GATE 2023
Civil Engineering
Exam Held on : 12-2-2023
Afternoon Session

SECTION - A GENERAL APTITUDE

Q.1 - Q.5 carry ONE mark Each

Q.1 The line ran _______ the page, right through the centre, and divided the page into two.

(a) across (b) of
(c) between (d) about

[MCQ]

Ans. (a)

End of Solution

Q.2 Kind : ________ : : Often : Seldom
(By word meaning)

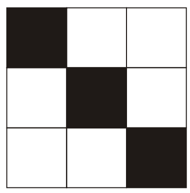
(a) Cruel (b) Variety
(c) Type (d) Kindred

[MCQ]

Ans. (a)

End of Solution

Q.3 In how many ways can cells in a 3 × 3 grid be shaded, such that each row and each column have exactly one shaded cell? An example of one valid shading is shown.



(a) 2 (b) 9
(d) 3 (d) 6

[MCQ]

Ans. (d)

Possible ways = $3 \times 2 \times 1 = 6$

End of Solution

Q.4 There are 4 red, 5 green, and 6 blue balls inside a box. If *N* number of balls are picked simultaneously, what is the smallest value of *N* that guarantees there will be at least two balls of the same colour?
One cannot see the colour of the balls until they are picked.

(a) 4 (b) 15
(c) 5 (d) 2

[MCQ]

Ans. **(a)**

Given: 4 Red, 5 Green and 6 Blue

We select three balls in worst case

1 Red, 1 Green and 1 Blue

If we select fourth ball then we found two balls are of same colour.

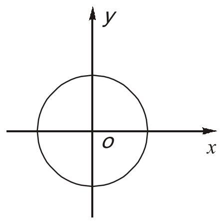
End of Solution

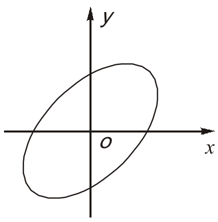
Q.5 Consider a circle with its centre at the origin (O), as shown. Two operations are allowed on the circle.

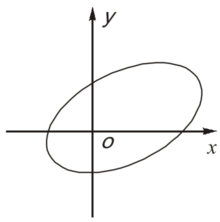
Operation 1: Scale independently along the x and y axes.

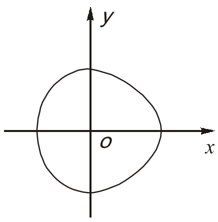
Operation 2: Rotation in any direction about the origin.

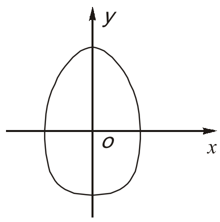
Which figure among the options can be achieved through a combination of these two operations on the given circle?



(a) 

(b) 

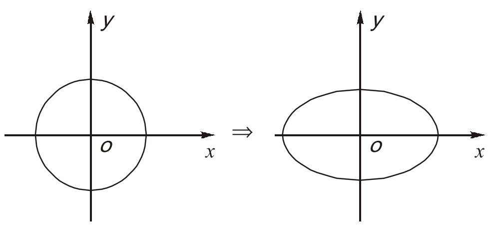
(c) 

(d) 

[MCQ]

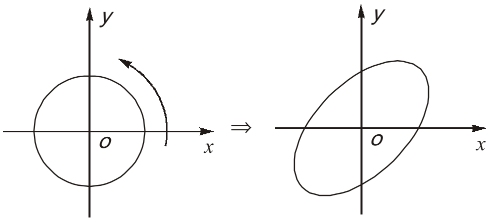
Ans. **(a)**

Operation 1:



Suppose, $s_{x\text{-axis}} > S_{y\text{-axis}}$

Operation 2:



End of Solution

Q.6 - Q.10 carry TWO mark Each

Q.6 Elvesland is a country that has peculiar beliefs and practices. They express almost all their emotions by gifting flowers. For instance, if anyone gifts a white flower to someone, then it is always taken to be a declaration of one's love for that person. In a similar manner, the gifting of a yellow flower to someone often means that one is angry with that person.

Based only on the information provided above, which one of the following sets of statement(s) can be logically inferred with certainty?

(i) In Elvesland, one always declares one's love by gifting a white flower.
(ii) In Elvesland, all emotions are declared by gifting flowers.
(iii) In Elvesland, sometimes one expresses one's anger by gifting a flower that is not yellow.
(iv) In Elvesland, sometimes one expresses one's love by gifting a white flower.

(a) only (ii) (b) (i), (ii) and (iii)
(c) (i), (iii) and (iv) (d) only (iv)

[MCQ]

Ans. (d)

End of Solution

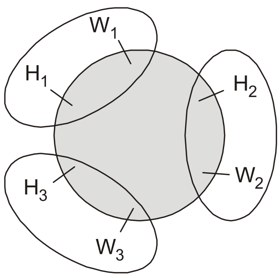
Q.7 Three husband-wife pairs are to be seated at a circular table that has six identical chairs. Seating arrangements are defined only by the relative position of the people. How many seating arrangements are possible such that every husband sits next to his wife?

(a) 16 (b) 4
(c) 120 (d) 720

[MCQ]

Ans. (a)

H_1 W_1, H_2 W_2, H_3 W_3



Treat every husband-wife pairs as one block
There are three pairs
So all three pairs can be arranged by (3 – 1)! ways and in all pairs husband and wife can be arranged by 2! ways.
So required number of ways are $2! \times 2^3 = 16$

End of Solution

Q.8 Based only on the following passage, which one of the options can be inferred with certainty?

When the congregation sang together, Apenyo would also join, though her little screams were not quite audible because of the group singing. But whenever there was a special number, trouble would begin; Apenyo would try singing along, much to the embarrassment of her mother. After two or three such mortifying Sunday evenings, the mother stopped going to church altogether until Apenyo became older and learnt to behave.

At home too, Apenyo never kept quiet; she hummed or made up silly songs to sing by herself, which annoyed her mother at times but most often made her become pensive. She was by now convinced that her daughter had inherited her love of singing from her father who had died unexpectedly away from home.

[Excerpt from These Hills Called Home by Temsula Ao]

(a) The mother was embarrassed about her daughter's singing at home.
(b) The mother's feelings about her daughter's singing at home were only of annoyance.
(c) The mother was not sure if Apenyo had inherited her love of singing from her father.
(d) When Apenyo hummed at home, her mother tended to become thoughtful.

[MCQ]

Ans. (d)

End of Solution

Q.9 If x satisfies the equation $4^{8^x} = 256$, then x is equal to ______.

(a) $\frac{1}{2}$
(b) $\log_{16} 8$
(c) $\frac{2}{3}$
(d) $\log_4 8$

[MCQ]

Ans. (c)

$$4^{8^x} = 256$$

$$\Rightarrow \quad 4^{8^x} = 4^4$$

$$\Rightarrow \quad 8^x = 4$$

$$\Rightarrow \quad \left(2^3\right)^x = 2^2$$

So, $$x = \frac{2}{3}$$

End of Solution

Q.10 Consider a spherical globe rotating about an axis passing through its poles. There are three points P, Q and R situated respectively on the equator, the north pole, and midway between the equator and the north pole in the northern hemisphere. Let P, Q and R move with speeds v_P, v_Q, and v_R, respectively.

Which one of the following options is CORRECT?

(a) $v_P < v_R < v_Q$

(b) $v_P < v_Q < v_R$

(c) $v_P > v_R > v_Q$

(d) $v_P = v_R \neq v_Q$

[MCQ]

Ans. (c)

End of Solution

■■■■

Q.11 - Q.35 carry ONE mark Each

Q.11 Let ϕ be a scalar field, and $\boldsymbol{u}$ be a vector field. Which of the following identities is true for div($\phi\boldsymbol{u}$)?

(a) $\text{div}(\phi\boldsymbol{u}) = \phi\text{div}(\boldsymbol{u}) + \boldsymbol{u}\cdot\text{grad}(\phi)$

(b) $\text{div}(\phi\boldsymbol{u}) = \phi\text{div}(\boldsymbol{u}) + u\times\text{grad}(\phi)$

(c) $\text{div}(\phi\boldsymbol{u}) = \phi\text{grad}(\boldsymbol{u}) + \boldsymbol{u}\cdot\text{grad}(\phi)$

(d) $\text{div}(\phi\boldsymbol{u}) = \phi\text{grad}(\boldsymbol{u}) + u\times\text{grad}(\phi)$

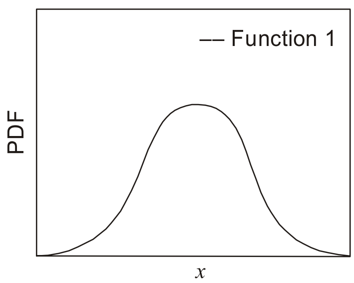
[MCQ]

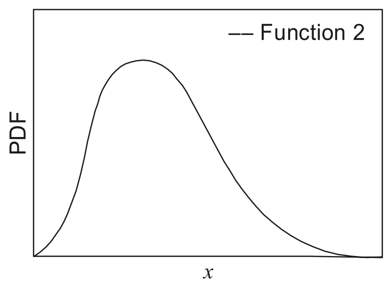
Ans. (a)

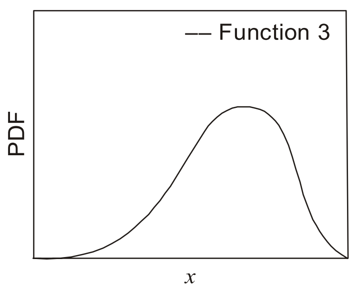
It is special property of vector, $\text{div}(\phi\boldsymbol{u}) = \phi\text{div}(\boldsymbol{u}) + \boldsymbol{u}\cdot\text{grad}(\phi)$

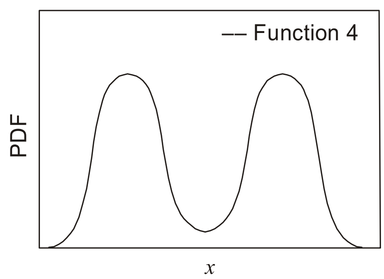
End of Solution

Q.12 Which of the following probability distribution functions (PDFs) has the mean greater than the median?



(b)




(d)


(a) Function 1

(b) Function 2

(c) Function 3

(d) Function 4

[MCQ]

Ans. (b)

End of Solution

Q.13 A remote village has exactly 1000 vehicles with sequential registration numbers starting from 1000. Out of the total vehicles, 30% are without pollution clearance certificate. Further, even- and odd-numbered vehicles are operated on even- and odd-numbered dates, respectively.

If 100 vehicles are chosen at random on an even-numbered date, the number of vehicles expected without pollution clearance certificate is _______ .

(a) 15 (b) 30

(c) 50 (d) 70

[MCQ]

Ans. **(b)**

Probability of selecting even vehicles on even numbered dates = 1

$\therefore$ The no. of vehicles expected without pollution clearance certificate

$= 100 \times 1 \times 0.3$

$= 30$

End of Solution

Q.14 A circular solid shaft of span L = 5 m is fixed at one end and free at the other end. A torque T = 100 kN.m is applied at the free end. The shear modulus and polar moment of inertia of the section are denoted as G and J, respectively. The torsional rigidity GJ is 50,000 kN.m^2/rad. The following are reported for this shaft:

Statement i) The rotation at the free end is 0.01 rad

Statement ii) The torsional strain energy is 1.0 kN.m

With reference to the above statements, which of the following is true?

(a) Both the statements are correct

(b) Statement i) is correct, but Statement ii) is wrong

(c) Statement i) is wrong, but Statement ii) is correct

(d) Both the statements are wrong

[MCQ]

Ans. **(b)**

Statement (i):

$$\text{Rotation at free end } (\theta) = \frac{\tau L}{GJ} = \frac{100 \times 5\left(\text{kNm}^2\right)}{50000\left(\text{kNm}^2/\text{rad}\right)}$$

$$= 0.01 \text{ rad}$$

Statement (ii):

$$\text{Torsional strain energy} = \frac{1}{2} \times T \times \theta = \frac{1}{2} \times 100 \times 0.01$$

$$= 0.5 \text{ kN-m}$$

End of Solution

Q.15 M20 concrete as per IS 456: 2000 refers to concrete with a design mix having ______

(a) an average cube strength of 20 MPa

(b) an average cylinder strength of 20 MPa

(c) a 5-percentile cube strength of 20 MPa

(d) a 5-percentile cylinder strength of 20 MPa

[MCQ]

Ans. (c)

End of Solution

Q.16 When a simply-supported elastic beam of span L and flexural rigidity EI (E is the modulus of elasticity and I is the moment of inertia of the section) is loaded with a uniformly distributed load w per unit length, the deflection at the mid-span is

$$\Delta_0 = \frac{5}{384}\frac{wL^4}{EI}$$

If the load on one half of the span is now removed, the mid-span deflection

(a) reduces to $\Delta_0/2$

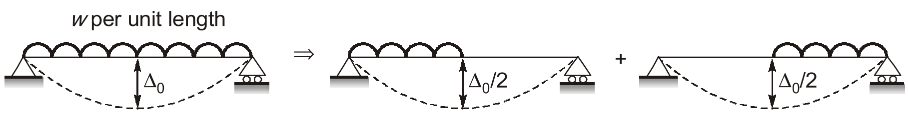
(b) reduces to a value less than $\Delta_0/2$

(c) reduces to a value greater than $\Delta_0/2$

(d) remains unchanged at Δ_0

[MCQ]

Ans. (a)



$$\Delta_0 = \frac{5}{384}\frac{WL^4}{EI} \qquad \Delta' = \frac{\Delta_0}{2} = \frac{1}{2}\left(\frac{5}{384}\frac{WL^4}{EI}\right)$$

So, midspan deflection reduced to $\Delta_0/2$

End of Solution

Q.17 Muller-Breslau principle is used in analysis of structures for ________ .

(a) drawing an influence line diagram for any force response in the structure

(b) writing the virtual work expression to get the equilibrium equation

(c) superposing the load effects to get the total force response in the structure

(d) relating the deflection between two points in a member with the curvature diagram in-between

[MCQ]

Ans. (a)

End of Solution

Q.18 A standard penetration test (SPT) was carried out at a location by using a manually operated hammer dropping system with 50% efficiency. The recorded SPT value at a particular depth is 28. If an automatic hammer dropping system with 70% efficiency is used at the same location, the recorded SPT value will be

(a) 28 (b) 20
(c) 40 (d) 25

[MCQ]

Ans. (b)

$$\text{Efficiency} \propto \frac{1}{\text{Number of blows}} \propto \frac{1}{SPT \text{ value}}$$

$$\Rightarrow \quad \eta_1 N_1 = \eta_2 N_2$$

$$\Rightarrow \quad 0.5 \times 28 = 0.7 \times N_2$$

$$\Rightarrow \quad N_2 = \frac{0.5 \times 28}{0.7} = 20$$

End of Solution

Q.19 A vertical sheet pile wall is installed in an anisotropic soil having coefficient of horizontal permeability, k_H and coefficient of vertical permeability, k_V. In order to draw the flow net for the isotropic condition, the embedment depth of the wall should be scaled by a factor of ______, without changing the horizontal scale.

(a) $\sqrt{\frac{k_H}{k_V}}$ (b) $\sqrt{\frac{k_V}{k_H}}$

(c) 1.0 (d) $\frac{k_H}{k_V}$

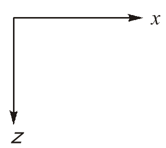
[MCQ]

Ans. (a)

A steady-state, homogeneous, anisotropic sysem can be mathematically transformed into an isotropic system by coordinate transformation, creating what is sometimes called a transformed section.

The coordinates in the true anisotropic sysem are x and z. In the tranformed isotropic system the coordinates are X and Z, where

$$X = x$$



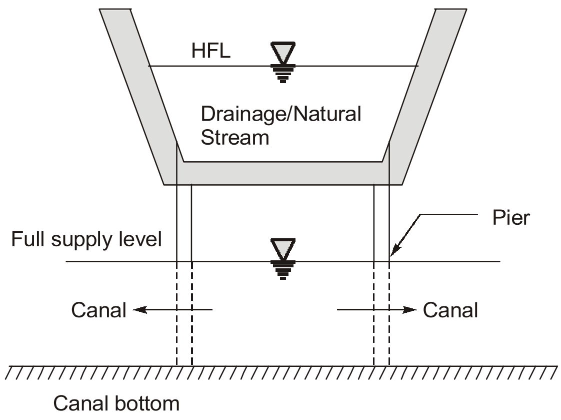
$$Z = z\sqrt{\frac{k_x}{k_z}}$$

$$Z = z\sqrt{\frac{k_H}{k_V}}$$

So, the embedment depth of wall should be scaled by factor of $\sqrt{\frac{k_H}{k_V}}$.

End of Solution

Q.20 Identify the cross-drainage work in the figure.



(a) Super passage
(b) Aqueduct
(c) Siphon aqueduct
(d) Level crossing

[MCQ]

Ans. (a)

End of Solution

Q.21 Which one of the following options provides the correct match of the terms listed in Column-1 and Column-2?

Column-1	Column-2
P. Horton equation	I. Precipitation
Q. Muskingum method	II. Flood frequency
R. Penman method	III. Evapotranspiration
	IV. Infiltration
	V. Channel routing

(a) P-IV, Q-V, R-III
(b) P-III, Q-IV, R-I
(c) P-IV, Q-III, R-II
(d) P-III, Q-I, R-IV

[MCQ]

Ans. (a)

End of Solution

Q.22 In the context of Municipal Solid Waste Management, 'Haul' in 'Hauled Container System operated in conventional mode' includes the ________ .

(a) time spent by the transport truck at the disposal site
(b) time spent by the transport truck in traveling between a pickup point and the disposal site with a loaded container
(c) time spent by the transport truck in picking up a loaded container at a pickup point
(d) time spent by the transport truck in driving from the depot to the first pickup point

[MCQ]

Ans. **(b)**

'Haul' in handed container system operated in conventional mode includes the time required to reach the disposal site, starting ater a container whose contents are to be emplied has been loaded on the truck, plus the time after leaving the disposal site until the truck arrives at the location where the empty container is to be redeposited.

End of Solution

Q.23 Which of the following is equal to the stopping sight distance?

(a) (braking distance required to come to stop) + (distance travelled during the perception-reaction time)

(b) (braking distance required to come to stop) – (distance travelled during the perception-reaction time)

(c) (braking distance required to come to stop)

(d) (distance travelled during the perception-reaction time)

[MCQ]

Ans. **(a)**

$$\text{SSD} = \underbrace{0.278Vt_R}_{\text{Lag distance}} + \underbrace{\frac{V^2}{254(f \pm s)}}_{\text{Braking distance}}$$

End of Solution

Q.24 The magnetic bearing of the sun for a location at noon is 183° 30′. If the sun is exactly on the geographic meridian at noon, the magnetic declination of the location is ______.

(a) 3° 30′ W
(b) 3° 30′ E
(c) 93° 30′ W
(d) 93° 30′ E

[MCQ]

Ans. **(a)**

$$\begin{aligned} \text{Declination} &= \text{True bearing} - \text{Magnetic bearing} \\ &= 180° - 183°30' \\ &= -3°30' = 3°30'\text{ W} \end{aligned}$$

End of Solution

Q.25 For the matrix

$$[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

which of the following statements is/are TRUE?

(a) $[A]\{x\} = \{b\}$ has a unique solution

(b) $[A]\{x\} = \{b\}$ does not have a unique solution

(c) $[A]$ has three linearly independent eigenvectors

(d) $[A]$ is a positive definite matrix

[MSQ]

Ans. **(b, c)**

As $|A| = 0$

So, one of the eigen value is zero.

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)\left[(2-\lambda)(1-\lambda)-1\right]+1[\lambda-1] = 0$$

$$(1 - \lambda)[(2 - \lambda)(1 - \lambda) - 1 - 1] = 0$$

$$\lambda(1 - \lambda)(3 - \lambda) = 0$$

$$\lambda = 0,\ 1,\ 3$$

As there are three eigen values so number of linearly independent eigen vector are 3.

Since, $|A| = 0$

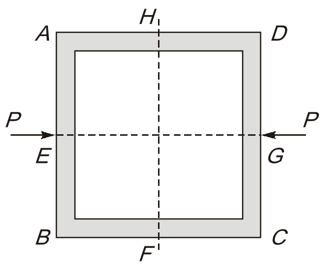
So, $[A]\{x\} = \{b\}$ does not have a unique solution

For the positive definite matrix, all the eigen values must be positive.

But, here one eigen value i.e. $\lambda = 0$, so, A is not a positive definite matrix.

End of Solution

Q.26 In the frame shown in the figure (not to scale), all four members (AB, BC, CD, and AD) have the same length and same constant flexural rigidity. All the joints A, B, C, and D are rigid joints. The midpoints of AB, BC, CD, and AD, are denoted by E, F, G, and H, respectively. The frame is in unstable equilibrium under the shown forces of magnitude P acting at E and G. Which of the following statements is/are TRUE?



(a) Shear forces at H and F are zero

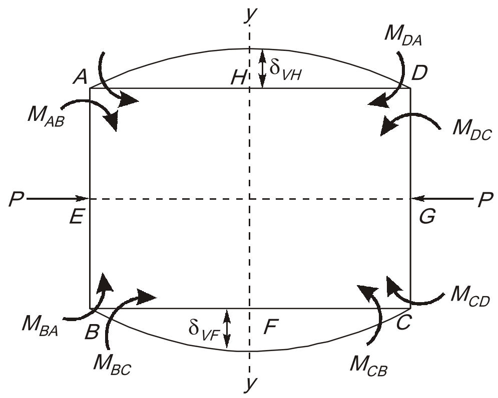
(b) Horizontal displacements at H and F are zero

(c) Vertical displacements at H and F are zero

(d) Slopes at E, F, G, and H are zero

[MSQ]

Ans. (a, b, d)



Shear forces at H and F are zero since it is subjected to pure bending.
Horizontal displacements at H and F are zero since axial deformations are neglected.
At the axis of symmetry, slopes must be zero for the deflection profile to be symmetrical.
$\therefore$ Slopes at E, F, G, H are zero.

Note: In actual question unstable equilibrium is given but in the figure equal force P is acting so there should be stable equilbrium as forces are balancing each other.

End of Solution

Q.27 With regard to the shear design of RCC beams, which of the following statements is/are TRUE?

(a) Excessive shear reinforcement can lead to compression failure in concrete
(b) Beams without shear reinforcement, even if adequately designed for flexure, can have brittle failure
(c) The main (longitudinal) reinforcement plays no role in the shear resistance of beam
(d) As per IS456 : 2000, the nominal shear stress in the beams of varying depth depends on both the design shear force as well as the design bending moment

[MSQ]

Ans. (a, b, d)

End of Solution

Q.28 The reason(s) of the nonuniform elastic settlement profile below a flexible footing, resting on a cohesionless soil while subjected to uniform loading, is/are:

(a) Variation of friction angle along the width of the footing
(b) Variation of soil stiffness along the width of the footing
(c) Variation of friction angle along the depth of the footing
(d) Variation of soil stiffness along the depth of the footing

[MSQ]

Ans. (b)

End of Solution

Q.29 Which of the following is/are NOT active disinfectant(s) in water treatment?

(a) •OH (hydroxyl radical)
(b) O_3 (ozone)
(c) OCl^- (hypochlorite ion)
(d) Cl^- (chloride ion)

[MSQ]

Ans. (d)

Ozone quickly decomposes to generate highly reactive free radicals. The ozone's oxidation potential (–2.7 V) is greater than that of the chlorine (–1.36 V) or hypochlorite ion (–1.49 V), substances widely used in wastewater treatment such as oxidants. Ozone is surpassed only by the hydroxyl radical (•OH) and fluoride in its oxidation capacity.

End of Solution

Q.30 As per the Indian Roads Congress guidelines (IRC 86 : 2018), extra widening depends on which of the following parameters?

(a) Horizontal curve radius
(b) Superelevation
(c) Number of lanes
(d) Longitudinal gradient

[MSQ]

Ans. (a, c)

$$W_E = \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}$$

$$\therefore \quad W_E \propto n$$

$$W_E \propto \frac{1}{R}$$

End of Solution

Q.31 The steady-state temperature distribution in a square plate ABCD is governed by the 2-dimensional Laplace equation. The side AB is kept at a temperature of 100 °C and the other three sides are kept at a temperature of 0 °C. Ignoring the effect of discontinuities in the boundary conditions at the corners, the steady-state temperature at the center of the plate is obtained as T_0 °C. Due to symmetry, the steady-state temperature at the center will be same (T_0 °C), when any one side of the square is kept at a temperature of 100 °C and the remaining three sides are kept at a temperature of 0 °C. Using the principle of superposition, the value of T_0 is ________ (rounded off to two decimal places).

[NAT]

Ans. (25.1) (24.9 – 25.1)

Laplace equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ...(i)

Its solution using separation of variables method is

$$\therefore \quad u(x, y) = (C_1 \cos\lambda x + C_2 \sin\lambda x)(C_3 e^{\lambda y} + C_4 e^{-\lambda y}) \quad ...(ii)$$

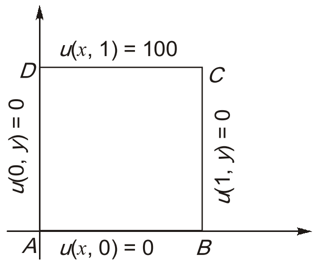
Let $AB = CD = 1$

Using $u(x, 0) = 0$

$\Rightarrow \quad C_3 = -C_4$

Using $(0, y) = 0$

$\Rightarrow \quad C_1 = 0$



Substituting constants values in equation (ii)

$$u(x, y) = (0 + C_2 \sin\lambda x)\left(C_3 e^{\lambda y} - C_3 e^{-\lambda y}\right)$$

$$u(x, y) = a_n \sin\lambda x(e^{\lambda y} - e^{-\lambda y})$$

Now, using $u(1, y) = 0$,

$\sin\lambda = 0 \Rightarrow \lambda = n\pi$

Hence $$u(x, y) = \sum a_n \sin(n\pi x)\left[e^{n\pi y} - e^{-n\pi y}\right] \quad \ldots\text{(iii)}$$

Using $u(x, 1) = 100$

$$\Rightarrow \quad a_n = \frac{100}{\sin n\pi x\left[e^{n\pi} - e^{-n\pi}\right]}$$

Using equation (iii),

$$u(x, y) = \sum_{n=1}^{\infty} \frac{100}{e^{n\pi} - e^{-n\pi}}\left(e^{n\pi y} - e^{-n\pi y}\right)$$

At mid point,

$$u\left(\frac{1}{2}, \frac{1}{2}\right) = T_0$$

So, $$T_0 = \sum_{n=1}^{\infty} \frac{100}{\left(e^{n\pi} - e^{-n\pi}\right)}\left(e^{\frac{n\pi}{2}} - e^{\frac{-n\pi}{2}}\right)$$

$$= 100\left[\left(\frac{e^{\frac{\pi}{2}} - e^{-\frac{\pi}{2}}}{e^{\pi} - e^{-\pi}}\right) + \left(\frac{e^{\pi} - e^{-\pi}}{e^{2\pi} - e^{-2\pi}}\right) + \frac{e^{\frac{3\pi}{2}} - e^{-\frac{3\pi}{2}}}{e^{3\pi} - e^{-3\pi}} + \ldots\ldots\right]$$

$$= 100\left[\frac{1}{e^{\frac{\pi}{2}} - e^{-\frac{\pi}{2}}} + \frac{1}{e^{\pi} - e^{-\pi}} + \frac{1}{e^{\frac{3\pi}{2}} - e^{-\frac{3\pi}{2}}} + \ldots\ldots\right]$$

$$= 50\left[\frac{1}{\cosh\left(\frac{\pi}{2}\right)} + \frac{1}{\cosh(\pi)} + \frac{1}{\cosh\left(\frac{3\pi}{2}\right)} + \ldots\ldots\right]$$

$$= 50\left[\frac{1}{2.509} + \frac{1}{11.59} + \frac{1}{55.66} + \ldots\ldots\right]$$

$= 19.928 + 4.314 + 0.903 +$ Neglecting

$= 25.145 \approx 25.1$

End of Solution

Q.32 An unconfined compression strength test was conducted on a cohesive soil. The test specimen failed at an axial stress of 76 kPa. The undrained cohesion (in kPa, *in integer*) of the soil is ________ .

[NAT]

Ans. **(38) (38 – 38)**

$\because \quad \sigma_3 = 0 \quad$ (UCS test)

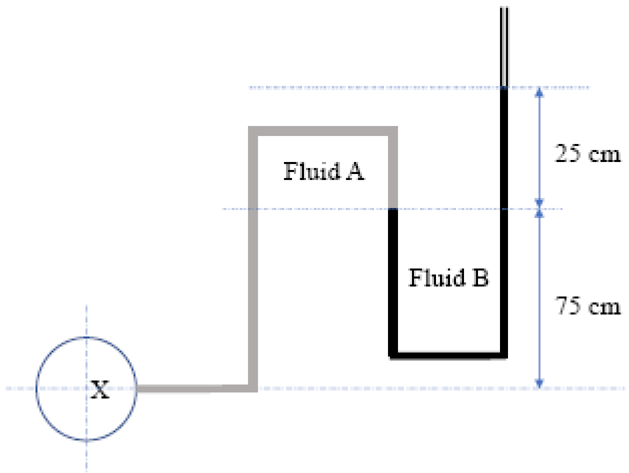
So, $$\sigma_1 = \sigma_3 \tan^2\left(45+\frac{\phi}{2}\right)+2c\tan\left(45+\frac{\phi}{2}\right)$$

$\Rightarrow$ $$76 = 2c\tan\left(45+\frac{0}{2}\right)$$

$\Rightarrow$ $$c = \frac{76}{2} = 38 \text{ MPa}$$

End of Solution

Q.33 The pressure in a pipe at X is to be measured by an open manometer as shown in figure. Fluid A is oil with a specific gravity of 0.8 and Fluid B is mercury with a specific gravity of 13.6. The absolute pressure at X is ________ kN/m^2 (round off to one decimal place). [Assume density of water as 1000 kg/m^3 and acceleration due to gravity as 9.81 m/s^2 and atmospheric pressure as 101.3 kN/m^2]



[Note: Figure is not to scale]

[NAT]

Ans. **(140.5) (140 – 141)**

$P_x - (800 \times 9.81 \times 0.75) - (13600 \times 9.81 \times 0.25) = P_{atm}$

$P_x = (101.3 \times 10^3) + (800 \times 9.81 \times 0.75) + (13600 \times 9.81 \times 0.25)$

$P_x = 140540 \text{ N/m}^2$

$P_x = 140.54 \text{ kN/m}^2$

End of Solution

Q.34 For the elevation and temperature data given in the table, the existing lapse rate in the environment is ________ °C/100 m (round off to two decimal places).

Elevation from ground level (m)	Temperature (°C)
5	14.2
325	16.9

[NAT]

Ans. **(0.84) (0.84 – 0.85)**

The existing lapse rate in the environment $= -\dfrac{\Delta T}{\Delta H} = -\dfrac{16.9-14.2}{3.25-5}\,°\text{C/m}$

$$= 2.7\times\frac{100}{320}\,°\text{C}/100\text{ m}$$

$$= 0.84°\text{C}/100\text{ m}$$

End of Solution

Q.35 If the size of the ground area is 6 km × 3 km and the corresponding photo size in the aerial photograph is 30 cm × 15 cm, then the scale of the photograph is 1 : ________ (in integer)

[NAT]

Ans. **(20000) (20000 – 20000)**

$$\text{Scale} = \frac{\text{Photodistance}}{\text{Ground distance}} = \frac{30\text{cm}}{6\text{km}} = \frac{30\text{cm}}{600000\text{cm}} = \frac{1}{20000}$$

Alternatively, $(\text{Scale})^2 = \left(\dfrac{\text{Area of map}}{\text{Area on ground}}\right)$

$$= \frac{(30\times 15)}{6\times 3} = \frac{30\times 15}{(6\times 3)\times(100000)^2}$$

$\Rightarrow$ $$\text{Scale} = \sqrt{\frac{25}{(100000)^2}}$$

$\therefore$ $$S = \frac{1}{20000}$$

End of Solution

Q.36 - Q.65 carry TWO mark Each

Q.36 The solution of the differential equation

$$\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$$

is expressed as $y = C_1e^{2.5x} + C_2e^{\alpha x} + C_3e^{\beta x}$, where C_1, C_2, C_3, α, and β are constants, with α and β being distinct and not equal to 2.5. Which of the following options is correct for the values of α and β?

(a) 1 and 2
(b) –1 and –2
(c) 2 and 3
(d) –2 and –3

[MCQ]

Ans. (a)

Auxillary equation is,

$m^3 - 5.5m^2 + 9.5m - 5 = 0$

By solving above equation, we get

$$m = 2.5,\ 1,\ 2$$

So, m_1 and m_2 are 1 and 2.

End of Solution

Q.37 Two vectors $[2\ \ 1\ \ 0\ \ 3]^T$ and $[1\ \ 0\ \ 1\ \ 2]^T$ belong to the null space of a 4 × 4 matrix of rank 2. Which one of the following vectors also belongs to the null space?

(a) $[1\ 1\ -1\ 1]^T$
(b) $[2\ \ 0\ \ 1\ \ 2]^T$
(c) $[0\ \ -2\ \ 1\ \ -1]^T$
(d) $[3\ \ 1\ \ 1\ \ 2]^T$

[MCQ]

Ans. (a)

$$\rho(A_{4\times 4}) = 2$$

$$N(A) = \text{Number of column} - \text{Rank}$$

$$= 4 - 2 = 2$$

i.e. Null space of A will consist only two linearly independent vectors which is given as x and y.

Eigen vectors of matrix A, $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

As these are linearly independent eigen vectors so remaining eigen vectors of null space must be linearly dependent.

Hence, $X - Y = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

End of Solution

Q.38 Cholesky decomposition is carried out on the following square matrix [*A*].

$$[A] = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

Let l_{ij} and a_{ij} be the (i, j)th elements of matrices [*L*] and [*A*], respectively. If the element l_{22} of the decomposed lower triangular matrix [*L*] is 1.968, what is the value (rounded off to the nearest integer) of the element a_{22}?

(a) 5 (b) 7

(c) 9 (d) 11

[MCQ]

Ans. (b)

$$LL^T = A$$

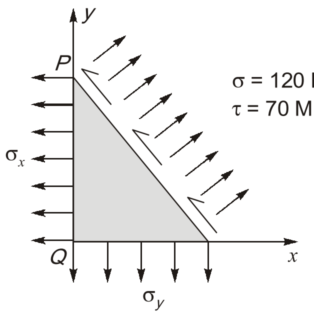
$$\begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}\begin{bmatrix} L_{11} & L_{21} \\ 0 & L_{22} \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

$$L_{11} = 2\sqrt{2}, \ L_{21} = -\frac{5}{2\sqrt{2}} \text{ and } L_{21}^2 + L_{22}^2 = a_{22}$$

$$a_{22} = \left(-\frac{5}{2\sqrt{2}}\right)^2 + 1.968 = 6.99 \simeq 7$$

End of Solution

Q.39 In a two-dimensional stress analysis, the state of stress at a point is shown in the figure. The values of length of *PQ*, *QR*, and *RP* are 4, 3, and 5 units, respectively. The principal stresses are _________ . (round off to one decimal place)

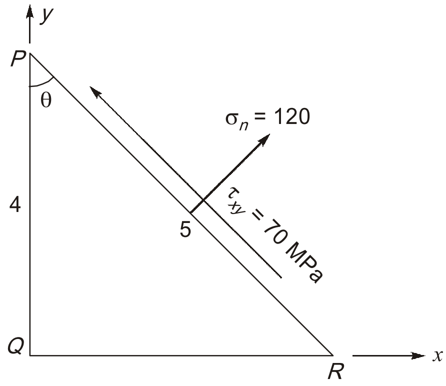


(a) σ_x = 26.7 MPa, σ_y = 172.5 MPa (b) σ_x = 54.0 MPa, σ_y = 128.5 MPa

(c) σ_x = 67.5 MPa, σ_y = 213.3 MPa (d) σ_x = 16.0 MPa, σ_y = 138.5 MPa

[MCQ]

Ans. (c)



$$\cos\theta = \frac{4}{5} \text{ and } \sin\theta = \frac{3}{5};\ \theta = 36.87°$$

$\because \quad \sigma_n = \sigma_x \cos 2\theta + \sigma_y \sin^2\theta$

$$\Rightarrow \quad 120 = \sigma_x \times \left(\frac{4}{5}\right)^2 + \sigma_y\left(\frac{3}{5}\right)^2$$

$$25 \times 120 = 16\sigma_x + 9\sigma_y \qquad \text{... (i)}$$

Also, $\tau_{xy} = (\sigma_y - \sigma_x) \sin\theta \cos\theta$

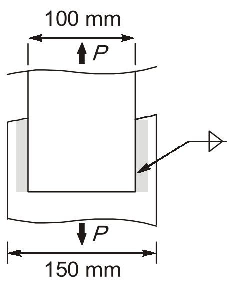
$$\Rightarrow \quad 70 = (\sigma_y - \sigma_x)\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$70 \times 25 = 12\sigma_y + 12\sigma_x \qquad \text{... (ii)}$$

Solving (i) and (ii), $\sigma_x = 67.5$ MPa and $\sigma_y = 213.3$ MPa

End of Solution

Q.40 Two plates are connected by fillet welds of size 10 mm and subjected to tension, as shown in the figure. The thickness of each plate is 12 mm. The yield stress and the ultimate stress of steel under tension are 250 MPa and 410 MPa, respectively. The welding is done in the workshop (partial safety factor, y_{mw} = 1.25). As per the Limit State Method of IS 800: 2007, what is the minimum length (in mm, rounded off to the nearest higher multiple of 5 mm) required of each weld to transmit a factored force *P* equal to 275 kN?



(a) 100
(b) 105
(c) 110
(d) 115

[MCQ]

Ans. **(b)**

$$P = L_w (ks) \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

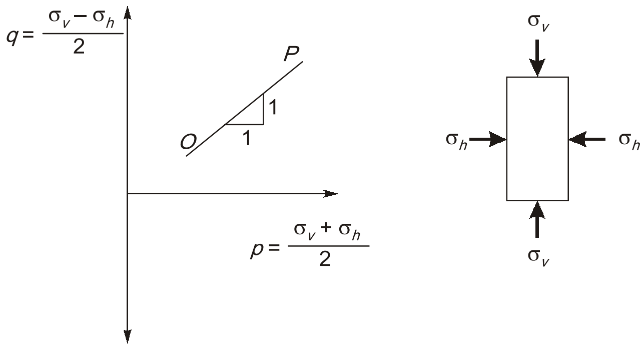
$$275 \times 10^3 = L_w \times 0.7 \times 10 \times \frac{410}{\sqrt{3} \times 1.25}$$

$$L_w = 207.45 \text{ mm}$$

$$\text{Length of each weld} = \frac{207.45}{2} = 103.73 \text{ mm} \simeq 105 \text{ mm}$$

End of Solution

Q.41 In the given figure, Point O indicates the stress point of a soil element at initial non-hydrostatic stress condition. For the stress path (OP), which of the following loading conditions is correct?

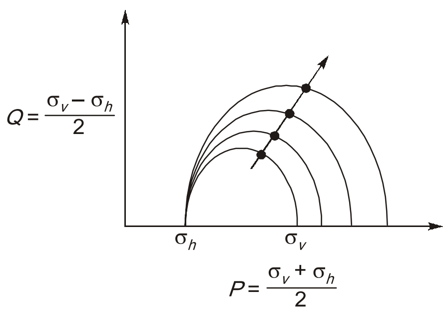


(a) σ_v is increasing and σ_h is constant
(b) σ_v is constant and σ_h is increasing
(c) σ_v is increasing and σ_h is decreasing
(d) σ_v is decreasing and σ_h is increasing

[MCQ]

Ans. **(a)**

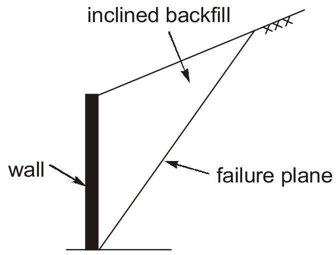
σ_v = Major principal stress
σ_h = Minor principal stress

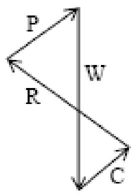


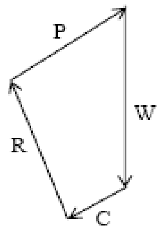
So, σ_h is constant and σ_v is increasing.

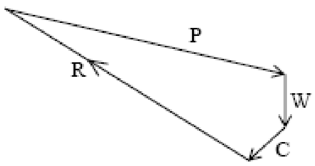
End of Solution

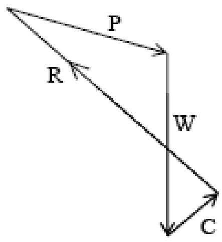
Q.42 The figure shows a vertical retaining wall with backfill consisting of cohesive-frictional soil and a failure plane developed due to passive earth pressure. The forces acting on the failure wedge are: *P* as the reaction force between the wall and the soil, *R* as the reaction force on the failure plane, *C* as the cohesive force along the failure plane and *W* as the weight of the failure wedge. Assuming that there is no adhesion between the wall and the wedge, identify the most appropriate force polygon for the wedge.



(a) 

(b) 

(c) 

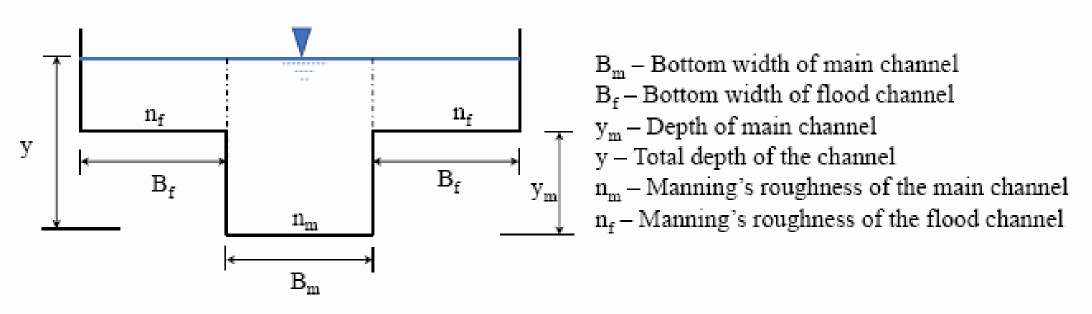
(d) 

[MCQ]

Ans. (c)

End of Solution

Q.43 A compound symmetrical open channel section as shown in the figure has a maximum of critical depth(s).



(a) 3 (b) 2

(c) 1 (d) 4

[MCQ]

Ans. (a)

Case 1:

For $Q = Q_1$ and depth of flow $(y) < y_m$

then, y_{c1} is the critical depth.

Case 2:

For $Q = Q_2$ and depth of flow $(y) > y_m$

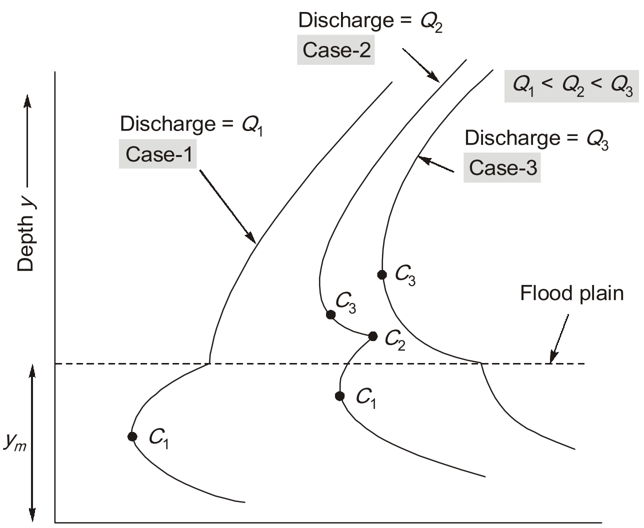
then, y_{c1} , y_{c2} and y_{c3} be the critical depth.

Case 3:

Only y_{c3} will exist at any discharge Q_3

such that $y_{c3} > y_3$

So, the maximum number of critical depth would be three.



End of Solution

Q.44 The critical flow condition in a channel is given by ______ .

[Note: α – kinetic energy correction factor; Q – discharge; A_c – cross-sectional area of flow at critical flow condition; T_c – top width of flow at critical flow condition; g – acceleration due to gravity]

(a) $\frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c}$

(b) $\frac{\alpha Q}{g} = \frac{A_c^3}{T_c^2}$

(c) $\frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c^2}$

(d) $\frac{\alpha Q}{g} = \frac{A_c^3}{T_c}$

[MCQ]

Ans. (a)

Froude number, $F_r = \frac{v}{\sqrt{gD}}$; where D = hydraulic depth = $\frac{A}{T}$

$$\Rightarrow \quad F_r^2 = \frac{v^2}{gD}$$

$$\Rightarrow \quad F_r^2 = \frac{Q^2 T}{gA^2 A} = \frac{Q^2 T}{gA^3}$$

For critical condition, $F_r = 1$

$$\Rightarrow \quad \frac{Q^2 T_c}{gA^3} = 1$$

$$\Rightarrow \quad \frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

Now, taking into account, kinetic energy correction factor,

$$\frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c}$$

End of Solution

Q.45 Match the following air pollutants with the most appropriate adverse health effects:

Air pollutant		Health effect to human and/or test animal
(P) Aromatic hydrocarbons	(I)	Reduce the capability of the blood to carry oxygen
(Q) Carbon monoxide	(II)	Bronchitis and pulmonary emphysema
(R) Sulfur oxides	(III)	Damage of chromosomes
(S) Ozone	(IV)	Carcinogenic effect

(a) (P) – (II), (Q) – (I), (R) – (IV), (S) – (III)

(b) (P) – (IV), (Q) – (I), (R) – (III), (S) – (II)

(c) (P) – (III), (Q) – (I), (R) – (II), (S) – (IV)

(d) (P) – (IV), (Q) – (I), (R) – (II), (S) – (III)

[MCQ]

Ans. (d)

End of Solution

Q.46 A delivery agent is at a location R. To deliver the order, she is instructed to travel to location P along straight-line paths of RC, CA, AB and BP of 5 km each. The direction of each path is given in the table below as whole circle bearings. Assume that the latitude (L) and departure (D) of R is (0, 0) km. What is the latitude and departure of P (in km, rounded off to one decimal place)?

Paths	RC	CA	AB	BP
Directions (in degrees)	120	0	90	240

(a) $L = 2.5;\ D = 5.0$
(B) $L = 0.0;\ D = 5.0$
(C) $L = 5.0;\ D = 2.5$
(D) $L = 0.0;\ D = 0.0$

[MCQ]

Ans. **(b)**

For location R,

Latitude, $L_R = 0$

Departure, $D_R = 0$

For location P,

Latitude, $L_P = L_R + \Sigma\Delta_L$

$= 0 + 5\cos 120° + 5\cos 0° + 5\cos 90° + 5\cos 240° = 0$

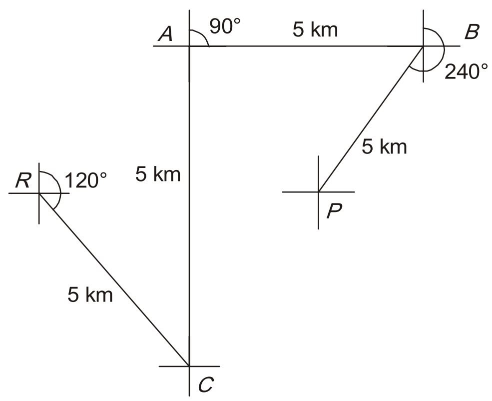
Departure, $D_P = D_R + \Sigma\Delta_D$

$= 0 + 5\sin 120° + 5\sin 0° + 5\sin 90° + 5\sin 240° = 5$

So, $L_P = 0,\quad D_P = 5$

Alternatively,

Path (l)	Direction (θ)	Latitude ($l\cos\theta$)	Departure ($l\sin\theta$)
RC	120°	–2.5	4.33
CA	0°	5	0
AB	90°	0	5
BP	240°	–2.5	–4.33



Length of each path (l) = 5 km

Latitude of P in km = Σlatitude = 0

Departure of P in km = Σdeparture = 5

End of Solution

Q.47 Which of the following statements is/are TRUE?

(a) The thickness of a turbulent boundary layer on a flat plate kept parallel to the flow direction is proportional to the square root of the distance from the leading edge

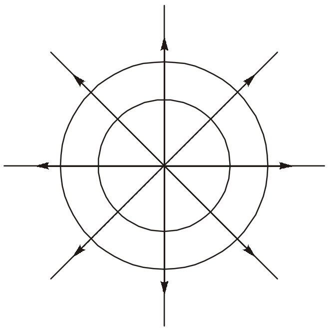
(b) If the streamlines and equipotential lines of a source are interchanged with each other, the resulting flow will be a sink

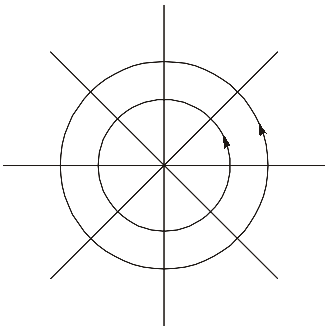
(c) For a curved surface immersed in a stationary liquid, the vertical component of the force on the curved surface is equal to the weight of the liquid above it

(d) For flow through circular pipes, the momentum correction factor for laminar flow is larger than that for turbulent flow

[MSQ]

Ans. (c, d)


Source


Free-vortex flow

Thickness of boundary layer in turbulent flow:

$$\frac{\delta}{x} = \frac{0.376}{\text{Re}_x^{1/5}} = \frac{0.376}{\left(\frac{\rho u_\infty x}{\mu}\right)^{1/5}}$$

$$\Rightarrow \qquad \delta = \frac{0.376 x^{4/5}}{\left(\frac{\rho u_\infty}{\mu}\right)^{1/5}}$$

$$\Rightarrow \qquad \delta \propto x^{4/5}$$

End of Solution

Q.48 In the context of water and wastewater treatments, the correct statements are:

(a) particulate matter may shield microorganisms during disinfection

(b) ammonia decreases chlorine demand

(c) phosphorous stimulates algal and aquatic growth

(d) calcium and magnesium increase hardness and total dissolved solids

[MSQ]

Ans. (a, c, d)

End of Solution

Q.49 Which of the following statements is/are TRUE for the aerobic composting of sewage sludge?

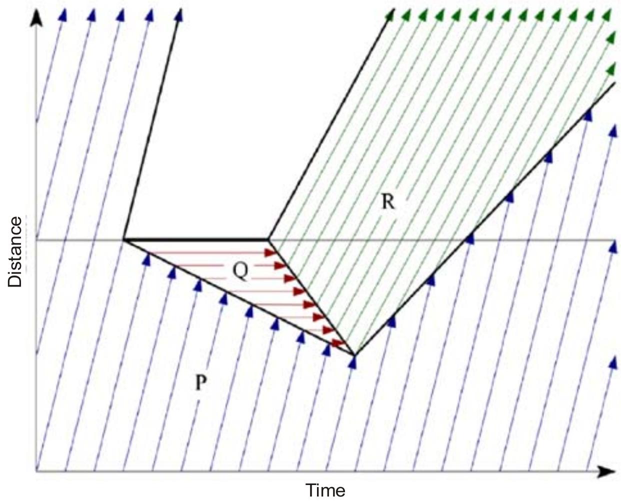
(a) Bulking agent is added during the composting process to reduce the porosity of the solid mixture
(b) Leachate can be generated during composting
(c) Actinomycetes are involved in the process
(d) In-vessel composting systems cannot be operated in the plug-flow mode

[MSQ]

Ans. (b, c)

End of Solution

Q.50 The figure presents the time-space diagram for when the traffic on a highway is suddenly stopped for a certain time and then released. Which of the following statements are true?



(a) Speed is higher in Region R than in Region P
(b) Volume is lower in Region Q than in Region P
(c) Volume is higher in Region R than in Region P
(d) Density is higher in Region Q than in Region R

[MSQ]

Ans. (b, c, d)

- Option (a) is incorrect because, slope of distance time graph is more in region P than in region R.
- Volume in region Q is lower than in region P because in region Q, vehicles are at halted conditions. So, in region Q, velocity of vehicle is zero and hence, volume is zero. Option (b) is correct.
- Option (c) is correct because slope of shock wave is higher from Q to R in comparision from P to Q.
- Density is higher in region Q than in region R and region P because in region Q, they are practically at Jam density that is maximum. So, option (d) is correct.

End of Solution

Q.51 Consider the Marshall method of mix design for bituminous mix. With the increase in bitumen content, which of the following statements is/are TRUE?

(a) the Stability decreases initially and then increases

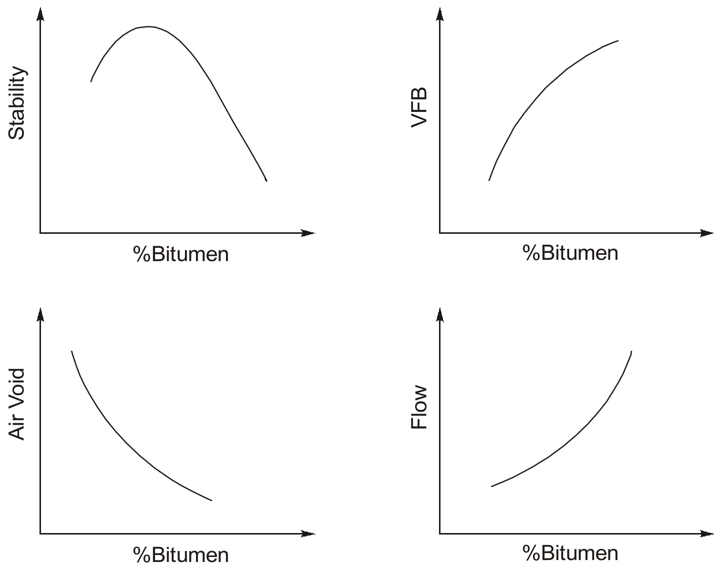
(b) the Flow increases monotonically

(c) the air voids (*VA*) increases initially and then decreases

(d) the voids filled with bitumen (*VFB*) increases monotonically

[MSQ]

Ans. (b, d)



End of Solution

Q.52 A 5 cm long metal rod *AB* was initially at a uniform temperature of T_0 °C. Thereafter, temperature at both the ends are maintained at 0 °C. Neglecting the heat transfer from the lateral surface of the rod, the heat transfer in the rod is governed by the one-dimensional diffusion equation $\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$, where D is the thermal diffusivity of the metal, given as 1.0 cm^2/s.

The temperature distribution in the rod is obtained as

$$T(x,\ t) = \sum_{n=1,3,5...}^{\infty} C_n \sin\frac{n\pi x}{5} e^{-\beta n^2 t},$$

where x is in cm measured from A to B with $x = 0$ at A, t is in s, C_n are constants in °C, T is in °C, and β is in s^{-1}. The value of β (in s^{-1}, rounded off to three decimal places) is _______ .

[NAT]

Ans. (0.395) (0.394 – 0.396)

$$\frac{\partial T}{\partial t} = \Delta\frac{\partial^2 T}{\partial x^2};\ T(0, t) = 0,\ T(5, t) = 0,\ T(x, 0) = T_0,\ D = 1 \qquad ...(1)$$

Put $D = 1$, it's general solution using separation of variables methods is

$$T(x, t) = (C_1 \cos px + C_2 \sin px)C_3 e^{-p^2 t} \qquad ...(2)$$

Using $T(0, t) = 0 \Rightarrow C_1 = 0$

Using $T(5, t) = 0 \Rightarrow C_2 C_3 \sin 5p e^{-p^2 t} = 0$

or $\quad \sin 5p = 0 \Rightarrow p = \frac{n\pi}{5}, n \in I$

Hence, by (2), $\quad T(x, t) = b_n \sin\left(\frac{n\pi x}{5}\right)\cdot e^{\frac{-n^2\pi^2 t}{25}}$ where $b_n = C_2 C_3$

Hence most general solution is

$$T(x, t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{5}\right) e^{\frac{-n^2\pi^2 t}{25}} \qquad ...(3)$$

Now using $T(x, 0) = T_0$

$$\Rightarrow \quad T_0 = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{5}\right)$$

which is half range fourier sine series for T_0

Hence,
$$b_n = \frac{2}{5}\int_0^5 T_0 \sin\left(\frac{n\pi x}{5}\right) dx = \begin{cases} 0, & n \text{ even} \\ \frac{4T_0}{n\pi}, & n \text{ odd} \end{cases}$$

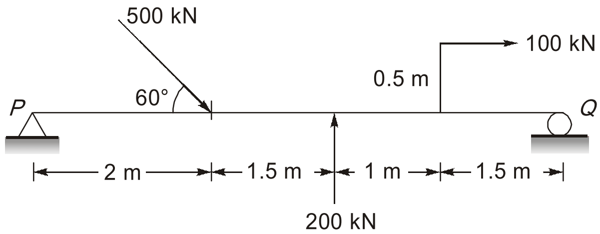
By (3),
$$T(x, t) = \sum_{n = \text{odd}} \frac{4T_0}{n\pi} \sin\left(\frac{n\pi x}{5}\right) e^{\frac{-n^2\pi^2 t}{25}} \qquad ...(4)$$

On comparision with, $T(x, t) = \sum_{n = 1, 3, 5} C_n \sin\left(\frac{n\pi x}{5}\right) e^{-\beta n^2 t}$

$$\Rightarrow \quad C_n = \frac{4T_0}{n\pi} \text{ and } \beta = \frac{\pi^2}{25} = 0.3947 \approx 0.395$$

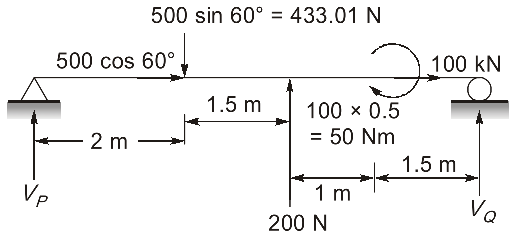
End of Solution

Q.53 A beam is subjected to a system of coplanar forces as shown in the figure. The magnitude of vertical reaction at Support P is ________ N. (round off to one decimal place).



[NAT]

Ans. (197.0) (195 – 200)



Taking moment at P

$\Sigma M_P = 0$

$\Rightarrow 433.01 \times 2 - 200 \times 3.5 + 50 - V_Q \times 6 = 0$

$\Rightarrow \quad V_Q = 36.00$ N

Also, $\quad \Sigma F_y = 0$

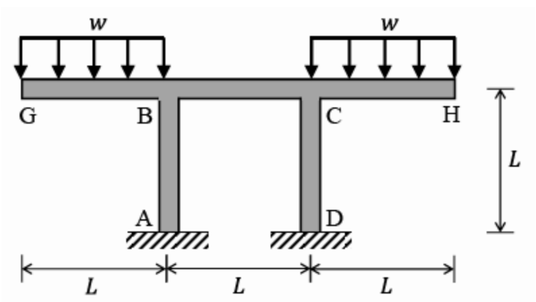
$\Rightarrow \quad V_P + 200 + V_Q = 433.01$

$V_P = 433.01 - 200 - 36.00$

$V_P = 197.01$ N

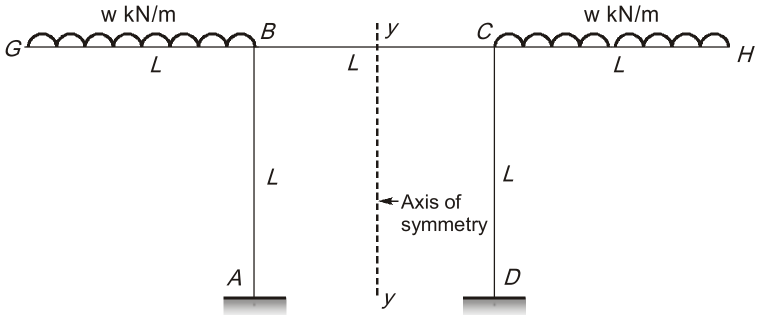
End of Solution

Q.54 For the frame shown in the figure (not to scale), all members (AB, BC, CD, GB, and CH) have the same length, L and flexural rigidity, EI The joints at B and C are rigid joints, and the supports A and D are fixed supports. Beams GB and CH carry uniformly distributed loads of w per unit length. The magnitude of the moment reaction at A is wL^2/k. What is the value of k (in integer)?

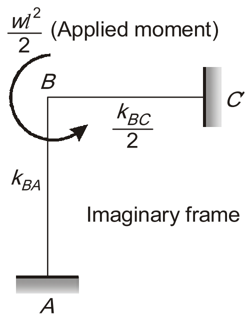


[NAT]

Ans. (6) (6 – 6)



Distribution factors



Joint	Member	k	Σk	D.F = $k/\Sigma k$
B	BA	$\frac{1}{L}$	$\frac{3I}{2L}$	$\frac{2}{3}$
	BC	$\frac{1}{2}\left(\frac{1}{L}\right)$		$\frac{1}{3}$

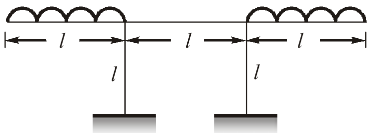
End moment distribution

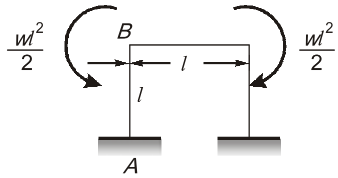
Joint	A	B		C
D.F.	0	$\frac{2}{3}$	$\frac{1}{3}$	—
Balance		$\frac{-wl^2}{3}$	$\frac{-wl^2}{6}$	
C.O.M.	$\frac{-wl^2}{6}$			—
Final end moments	$\frac{-wl^2}{6}$	$\frac{-wl^2}{3}$	$\frac{-wl^2}{6}$	—

$\therefore$ Moment at $A = \frac{wl^2}{6}$

$\therefore$ $\alpha = 6$

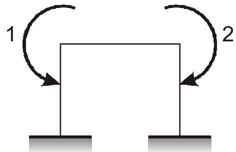
Alternatively,
1st method



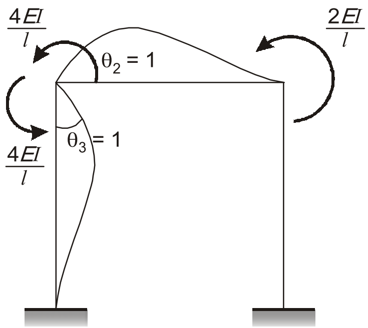


$D_k = 1$ (i.e. θ_3)

$\theta_c = f(\theta_B)$



Stiffness matrix



$$[k] = \begin{bmatrix} \frac{8EI}{l} & \frac{2EI}{l} \\ \frac{2EI}{l} & \frac{8EI}{l} \end{bmatrix} = \frac{2EI}{l}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

Now, $\quad [P] = [P_L] + [k][\Delta]$

$$\Rightarrow \qquad 0 = \begin{bmatrix} \frac{-wl^2}{2} \\ \frac{wl^2}{2} \end{bmatrix} + \frac{2EI}{l}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$

$$\frac{wl^2}{2} = \frac{8EI}{l}\theta_B + \frac{2EI}{l}\theta_C$$

$$\frac{wl^2}{2} = \frac{6EI}{l}\theta_B \qquad (\because \theta_B = -\theta_C)$$

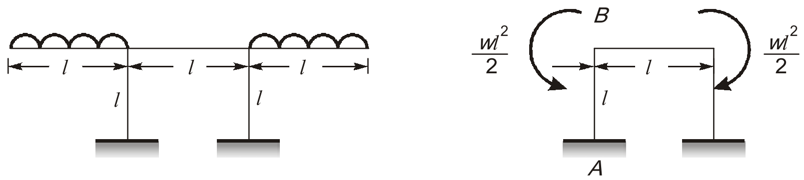
$$\therefore \qquad \theta_B = \frac{wl^2}{12EI}$$

$$\therefore \qquad M_{BA} = \frac{4EI}{l}\theta_B = \frac{4EI}{l}\frac{wl^2}{12EI} = \frac{wl^2}{3}$$

$\therefore \quad M_{BA} = \frac{M_{BA}}{2} = \frac{wl^2}{6}$

2nd method

Slope deflection method



$$M_{BA} = \frac{2EI}{l}(2\theta_B + \theta_A) = \frac{4EI}{l}\theta_B \quad \text{...(1)}$$

$$M_{BC} = \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$\theta_B = -\theta_C \qquad \text{due to symmetry}$$

$$\therefore \quad M_{BC} = \frac{2EI}{l}\theta_B + \frac{4EI}{l}\theta_B$$

Joint equation comliting

$$M_{BA} + M_{BC} = \frac{wl^2}{2}$$

$$\frac{6EI}{l}\theta_B = \frac{wl^2}{2}$$

$$\Rightarrow \quad \theta_B = \frac{+wl^2}{12EI}$$

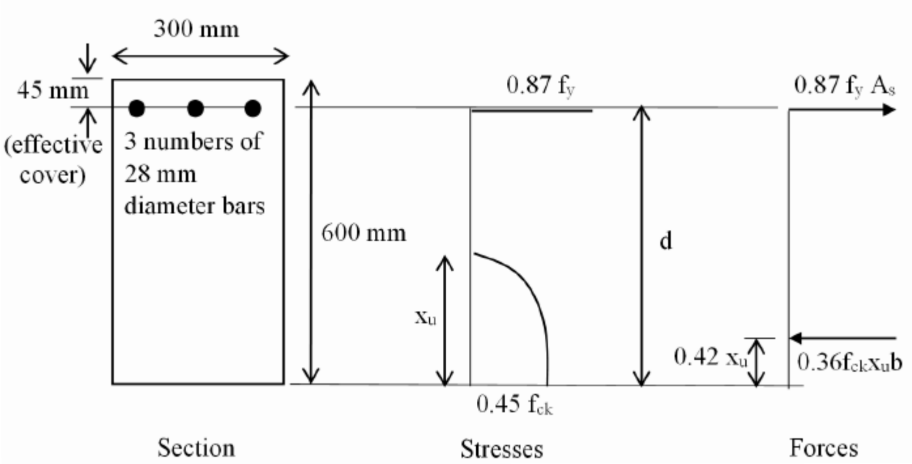
$$M_{BA} = \frac{4EI}{l} + \frac{wl^2}{12EI} = \frac{+wl^2}{3EI}$$

$$\therefore \quad M_B = \frac{M_{BA}}{2} = \frac{M_{BA}}{2} = \frac{wl^2}{6EI}$$

End of Solution

Q.55 Consider the singly reinforced section of a cantilever concrete beam under bending, as shown in the figure (M25 grade concrete, Fe415 grade steel). The stress block parameters for the section at ultimate limit state, as per IS 456 : 2000 notations, are given. The ultimate moment of resistance for the section by the Limit State Method is ________ kN.m (round off to one decimal place).

[Note: Here, As is the total area of tension steel bars, b is the width of the section, d is the effective depth of the bars, f_{ck} is the characteristic compressive cube strength of concrete, f_y is the yield stress of steel, and x_u is the depth of neutral axis.]



[NAT]

Ans. (301) (295 – 305)

Given: B = 300 mm, d = 600 – 45 = 555 mm

$$\text{Area of steel} = 3 \times \frac{\pi}{4} \times 28^2 = 1847.256 \text{ mm}^2$$

Depth of N.A.:

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$0.36 \times 25 \times 300 \times x_u = 0.87 \times 415 \times 1847.256$$

$$x_u = \frac{0.87 \times 415 \times 1847.256}{0.36 \times 25 \times 300} = 247.02 \text{ mm}$$

$$x_{u,\lim} = 0.48d = 0.48 \times 555 = 266.4 \text{ mm} > x_u$$

∴ Under reinforced section.

$$\text{M.R.} = 0.36\ f_{ck} b x_u\ (d - 0.42 x_u)$$

$$= 0.36 \times 25 \times 300 \times 247.02\ (555 - 0.42 \times 247.02)$$

$$= 300964059.6 \text{ N-mm}$$

∴ $$\text{M.R.} = 300.96 \text{ kN-m}$$

End of Solution

Q.56 A 2D thin plate with modulus of elasticity, E =1.0 N/m^2, and Poisson's ratio, μ = 0.5, is in plane stress condition. The displacement field in the plate is given by $u = Cx^2y$. and v = 0, where u and v are displacements (in m) along the X and Y directions, respectively, and C is a constant (in m^{-2}). The distances x and y along X and Y, respectively, are in m. The stress in the X direction is $\sigma_{XX} = 40xy$ N/m^2, and the shear stress is $\tau_{XY} = \alpha x^2$ N/m^2. What is the value of α (in N/m^4, in integer)? _______

[NAT]

Ans. (5) (5 – 5)

We know that,

$$\sigma_x = \frac{E}{(1-\mu^2)}(\epsilon_x + \mu \epsilon_y)$$

$$\sigma_x = \frac{E}{(1-\mu^2)}\left(\frac{\partial u}{\partial x} + \mu \frac{\partial v}{\partial y}\right)$$

$$\sigma_x = \frac{E}{(1-\mu^2)}(2Cxy + 0)$$

$$\sigma_x = \frac{2ECxy}{(1-\mu^2)}$$

Given, $\sigma_x = 40xy \text{ N/m}^2$

$$\Rightarrow \quad \frac{2EC\cancel{xy}}{(1-\mu^2)} = 40\cancel{xy}$$

$$\Rightarrow \quad C = \frac{40(1-\mu^2)}{2E} = \frac{40(1-0.5^2)}{2\times 1} = 15$$

Now, $$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\gamma_{xy} = Cx^2 + 0$$

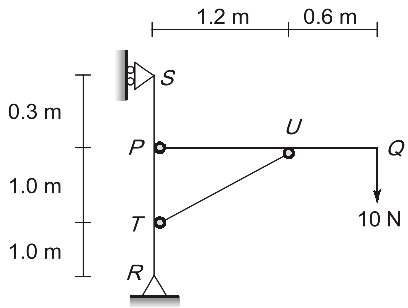
Also, $$G = \frac{E}{2(1+\mu)} = \frac{\tau_{xy}}{\gamma_{xy}}$$

$$\Rightarrow \quad \tau_{xy} = \frac{Cx^2 \times 1}{2(1+0.5)} = \left(\frac{15}{2\times 1.5}\right)x^2 = 5x^2$$

$\therefore$ On comparing with $\tau_{xy} = \alpha x^2$, the value of α is 5.

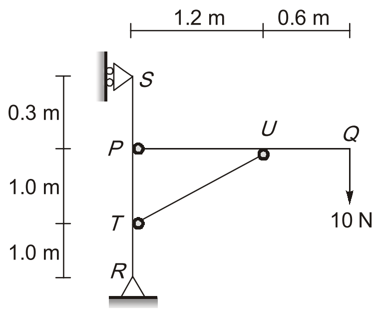
End of Solution

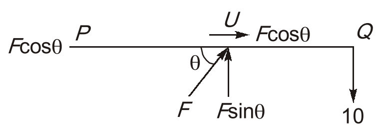
Q.57 An idealised frame supports a load as shown in the figure. The horizontal component of the force transferred from the horizontal member PQ to the vertical member RS at P is _______ N (round off to one decimal place).



[NAT]

Ans. (18) (17.5 – 18.5)





$\Sigma M_A = 0$

$\Rightarrow$ $F\sin\theta \times 1.2 = 10 \times 1.8$

$$F = \frac{18}{1.2\sin\theta} = 23.4307$$

Horizontal force at P = 18 N

End of Solution

Q.58 A square footing is to be designed to carry a column load of 500 kN which is resting on a soil stratum having the following average properties: bulk unit weight = 19 kN/m^3; angle of internal friction = 0° and cohesion = 25 kPa. Considering the depth of the footing as 1 m and adopting Meyerhoff's bearing capacity theory with a factor of safety of 3, the width of the footing (in m) is ________ (round off to one decimal place)

[Assume the applicable shape and depth factor values as unity; ground water level at greater depth.]

[NAT]

Ans. **(3.4) (3 – 3.4)**

As per Meyerhoff's theory,

$$q_u = CN_cS_cd_ci_c + qN_qS_qd_qi_q + 0.5\gamma BN_\gamma S_\gamma d_\gamma i_\gamma$$

For $\phi = 0$, $N_c = 5.14$, $N_q = 1$ and $N_\gamma = 0$

Also, considering shape, depth and inclination factor as 1, we get

$$\Rightarrow \quad q_u = 5.14 \times 25 + \gamma D_f$$

$$\Rightarrow \quad q_{nu} = q_u - \gamma D_f = 5.14 \times 25$$

$$\Rightarrow \quad q_{ns} = \frac{q_{nu}}{\text{FOS}} = \frac{5.14 \times 25}{3} = 42.83 \text{ kN/m}^2$$

Also, $Q_{ns} = 500 \text{ kN} = q_{ns} \times B^2$

$$\Rightarrow \quad 42.83 \times B^2 = 500$$

$$\Rightarrow \quad B = 3.41 \text{ m}$$

End of Solution

Q.59 A circular pile of diameter 0.6 m and length 8 m was constructed in a cohesive soil stratum having the following properties: bulk unit weight = 19 kN/m^3; angle of internal friction = 0° and cohesion = 25 kPa.

The allowable load the pile can carry with a factor of safety of 3 is _______ kN (round off to one decimal place).

[Adopt: Adhesion factor, $\alpha = 1.0$ and Bearing capacity factor, $N_c = 9.0$]

[NAT]

Ans. **(146.9) (145 – 149)**

$$Q_{\text{up}} = 9c\left(\frac{\pi}{4}D^2\right) + \alpha\bar{c}(\pi DL)$$

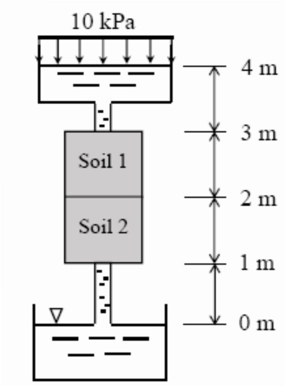
$$= 9 \times 25\left(\frac{\pi}{4} \times (0.6)^2\right) + 1 \times 25(\pi \times 0.6 \times 8)$$

$$\therefore \quad Q_{\text{up}} = 440.608 \text{ kN}$$

$$\therefore \quad \text{Allowable load} = Q_{\text{ap}} = \frac{Q_{up}}{\text{FOS}} = \frac{440.608}{3} = 146.87 \text{ kN}$$

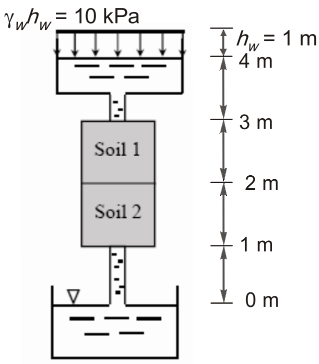
End of Solution

Q.60 For the flow setup shown in the figure (not to scale), the hydraulic conductivities of the two soil samples, Soil 1 and Soil 2, are 10 mm/s and 1 mm/s, respectively. Assume the unit weight of water as 10 kN/m^3 and ignore the velocity head. At steady state, what is the total head (in m, rounded off to two decimal places) at any point located at the junction of the two samples?



[NAT]

Ans. (4.54) (4.5 – 4.6)



$$\gamma_w h_w = 10 \text{ kPa}$$

$$\Rightarrow \quad h_w = 1 \text{ m}$$

$$(\text{TH})_{\text{entry}} = \text{D.H} + \text{P.H} = 3 + 2 = 5 \text{ m}$$

$$(\text{T·H})_{\text{middle}} = (\text{TH})_{\text{entry}} - h_{L1}$$

Now, soil is in series,

$$\text{So,} \quad \frac{\Sigma z_i}{\Sigma \frac{z_i}{k_i}} \times \left(\frac{H_L}{L}\right)_{Total} \times A = k_1 \times \frac{h_{L_1}}{L_1} \times A$$

$$\Rightarrow \quad \frac{1+1}{\frac{1}{10}+\frac{1}{1}} \times \left(\frac{5}{2}\right) \times A = 10 \times \frac{h_{L_1}}{1} \times A$$

$$\Rightarrow \quad h_{L_1} = 0.4545$$

$$\therefore \quad (\text{T.H})_{\text{middle}} = 5 - 0.4545 = 4.54 \text{ m}$$

End of Solution

Q.61 A consolidated drained (CD) triaxial test was carried out on a sand sample with the known effective shear strength parameters, $c' = 0$ and $\phi' = 30$. In the test, prior to the failure, when the sample was undergoing axial compression under constant cell pressure, the drainage valve was accidentally closed. At the failure, 360 kPa deviatoric stress was recorded along with 70 kPa pore water pressure. If the test is repeated without such error, and no back pressure is applied in either of the tests, what is the deviatoric stress (in kPa, in integer) at the failure? ________.

[NAT]

Ans. (500) (500 – 500)

$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c' \tan\left(45 + \frac{\phi}{2}\right)$$

$\because$ $\bar{\sigma}_3 = \sigma_3 - 70$

$\bar{\sigma}_1 = \sigma_1 - 70 = \sigma_3 + \sigma_d - 70 = (\sigma_3 + 360 - 70)$

$\Rightarrow$ $(\sigma_3 + 360 - 70) = (\sigma_3 - 70) \tan(45 + 30/2)$ $[\because c = 0]$

$\Rightarrow$ $\sigma_3 = 250$ kPa

Now, $\sigma_1 = \sigma_3 \tan^2(45 + \phi/2)$ $[\because c = 0]$

$\Rightarrow$ $(\sigma_3 + \sigma_d) = \sigma_3 \tan^2 (45 + 30/2)$

$\Rightarrow$ $(250 + \sigma_d) = 250 \times 3$

$\Rightarrow$ $\sigma_d = 500$ kPa

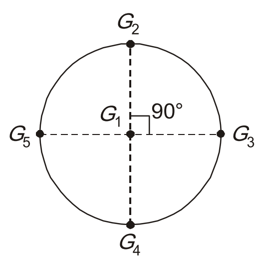
End of Solution

Q.62 A catchment may be idealized as a circle of radius 30 km. There are five rain gauges, one at the center of the catchment and four on the boundary (equi-spaced), as shown in the figure (not to scale).

The annual rainfall recorded at these gauges in a particular year are given below.

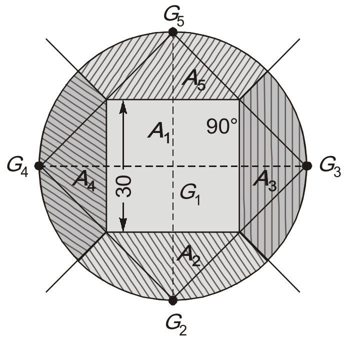
Gauge	G_1	G_2	G_3	G_4	G_5
Rainfall (mm)	910	930	925	895	905

Using the Thiessen polygon method, what is the average rainfall (in mm, rounded off to two decimal places) over the catchment in that year? _________



[NAT]

Ans. (912.55) (912.3 – 912.8)



$$\text{Area of circle } (A) = \pi \times (30)^2 \text{ km}^2$$

$$A_1 = 30 \times 30 = 900 \text{ km}^2$$

$$A_2 = A_3 = A_4 = A_5 = \frac{\pi \times 30^2 - 900}{4} = 481.858 \text{ km}^2$$

$$\therefore \quad \text{Average rainfall, } \overline{P} = \frac{G_1A_1 + G_2A_2 + G_3A_3 + G_4A_4 + G_5A_5}{A}$$

$$= \frac{910 \times 900 + 481.858(930 + 925 + 895 + 905)}{\pi \times 30^2}$$

$$= 912.555 \text{ mm}$$

End of Solution

Q.63 The cross-section of a small river is sub-divided into seven segments of width 1.5 m each. The average depth, and velocity at different depths were measured during a field campaign at the middle of each segment width. The discharge computed by the velocity area method for the given data is ________ m^3/s (round off to one decimal place).

Segment	Average depth (D) (m)	Velocity (m/s) at different depths		
		0.2 D	0.6 D	0.8 D
1	0.40	--	0.40	--
2	0.70	0.76	--	0.70
3	1.20	1.19	--	1.13
4	1.40	1.25	--	1.10
5	1.10	1.13	--	1.09
6	0.80	0.69	--	0.65
7	0.45	--	0.42	--

[NAT]

Ans. (8.5) (8.4 – 8.6)

Segment (1)	Average depth (D) m (1)	Velocity of depths 0.2 D (3)	0.6 D (4)	0.8 D (5)	$\frac{0.2D + 0.8D}{2}$ (6)	Average Velocity
1	0.40	--	0.40	--	--	0.4
2	0.70	0.76	--	0.70	0.73	0.13
3	1.20	1.19	--	1.13	1.16	1.16
4	1.40	1.25	--	1.10	1.175	1.175
5	1.10	1.13	--	1.09	1.11	1.11
6	0.80	0.69	--	0.65	0.67	0.67
7	0.45	--	0.42	--	--	0.42

Effective width of 1st section = $(\bar{W}_1) = \dfrac{\left(W_1 + \dfrac{W_2}{2}\right)^2}{2W_1} = \dfrac{\left(1.5 + \dfrac{1.5}{2}\right)^2}{2 \times 1.5} = 1.6875\text{ m}$

Effective width of 2nd to 6th section ($W_2 = W_2 \ldots = W_6$)

$$= \left(\frac{1.5 + 1.5}{2}\right) = 1.5\text{ m}$$

Effective width of 7th section = $\dfrac{\left(1.5 + \dfrac{1.5}{2}\right)^2}{2 \times 1.5} = 1.6875\ m$

∴ Discharge = $\sum_{i=1}^{7} \Delta Q_i \;\sum_{i=1}^{7} \Delta Q_i = \sum_{i=1}^{7} W_i \times y_1 \times \bar{V}_i$

= (1.6875 × 0.4 × 0.4) + (1.5 × 0.7 × 0.73) + (1.5 × 1.2 × 1.16) + (1.5 × 1.4 × 1.175)
+ (1.5 × 1.1 × 1.11) + (1.5 × 0.8 × 0.67) + (1.6875 × 0.45 × 0.42)
= 8.564 m^3/sec

End of Solution

Q.64 The theoretical aerobic oxidation of biomass ($C_5H_7O_2N$) is given below:

$$C_5H_7O_2N + 5O_2 \rightarrow 5CO_2 + NH_3 + 2H_2O$$

The biochemical oxidation of biomass is assumed as a first-order reaction with a rate constant of 0.23/d at 20°C (logarithm to base *e*). Neglecting the second-stage oxygen demand from its biochemical oxidation, the ratio of BOD_5 at 20°C to total organic carbon (TOC) of biomass is ________ (round off to two decimal places).

[Consider the atomic weights of C, H, O and N as 12 g/mol, 1 g/mol, 16 g/mol and 14 g/mol, respectively]

[NAT]

Ans. **(1.82) (1.8 – 2)**

$$C_5H_7O_2N + 5O_2 \rightarrow 5CO_2 + NH_3 + 2H_2O$$

Molar mass of $C_5H_7O_2N$ = 113g

$$\text{TOC} = 5 \times 12 = 60\text{g}$$

$$\text{Ultimate BOD} = 5 \times 32 = 160\text{g}$$

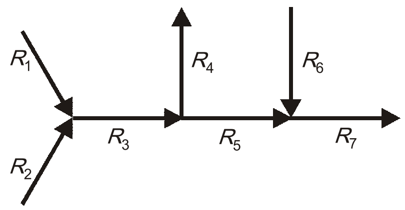
$$BOD_5(20°C) = 160(1 - e^{-0.23 \times 5})$$

$$BOD_5(20°C) = 109.34\text{g}$$

$$\frac{BOD_5(20°C)}{TOC} = \frac{109.34}{60} = 1.822$$

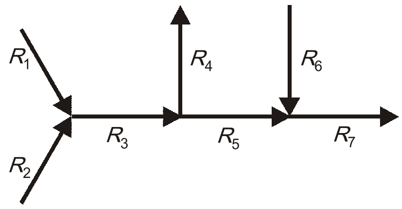
End of Solution

Q.65 A system of seven river segments is shown in the schematic diagram. The R_i's, Q_i's, and C_i's (i = 1 to 7) are the river segments, their corresponding flow rates, and concentrations of a conservative pollutant, respectively. Assume complete mixing at the intersections, no additional water loss or gain in the system, and steady state condition. Given: $Q_1 = 5\ m^3/s$; $Q_2 = 15\ m^3/s$; $Q_4 = 3\ m^3/s$; $Q_6 = 8\ m^3/s$; $C_1 = 8\ kg/m^3$; $C_2 = 12\ kg/m^3$; $C_6 = 10\ kg/m^3$. What is the steady state concentration (in kg/m^3, rounded off to two decimal place) of the pollutant in the river segment 7 ? _______



[NAT]

Ans. (10.68) (10.58 – 10.78)



Given,

$Q_1 = 5\ m^3/s$ $\qquad Q_2 = 15\ m^3/s$

$Q_4 = 3\ m^3/s$ $\qquad Q_6 = 8\ m^3/s$

$C_1 = 8\ kg/m^3$ $\qquad C_2 = 12\ kg/m^3$

$C_6 = 10\ kg/m^3$

$Q_3 = Q_1 + Q_2 = 5 + 15 = 20\ m^3/s$

$$C_3 = \frac{(5)(8)+(15)(12)}{5+15} = 11 kg/m^3$$

$Q_5 = Q_3 - Q_4 = 20 - 3 = 17\ m^3/s$

$C_5 = C_3 = 11\ kg/m^3$

$Q_7 = Q_5 + Q_6 = 17 + 8 = 25\ m^3/s$

$$C_7 = \frac{(17)(11)+(8)(10)}{25} = 10.68\ kg/m^3$$

End of Solution

■■■■