Total No. of Questions - 24

Total No. of Printed Pages - 4

Regd. No.

Part - III

MATHEMATICS, Paper - II (B)

(Co-ordinate Geometry and Calculus)

(English Version)

Time : 3 hours *Max. Marks : 75*

Note : This question paper consists of **three** sections A, B and C.

SECTION A

I. **Very short answer** type questions. **10 × 2 = 20**

i) Answer **all** questions.

ii) Each question carries **two** marks.

1. Find the other end of the diameter of the circle $x^2 + y^2 - 8x - 8y + 27 = 0$ if one end of it is (2, 3).

2. Find the equation of the sphere whose center is (2, –3, 4) and radius is 5.

3. Find the equation of the parabola whose focus is S (1, –7) and vertex is A (1, –2).

4. Show that the angle between the two asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\,Tan^{-1}\left(\frac{b}{a}\right)$ or $2\,Sec^{-1}(e)$.

5. Find the nth derivative of $f(x) = Log\left(8x^3 + 36x^2 + 54x + 27\right)$ for all $x > -\frac{3}{2}$.

6. Evaluate $\int \left(1 - \dfrac{1}{x^2}\right) e^{\left(x + \frac{1}{x}\right)} dx$.

7. Evaluate : $\int \dfrac{1}{(x + 3)\sqrt{x + 2}} dx$ on $I \subset (-2, \infty)$.

8. Evaluate : $\int\limits_{0}^{\pi/2} \dfrac{Sin^2x - Cos^2x}{Sin^3x + Cos^3x} dx$.

9. Find the area bounded between the curves $y^2 - 1 = 2x$ and $x = 0$.

10. Find the order and degree of the differential equation

$$\left[\left(\dfrac{dy}{dx}\right)^{1/2} + \left(\dfrac{d^2y}{dx^2}\right)^{1/3}\right]^{1/4} = 0.$$

SECTION B

II. **Short answer** type questions. $5 \times 4 = 20$

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. Find the condition that the tangents drawn from the exterior point (g, f) to $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ are perpendicular to each other.

12. Find the equation of the parabola whose axis is parallel to Y-axis and which passes through the points $(4, 5)$, $(-2, 11)$ and $(-4, 21)$.

13. Find the eccentricity, foci and equation of the directrices of the hyperbola $5x^2 - 4y^2 + 20x + 8y = 4$.

14. If PP' and QQ' are two perpendicular focal chords of a conic, prove that $\dfrac{1}{(SP)(SP')} + \dfrac{1}{(SQ)(SQ')}$ is constant.

15. Evaluate $\int x \sqrt{1 + x - x^2}\, dx$.

16. Solve $\dfrac{dy}{dx} - x\, Tan\,(y - x) = 1$.

17. Solve $\left(x + 2y^3\right)\dfrac{dy}{dx} = y$.

SECTION C

III. **Long answer** type questions.   **5 × 7 = 35**

   i)   Attempt **any five** questions.

   ii) Each question carries **seven** marks.

18. Find the equation and center of the circle passing through the points $(-2, 3)$, $(2\ -1)$ and $(4, 0)$.

19. Find the equation of the circle which cuts the circles $x^2 + y^2 + 2x + 4y + 1 = 0$, $2x^2 + 2y^2 + 6x + 8y - 3 = 0$ and $x^2 + y^2 - 2x + 6y - 3 = 0$ orthogonally.

20. The tangent and normal to the ellipse $x^2 + 4y^2 = 4$ at a point $P\,(\theta)$ on it meets the major axis in Q and R respectively. If $0 < \theta < \dfrac{\pi}{2}$ and $QR = 2$, then show that $\theta = Cos^{-1}\left(\dfrac{2}{3}\right)$.

21. If $y = \dfrac{Sinh^{-1}x}{\sqrt{1 + x^2}}$ then show that $\left(1 + x^2\right)y_2 + 3xy_1 + y = 0$ and hence by using Leibnitz theorem, deduce that

$$\left(1 + x^2\right)y_{n+2} + (2n + 3)\,xy_{n+1} + (n + 1)^2\,y_n = 0.$$

22. Evaluate $\int \dfrac{1}{Sinx + \sqrt{3}\,Cosx}\, dx$.

23. Evaluate $\int\limits_0^1 \dfrac{Log\,(1+x)}{1+x^2}\,dx$.

24. The velocity of a train which starts from rest is given by the following table, the time being recorded in minutes from the start and the speed in kilometers.

Minutes	2	4	6	8	10	12	14	16	18	20
Kmph	10	18	25	29	32	20	11	5	2	0

Estimate approximately the total distance run in 20 minutes by Simpson's rule and Trapezoidal rule.

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