3+\frac{1}{2}y^2}{3y} = \frac{y^4}{3y} - \frac{3y^3}{3y} + \frac{1}{2}.\frac{y^2}{3.y}$$

$$= \frac{1}{3}.y^3 - y^2 + \frac{1}{6}.y.$$

$\boxed{3.\ -2a^2+2a+\frac{1}{2}}$

3. $-4a^3+4a^2+a$ by $2a$.

$$\frac{-4a^3+4a^2+a}{2a} = \frac{-4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a}$$

$$= -2a^2+2a+\frac{1}{2}.$$

4. $-x^6+2x^4+4x^3+2x^2$ by $\sqrt{2}\ x^2$.

$$\frac{-x^6+2x^4+4x^3+2x^2}{\sqrt{2}\ x^2} = \frac{-x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} + \frac{2x^2}{\sqrt{2}x^2}$$

$$= \frac{-1}{\sqrt{2}}x^4 + \sqrt{2}x^2 + 2\sqrt{2}x + \sqrt{2}.$$

5. $5z^3-6z^2+7z$ by $2z$.

$$\frac{5z^3-6z^2+7z}{2z} = \frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z}$$

$$= \frac{5}{2}z^2 - 3z + \frac{7}{2}.$$

6. $\sqrt{3}a^4+2\sqrt{3}\ a^3+3a^2-6a$ by $3a$.

$$\frac{\sqrt{3}a^4+2\sqrt{3}\ a^3+3a^2-6a}{3a} = \frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{3a^2}{3a} - \frac{6a}{3a}$$

$$= \frac{1}{\sqrt{3}}\ a^2 + \frac{2}{\sqrt{3}}a^2 + a - 2.$$

Division Of Algebraic Expressions

Ex 8.4

Exercise-8.4.

Divide.

1. $5x^3 - 15x^2 + 25x$ by $5x$.

$$\frac{5x^3 - 15x^2 + 25x}{5x} = \frac{5x^3}{5x} + \left(\frac{-15}{5}\right)\cdot\frac{x^2}{x} + \frac{25}{5}\cdot\frac{x}{x}$$

$$= x^2 - 3x + 5.$$

2. $4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$

$$\frac{4z^3 + 6z^2 - z}{-\frac{1}{2}z} = \frac{4z^3.(2)}{-z} - \frac{6z^2.2}{z} + \frac{z.2}{z}$$

$$= -8z^2 - 12z + 2.$$

3. $9x^2y - 6xy + 12xy^2$ by $\frac{-3}{2}xy$.

$$\frac{9x^2y - 6xy + 12xy^2}{-\frac{3}{2}xy} = \frac{9x^2y}{-3x.y}\cdot 2 + \frac{6xy.2}{3.x.y} + \frac{12x.y^2.2}{-3xy}$$

$$= -6x + 4 - 8y.$$

4. $3x^3.y^2 + 2x^2.y + 15xy$ by $3xy$.

$$\frac{3x^3.y^2 + 2x^2.y + 15xy}{3xy} = \frac{3.x^3.y^2}{3xy} + \frac{2x^2.y}{3xy} + \frac{15xy}{3xy}$$

$$= x^2y + \frac{2}{3}x + 5.$$

5. $x^2+7x+12$ by $x+4$.

step 1:-

we devide the first term x^2 of the dividend by the first term x of the divisor and obtain $\frac{x^2}{x}=x$ as the first term of the quotient.

$$
\begin{array}{r|l}
 & \quad x+3 \\
\hline
x+4 & x^2+7x+12 \\
 & \underline{x^2+4x} \\
 & \quad 3x+12 \\
 & \quad \underline{3x+12} \\
 & \qquad 0
\end{array}
$$

step -2 :-

we multiply the divisor $x+4$ by the first term x of the quotient and subtract the result from the dividend $x^2+7x+12$. we obtain $3x+12$ as the remainder

step-3 :-

Now we treat $3x+12$ as the new dividend and divide the first term $3x$ by the first term x of the divisor to obtain $\frac{3x}{x}=3$ as the third term of the quotient.

step- IV:-

we multiply the divisor $x+4$ and the ~~third~~ Second term 3 of the quotient and subtract the resur $3x+12$ from the new dividena. we obtain 0 as the remainder.

Thus, we can say that

$$\frac{x^2+7x+12}{x+4} = x+3.$$

Solution-06:-

$4y^2+3y+\frac{1}{2}$ by $2y+1$

$$\begin{array}{r|l} & 2y+\frac{1}{2} \\ \hline 2y+1 & 4y^2+3y+\frac{1}{2} \\ & \underline{-4y^2 - 2y} \\ & \quad y+\frac{1}{2} \\ & \quad \underline{y+\frac{1}{2}} \\ & \quad (0) \end{array}$$

Solution-07.

$3x^3+4x^2+5x+18$ by $x+2$.

$$\begin{array}{r|l} & 3x^2-2x+9 \\ \hline x+2 & 3x^3+4x^2+5x+18 \\ & \underline{3x^3+6x^2} \\ & \quad -2x^2+5x \\ & \quad \underline{\mp 2x^2 \mp 4x} \\ & \qquad 9x+18 \\ & \qquad \underline{9x+18} \\ & \qquad (0) \end{array}$$

Solution-08:-

$$\begin{array}{r|l} & 2x-5 \\ \hline 7x-9 & 14x^2-53x+45 \\ & \underline{-14x^2 \mp 18x} \\ & \quad -35x+45 \\ & \quad \underline{\mp 35x \pm 45} \\ & \quad (0) \end{array}$$

Solution - 09.

$$\frac{-(-21+71x-31x^2-24x^3)}{-(3-8x)} = \frac{21-71x+31x^2+24x^3}{8x-3}$$

$$\begin{array}{r|l} & 3x^2+5x-7. \\ \hline 8x-3 & 24x^3+31x^2-71x+21 \\ & \underline{\overset{-}{2}4x^3 \overset{+}{-}9x^2} \\ & \quad 40x^2-71x \\ & \quad \underline{\overset{-}{4}0x^2 \overset{+}{-}15x} \\ & \qquad -56x+21 \\ & \qquad \underline{\overset{+}{-}56x \pm 21} \\ & \qquad\quad 0 \end{array}$$

Solution-10:-

$3y^4-3y^3-4y^2-4y$ by y^2-2y.

$$\begin{array}{r|l} & 3y^2+3y+2 \\ \hline y^2-2y & 3y^4-3y^3-4y^2-4y \\ & 3y^4-6y^3 \\ & \underline{-\quad +} \\ & \quad 3y^3-4y^2 \\ & \quad 3y^3-6y^2 \\ & \quad \underline{\quad +} \\ & \qquad 2y^2-4y \\ & \qquad 2y^2-4y \\ & \qquad \underline{-\quad +} \\ & \qquad\quad 0 \end{array}$$

$(y^2-2y)(3y^2+3y+2) = 3y^4-3y^3-4y^2-4y$.

Solution-11

$2y^5+10y^4+6y^3+y^2+5y+\cancel{y}+3$ by $2y^3+1$

$$
\begin{array}{r|l}
 & y^2+5y+3 \\
\hline
2y^3+1 & 2y^5+10y^4+6y^3+y^2+5y+3 \\
 & \underline{\underset{-}{2y^5}+0+0\underset{-}{+y^2}} \\
 & \quad 10y^4+6y^3+0+5y \\
 & \quad \underline{\underset{-}{10y^4}+0+0\underset{-}{+}5y} \\
 & \qquad 6y^3+0+0+3 \\
 & \qquad \underline{\underset{-}{6y^3}+0+0\underset{-}{+}3} \\
 & \qquad\quad (0)
\end{array}
$$

Solution-12:-

$x^4-2x^3+2x^2+x+4$ by x^2+x+1.

$$
\begin{array}{r|l}
 & x^2-3x+4 \\
\hline
x^2+x+1 & x^4-2x^3+2x^2+x+4. \\
 & \underline{\underset{-}{x^4}\underset{-}{+x^3}\underset{-}{+x^2}} \\
 & \quad -3x^3+x^2+x \\
 & \quad \underline{\underset{+}{-3x^3}\underset{+}{-3x^2}\underset{+}{-3x}} \\
 & \qquad 4x^2+4x+4 \\
 & \qquad \underline{\underset{-}{4x^2}\underset{-}{+4x}\underset{-}{+}4} \\
 & \qquad\quad (0).
\end{array}
$$

Solution-13:-

$m^3 - 14m^2 + 37m - 26$ by $m^2 - 12m + 13$.

$$
\begin{array}{r|l}
 & m - 2 \\
\hline
m^2 - 12m + 13 & m^3 - 14m^2 + 37m - 26 \\
 & \underline{\overset{-}{m^3} \overset{+}{- 12m^2} \overset{-}{+ 13m}} \\
 & \quad -2m^2 + 24m - 26 \\
 & \quad \underline{\overset{+}{-2m^2} \overset{-}{+ 24m} \overset{+}{- 26}} \\
 & \qquad (0)
\end{array}
$$

Solution-014:-

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x^2 + x + 1 & x^4 + x^2 + 1 + 0 \\
 & \underline{x^4 + x^2 + 0 + x^3}
\end{array}
$$

Solution-14:-

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x^2 + x + 1 & x^4 + 0 + x^2 + 0 + 1 \\
 & \underline{x^4 + x^3 + x^2} \\
 & \quad x^3 + 0 + 0 + 1 \\
 & \quad \underline{-x^3 - x^2 - x + 0} \\
 & \qquad x^2 + x + 1 \\
 & \qquad \underline{x^2 + x + 1} \\
 & \qquad\quad (0)
\end{array}
$$

Solution-15:-

$$
\begin{array}{r|l}
 & x^2+x+1 \\
\hline
x^3+1 & x^5+x^4+x^3+x^2+x+1 \\
 & \underline{x^5+0+0+x^2} \\
 & \quad x^4+x^3+0+x \\
 & \quad \underline{\overset{-}{x^4}\overset{-}{+}0\overset{-}{\pm}0\overset{-}{+}x} \\
 & \qquad x^3+0+0+1 \\
 & \qquad \underline{\overset{-}{x^3}\overset{-}{+}0\overset{-}{+}0\overset{-}{+}1} \\
 & \qquad\quad (0)
\end{array}
$$

16:- Divide the following and find the quotient and remainder.

Solution-16:-

$14x^3-5x^2+9x-1$ by $2x-1$.

$$
\begin{array}{r|l}
 & 7x^2+x+5 \\
\hline
2x-1 & 14x^3-5x^2+9x-1. \\
 & \underline{\overset{-}{14x^3}\overset{+}{-}7x^2} \\
 & \qquad 2x^2+9x \\
 & \qquad \underline{\overset{-}{2x^2}\overset{+}{-}x} \\
 & \qquad\quad 10x-1 \\
 & \qquad\quad \underline{\overset{-}{10x}\mp 5} \\
 & \qquad\qquad (4).
\end{array}
$$

$\therefore\ Q = 7x^2+x+5\ ;\ R = 4.$

Solution -17 :-

$$
\begin{array}{r|l}
 & \quad 3x^2+4x+1 \\
\hline
2x-3 & 6x^3-x^2-10x-3 \\
 & \underline{\overset{-}{}6x^3\underset{+}{-}9x^2} \\
 & \qquad 8x^2-10x \\
 & \qquad \underline{8x^2-8x} \\
 & \qquad\qquad 2x-3 \\
 & \qquad\qquad \underline{\overset{-}{}2x\underset{+}{-}3} \\
 & \qquad\qquad\quad 0
\end{array}
$$

$\therefore$ Quotient $= 3x^2+4x+1$; Remainder $= 0$.

Solution -18 :-

$6x^3+11x^2-39x-65$ by $3x^2+13x+13$

$$
\begin{array}{r|l}
 & \qquad 2x-5 \\
\hline
3x^2+13x+13 & 6x^3+11x^2-39x-65. \\
 & \underline{\underset{-}{6x^3}\underset{-}{+26x^2}\underset{-}{+26x}} \\
 & \quad -15x^2-65x-65 \\
 & \quad \underline{\underset{+}{-15x^2}\underset{+}{-65x}\underset{+}{-65}} \\
 & \qquad\qquad 0 .
\end{array}
$$

$\therefore$ Quotient $= 2x-5$; Remainder $= 0$.

Solution-19:-

$30x^4+11x^3-82x^2-12x+48$ by $3x^2+2x-4$.

$$10x^2-3x-12$$

$$3x^2+2x-4 \,\Big)\, 30x^4+11x^3-82x^2-12x+48$$

$$30x^4+20x^3-40x^2$$
$$-\quad -\quad +$$

$$-9x^3-42x^2-12x$$

$$-9x^3-6x^2+12x$$
$$+\quad +\quad -$$

$$-36x^2-24x+48$$

$$-36x^2-24x+48$$
$$+\quad +\quad -$$

$$0$$

$\therefore$ Quotient $= 10x^2-3x-12$; Remainder $=0$.

Solution-20:-

$9x^4-4x^2+4$ by $3x^2-4x+2$.

$$3x^2+4x+2$$

$$3x^2-4x+2 \,\Big)\, 9x^4-0x^3+(-4x^2)+0x+4$$

$$9x^4-12x^3+6x^2$$
$$-\quad +\quad -$$

$$+12x^3-10x^2+0x+4$$

$$12x^3-16x^2+8x+0$$
$$-\quad +\quad -\quad -$$

$$6x^2-8x+4$$

$$6x^2-8x+4$$
$$-\quad +\quad -$$

$$0$$

Quotient $= 3x^2-4x+2$; Remainder $=0$.

Solution -21.

(i) Dividend = $14x^2+13x-15$.

Divisor = $7x-4$

$$
\begin{array}{r|l}
 & 2x+3 \\
\hline
7x-4 & 14x^2+13x-15 \\
 & 14x^2-8x \\
 & \quad\ \ + \\
\hline
 & \qquad\ \ 21x-15 \\
 & \qquad\ \ 21x-12 \\
 & \qquad\qquad\ + \\
\hline
 & \qquad\qquad (-3)
\end{array}
$$

$\therefore$ Quotient $= 2x+3$; Remainder $= -3$.

$$14x^2+13x-15 = (7x-4)(2x+3)-3$$

$$= 14x^2+21x-8x-12-3$$

$$= 14x^2+13x-15.$$

$\therefore$ LHS = RHS.

(ii)

$$
\begin{array}{r|l}
 & 5z^2+\frac{10}{3}z+11 \\
\hline
3z-6 & 15z^3-20z^2+13z-12 \\
 & 15z^3-30z^2 \\
\hline
 & \qquad 10z^2+13z \\
 & \qquad 10z^2-20z \\
 & \qquad\qquad + \\
\hline
 & \qquad\qquad 33z-12 \\
 & \qquad\qquad 33z-66 \\
 & \qquad\qquad\quad\ + \\
\hline
 & \qquad\qquad\ (54)
\end{array}
$$

$$\therefore \quad 15z^3-20z^2+13z-12 = (3z-6)\left(5z^2+\frac{10z}{3}+11\right)+54$$

$$= 15z^3-20z^2+13z-66+54$$

$$= 15z^3-20z^2+13z-12.$$

Solution - 21 :-

(iii)

$2y^2 - 6 \qquad 6y^5 - 28y^3 + 3y^2 + 30y - 9$

$\downarrow$ divisor $\qquad \downarrow$ dividend

$$
\begin{array}{r|l}
 & 3y^3 - 5y + \frac{3}{2} \\
\hline
2y^2 - 6 & 6y^5 + 0 - 28y^3 + 3y^2 + 30y - 9 \\
 & 6y^5 + 0 - 18y^3 \\
 & \qquad\quad + \\
\hline
 & \qquad - 10y^3 + 3y^2 + 30y \\
 & \qquad - 10y^3 + 0 + 30y \\
 & \qquad + \qquad - \\
\hline
 & \qquad\qquad 3y^2 - 9 \\
 & \qquad\qquad 3y^2 - 9 \\
 & \qquad\qquad - \quad + \\
\hline
 & \qquad\qquad (0)
\end{array}
$$

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = (2y^2 - 6)(3y^3 - 5y + \tfrac{3}{2}) + 0$$

$$= 6y^5 - 10y^3 + 3y^2 - 18y^3 + 30y - 9$$

$$= 6y^5 - 28y^3 + 3y^2 + 30y - 9$$

(iv)

$$
\begin{array}{r|l}
 & -4x^3 + 2x^2 - 8x + 30 \\
\hline
3x + 7 & -12x^4 - 22x^3 - 10x^2 + 34x - 75 \\
 & -12x^4 - 28x^3 \\
 & + \qquad\quad + \\
\hline
 & \qquad 6x^3 - 10x^2 + 34x \\
 & \qquad 6x^3 + 14x^2 + 0 \\
 & \qquad\quad - \\
\hline
 & \qquad\qquad - 24x^2 + 34x - 75 \\
 & \qquad\qquad - 24x^2 - 56x \\
 & \qquad\qquad\qquad + \\
\hline
 & \qquad\qquad\qquad 90x - 75 \\
 & \qquad\qquad\qquad 90x + 210 \\
\hline
 & \qquad\qquad\qquad (-285)
\end{array}
$$

Quotient $= -4x^3 + 2x^2 - 8x + 30$

$R = -285$

$\therefore \quad 34x - 22x^3 - 12x^4 - 10x^2 - 75 = (3x+7)(-4x^3 + 2x^2 - 8x + 30) - 285$

$= -12x^4 - 22x^4 - 10x^2 + 34x - 75.$

(v)

$$
\begin{array}{r|l}
 & \cancel{15y^4}\ \cancel{16y}\quad 5y^3 - 2y^2 + \frac{5}{3}y \\
\hline
3y-2 & 15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6 \\
 & -\ 15y^4 \underset{+}{-} 10y^3 \\
\cline{2-2}
 & \quad -6y^3 + 9y^2 \\
 & \quad \underset{+}{-}6y^3 \underset{-}{+} 4y^2 \\
\cline{2-2}
 & \qquad 5y^2 - \frac{10}{3}y + 6 \\
 & \qquad 5y^2 - \frac{10}{3}y + 0 \\
\cline{2-2}
 & \qquad\qquad (6)
\end{array}
$$

$\therefore \quad 15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6 = (3y-2)(5y^3 - 2y^2 + \frac{5}{3}y) + 6$

$= 15y^4 - 6y^3 + 5y^2 - 10y^3 + 4y^2 - \frac{10y}{3} + 6$

$= 15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6.$

$\therefore$ LHS = RHS.

(vi)

$$2y^2-y+1 \;\overline{\big)\; 4y^3+8y^2+8y+7}^{\;2y+5}$$

$$4y^3-2y^2+2y$$
$$- \quad + \quad -$$
$$\overline{\quad 10y^2+6y+7 \quad}$$
$$10y^2-5y+5$$
$$- \quad + \quad -$$
$$\overline{\quad 11y+2. \quad}$$

$(2y^2-y+1)(2y+5)+11y+2 = 4y^3+10y^2-2y^2-5y$
$+2y+5+11y+2$

$= 4y^3+8y^2+8y+7.$

(vii)

$$2y^3+1 \;\overline{\big)\; 6y^5+4y^4+4y^3+7y^2+27y+6}^{\;3y^2+2y+2}$$

$$6y^5+0+0+3y^2$$
$$- \qquad\qquad -$$
$$\overline{\quad 4y^4+4y^3+4y^2+27y+6 \quad}$$
$$4y^4+0+0+2y$$
$$- \qquad\qquad -$$
$$\overline{\quad 4y^3+4y^2+25y+6 \quad}$$
$$4y^3+0+0+2$$
$$- \qquad\qquad -$$
$$\overline{\quad 4y^2+25y+4. \quad}$$

$\therefore$ Quotient $= 4y^2+25y+4$; Divisor $= 2y^3+1.$

Remainder $= 4y^2+25y+4.$

$\Rightarrow$ $6y^5+4y^4+4y^3+7y^2+27y+6 = (2y^3+1)(4y^2+25y+4)+$
$4y^2+25y+4$

$= 6y^5+4y^4+4y^3+7y^2+27y+6.$

Solution-22 :-

$$Q = 5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$$

$3y-2$) $15y^4 + 16y^3 + \frac{10}{3}y - 9y^2 - 6$

can be written as

$$15y^4 + 16y^3 - 9y^2 + \frac{10}{3}y - 6$$

$$15y^4 \mp 10y^3$$

$$26y^3 - 9y^2$$

$$26y^3 - \frac{52}{3}y^2$$

$$- \quad +$$

$$\frac{25}{3}y^2 + \frac{10}{3}y$$

$$\frac{25y^2}{3} - \frac{50}{9}y$$

$$- \quad +$$

$$\frac{80}{9}y - 6$$

$$\frac{80}{9}y - 6$$

$$+ \quad +$$

---

(0).

$\therefore$ Quotient $= 5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$.

coefficient of $y^3 = 5$

" $y^2 = \frac{26}{3}$

" $y = \frac{25}{9}$.

constant term $= \frac{80}{27}$.

23. (i)

$$\begin{array}{r|l} & x-7 \\ \hline x+6 & x^2-x-42 \\ & x^2+6x \\ & \underline{-\quad -} \\ & -7x-42 \\ & -7x-42 \\ & \underline{+\quad\ +} \\ & 0 \end{array}$$

Yes, $x+6$ is a factor of x^2-x-42.

(ii)

$$\begin{array}{r|l} & x-3 \\ \hline 4x-1 & 4x^2-13x-12 \\ & 4x^2-x \\ & \underline{-\quad +} \\ & -12x-12 \\ & -12x+3 \\ & \underline{+\quad -} \\ & -15 \end{array}$$

$4x-1$ is not a factor of $4x^2-13x-12$.

23. (iii)

$$\begin{array}{r|l} & 2y^3+5y \text{ [illegible]} \\ \hline 2y-5 & 4y^4-10y^3-10y^2+30y-15 \\ & 4y^4-10y^3 \\ & \underline{-\quad +} \\ & -10y^2+30y \\ & 10y^2-25y \\ \hline & 5y-15 \end{array}$$

$2y-5$ is not a factor of $4y^4-10y^3-10y^2+30y-15$

(iv)

$$\begin{array}{r|l} & 2y^3+5y^2+2y-7 \\ \hline 3y^2+5 & 6y^5+15y^4+16y^3+4y^2+10y-35 \\ & 6y^5+10y^4+10y^3 \\ & \underline{-\quad -\quad -} \\ & 15y^4+6y^3+4y^2 \\ & 15y^4+0+25y^2 \\ & \underline{-\quad -\quad -} \\ & 6y^3-21y^2+10y \\ & 6y^3+0+10y \\ & \underline{-\quad\quad -} \\ & -21y^2-35 \\ & -21y^2-35 \\ & \underline{+\quad +} \\ & 0 \end{array}$$

$\therefore$ $3y^2+5$ is a factor of given polynamial.

(v)

$$z^2+3 \,\big)\, z^5-9z = z^5+0+0+0-9z+0 \;\big(\, z^3+3z$$

$$\underline{-z^5+0\pm 3z^3}$$

$$-3z^3-9z$$

$$\underline{3z^3+9z}$$

$$(0)$$

z^2+3 is a factor of polynamial z^5-9z.

(vi)

$$2x^2-x+3 \,\big)\, 6x^5-x^4+4x^3-5x^2-x-15 \;\big(\, 3x^3+x^2-2x+5$$

$$\underline{6x^5-3x^4+9x^3}$$

$$2x^4-5x^3-5x^2$$

$$\underline{2x^4-x^3+3x^2}$$

$$-4x^3-8x^2-x$$

$$\underline{-4x^3+2x^2-6x}$$

$$-10x^2+5x-15$$

$$\underline{10x^2-5x+15}$$

$$(0)$$

$2x^2-x+3$ is factor of given polynamial.

26. $$x^2+2x-3 \,\big)\, x^4+2x^3-2x^2+x+1 \;\big(\, x^2+1$$

$$\underline{x^4+2x^3-3x^2}$$

$$x^2+x+1$$

$$\underline{x^2+2x-3}$$

$$-x+4$$

$x-4$ added to $x^4+2x^3-2x^2+x-1$ Sothat the resulting Polynomial exactly divisible by x^2+2x-3.

24. $$x+2 \,\big)\, 4x^4+2x^3-3x^2+8x+5a \;\big(\, 4x^3-6x^2+9x+10$$

$$\underline{4x^4+8x^3}$$

$$-6x^3-3x^2$$

$$\underline{-6x^3-12x^2}$$

$$9x^2+8x$$

$$\underline{9x^2+18x}$$

$$-10x+5a$$

$$\underline{-10x-20}$$

$$5a+20$$

$$5a+20=0$$

$$a=\frac{-20}{5}$$

$$\boxed{a=-4}$$

Division Of Algebraic Expressions

Ex 8.5

Exercise - 8.5.

1. Divide the first polynomial by the second polynomial in each of the following. Also, write the quotient and the remainder:

(i) $3x^2+4x+5$, $x-2$.

we have,

$$3x^2+4x+5 = x(3x+10)-2(3x+10)+5+20$$
$$= (x-2)(3x+10)+20+5$$

Quotient $= 3x+10$, Remainder $= 25$.

(ii) $10x^2-7x+8$, $5x-3$

we have

$$10x^2-7x+8 = 5x\left(2x-\frac{1}{5}\right)-3\left(2x-\frac{1}{5}\right)+8-\frac{3}{5}.$$
$$= (5x-3)\left(2x-\frac{1}{5}\right)+\frac{40-3}{5}$$
$$= (5x-3)\left(2x-\frac{1}{5}\right)+\frac{37}{5}$$

Quotient $= \left(2x-\frac{1}{5}\right)$, Remainder $= \frac{37}{5}$.

(iii) $5y^3-6y^2+6y-1$, $5y-1$.

$$5y^3-6y^2+6y-1 = 5y(y^2-y+1)-1(y^2-y+1)-1+1.$$
$$= (5y-1)(y^2-y+1)+0.$$

$\therefore$ Quotient $= y^2-y+1$, Remainder $= 0$.

② (iii)

$$4x^2-5 \overline{\big)\, 4x^4+0+7x^2+15}$$ quotient $2x^2+3$

$4x^4+0-5x^2$

$\underline{- \quad +}$

$12x^2+15$

$\underline{-12x^2+15}$

30.

No, $4x^2-5$ is not a factor of $4x^4+7x^2+15$.

② ⓥ

$10a^2-9a-5 / 2a-5 = 5a+3$

$$2a-3 \overline{\big)\, 10a^2-9a-5}$$

$\underline{10a^2-15a}$

$6a-5$

$\underline{6a-9}$

(4)

No, $(2a-3)$ is not a factor of $10a^2-9a-5$

1>

(iv) $x^4-x^3+5x,\ x-1$.

we have,

$x^4-x^3+5x = x(x^3+5)-1(x^3+5)+5$

$= (x-1)(x^3+5)+5$.

$\therefore$ Quotient $= (x^3+5)$, Remainder $= 5$

ⓥ $y^4+y^2,\ y^2-2$.

$\boxed{y^2(y^2-2)-2(y^2-2)+4.}$

we have,

$y^4+y^2 = y^2(y^2+3)-2(y^2+3)+6$

$= (y^2-2)(y^2+3)+6$.

$\therefore$ Quotient $= y^2+3$, Remainder $= 6$.

②

ⓘ

$$x+1 \overline{\big)\, 2x^2+5x+4}$$ quotient $2x+3$

$\underline{2x^2+2x}$

$3x+4$

$\underline{3x+3}$

1.

NO, $(x+1)$ is not a factor of the second.

ⓘⓘ

$$y-2 \overline{\big)\, 3y^3+5y^2+5y+2}$$ quotient $3y^2+11y+27$

$\underline{3y^3-6y^2}$

$11y^2+5y$

$\underline{11y^2-22y}$

$27y+2$

$\underline{27y-54}$

-52

No, $(y-2)$ is not a factor of the second.

(2)

(vi)

$$
\begin{array}{r|l}
 & \;\; 2y + 1 \\
\hline
4y+1 & 8y^2 - 2y + 1 \\
 & \underline{8y^2 + 2y} \\
 & \quad\;\; -4y + 1 \\
 & \quad\;\; \underline{\;\;4y + 1} \\
 & \qquad\quad (2)
\end{array}
$$

No, 4y+1 is not a factor of given Polynomial

Division Of Algebraic Expressions

Ex 8.6

Exercise - 8.6.

1. $x^2 - 5x + 6$ by $x - 3$.

$$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$$

$$= x(x-2) - 3(x-2)$$

$$= (x-3)(x-2)$$

$$\therefore \frac{x^2 - 5x + 6}{(x-3)} = \frac{(x-3)(x-2)}{(x-3)} = x - 2.$$

2. $ax^2 - ay^2$ by $ax + ay$.

$$ax^2 - ay^2 = a(x^2 - y^2)$$

$$= a(x+y)(x-y)$$

$$\therefore \frac{ax^2 - ay^2}{ax + ay} = \frac{a(x+y)(x-y)}{a(x+y)} = (x-y)$$

3. $x^4 - y^4$ by $x^2 - y^2$

$$x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2+y^2)(x^2-y^2)$$

$$\therefore \frac{x^4 - y^4}{x^2 - y^2} = \frac{(x^2+y^2)(x^2-y^2)}{(x^2-y^2)} = (x^2+y^2).$$

4. $\left[acx^2+(bc+ad)x+bd\right]$ by $\left[(ax+b)\right]$

$$acx^2+(bc+ad)x+bd = (ax+b)(cx+d)$$

$$\therefore \quad \frac{acx^2+(bc+ad)x+bd}{(ax+b)} = \frac{(ax+b)(cx+d)}{(ax+b)}$$

$$= cx+d.$$

5. $(a^2+2ab+b^2)-(a^2+2ac+c^2)$ by $2a+b+c$.

$$\cancel{a^2}+2ab+b^2-\cancel{a^2}-2ac-c^2 = (b^2-c^2)+2ab(1)-2ac$$

$$= (b-c)(b+c)+2a(b-c)$$

$$= (b-c)(2a+b+c)$$

$$\therefore \quad \frac{(a^2+2ab+b^2)-(a^2+2ac+c^2)}{(2a+b+c)} = \frac{(2a+b+c)(b-c)}{(2a+b+c)}$$

$$= b-c.$$

6. $\frac{1}{4}x^2-\frac{1}{2}x-12$ by $\frac{1}{2}x-4$.

$$\frac{1}{4}x^2-\frac{1}{2}x-12 = \frac{1}{2}x\left(\frac{1}{2}x+3\right)-4\left(\frac{1}{2}x+3\right)$$

$$= \frac{1}{2}x\left(\frac{1}{2}x+3\right)-4\left(\frac{x}{2}+3\right)$$

$$= \left(\frac{x}{2}-4\right)\left(\frac{x}{2}+3\right)$$

$$\therefore \quad \frac{\frac{1}{4}x^2-\frac{1}{2}x-12}{\frac{x}{2}-4} = \frac{\left(\frac{x}{2}-4\right)\left(\frac{x}{2}+3\right)}{\frac{x}{2}-4} = \frac{x}{2}+3.$$