# Mathematics

**Time: 3 Hours** **Max. Marks: 80**

| S. No. | Typology of Question | Very Short Answer (VSA) 1 Mark | Short Answer– I (SA I) 2 Marks | Short Answer– II (SA II) 2 Marks | Long Answer (LA) 5 Marks | Total Marks | % Weightage |
|---|---|---|---|---|---|---|---|
| 1. | **Remembering** | 2 | 2 | 2 | 2 | 20 | 25% |
| 2. | **Understanding** | 2 | 1 | 1 | 4 | 23 | 29% |
| 3. | **Application** | 2 | 2 | 3 | 1 | 19 | 24% |
| 4. | **High Order Thinking Skills** | - | 1 | 4 | - | 14 | 17% |
| 5. | **Inferential and Evaluative** | - | - | - | 1 | 4 | 5% |
| | **Total** | 6 × 1 = 6 | 6 × 2 = 12 | 10 × 3 = 30 | 8 × 4 = 32 | 80 | 100% |

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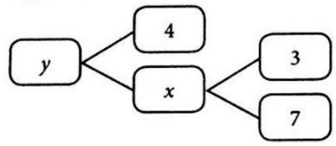
**General Instructions:**

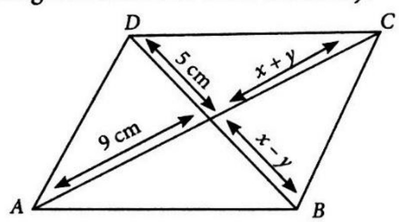
**(i)** All question are compulsory
**(ii)** The question paper consists of 30 question divided into four section A, B, C and D.
**(iii)** Section A contains 6 questions of I mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
**(iv)** There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

## SECTION - A

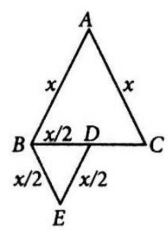
1. Find the values of $x$ and $y$ in the given figure.




2. In the given figure, $ABCD$ is parallelogram. Find the value of $x$ and $y$.




3. $\Delta ABC$ and $\Delta BDE$ are two equilateral triangles such that $D$ is the mid-point of $BC$. Find the ratio of the areas of $\Delta ABC$ and $\Delta BDE$




4. Write the polynomial whose zeroes are $3+\sqrt{2}$ and $3-\sqrt{2}$.

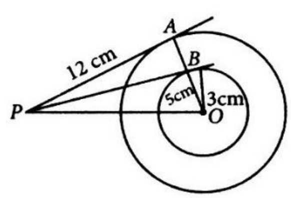
5. Find the value of $k$, if $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots.

6. If $\left(3, \frac{3}{4}\right)$ is the mid-point of the line segment joining the points $(k, 0)$ and $\left(7, \frac{3}{2}\right)$, then find the value of $k$.

## SECTION - B

7. If $p$, $q$ are zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

8. Two concentric circles with centre $O$ are of radii 5 cm and 3 cm. From an external point $P$, two tangents $PA$ and $PB$ are drawn to these circles, respectively. If $PA = 12$ cm, then find length of $PB$.




9. Write the median class of the following distribution:

| Class | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|---|---|---|---|---|---|---|---|
| Frequency | 4 | 4 | 8 | 10 | 12 | 8 | 4 |

**10.** Find the nature of the roots of the quadratic equation $2x^2 - 2\sqrt{6}x + 3 = 0$, if the real roots exist, then find them.

**11.** A cylinder and a cone have same base area. But the volume of cylinder is twice the volume of cone. Find the ratio between their heights.

**12.** Find the sum of all two-digit numbers greater than 50 which when divided by 7 leaves remainder 4.

## SECTION - C

**13.** Prove that $\frac{7}{5}\sqrt{2}$ is not a rational number.

**14.** In a $\Delta ABC$, let $P$ and $Q$ be points on $AB$ and $AC$ respectively such that $PQ \| BC$. Prove that median $AD$ bisects $PQ$.

**15.** A vertical straight tree, 15 m high, is broken by the wind, in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did it break ? (Use $\sqrt{3} = 1.73$)

**OR**

A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point $A$ on the ground is 60 degree and the angle of depression of point $A$ from the top of the tower is 45°. Find the height of the tower. (take $\sqrt{3}$ = 1.732)

**16.** The sum of remainders obtained when $x^3 + (k + 8)x + k$ is divided by $x - 2$ and when it is divided by $x + 1$, is 0. Find the value of $k$.

**17.** A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it ?    (ii) She will not buy it?

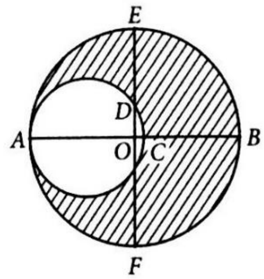
**OR**

All the three face cards of spades are removed from a well shuffled pack of 52 cards. A card is drawn at random from the remaining pack. Find the probability of getting.

(i) a black face card    (ii) a queen    (iii) a black card    (iv) a spade

**18.** $PB$ and $QA$ are the perpendiculars to segment $AB$. If $PO$ = 5 cm, $QO$ = 7 cm and $ar(\Delta BOP)$ = 150 cm$^2$, find $ar(\Delta QOA)$.

**19.** In the figure alongside, crescent is formed by two circles which touch at the point $A$, $O$ is the centre of the bigger circle. If $CB = 9$cm and $ED = 5$ cm, find the area of the shaded region. [Take $\pi = 3.14$]

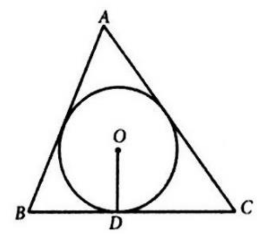



**20.** If $\sec A = \frac{2}{\sqrt{3}}$, find the value of $\frac{\tan A}{\cos A} + \frac{1+\sin A}{\tan A}$.

**OR**

Prove that : $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$.

**21.** In the given figure, a triangle $ABC$ is drawn to circumscribe a circle of radius 3 cm such that the segments $BD$ and $DC$ into which $BC$ is divided by the point of contact $P$, are of lengths 6 cm and 9 cm respectively. If the area of $\Delta ABC = 54$ cm$^2$, then find the lengths of sides $AB$ and $AC$.




**OR**

A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal.

**22.** In an acute angled triangle $ABC$, if $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$, find $\angle A$, $\angle B$, and $\angle C$,

## SECTION - D

**23.** If $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, then prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2.$

**OR**

If $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, then find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

**24.** A survey was conducted to give the percentage distribution of doctors in hospitals of rural areas of various states and Union Territories (UT) of India are given in the following table:

| Percentage of doctors | Number of states/UT |
|---|---|
| 15-25 | 6 |
| 25-35 | 11 |
| 35-45 | 7 |
| 45-55 | 4 |
| 55-65 | 4 |
| 65-75 | 2 |
| 75-85 | 1 |

(i) Find the mean percentage of doctors of rural areas of various states and union territories.

(ii) Suppose there are two persons Ram and Shyam. If Ram find out the mean by direct method and Shyam find out the mean by step deviation method, then whether both of them get the same value. Explain the reason.

(iii) Give the advantages of conducting health programme.

**25.** From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower.

**OR**

Tha angle of elevation of the top of a tower from certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top increased by 15°. Find the height of the tower.

**26.** In a right triangle *ABC*, right angled at *C*, *P* and *Q* are the points on the sides *CA* and *CB* respectively, which divide these sides in the ratio 2 : 1. Prove that

(i) $9AQ^2 = 9AC^2 + 4BC^2$

(ii) $9BP^2 = 9BC^2 + 4AC^2$

(iii) $9(AQ^2 + BP^2) = 13AB^2$

**27.** A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 16 cm and 12 cm. Find the capacity of the glass. $\left[\text{Use } \pi = \frac{22}{7}\right]$

**OR**

The radii of internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid cylinder of diameter 14 cm. Find the height of the cylinder.

**28.** If cosec $\theta$ – sin $\theta = m$ and sec $\theta$ – cos $\theta = n$, then prove that $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$.

**29.** Prove that the length of tangents drawn from an external point to a circle are equal.

**30.** A piggy bank contains hundred 50 p coins, seventy ₹ 1 coins, fifty ₹ 2 coins and thirty ₹ 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a ₹ 1 coin ?

(ii) will not be a ₹ 5 coin ?

(iii) will be a 50 p or ₹ 2 coin ?

## Solution

**1.** Here, $x = 3 \times 7 = 21$

and $y = x \times 4 = 21 \times 4 = 84$

**2.** We know that, diagonals of a parallelogram bisect each other.

$\therefore \quad x + y = 9$ ...(i)

and $x - y = 5$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2x = 14 \Rightarrow x = 7$

From Eq. (i), $7 + y = 9 \Rightarrow y = 2$

Hence, $x = 7$ and $y = 2$

**3.** Since $\Delta ABC$ and $\Delta BDE$ are two equilateral triangles.

$\therefore \quad \Delta ABC \sim \Delta BDE$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{BC^2}{BD^2} = \frac{x^2}{\frac{x^2}{4}} = \frac{4}{1}$$

**4.** Since $3+\sqrt{2}$ and $3-\sqrt{2}$ are the zeroes of the required polynomial.

$\therefore$ Sum of zeroes $= 3+\sqrt{2}+3-\sqrt{2} = 6$

Product of zeroes $= (3+\sqrt{2})(3-\sqrt{2}) = 9-2 = 7$

Hence, the required polynomial is $p(x) = x^2$ – (Sum of zeroes)$x$ + product of zeroes

$= x^2 - 6x + 7$

**5.** For equal roots, we have $b^2 - 4ac = 0$

$(k+1)^2 - 4(k+4)(1) = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow (k-5)(k+3) = 0$

$\Rightarrow k = 5$ or $k = -3$

6. Given points are $(k, 0)$ and $\left(7, \frac{3}{2}\right)$

Now, coordinates of mid-point

$$= \left(\frac{k+7}{2}, \frac{0+\frac{3}{2}}{2}\right) = \left(\frac{k+7}{2}, \frac{3}{4}\right)$$

$$\left[\because \text{coordinates of mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

But coordinates of mid-point are given $\left(3, \frac{3}{4}\right)$

$$\therefore \left(3, \frac{3}{4}\right) = \left(\frac{k+7}{2}, \frac{3}{4}\right)$$

On comparing $x$-coordinate of both sides, we get

$$3 = \frac{k+7}{2} \Rightarrow k+7=6 \Rightarrow k=-1$$

7. Here, $p$ and $q$ are zeroes of polynomial $f(x) = 2x^2 - 7x + 3$

$$\therefore p+q = \frac{7}{2} \text{ and } pq = \frac{3}{2}$$

Now, $p^2 + q^2 = (p+q)^2 - 2pq$

$$= \left(\frac{7}{2}\right)^2 - 2 \times \frac{3}{2}$$

$$= \frac{49}{4} - 3 = \frac{49-12}{4} = \frac{37}{4}$$

8. Here, distance between $A(-3, -14)$ and $B(a, -5)$ is 9 units.

$\Rightarrow |AB| = 9 \Rightarrow AB^2 = 81$

$\Rightarrow (a+3)^2 + (-5+14)^2 = 81$

$\Rightarrow a^2 + 6a + 9 + 81 - 81 = 0$

$\Rightarrow a^2 + 6a + 9 = 0 \Rightarrow (a+3)^2 = 0 \Rightarrow a = -3$

**9.** The frequency distribution table for the given data can be drawn as:

| Class | Frequency ($f_i$) | Cumulative frequency ($cf$) |
|---|---|---|
| 0 – 10 | 4 | 4 |
| 10 – 20 | 4 | 4 + 4 = 8 |
| 20 – 30 | 8 | 8 + 8 = 16 |
| 30 – 40 | 10 | 16 + 10 = 26 |
| 40 – 50 | 12 | 26 + 12 = 38 |
| 50 – 60 | 8 | 38 + 8 = 46 |
| 60 – 70 | 4 | 46 + 4 = 50 |

Since, $\Sigma f_i = n = 50 \Rightarrow \frac{n}{2} = \frac{50}{2} = 25$

Median class is that class whose cumulative frequency is just greater than or nearest to $\frac{n}{2}$.

Thus, the median class is 30–40.

**10.** Given quadratic equation is

$2x^2 - 2\sqrt{6}x + 3 = 0$

Discriminant $(D) = b^2 - 4ac$

$= (-2\sqrt{6})^2 - 4(2)(3) = 24 - 24 = 0$

Thus, real and coincident roots exist.

Now, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-2\sqrt{6})}{2 \times 2} = \frac{\sqrt{6}}{2}$

**11.** Since, the base area of the cylinder and the cone are the same.

So their radius are equal (same).

Let the radius of their base be $r$ and their heights be $h_1$ and $h_2$ respectively.

Clearly, volume of the cylinder $= \pi r^2 h_1$

and, volume of the cone $= \frac{1}{3}\pi r^2 h_2$

Given : Volume of cylinder = 2 × volume of cone

$$\Rightarrow \pi r^2 h_1 = 2 \times \frac{1}{3} \pi r^2 h_2$$

$$\Rightarrow h_1 = \frac{2}{3} h_2 \Rightarrow \frac{h_1}{h_2} = \frac{2}{3}$$

*i.e.*, $h_1 : h_2 = 2 : 3$

**12.** All two-digit numbers greater than 50 which when divided by 7 leaves remainder 4, are

53, 60, 67, ...... 95

which forms an A.P. with first term, $a = 53$.

common difference, $d = 60 - 53 = 7$

and last term, $l = 95$

Let last term be the $n^{th}$ term of given A.P.

Then, $l = a + (n - 1)d$

$\therefore$ $95 = 53 + (n - 1)7$

$\Rightarrow 7(n - 1) = 95 - 53 \Rightarrow 7(n - 1) = 42$

$\Rightarrow n - 1 = 6 \Rightarrow n = 7$

Now, required sum = 53 + 60 + 67 + .... + 95

$$= \frac{7}{2} \times (53 + 95) \qquad \left[\because S_n = \frac{n}{2}(a + l)\right]$$

$$= \frac{7}{2} \times 148 = 518$$

**13.** Let us assume, to the contrary that $\frac{7}{5}\sqrt{2}$ is rational.

$$\therefore \frac{7}{5}\sqrt{2} = \frac{p}{q}, q \neq 0 \text{ and } p, q \in Z$$

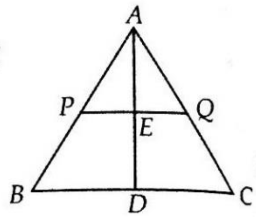
$$\Rightarrow \sqrt{2} = \frac{5p}{7q} \Rightarrow \sqrt{2} = \text{a rational number.}$$

But this contradicts the fact that $\sqrt{2}$ is irrational.

Hence, $\frac{7}{5}\sqrt{2}$ is not a rational number.

**14.** Suppose the median $AD$ intersects $PQ$ at $E$.

$PQ \| BC$ (given)




$\Rightarrow \angle APE = \angle B$ and $\angle AQE = \angle C$

(Corresponding angles)

So, in $\Delta$'s $APE$ and $ABD$

$\angle APE = \angle ABD$ (Proved)

$\angle PAE = \angle BAD$ (common)

$\therefore \Delta APE \sim \Delta ABD$

$$\Rightarrow \frac{PE}{BD} = \frac{AE}{AD} \quad \text{...(i)}$$

Similarly, $\Delta AQE \sim \Delta ACD$

$$\therefore \frac{QE}{CD} = \frac{AE}{AD} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{PE}{BD} = \frac{QE}{CD} \Rightarrow \frac{PE}{BD} = \frac{QE}{BD} \quad [\because BD = CD]$$

$\Rightarrow PE = QE$

$\therefore$ $AD$ bisects $PQ$.

**15.** Let the height of the tree $AB = 15$ m

It broke at $C$, its top $A$ touches the ground at $D$

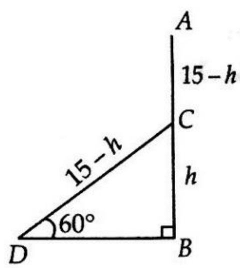
Now, $AC = CD$, $\angle BDC = 60°$

Let $BC = h$ m

$AC = AB - BC$

$= (15 - h)$ m

$\therefore AC = CD = 15 - h$




In $\Delta BCD$, $\sin 60° = \dfrac{h}{15-h}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{15-h} \Rightarrow \sqrt{3}(15-h) = 2h$$

$$\Rightarrow 15\sqrt{3} - \sqrt{3}h = 2h \Rightarrow 2h + \sqrt{3}h = 15\sqrt{3}$$

$$\Rightarrow h(2+\sqrt{3}) = 15\sqrt{3} \Rightarrow h = \frac{15\sqrt{3}}{2+\sqrt{3}}$$

$$\Rightarrow h = \frac{15\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$\Rightarrow h = \frac{30\sqrt{3} - 45}{4-3} = 30 \times 1.73 - 45$$

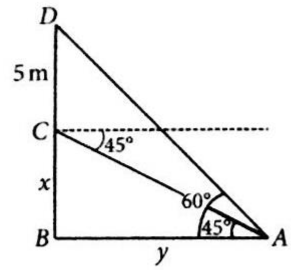
$$= 51.9 - 45 = 6.9$$

Hence, the tree broke at the height of 6.9 m from the ground.

**OR**

Let $BC$ be the height of the tower and $CD$ be the height of the pole. Let $BC = x$ m and $AB = y$ m

Now, in $\Delta ABC$,




$$\tan 45^\circ = \frac{BC}{AB} \Rightarrow 1 = \frac{x}{y}$$

$$\Rightarrow y = x \qquad \text{... (i)}$$

Now, in $\Delta ABD$,

$$\tan 60^\circ = \frac{BD}{AB} \Rightarrow \sqrt{3} = \frac{x+5}{y}$$

$$\Rightarrow x+5 = \sqrt{3}y \Rightarrow \sqrt{3}x = x+5 \qquad (\because x = y)$$

$$\Rightarrow (\sqrt{3}-1)x = 5$$

$$\Rightarrow x = \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{5(\sqrt{3}+1)}{2}$$

$$= \frac{5(2.732)}{2} = 6.83 \text{ m}$$

Height of tower = 6.83 m

**16.** let $f(x) = x^3 + (k+8)x + k$

Now, divide $f(x)$ by $x - 2$ and $x + 1$, respectively. Then, the division process is

$$\begin{array}{l} \qquad\qquad x^2 + 2x + (k+12) \\ x-2 \overline{\big) \; x^3 + (k+8)x + k} \\ \qquad\; x^3 - 2x^2 \\ \qquad\; \underline{- \quad +} \\ \qquad\; 2x^2 + (k+8)x + k \\ \qquad\; 2x^2 - 4x \\ \qquad\; \underline{- \quad +} \\ \qquad\qquad (k+12)x + k \\ \qquad\qquad (k+12)x - (2k+24) \\ \qquad\qquad \underline{- \qquad\qquad +} \\ \qquad\qquad 3k + 24 \end{array}$$

$$\begin{array}{l} \qquad\qquad x^2 - x + (k+9) \\ x+1 \overline{\big) \; x^3 + (k+8)x + k} \\ \qquad\; x^3 + x^2 \\ \qquad\; \underline{- \quad -} \\ \qquad\qquad -x^2 + (k+8)x + k \\ \qquad\qquad -x^2 - x \\ \qquad\qquad \underline{+ \quad +} \\ \qquad\qquad\qquad (k+9)x + k \\ \qquad\qquad\qquad (k+9)x + (k+9) \\ \qquad\qquad\qquad \underline{- \qquad\quad -} \\ \qquad\qquad\qquad\qquad -9 \end{array}$$

Thus, on dividing $f(x)$ by $(x - 2)$, we get
Remainder = $3k + 24$ and on dividing $f(x)$ by $(x + 1)$, we get
Remainder = $-9$
Now, according to the given condition,
Sum of remainders = 0

$\therefore \quad 24 + 3k + (-9) = 0$

$\Rightarrow \quad 3k + 15 = 0$

$\Rightarrow \quad k = -5$

**17.** Total number of ball pens = 144

$\Rightarrow$ All possible outcomes = 144

(i) Since, there are 20 defective pens

$\therefore$ Number of good pens = 144 – 20 = 124

$\Rightarrow$ Number of favourable outcomes = 124

$\therefore$ Probability that she will buy it $= \frac{124}{144} = \frac{31}{36}$

(ii) Probability that she will not buy it

= 1 – [probability that she will buy it]

$= 1 - \frac{31}{36} = \frac{36 - 31}{36} = \frac{5}{36}$

**OR**

Total cards = 52

Since, three face cards of spades are removed, then number of remaining cards = 52 – 3 = 49

$\therefore$ Total number of possible outcomes, $n(S) = 49$.

(i) Let $E_1$ = Event of getting a black face card.

Then, total number of favourable outcomes = 3

*i.e.*, $n(E_1) = 3$

$\therefore$ $P$ (getting a black face card) $= \frac{n(E_1)}{n(S)} = \frac{3}{49}$

(ii) Let $E_2$ = Event of getting a queen

Then, total number of favourable outcomes = 3

*i.e.*, $n(E_2) = 3$

$\therefore$ $P$ (getting a queen) $= \frac{n(E_2)}{n(S)} = \frac{3}{49}$

(iii) Remaining black cards = 13 of club + 10 of spade

= 23.

Then, total number of favourable outcomes = 23

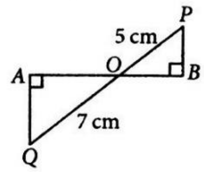
$\therefore$ $P$ (getting a black card) $= \frac{23}{49}$

(iv) Remaining cards of spade = 13– 3 = 10

So, favourable outcomes = 10

$\therefore$ Required probability $= \frac{10}{49}$

**18.** In $\Delta BOP$ and $\Delta AOQ$,




$\angle OBP = \angle OAQ$ [Each 90°]
$\angle POB = \angle QOA$ [Vertically opposite angle]
$\therefore \Delta BOP \sim \Delta AOQ$ [By $AA$ similarity criteria]

Now, $\dfrac{ar(\Delta BOP)}{ar(\Delta AOQ)} = \dfrac{PO^2}{QO^2}$

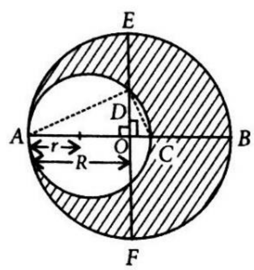
$\Rightarrow \dfrac{150}{ar(\Delta AOQ)} = \dfrac{5^2}{7^2} = \dfrac{25}{49}$

$\therefore ar(\Delta AOQ) = \dfrac{150 \times 49}{25}$

$= 294 \text{ cm}^2$

**19.** Suppose $R$ and $r$ be the radii of bigger and smaller circles, respectively

$\Rightarrow 2R - 2r = 9$




$\Rightarrow R - r = \dfrac{9}{2} = 4.5$ cm ... (i)

Join $AD$ and $CD$.

$\Delta AOD \sim \Delta DOC$

$\Rightarrow \dfrac{OD}{OA} = \dfrac{OC}{OD}$

$\Rightarrow OD^2 = OA \times OC$

$\Rightarrow (R - 5)^2 = R \times (R - 9)$

[$\because DE = 5$ cm and $CB = 9$ cm]

$\Rightarrow R^2 + 25 - 10R = R^2 - 9R$

$\Rightarrow R = 25$

From (i), we have $R - r = 4.5$

$\Rightarrow r = R - 4.5 = 25 - 4.5 = 20.5$ cm

Now, area of the shaded portion $= \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2) = \pi (R + r) R - r)$

$= 3.14 \times (25 + 20.5) (25 - 20.5)$

$= 3.14 \times 45.5 \times 4.5 = 642.915 \text{ cm}^2$

Hence, the required area of the shaded portion is $642.915 \text{ cm}^2$.

**20.** Given that $\sec A = \frac{2}{\sqrt{3}} \quad \Rightarrow \cos A = \frac{\sqrt{3}}{2}$

$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{1}{\sqrt{3}}$

Now, $\frac{\tan A}{\cos A} + \frac{1+\sin A}{\tan A} = \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}} + \frac{1+\frac{1}{2}}{\frac{1}{\sqrt{3}}}$

$= \frac{2}{3} + \frac{3\sqrt{3}}{2} = \frac{4+9\sqrt{3}}{6}$

**OR**

L.H.S. $= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A}\frac{\sin^2 A}{\cos A} = \cos A \sin A$

R.H.S. $= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{\cos A \sin A}{\sin^2 A + \cos^2 A}$

$= \cos A \sin A$

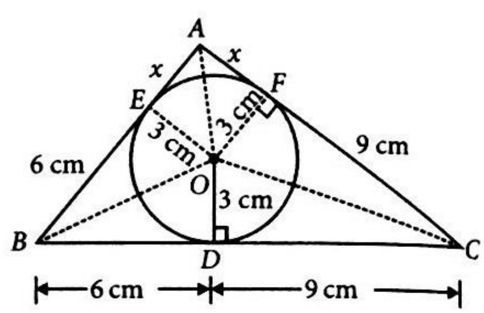
L.H.S. = R.H.S.

**21.** Let $E$ and $F$ be the points where the tangents $AB$ and $AC$ touches the circle respectively. Join $OE$ and $OF$.

Now, radius is perpendicular the to tangent at the point of contact.

So, $OD \perp BC$, $OE \perp AB$ and $OF \perp AC$.

Join $OA$, $OB$ and $OC$.



Since, tangents drawn from an external point to a circle are equal.

$\therefore$ $BD = BE = 6$ cm, $CD = CF = 9$ cm and

$AE = AF = x$ cm (say)

Now, area of $\Delta ABC$ = Area of $\Delta AOB$ + Area of $\Delta BOC$ + Area of $\Delta AOC$

$$\Rightarrow 54 = \frac{1}{2}\times AB\times OE + \frac{1}{2}\times BC\times OD + \frac{1}{2}\times AC\times OF$$

$$\Rightarrow 54 = \frac{1}{2}\times(x+6)\times 3 + \left(\frac{1}{2}\times 15\times 3\right) + \left(\frac{1}{2}\times(x+9)\times 3\right)$$

$$\Rightarrow 54 = \frac{1}{2}\times(3x+18+45+3x+27)$$

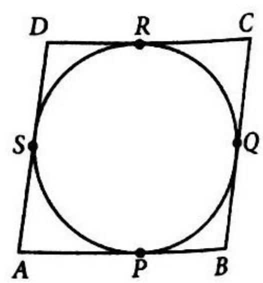
$$\Rightarrow 6x + 90 = 108 \Rightarrow 6x = 108 - 90 = 18$$

$$\Rightarrow x = \frac{18}{6} = 3$$

$\therefore$ $AB = 3 + 6 = 9$ cm

and $AC = 3 + 9 = 12$ cm

**OR**




Let the circle touches the sides $AB$, $BC$, $CD$ and $DA$ of quadrilateral $ABCD$ at $P$, $Q$, $R$ and $S$ respectively. Since, lengths of tangents drawn from an external point to the circle are equal.

$\therefore$ $AP = AS$ ...(1) (Tangents drawn from $A$)

$BP = BQ$ ...(2) (Tangents drawn from $B$)

$CR = CQ$ ...(3) (Tangents drawn from $C$)

$DR = DS$ ...(4) (Tangents drawn from $D$)

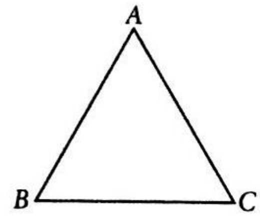
Adding (1), (2), (3) and (4), we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + PB) + (CR + RD) = (AS + SD) + (BQ + QC)$$

$$\Rightarrow AB + CD = AD + BC$$

22. Here, $\Delta ABC$ is an acute angled triangle and




$\sin(A+B-C)=\frac{1}{2}$

$\Rightarrow \sin(A+B-C)=\sin 30°$

$\Rightarrow A+B-C=30°$ ... (i)

And $\cos(B+C-A=\frac{1}{\sqrt{2}}$

$\Rightarrow \cos(B+C-A)=\cos 45°$

$\Rightarrow B+C-A=45°$ ... (ii)

Also, $A+B+C=180°$ ... (iii)

(by angle sum property of a triangle)

From (i) and (iii) we have,

$2C=150°$

$\Rightarrow C=75°$

From (ii) and (iii) we have $2A=135°$

$\Rightarrow A=67.5°$

Now, from (iii) we have

$67.5°+B+75°=180°$

$\Rightarrow B=180°-67.5°-75°=37.5°$

Hence, $\angle A=67.5°$, $\angle B=37.5°$ and $\angle C=75°$

23. We have to prove, $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2.$

Given, $\alpha$ and $\beta$ are the zeroes of the polynomial

$f(x)=x^2-px+q.$

$\therefore$ Sum of zeroes, $\alpha+\beta=\frac{-(-p)}{1}=p$

and product of zeroes, $\alpha\beta=\frac{q}{1}=q$

Now, $\text{LHS}=\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\alpha^4+\beta^4}{\alpha^2\beta^2}=\frac{(\alpha^2)^2+(\beta^2)^2}{\alpha^2\beta^2}$

$=\frac{(\alpha^2+\beta^2)^2-2\alpha^2\beta^2}{\alpha^2\beta^2}$

$[\because a^2+b^2=(a+b)^2-2ab]$

$=\frac{[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$

On substituting $\alpha + \beta = p$ and $\alpha\beta = q$, we get

$$\text{LHS} = \frac{[p^2 - 2q]^2 - 2q^2}{q^2}$$

$$= \frac{(p^2)^2 + (2q)^2 - 2 \times p^2 \times 2q - 2q^2}{q^2}$$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$$= \frac{p^4 + 4q^2 - 4p^2q - 2q^2}{q^2}$$

$$= \frac{p^4 + 2q^2 - 4p^2q}{q^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2 = \text{R.H.S.}$$

**OR**

Given, $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$.

$$\therefore \quad \alpha + \beta = -\frac{\text{Coefficient of } s}{\text{Coefficient of } s^2} = \frac{-(-6)}{3} = \frac{6}{3} = 2$$

and $$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } s^2} = \frac{4}{3}$$

Now, $$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$[\because a^2 + b^2 = (a + b)^2 - 2ab]$

$$= \frac{(2)^2 - 2(4/3)}{4/3} + 2\left(\frac{2}{4/3}\right) + 3(4/3)$$

$[\because \alpha + \beta = 2 \text{ and } \alpha\beta = 4/3]$

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2 \times 2 \times \frac{3}{4} + 4 = \frac{12 - 8}{3} \times \frac{3}{4} + 3 + 4$$

$$= \frac{4}{3} \times \frac{3}{4} + 7 = 1 + 7 = 8$$

**24.** (i) Table for given data is

| $x_i$ | $f_i$ | $f_i x_i$ |
|---|---|---|
| 20 | 6 | 120 |
| 30 | 11 | 330 |
| 40 | 7 | 280 |
| 50 | 4 | 200 |
| 60 | 4 | 240 |
| 70 | 2 | 140 |
| 80 | 1 | 80 |

Here, $\Sigma f_i = 35$ and $\Sigma f_i x_i = 1390$

$\therefore$ Mean $(\bar{x}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$= \dfrac{1390}{35} = 39.72$

Hence, mean of the given data is 39.72.

(i) The values like humanity, social service and honestly in performing duty are depicted here.

Yes, the mean will be same *i.e.* 39.72 Because, step deviation method is just simplified forms of direct methods.

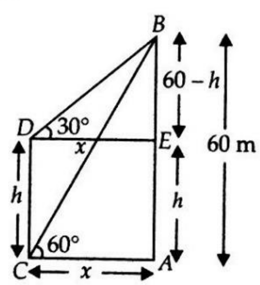
(iii) Health programmes aware the people about various types of infection and diseases.

**25.** Let $AB$ be the building and $CD$ be the tower. Let $CD = h$ metres. It is given that from the top of the building $B$, the angles of depression of the top $D$ and the bottom $C$ of the tower $CD$ are 30° and 60° respectively.

$\therefore$ $\angle EDB = 30°$ and $\angle ACB = 60°$

Let $AC = DE = x$

In $\Delta DEB$, right angled at $E$, we have




$\tan 30° = \dfrac{BE}{DE}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{60-h}{x}$

$\Rightarrow x = \sqrt{3}\,(60-h)$ ...(1)

In $\Delta CAB$, right-angled at $A$, we have

$\tan 60° = \dfrac{AB}{CA}$

$\Rightarrow \sqrt{3} = \dfrac{60}{x} \Rightarrow x = \dfrac{60}{\sqrt{3}} = \dfrac{60\sqrt{3}}{3} = 20\sqrt{3}$

Putting the value of $x$ in (1), we get

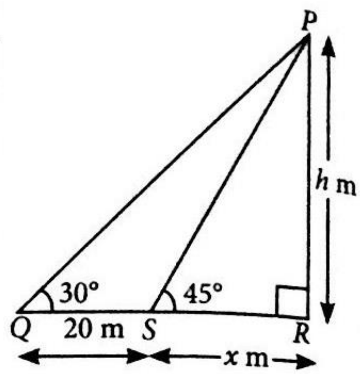
$20\sqrt{3} = \sqrt{3}(60-h)$

$\Rightarrow 20 = 60 - h \Rightarrow h = 60 - 20 = 40$

Thus, the height of the tower is 40 metres.

**OR**

Let the height of the tower $PR$ be $h$ m, the angle of elevation at point $Q$ is $30°$ i.e., $\angle PQR = 30°$ and $S$ be the position of observer after moving 20 m towards the tower.




According to the question,

$\angle PSR = \angle PQR + 15°$

$\Rightarrow \angle PSR = 30° + 15° \Rightarrow \angle PSR = 45°$ ...(i)

Now, in right angle $\Delta PRS$,

$$\tan \angle PSR = \frac{PR}{SR} = \frac{h}{x}$$

$$\Rightarrow \tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \quad \text{[from (i)]}$$

$\Rightarrow x = h$ ...(ii)

and in right angle $\Delta PRQ$,

$$\Rightarrow \tan 30° = \frac{PR}{QR} = \frac{PR}{QS + SR} \quad [\because QR = QS + SR]$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20 + x}$$

$$\Rightarrow 20 + x = \sqrt{3}h \Rightarrow 20 + h = \sqrt{3}h \quad \text{[from eq. (ii)]}$$

$$\Rightarrow \sqrt{3}h - h = 20 \Rightarrow h(\sqrt{3} - 1) = 20$$

$$\Rightarrow h = \frac{20}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow h = \frac{20(\sqrt{3} + 1)}{3 - 1} = \frac{20(\sqrt{3} + 1)}{2} = 10(\sqrt{3} + 1) \text{ m}$$

Hence, the required height of the tower is $10(\sqrt{3} + 1)$ m.

**26. Given :** In right $\Delta ACB$, $P$ and $Q$ are points on sides $CA$ and $CB$ dividing them in 2 : 1.

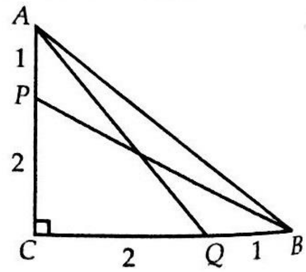
**To prove :** (i) $9AQ^2 = 9AC^2 + 4BC^2$

(ii) $9BP^2 = 9BC^2 + 4AC^2$

(iii) $9(AQ^2 + BP^2) = 13AB^2$.

**Proof :** Since, $P$ divides $CA$ in the ratio of 2 : 1.

$\therefore\ CP = \frac{2}{3}AC$ ...(1)




Since, $Q$ divides $CB$ in the ratio of 2 : 1

$\therefore\ CQ = \frac{2}{3}BC$ ...(2)

(i) In right $\Delta ACQ$

$AQ^2 = AC^2 + CQ^2$

[By Pythagoras theorem]

$= AC^2 + \left(\frac{2}{3}BC\right)^2 = AC^2 + \frac{4BC^2}{9}$ [From (2)]

$\Rightarrow\ AQ^2 = \frac{9AC^2 + 4BC^2}{9}$

$\Rightarrow\ 9AQ^2 = 9AC^2 + 4BC^2$ ...(3)

(ii) Now, in right $\Delta PCB$,

$BP^2 = BC^2 + CP^2$ [By Pythagoras theorem]

$\Rightarrow\ BP^2 = BC^2 + \left(\frac{2AC}{3}\right)^2$

$= BC^2 + \frac{4AC^2}{9} = \frac{9BC^2 + 4AC^2}{9}$ [From (1)]

$\Rightarrow\ 9BP^2 = 9BC^2 + 4AC^2$ ...(4)

Adding (3) and (4), we get

$9AQ^2 + 9BP^2 = 9AC^2 + 4BC^2 + 9BC^2 + 4AC^2$

$\Rightarrow\ 9(AQ^2 + BP^2) = 13AC^2 + 13BC^2$

$\Rightarrow\ 9(AQ^2 + BP^2) = 13(AC^2 + BC^2)$

$\Rightarrow\ 9(AQ^2 + BP^2) = 13AB^2$

[In right $\Delta ABC$, by Pythagoras theorem]

**27.** Given, height ($h$) of frustum = 14 cm

Lower radius ($r$) = 6 cm

Upper radius ($R$) = 8 cm

$\therefore$ Capacity (Volume) of the glass

$= \frac{1}{3}\pi\left(r^2 + R^2 + rR\right)h$

$= \frac{1}{3} \times \frac{22}{7}\left[(6)^2 + (8)^2 + 6 \times 8\right] \times 14$

$= \frac{1}{3} \times \frac{22}{7}[36 + 64 + 48] \times 14$

$= \frac{1}{3} \times \frac{22}{7} \times 148 \times 14 = \frac{6512}{3} = 2170\frac{2}{3}\text{ cm}^3$

**OR**

Internal radii ($r$) of spherical shell = 3 cm
External radii ($R$) of spherical shell = 5 cm
Volume of metal used in spherical shell

$= \frac{4}{3}\pi\left(R^3 - r^3\right) = \frac{4}{3}\pi\left[(5)^3 - (3)^3\right] = \frac{98 \times 4\pi}{3}$

Diameter of solid cylinder = 14 cm (Given)
∴ Radius ($r$) of solid cylinder = 7 cm
Let height of cylinder be $h$ cm.
Then, volume of metal used in spherical shell = Volume of metallic solid cylinder = $\pi r^2 h$

$\Rightarrow \frac{98 \times 4\pi}{3} = \pi \times (7)^2 \times h \Rightarrow h = \frac{8}{3}.$

**28.** Given, cosec $\theta$ – sin $\theta$ = $m$ and sec $\theta$ – cos $\theta$ = $n$

$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = m$ and $\frac{1}{\cos\theta} - \cos\theta = n$

$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = m$ and $\frac{1-\cos^2\theta}{\cos\theta} = n$

$\Rightarrow m = \frac{\cos^2\theta}{\sin\theta}$ and $n = \frac{\sin^2\theta}{\cos\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1]$

Now, $m^2 n = \left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \times \left(\frac{\sin^2\theta}{\cos\theta}\right)$

$= \frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos\theta} = \cos^3\theta$

and $mn^2 = \left(\frac{\cos^2\theta}{\sin\theta}\right) \times \left(\frac{\sin^2\theta}{\cos\theta}\right)^2$

$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^4\theta}{\cos^2\theta} = \sin^3\theta$

$\therefore \; (m^2n)^{2/3} = (\cos^3\theta)^{2/3} = \cos^2\theta$

$\therefore \; (mn^2)^{2/3} = (\sin^3\theta)^{2/3} = \sin^2\theta$

Now, $(m^2n)^{2/3} + (mn^2)^{2/3} = \cos^2\theta + \sin^2\theta$

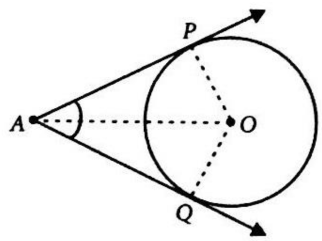
$\therefore \; (m^2n)^{2/3} + (mn^2)^{2/3} = 1 \quad [\because \cos^2\theta + \sin^2\theta = 1]$

Hence proved.

**29.** Given : $AP$ and $AQ$ are two tangents from a point $A$ to a circle $C(O, r)$.

To Prove : $AP = AQ$

Construction : Join $OP$, $OQ$ and $OA$.




Proof : In order to prove that $AP = AQ$, we shall first prove that $\Delta OPA \cong \Delta OQA$

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \; OP \perp AP$ and $OQ \perp AQ$

$\Rightarrow \; \angle OPA = \angle OQA = 90°$ ...(i)

Now, in right triangle $OPA$ and $OQA$, we have

$OP = OQ$ [Radii of circle]

$\angle OPA = \angle OQA$ [Each 90°]

and $OA = OA$ [Common]

So, by RHS-criterion of congruence, we get

$\Delta OPA \cong \Delta OQA \Rightarrow AP = AQ$ [By CPCT]

Hence, lengths of two tangents drawn from an external point are equal.

**30.** Total coins in the piggy bank

$= 100 + 70 + 50 + 30 = 250$

$\therefore$ Total number of possible outcomes = 250

(i) Number of ₹1 coins = 70

$\therefore$ Number of favourable outcomes = 70

$\therefore \; P(\text{getting a ₹1 coin}) = \frac{70}{250} = \frac{7}{25}$

(ii) Number of ₹5 coins = 30

$\therefore \; P(\text{getting a ₹5 coin}) = \frac{30}{250} = \frac{3}{25}$

Now, required probability $= 1 - \frac{3}{25} = \frac{22}{25}$

(iii) Total number of coins which are either of 50 p or ₹ 2 = 100 + 50 = 150

$\therefore$ Number of favourable outcomes = 150

Required probability $= \frac{150}{250} = \frac{3}{5}$