Long Answer Questions [4 Marks]

Que 1. In Fig. 8.41, ABCD is a parallelogram and ∠DAB = 60°. If the bisector of angles A and B meet at M on CD, Prove that M is the mid-point of CD.

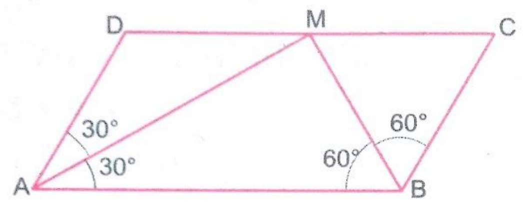

Fig. 8.41

Sol. We have, ∠DAB = 60°

Since, AD||BC and AB is the transversal.

∴ ∠A + ∠B = 180°

60° + ∠B = 180°

⇒ ∠B = 120°

Also, AM and BM are angle bisectors of ∠A and ∠B respectively.

∴ ∠DAM = ∠MAB = 30° and ∠CBM = ∠MBA = 60°

Now, AB||DC and transversal AM cuts them.

∴ ∠MAB = ∠DMA (Alternate interior angles)

⇒ ∠DMA = 30°

Thus, in ΔAMD, we have

∠MAD = ∠AMD (Each equals to 30°)

⇒ MD = AD (Sides opposite to equal angles) ..(i)

Again, AB||DC and transversal BM cuts them.

∴ ∠CMB = ∠MBA (alternate interior angles)

⇒ ∠CMB = 60°

Thus, in ΔCMB, we have

$\angle CBM = \angle CMB$ (Each equals to 60^o)

$\Rightarrow$ $CM = BC$ (Sides opposite to equals angles)

$\Rightarrow$ $CM = AD$ ($\because BC = AD$)

From (i) and (ii), we get

$$MD = CM$$

$\Rightarrow$ M is the mid-point of CD.

Que 2. ABC is a triangle right-angled at C. A line through the mid-point of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) MD $\perp$ AC
(ii) D is the mid-point of AC
(iii) MC = MA = $\frac{1}{2}$ AB.

Sol. Given : A triangle ABC right-angled at C. M is the mid-point of AB and MD||BC.

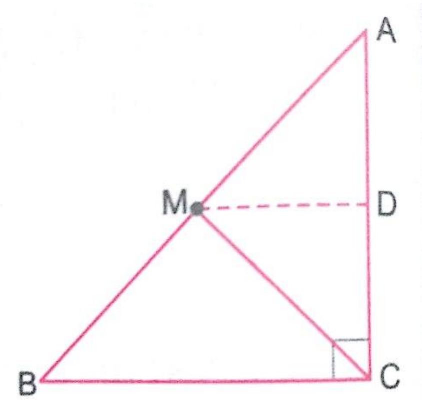

Fig. 8.42

To Prove: (i) MD $\perp$ AC

(ii) D is the mid-point of AC. (iii) MC = MA = $\frac{1}{2}$ AB.

Proof: (i) Since MD||BC.

Therefore, $\angle ADM = \angle ACB$ (Corresponding angles)

But $\angle ACB = 90^o$

$\therefore$ $\angle ADM = 90^o$

$\Rightarrow$ $90^o + \angle CDM = 180^o$ (Linear pair) or $\angle CDM = 90^o$

Hence, MD $\perp$ AC

(ii) In ΔABC, M is the mid-point of AB and $MD || BC$.

Therefore, D is the mid-point of AC,

i.e., $AD = CD$ (By the converse of mid-point Theorem)

(iii) In triangle AMD and CMD, we have

$AD = CD$ (Proved above)

$\angle ADM = \angle CDm$ (Each 90^o)

And $MD = MD$ (Common)

$\therefore$ $\Delta AMD \cong \Delta CMD$ (By SAS congruence criterion)

$\Rightarrow$ $MA = MC$ (CPCT)

Also $M4 = \frac{1}{2} AB$

Hence, $MC = MA = \frac{1}{2} AB$

Que 3. Prove that the diagonal divides a parallelogram into two congruent triangles.

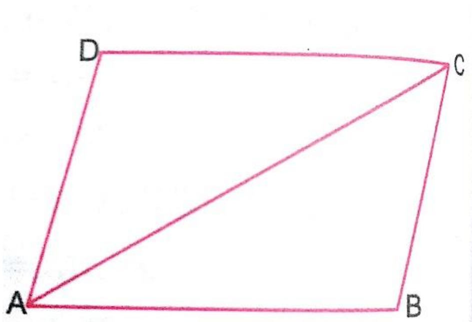

Fig. 8.43

Sol. Given: A parallelogram ABCD.

To Prove: $\Delta ABC \cong \Delta CDA$

Construction: Join AC.

Proof: Since ABCD is a parallelogram.

Therefore, $AB || DC$ and $AD || BC$

Now, in $AB || DC$ and transversal AC cuts them at A and C respectively.

$\therefore$ $\angle DCA = \angle BAC$ (Alternate interior angles)

Now, in ΔABC and ΔCDA; we have

$\angle BAC = \angle DCA$

$AC = AC$ (Common)

$\angle DAC = \angle ACB$

$\therefore$ $\Delta ABC \cong \Delta CDA$ (By ASA Congruence criterion)

Que 4. Prove that the figure formed by joining the mid-point of the adjacent sides of a quadrilateral is a parallelogram.

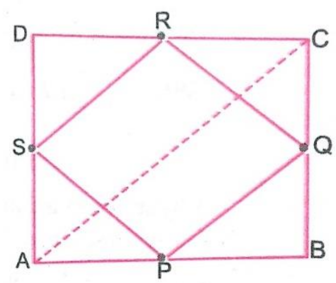

Fig. 8.44

Sol. Let ABCD be a quadrilateral which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively.

Join AC.

In ΔABC, the points P and Q are the mid-points of sides AB and BC respectively.

$\therefore$ $PQ || AC$ and $PQ = \frac{1}{2}AC$ (By mid-point theorem)

Again, in ΔDAC, the points S and R are the mid-points of AD and DC respectively.

$\therefore$ $PQ || AC$ and $PQ = \frac{1}{2}AC$ (By mid-point theorem) ... (i)

Again, in ΔDAC, the points S and R are the mid-points of AD and DC respectively.

$\therefore$ $SR || AC$ and $SR = \frac{1}{2}AC$ ... (ii)

From (i) and (ii)

$PQ || SR$ and $PQ = SR$

Hence, quadrilateral PQRS is a parallelogram.

Que 5. Prove that the bisector of the angles of a parallelogram enclose a rectangle.

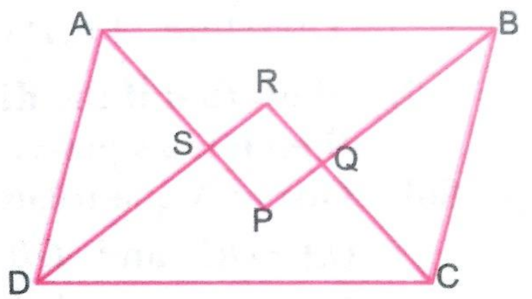

Fig. 8.45

Sol. Given: A parallelogram in which bisector of angles A, B, C, D intersect at P, Q, R, S to form a quadrilateral PQRS.

To Prove: Since ABCD is parallelogram. Therefore, AB||DC

Now, AB||DC, and transversal AD cuts them, so we have

$$\angle A + \angle D = 180^o$$

$$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle D = \frac{180^o}{2}$$

$$\angle DAS + \angle ADS = 90^o$$

But, in ΔASD, we have

$$\angle ADS + \angle DAS + \angle ASD = 180^o$$

$$\Rightarrow \quad \angle 90^o + \angle ASD = 180^o$$

$$\Rightarrow \quad \angle ASD = 90^o$$

$$\angle RSP = \angle ASD \qquad \text{(Vertically opposite angles)}$$

$$\therefore \quad \angle RSP = 90^o$$

Similarly, we can prove that

$$\angle SRQ = 90^o, \angle RQP = 90^o \text{ and } \angle QPS = 90^o$$

Thus, PQRS is a quadrilateral each of whose angle is 90°

Hence, PQRS is a rectangle.

Que 6. Two parallel lines l and m are interacted by a transversal p (see Fig. 8.46). Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

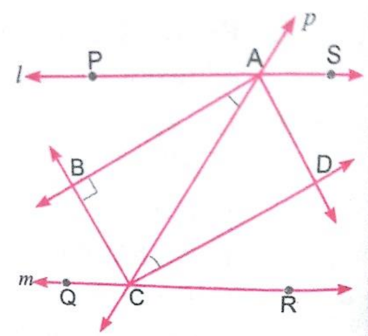

Fig. 8.46

Sol. It is given that PS||QR and transversal p intersects them at points A and C respectively.

The bisectors of ∠PAC and ∠ACQ intersect at B and bisectors of ∠ACR and ∠SAC intersect at D.

We are to show that quadrilateral ABCD is a rectangle.

Now, ∠PAC = ∠ACR (Alternate angles as l||m and p is a transversal)

So, $\frac{1}{2}\angle PAC = \frac{1}{2}\angle ACR$

i.e., ∠BAC = ∠ACD

These form a pair of alternate angles for lines AB and DC with AC as transversal and they are equal also.

So, AB||DC

Similarly BC||AD (Considering ∠ACB and ∠CAD)

Therefore, quadrilateral ABCD is a parallelogram

Also, $\angle PAC + \angle CAS = 180^o$ (Linear Pair)

So, $\frac{1}{2}\angle PAC + \frac{1}{2}\angle CAS = \frac{1}{2} \times 180^o = 90^o$

Or, $\angle BAC + \angle CAD = 90^o$

Or, $\angle BAD = 90^o$

So, ABCD is parallelogram in which one angle is 90°.

Therefore, ABCD is a rectangle.

Que 7. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

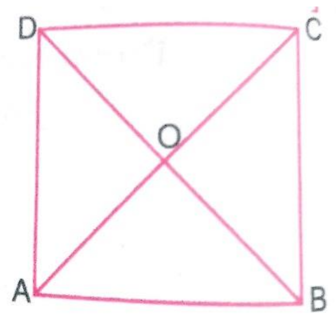

Fig. 8.47

Sol. Given: A quadrilateral ABCD in which in which diagonals AC = BD and AC ⊥ BD.

OA = OC and OB = OD.

To Proof: Quadrilateral ABCD is a square.

Proof: First, we shall prove that quadrilateral ABCD is parallelogram.

In triangles AOD and COB, we have

OA = OC (Given)

∠AOD = ∠BOC (Vertically opposite angles)

OD = OB (Given)

∴ ΔAOD ≅ ΔCOB (By SAS congruence criterion)

⇒ ∠OAD = ∠OCB (CPCT)

But these are alternate interior angles

∴ AD||BC

Similarly, AB||CD

Since both pair of opposite sides are parallel in quadrilateral ABCD.

Therefore, quadrilateral ABCD is a parallelogram.

Now, we shall prove that it is a square.

In triangles AOB and AOD, we have

OA = OA (Common)

$\angle AOB = \angle AOD$ (Each 90^o)

$OB = OD$ (Given)

$\therefore$ $\Delta AOB \cong \Delta AOD$ (By SAS congruence criterion)

$\Rightarrow$ $AB = AD$ (CPCT)

As opposite sides of a of a parallelogram are equal,

$\therefore$ $AB = CD$ and $AD = BC$

But $AB = AD$ (Proved above)

$\therefore$ $AB = BC = CD = DA$ ... (i)

Now, in triangles ABD and BAC, we have

$AD = BC$ (Opposite sides of parallelogram)

$AB = AB$ (Common)

$BD = AC$ (Given)

$\therefore$ $\Delta ABD \cong \Delta BAC$ (By SSS congruence criterion)

$\Rightarrow$ $\angle DAB = \angle CBA$ (CPCT)

Or $\angle A = \angle B$

But $\angle DAB + \angle CBA = 180^o$

(Interior angles on the same of transversal are supplementary)

$\Rightarrow$ $2\angle DAB = 180^o$ ($\because$ $\angle CBA = \angle DAB$)

$\Rightarrow$ $\angle DAB = 90^o$ i.e., $\angle A = 90^o$

As opposite angles of parallelogram are equal,

$\angle A = \angle C$ and $\angle B = \angle D$

But $\angle A = \angle B$ and $\angle A = 90^o$ (Proved above)

$\therefore$ $\angle A = \angle B = \angle C = \angle D = 90^o$ ... (ii)

From (i) and (ii), we have

Quadrilateral ABCD is a square.

Que 8. ABCD is a parallelogram in which P and Q are the mid-points of opposite sides AB and CD (Fig. 8.48). If AQ intersects DP at S and BQ intersects CP at R, show that

(i) APCQ is a parallelogram
(ii) DPBQ is a parallelogram
(iii) PSQR is a parallelogram

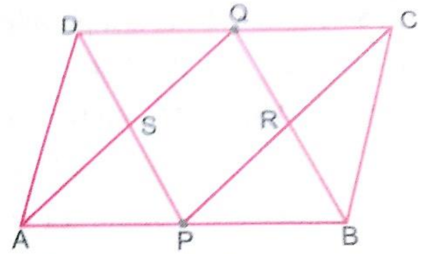

Fig. 8.48

Sol. (i) in quadrilateral APCQ,

AP||QC (∵ AB||CD) ... (i)

$AP = \frac{1}{2}AB$, $CQ = \frac{1}{2}DC$ (Given)

Also, AB = CD

So, AP = QC ... (ii)

From (i) and (ii), we have AP||QC and AP = QC.

Therefore, APCQ is a parallelogram.

(ii) Similarly, quadrilateral DPBQ is parallelogram, because

DQ||PB and DQ = PB

(iii) In quadrilateral PSQR,

SP||QR (SR is a part of DP and QR is part QB)

Similarly, SQ||PR

So, PSQR is a parallelogram.

Que 9. Prove that the line segment joining the mid- points of the diagonals of a trapezium is parallel to the parallel sides and equal to half of their difference.

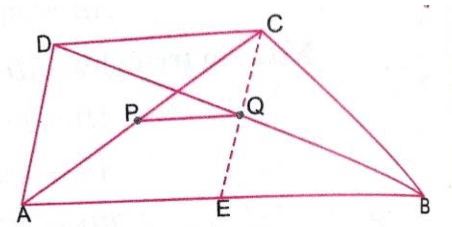

Fig. 8.49

Sol. Let ABCD be a trapezium in which AB||DC, and let P and Q be the mid-points of the diagonals AC and BD respectively.

Join CQ and produce it to meet AB at E.

In ΔCDQ and ΔEBQ, we have

DQ = BQ (∵ Q is mid-point of BD)

∠DCQ = ∠BEQ (Alternate interior angles)

∠CDQ = ∠EBQ (Alternate interior angles)

∴ ΔCDQ ≅ ΔEBQ (AAS congruence criterion)

⇒ CQ = QE and CD = EB (CPCT)

Thus in ΔCAE, the points P and Q are the mid-point of AC and CE respectively.

∴ $PQ || AE$ and $PQ = \frac{1}{2} AE$

⇒ $PQ || AB || DC$

And $PQ = \frac{1}{2} AE = \frac{1}{2}(AB - ED)$

$= \frac{1}{2}(AB - DC)$ (∵ EB = DC)

Que 10. In a parallelogram ABCD, the bisector of ∠A also bisects BC at X. Prove that AD = 2AB.

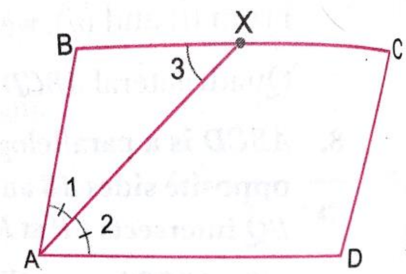

Fig. 8.50

Sol. As ABCD is a parallelogram,

∴ AD||BC

Also, AX is a transversal

∴ $\angle 3 = \angle 2$ (Alternate angles)

$\angle 1 = \angle 2$ (∵ AX is bisector of ∠A)

⇒ $\angle 1 = \angle 3$

In ΔABX, we have

$\angle 1 = \angle 3$ (Proved above)

⇒ AB = BX (Sides opposite to equal angles are equal)

⇒ $AB = \frac{1}{2} BC$ (X is the mid-point of BC)

Also, AD = BC (Opposite sides of parallelogram are equal)

⇒ $AB = \frac{1}{2} AD$ ⇒ 2AB = AD Hence Proved.

Que 11. AD is the median of ΔABC. E is mid-point of AD. BE produced to meet AC at F. Show that AF = $\frac{1}{3}$ AC.

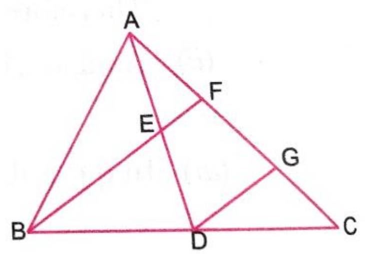

Fig. 8.51

Sol. Draw DG||BF

In ΔBFC,

D is the mid-point of BC (∵ AD is median)

DG||BF (By construction)

⇒ G is mid-pointy of FC (By the converse if mid-point theorem)

⇒ CG = GF … (i)

In ΔADG, E is mid-point of AD

DG||EF

⇒ F is mid-point of AG ⇒ AF = GF

From (i) and (ii), we have

AF = GF = CG

Now, AC = AF + FG + GC

⇒ AC = 3AF ⇒ AF = $\frac{1}{3}$ AC

Que 12. E and F are respectively the mid-points of the non-parallel sides AD and BC of A trapezium ABCD. Prove that EF||AB and EF = $\frac{1}{2}$ (AB + CD).

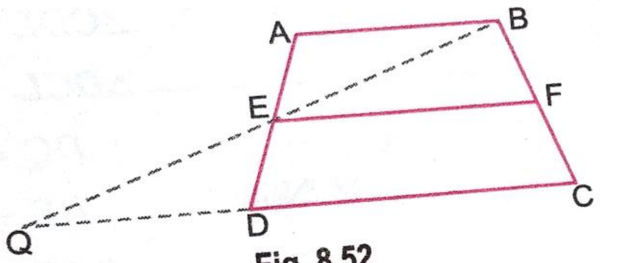

Fig. 8.52

Sol. Given: AB||Cd and E, F are the mid-point of sides AD and BC respectively.

To Prove: EF||AB, EF = $\frac{1}{2}$ (AB + CD)

Construction: Join BE and produce it to meet CD produced at Q.

Proof: In ΔBQC

Since E and F are the mid-point of sides BQ and BC respectively.

∴ By mid-point Theorem,

EF||QC and EF = $\frac{1}{2}$ QC … (i)

⇒ EF||DC

⇒ CD||AB (Given)

⇒ EF||AB

Now, in ΔAEB and ΔDEQ, we have

∠AEB = ∠DEQ (vertically opposite angles)

AE = ED (E is the mid-point of AD)

∠BAE = ∠EDQ (Alternate interior angles)

AB = QD (CPCT)

From (i) EF = $\frac{1}{2}$QC = $\frac{1}{2}$(QD + DC) = $\frac{1}{2}$(AB + CD) [Using (ii)]

Hence, EF = $\frac{1}{2}$(AB + CD)