Class-XII

Session - 2022-23

Subject - Mathematics (041)

Sample Question Paper - 21

With Solution

BLUE PRINT

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Relations and Functions	8		Q.19			Q.32		6
2	Inverse Trigonometry Functions		Q.2,4						2
3	Matrices	10	Q.1,5	Q.20			Q.33		8
4	Determinants				Q.21				2
5	Continuity and Differentiability	35	Q.3,7		Q.22				4
6	Applications of Derivatives		Q.6,8				Q.34	Q.36	11
7	Integrals		Q.9,12		Q.24	Q.26,29			10
8	Applications of Integrals		Q.10			Q.27			4
9	Differential Equations		Q.13,15		Q.23,25				6
10	Vector Algebra	14				Q.28		Q.37	7
11	Three Dimensional Geometry		Q.11,14				Q.35		7
12	Linear Programming	5	Q.16,18			Q.30			5
13	Probability	8	Q.17			Q.31		Q.38	8
	Total Marks (Total Questions)		18(18)	2(2)	10(5)	18(6)	20(4)	18(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$, then

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$ (c) $3 - \alpha^2 - \beta\gamma = 0$ (d) $3 + \alpha^2 + \beta\gamma = 0$

2. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is:

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $-\frac{2\pi}{3}$ (d) $\frac{2\pi}{3}$

3. If $f(x) = \begin{cases} x^k \cos(1/x), & x \neq 0 \\ 0 \quad , & x = 0 \end{cases}$ is continuous at x = 0, then

(a) $k < 0$ (b) $k > 0$ (c) $k = 0$ (d) $k \geq 0$

4. The range of the function $y = \left(\frac{\cos^{-1}(3x-1)}{\pi} + 1\right)^2$ is

(a) $[1, 4]$ (b) $[0, \pi]$ (c) $[1, \pi]$ (d) $[0, \pi^2]$

5. If $A = [a_{ij}]_{m \times n}$, then A' is equal to

(a) $[a_{ji}]_{n \times m}$ (b) $[a_{ij}]_{m \times n}$ (c) $[a_{ji}]_{m \times n}$ (d) $[a_{ij}]_{n \times m}$

6. The interval in which the function $f(x) = \frac{4x^2 + 1}{x}$ is decreasing is :

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $(-1, 1)$ (d) $[-1, 1]$

7. The derivative of e^{x^3} with respect to log x is

(a) e^{x^3} (b) $3x^2 2e^{x^3}$ (c) $3x^3 e^{x^3}$ (d) $3x^3 e^{x^3} + 3x^2$

8. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (c) $\left(0, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

9. $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx =$

(a) $-3(\tan x)^{1/3} + c$ (b) $-3(\tan x)^{-1/3} + c$ (c) $3(\tan x)^{-1/3} + c$ (d) $(\tan x)^{-1/3} + c$

10. The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is

(a) 1 (b) 2 (c) 3 (d) 4

11. The direction cosines *l*, *m*, *n* of two lines are connected by the relations $l + m + n = 0$, $lm = 0$, then the angle between them is:

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/2$ (d) 0

12. Value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $\frac{\pi}{4}$ (d) None of these

13. The order and degree of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter, is

(a) order = 4, degree = 4 (b) order = 4, degree = 1 (c) order = 1, degree = 4 (d) None of these

14. The points A(1, 2, 3), B (–1, –2, –3) and C(2, 3, 2) are three vertices of a parallelogram ABCD. The equation of CD is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{2}$ (b) $\frac{x+2}{1}=\frac{y+3}{2}=\frac{z-2}{2}$ (c) $\frac{x}{2}=\frac{y}{3}=\frac{z}{2}$ (d) $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-2}{2}$

15. The degree of the equation $e^x \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 3$ is

(a) 2 (b) 0 (c) not defined (d) 1

16. Minimum value of $Z = 3x + 5y$ subject to constraints $x + y \geq 2$, $x + 3y \geq 3$, $x, y \geq 0$

(a) 6 (b) 7 (c) 8 (d) 9

17. If $P(A \cap B) = 0.15$, $P(B') = 0.10$, then $P(A/B) =$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

18. The maximum value of $z = 2x + 5y$ subject to the constraints $2x + 5y \leq 10$, $x + 2y \geq 1$, $x - y \leq 4$, $x \geq y \geq 0$, occurs at

(a) exactly one point (b) exactly two points (c) infinitely many points (d) None of these

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion :** Every relation which is symmetric and transitive is also reflexive.
Reason : If aRb then bRa as R is symmetric. Now aRb and bRa $\Rightarrow$ aRa as R is transitive.

20. For any square matrix A with real number entries, consider the following statements.

Assertion : $A + A'$ is a symmetric matrix.

Reason: $A - A'$ is a skew-symmetric matrix.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write $\mathbf{A}^{-1}$.

22. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

OR

Differentiate the function cos x . cos 2x . cos 3x w.r.t.x.

23. Solve $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

24. Evaluate: $\int_0^{\pi/2} \cos^2 x\, dx$

25. Find the order and degree of the differential equation $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$.

OR

Find the order & degree of: $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Evaluate: $\int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}dx$

OR

Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$

27. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

28. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of the same magnitude, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

OR

If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form $\vec{b} = \vec{b_1} + \vec{b_2}$ where $\vec{b_1} \parallel \vec{a}$ and $\vec{b_2} \perp \vec{a}$.

29. Evaluate: $\int \frac{dx}{\sin^2 x + \tan^2 x}$

OR

Evaluate: $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is

30. Minimise $Z = -3x + 4y$ subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$

31. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probabililty that she reads English newspapers.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Let A = R – {3} and B = R – {1}. Let f : A → B defined as $f(x) = \frac{x-2}{x-3}\ \forall x \in A$. Then show that f is bijective.

33. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the identity matrix of order 2 and n ∈ N.

OR

Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

34. Prove that radius of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

35. A (0, 6, –9), B (–3, –6, 3) and C (7, 4, –1) are three points. Find the equation of the line AB. If D is the foot of the perpendicular drawn from the point C to the line AB, find the coordinates of the point D.

OR

Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case - Study 1:** Read the following passage and answer the questions given below.

Mr. Rakesh is the owner of a high rise residential society having 50 apartments. When he set rent at ₹ 10000/month, all apartements are rented. If he increases rent by ₹ 250/month, one fewer apartment is rented. The maintanance cost for each occupied unit is ₹ 500/month.

(i) If *P* is the rent price per apartment and *N* is the number of rented apartment, then find profit.

(ii) If *x* respected the number of apartments which are not rented, then find the profit.

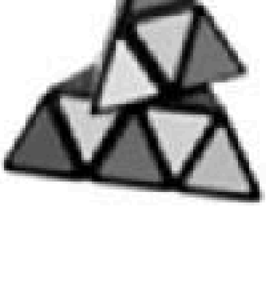
(iii) If P = 11,000, then find the profit.

OR

Find the rent that maximizes the total amount of profit.

37. **Case - Study 2:** Read the following passage and answer the questions given below.

A building is to be constructed in the form of a triangular pyramid, PQRS as shown in the figure.



Let its angular points are P(0, 1, 2), Q(3, 0, 1), R(4, 3, 6) and S(2, 3, 2) and G be the point of intersection of the medians of ΔQRS.

(i) Find the coordinates of point G.

(ii) Find the length of vector $\overline{PG}$.

(iii) Find area of ΔPQR (in sq. units).

OR

Find the sum of lengths of $\overline{PQ}$ and $\overline{PR}$.

38. **Case - Study 3:** Read the following passage and answer the questions given below.
Three students A, B and C are playing a dice game. The numbers rolled up by them in their first three chances were noted and given by A = {1, 5}, B = {2, 4, 5} and C = {1, 2, 5} as A reaches the cell 'SKIP YOUR NEXT TURN' in second throw.

(i) Find $P(A \cap B | C)$

(ii) Find $P(A \cup B | C)$

Solutions

SAMPLE PAPER-5

1. **(c)** $A^2 = 3I$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta r & 0 \\ 0 & \beta r + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 3 - \alpha^2 - \beta r = 0$$

2. **(a)** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \left[\sin^{-1}\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$

3. **(b)** Since f (x) is continuous at x = 0 $\therefore \lim_{x\to 0} f(x) = f(0)$

but f(0) = 0 (given) $\therefore \lim_{x\to 0} f(x) = \lim_{x\to 0} x^k \cos(1/x) = 0$, if $k > 0$

4. **(a)** $-1 \le 3x - 1 \le 1 \Rightarrow 0 \le x \le \frac{2}{3} \Rightarrow$ domain is $\left[0, \frac{2}{3}\right]$

when $x = 0$ then $y = 1$, $x = \frac{2}{3}$, $y = 4$. Hence range is [1, 4]

5. **(a)** If $A = [a_{ij}]_{m \times n}$, then $A' = [a_{ji}]_{n \times m}$

6. **(a)** Given $f(x) = \frac{4x^2 + 1}{x}$ Thus $f'(x) = 4 - \frac{1}{x^2}$

f(x) will be decreasing if $f'(x) < 0$

Thus $4 - \frac{1}{x^2} < 0 \Rightarrow \frac{1}{x^2} > 4 \Rightarrow \frac{-1}{2} < x < \frac{1}{2}$

Thus interval in which f(x) is decreasing, is $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

7. **(c)** Let $y = e^{x^3}$, $z = \log x$

On differentiating w.r.t.x, we get

$$\frac{dy}{dx} = e^{x^3}(3x^2) = 3x^2e^{x^3} \text{ and } \frac{dz}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{3x^2e^{x^3}}{\left(\frac{1}{x}\right)} = 3x^3e^{x^3}$$

8. **(b)** Since, $f(x) = \tan^{-1}(\sin x + \cos x)$

$$\therefore f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$$

$$= \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$$

f(x) is increasing if $f'(x) > 0 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) > 0$

$$\Rightarrow -\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2} \Rightarrow -\frac{3\pi}{2} < x + \frac{\pi}{4}$$

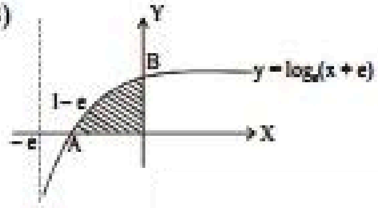
Hence, f(x) is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

9. **(b)** $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx = \int \frac{dx}{\sin^{4/3} x \cos^{2/3} x}$

Multiplying N^r and D^r by $\cos^2 x$, we get
{ Putting $\tan x = t \Rightarrow \sec^2 x\,dx = dt$}

$$= \int \frac{\sec^2 x\,dx}{\tan^{4/3} x} = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + c = -3(\tan x)^{-1/3} + c.$$

10. **(a)** Required area (OAB)



$$= \int_{1-e}^{0} \ln(x + e)dx$$

$$= \left[x \ln(x + e) - \int \frac{1}{x + e} x dx\right]_0^1 = 1.$$

11. **(a)** Given d'c's of two lines are ℓ, m, n connected by the relations $\ell + m + n = 0$ and $\ell m = 0$

Now, $\ell + m + n = 0 \Rightarrow \ell = -m - n \Rightarrow \ell = -(m + n)$

and $\ell m = 0 \Rightarrow -(m + n)m = 0 \Rightarrow -mm - mn = 0$

mm = –mn; Therefore m and m + n = 0

Then $\frac{\ell_1}{-1} = \frac{m_1}{0} = \frac{n_1}{1}$ and if $\ell + m + n = 0$ then

$$\frac{\ell_2}{0} = \frac{m_2}{-1} = \frac{n_2}{1}$$

$(\ell_1, m_1, n_1) = (-1, 0, 1)$ and $(\ell_2, m_2, n_2) = (0, -1, 1)$

We know that angle between them

$$\cos\theta = \frac{0 + 0 + 1}{\sqrt{1 + 0 + 1}\sqrt{0 + 1 + 1}} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2} = \cos 60^\circ \Rightarrow \theta = 60^\circ \Rightarrow \theta = \frac{\pi}{3}$$

12. **(c)** Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ...(i)

Then, $I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \int_0^{\pi/2} 1.dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$\Rightarrow \quad I = \frac{\pi}{4} \Rightarrow \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \frac{\pi}{4}$$

13. (c) $y = cx + c^2 - 3c^{3/2} + 2$ ... (i)

Differentiating above with respect to x, we get $\frac{dy}{dx} = c$.

Putting this value of c in (i), we get

$$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 - 3\left(\frac{dy}{dx}\right)^{3/2} + 2$$

Clearly its order is ONE and after removing the fractional power we get the degree FOUR.

14. (d) Given A(1, 2, 3), B(−1, −2, −1) and C(2, 3, 2), Let D be (α, β, γ). Since ABCD is a parallelogram, diagonals AC and BD bisect each other i.e., mid-point of segment AC is same as mid-point of segment BD.

$$\Rightarrow \left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = \left(\frac{\alpha-1}{2}, \frac{\beta-2}{2}, \frac{\gamma-1}{2}\right)$$

$\Rightarrow \alpha - 1 = 3, \beta - 2 = 5, \gamma - 1 = 5$

$\Rightarrow \alpha = 4, \beta = 7, \gamma = 6$

Hence, the point D is (4, 7, 6).

We have C(2, 3, 2) and D(4, 7, 6).

∴ Equation of line CD is

$$\frac{x-2}{4-2} = \frac{y-3}{7-3} = \frac{z-2}{6-2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-3}{4} = \frac{z-2}{4}$$

i.e., $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-2}{2}$ is the required equation of line.

15. (c) The given differential equation is

$$e^x \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 3$$

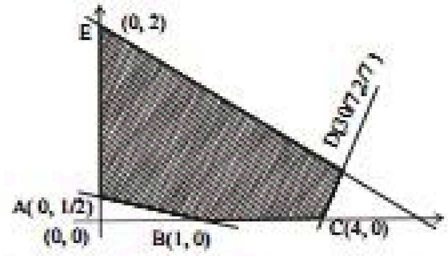
Since, this differential equation is not a polynomial in terms of its derivatives.

∴ Its degree is not defined.

16. (b) Z is 7 minimum at $\left(\frac{3}{2}, \frac{1}{2}\right)$

17. (c) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{1 - P(B')} = \frac{0.15}{1 - 0.1} = \frac{15}{90} = \frac{1}{6}$

18. (c) We find that the feasible region is on the same side of the line 2x + 5y = 10 as the origin, on the same side of the line x − y = 4 as the origin and on the opposite side of the line x + 2y = 1 from the origin. Moreover, the lines meet the coordinate axes at (5, 0), (0, 2); (1, 0), (0, 1/2) and (4, 0). The lines x − y = 4 and 2x + 5y = 10 intersect at $\left(\frac{30}{7}, \frac{2}{7}\right)$.



The values of the objective function at the vertices of the pentagon are:

(i) $Z = 0 + \frac{5}{2} = \frac{5}{2}$ (ii) $Z = 2 + 0 = 2$

(iii) $Z = 8 + 0 = 8$ (iv) $Z = \frac{60}{7} + \frac{10}{7} = 10$

(v) $Z = 0 + 10 = 10$

The maximum value 10 occurs at the points D(30/7, 2/7) and E(0, 2). Since D and E are adjacent vertices, the objective function has the same maximum value 10 at all the points on the lines DE.

19. (d) Let R be defined on the set {a, b, c}
then R = {(a, a), (b, b), (a, b), (b, a)}
is symmetric and transitive but not reflexive.

20. (b) Let $B = A + A'$, then $B' = (A + A')'$

$= A' + (A')' \quad [\text{as} (A + B)' = A' + B']$

$= A' + A \quad [\text{as} (A')' = A]$

$= A + A' = B$ (as A + B = B + A)

Therefore, B = A + A′ is a symmetric matrix.

Now let $C = A - A' \Rightarrow C' = (A - A')' = A' - (A')'$

$= A' - A = -(A - A') = -C$

Therefore C = A − A′ is a skew-skymmetric matrix.

21. $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19 \neq 0$ [½ Mark]

So, A is a non-singular matrix. Therefore, it is invertible. Now,

$\therefore \text{adj} A = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$ [½ Mark]

We know

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\therefore A^{-1} = \frac{1}{(-19)}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{19} & \frac{3}{19} \\ \frac{5}{19} & \frac{-2}{19} \end{bmatrix}$$ [1 Mark]

22. Given

$x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$

$$\Rightarrow \frac{dx}{dt} = a(-\sin t + 1.\sin t + t\cos t)$$

$$\Rightarrow \frac{dx}{dt} = at\cos t \quad \text{...(i)}$$ [½ Mark]

and $\frac{dy}{dt} = a(\cos t - 1.\cos t + t\sin t)$

$$\Rightarrow \frac{dy}{dt} = at\sin t \quad \text{...(ii)}$$ [½ Mark]

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{at\sin t}{at\cos t} = \tan t \text{ (using (i) and (ii))}$$

[½ Mark]

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}(\tan t) = \frac{d}{dt}(\tan t).\frac{dt}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2 t.\frac{dt}{dx} = \sec^2 t.\frac{1}{at\cos t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}.$$ [½ Mark]

OR

Let $y = \cos x . \cos 2x . \cos 3x$,

Taking log on both sides,

$\log y = \log\cos x + \log\cos 2x + \log\cos 3x$,

[½ Mark]

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = -(\tan x + 2\tan 2x + 3\tan 3x)$$ [½ Mark]

$$\therefore \frac{dy}{dx} = -y\,(\tan x + 2\tan 2x + 3\tan 3x)$$ [½ Mark]

$$\therefore \frac{dy}{dx} = -\cos x . \cos 2x . \cos 3x$$

$(\tan x + 2\tan 2x + 3\tan 3x)$ [½ Mark]

23. The given differential equation is

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$$

Integrating both sides, we get

$$\int \frac{1}{\sqrt{1-y^2}}dx = -\int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow \sin^{-1} y = -\sin^{-1} x + \sin^{-1} C$$ [1 Mark]

$$\Rightarrow \sin^{-1} y + \sin^{-1} x = \sin^{-1} C$$

$$\Rightarrow \sin^{-1}\left[y\sqrt{1-x^2} + x\sqrt{1-y^2}\right] = \sin^{-1} C$$

$$\Rightarrow y\sqrt{1-x^2} + x\sqrt{1-y^2} = C$$

This is defined for $1 - x^2 \geq 0$ i.e., $x \in [-1, 1]$

Hence, $y\sqrt{1-x^2} + x\sqrt{1-y^2} = C$, $x \in [-1, 1]$ is the general solution of the given differential equation.

[1 Mark]

24. $$\int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \frac{1+\cos 2x}{2}dx$$ [1 Mark]

$(\because 2\cos^2 A = 1 + \cos 2A)$

$$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)\Big]_0^{\frac{\pi}{2}} \qquad \left(\because \int \cos 2A \cdot dA = \frac{\sin 2A}{A}\right)$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{\sin\pi}{2}\right] - \frac{1}{2}\left[0 + \frac{\sin 0}{2}\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{2} + 0\right) - 0 = \frac{\pi}{4}.$$ [1 Mark]

25. Given differential equation can be written as

$$y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$ [1 Mark]

Clearly, it is a 1st order and 2nd degree differential equation. [1 Mark]

OR

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

It is a D.E. of order 2 and degree undefined. [2 Marks]

26. Let $I = \int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}dx$ .....(i)

$$I = \int_0^{\pi} \frac{4(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$$ [½ Mark]

$$I = \int_0^{\pi} \frac{(4\pi - 4x)\sin x}{1+\cos^2 x}dx \quad \text{.....(ii)}$$

On adding (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{4\pi\sin x}{1+\cos^2 x}dx = 4\pi\int_1^{-1} \frac{-dt}{1+t^2}$$

where $t = \cos x$

when $x = 0, t = 1$

$x = \pi, t = -1$ [1 Mark]

$$= 4\pi\int_{-1}^{1} \frac{dt}{1+t^2} = 4\pi\left|(\tan^{-1} t)\right|_{-1}^{1}$$ [½ Mark]

$= 4\pi\left[\tan^{-1}(1) - \tan^{-1}(-1)\right]$

$= 4\pi\left[\frac{\pi}{4} - \frac{3\pi}{4}\right] = -2\pi^2$ [1 Mark]

$\therefore \quad I = -\pi^2$

OR

Let $I = \int \frac{x+2}{\sqrt{x^2+5x+6}}\,dx$

$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+5x+6}}\,dx$

$= \frac{1}{2}\int \frac{2x+5-1}{\sqrt{x^2+5x+6}}\,dx$

$= \frac{1}{2}\int \frac{(2x+5)dx}{\sqrt{x^2+5x+6}} - \frac{1}{2}\int \frac{dx}{\sqrt{x^2+5x+6}}$ [1 Mark]

Consider

$I_1 = \frac{1}{2}\int \frac{(2x+5)dx}{\sqrt{x^2+5x+6}}$

Let $x^2+5x+6 = t^2$

$2x+5\,dx = 2 + \text{dt}$

$= \frac{1}{2}\int \frac{2t\,dt}{t} = \sqrt{x^2+5x+6}$ [1 Mark]

$I_2 = \int \frac{dx}{\sqrt{x^2+5x+6}}$ [½ Mark]

$= \int \frac{dx}{\sqrt{\left(x+\frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

$= \log\left|\left(x+\frac{5}{2}\right) + \sqrt{\left(x+\frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + C$

$= \log\left|\left(x+\frac{5}{2}\right) + \sqrt{x^2+5x+6}\right| + C$

$= \log\left|\left(\frac{2x+5}{2}\right) + \sqrt{x^2+5x+6}\right| + C$

$I = \sqrt{x^2+5x+6} - \frac{1}{2}\left[\log\left|\frac{2x+5}{2} + \sqrt{x^2+5x+6}\right|\right] + C$

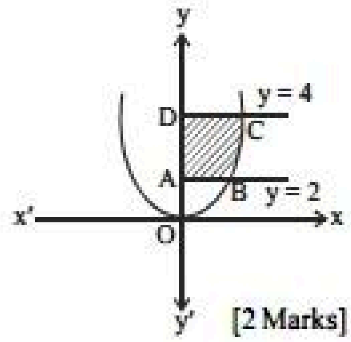
[1 Mark]

27. The given curve $x^2 = 4y$ is a parabola with vertex at (0, 0). Also since it contains only even powers of x, it is symmetrical about y-axis. y = 2 and y = 4 are straight lines parallel to x-axis at a positive distance of 2 and 4 from it respectively.

Required area = area ABCD

$= \int_2^4 x\,dy = \int_2^4 2\sqrt{y}\,dy$ [1 Mark]

$= 2\int_2^4 \sqrt{y}\,dy$



$= 2\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_2^4$

$= \left(\frac{32 - 8\sqrt{2}}{3}\right)$ sq. units [2 Marks]

28. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors and

$|\vec{a}| = |\vec{b}| = |\vec{c}| = k\,(\text{say})$ ...(i)

and $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ ...(ii) [1 Mark]

Now, $|\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})$

$= \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c}$
$+ \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c}$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$= k^2 + k^2 + k^2 + 2(0)$ [using (i) and (ii)]

$= 3k^2$

$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}k$ ...(iii) [1 Mark]

Let $\vec{a}+\vec{b}+\vec{c}$ makes angles α, β and γ with $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

$\therefore \ \vec{a}.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\alpha$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = k(\sqrt{3}k)\cos\alpha$

$k^2 = \sqrt{3}k^2\cos\alpha$

$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Similarly, $\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and $\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Thus α = β = γ

Hence, $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. [1 Mark]

OR

Here, given vectors are $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

Since $\vec{b_1} \parallel \vec{a} \Rightarrow \vec{b_1} = \lambda \vec{a}$ for some λ ......(i)

Let $\vec{b_2} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ ......(ii)

Since $\vec{b_2} \perp \vec{a} \therefore \vec{b_2}.\vec{a} = 0$

$\Rightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right).\left(3\hat{i} - \hat{j}\right) = 0$

$\Rightarrow 3a_1 - a_2 = 0$ ......(iii) [1 Mark]

Now, $\vec{b} = \vec{b_1} + \vec{b_2} = \lambda\vec{a} + \vec{b_2}$

$2\hat{i} + \hat{j} - 3\hat{k} = \lambda\left(3\hat{i} - \hat{j}\right) + a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$= (3\lambda + a_1)\hat{i} + (-\lambda + a_2)\hat{j} + a_3\hat{k}$

Comparing co-efficients, we obtain

$a_1 = 2 - 3\lambda$, $a_2 = 1 + \lambda$ and $a_3 = -3$ ......(iv)

From (iii), we have

$3(2 - 3\lambda) - 1 - \lambda = 0 \Rightarrow \lambda = \frac{1}{2}$

From (iv), we have

$a_1 = 2 - 3 \times \frac{1}{2} = \frac{1}{2}$, $a_2 = 1 + \frac{1}{2} = \frac{3}{2}$ and $a_3 = -3$ [1 Mark]

From (ii), we obtain

$\vec{b_2} = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

From (i), we obtain

$\vec{b_1} = \left(3\hat{i} - \hat{j}\right) = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$

Hence,

$\vec{b_1} = \frac{1}{2}\left(3\hat{i} - \hat{j}\right)$ and $\vec{b_2} = \frac{1}{2}(\hat{i} + 3\hat{j} - 6\hat{k})$ [1 Mark]

29. $I = \int \frac{dx}{\sin^2 x + \tan^2 x}$

Dividing numerator and denominator by $\cos^2 x$, we get

$I = \int \frac{\sec^2 x}{\tan^2 x + \tan^2 x \sec^2 x} dx$

$= \int \frac{\sec^2 x}{\tan^2 x(1 + \sec^2 x)} dx$ [1 Mark]

$= \int \frac{\sec^2 x}{\tan^2 x(2 + \tan^2 x)} dx$

Put $\tan x = t$, then $\sec^2 x\, dx = dt$

$\therefore \quad I = \int \frac{dt}{t^2(t^2 + 2)} = \frac{1}{2}\int \frac{2}{t^2(t^2 + 2)} dt$

$= \frac{1}{2}\int \left(\frac{1}{t^2} - \frac{1}{t^2 + 2}\right) dt$ [1 Mark]

$= \frac{1}{2}\int t^{-2} dt - \frac{1}{2}\int \frac{dt}{t^2 + (\sqrt{2})^2}$

$= \frac{-1}{2\tan x} - \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + C$ [1 Mark]

OR

Let $I = \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$

$= \int_{-\pi}^{\pi} \frac{2x\, dx}{1 + \cos^2 x} + 2\int_{-\pi}^{\pi} \frac{x \sin x\, dx}{1 + \cos^2 x}$

$= 0 + 4\int_0^{\pi} \frac{x \sin x\, dx}{1 + \cos^2 x}$; [½ Mark]

$I = 4\int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$ [½ Mark]

$I = 4\int_0^{\pi} \frac{(\pi - x)\sin x}{1 + \cos^2 x} dx$

$\Rightarrow I = 4\pi\int_0^{\pi} \frac{\sin x\, dx}{1 + \cos^2 x} - 4\int_0^{\pi} \frac{x \sin x\, dx}{1 + \cos^2 x}$;

$\Rightarrow 2I = 4\pi\int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$ [1 Mark]

put $\cos x = t \Rightarrow -\sin x\, dx = dt \Rightarrow \sin x\, dx = -dt$

when $x = \pi$ then $t = -1$

when $x = 0$ then $t = 1$

$\therefore 2I = 4\pi\int_1^{-1} \frac{-dt}{1 + t^2} = 4\pi\int_{-1}^{1} \frac{dt}{1 + t^2}$ [½ Mark]

$= 8\pi\int_0^1 \frac{dt}{1 + t^2} = 8\pi\left[\tan^{-1} t\right]_0^1$

$= 8\pi\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$= 8\pi\left[\frac{\pi}{4} - 0\right] = 2\pi$

$\Rightarrow I = \pi$ [½ Mark]

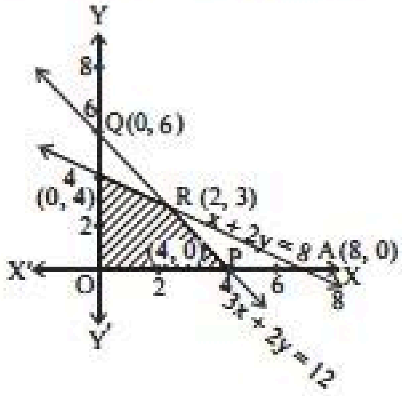
30. Objective function $Z = -3x + 4y$

constraints are $x + 2y \le 8$,

$3x + 2y \le 12, x \ge 0, y \ge 0$

(i) Consider the line $x + 2y = 8$. It pass through A (8, 0) and B (0, 4), putting $x = 0$, $y = 0$ in $x + 2y \le 8$, $0 \le 8$ which is true.

$\Rightarrow$ region $x + 2y \le 8$ lies on and below AB.



[2 Marks]

(ii) The line $3x + 2y = 12$ passes through P (4, 0), Q (0, 6) putting $x = 0, y = 0$ in

$3x + 2y \le 12$

$\Rightarrow 0 \le 12$, which is true.

$\therefore$ Region $3x + 2y \le 12$ lies on and below PQ.

(iii) $x \ge 0$ the region lies on and to the right of y-axis.

(iv) $y \ge 0$ lies on and above x-axis.

(v) Solving the equations $x + 2y = 8$ and $3x + 2y = 12$ we get $x = 2$, $y = 3$ $\Rightarrow$ R is (2,3) where AB and PQ. intersect.The shaded region OPRB is the feasible region.

At P (4,0) $\quad Z = -3x + 4y = -12 + 0 = -12$

At R (2,3) $\quad Z = -6 + 12 = 6$

At B (0,4) $\quad Z = 0 + 16 = 16$

At Q (0,0) $\quad Z = 0$ [1 Mark]

Thus minimum value of Z is -12 at P (4, 0)

31. (a) Let H and E represent the event that a student reads Hindi and English newspaper respectively

$P(H) = 0{\cdot}6, P(E) = 0{\cdot}4, P(H \cap E) = 0{\cdot}2$

Probability that the student reads at least one paper

$= P(H \cup E)$

Now $P(H) = 0{\cdot}6, P(E) = 0{\cdot}4, P(H \cap E) = 0{\cdot}2$

$\therefore \; P(H \cup E) = 0{\cdot}6 + 0{\cdot}4 - 0{\cdot}2 = 1 - 0{\cdot}2 = 0{\cdot}8$

$\therefore$ Probability that a student reads neither Hindi nor English newspaper $= 1 - P(H \cup E) = 1 - 0{\cdot}8 = 0{\cdot}2$

[1 Mark]

(b) The probability that the student reads English newspaper if she reads Hindi

$$= P(E/H) = \frac{P(E \cap H)}{P(H)}$$

Now $P(E \cap H) = 0{\cdot}2, P(H) = 0{\cdot}6$

$$\therefore \; P(E/H) = \frac{0{\cdot}2}{0{\cdot}6} = \frac{1}{3}$$ [1 Mark]

(c) The probability that she reads Hindi newspaper if she reads English newspaper

$= P(H/E) = P(H \cap E) = 0{\cdot}2, P(E) = 0{\cdot}4$

$$\therefore \quad P(H/E) = \frac{P(H \cap E)}{P(E)} = \frac{0{\cdot}2}{0{\cdot}4} = \frac{1}{2}$$ [1 Mark]

32. **One-one/Many-one :** Let $x_1, x_2 \in R - \{3\}$ are the elements such that

$$f(x_1) = f(x_2) : \text{then } f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$

$\Rightarrow -2x_2 - 3x_1 = -2x_1 - 3x_2$

$\Rightarrow x_2 = x_1, \; \therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one-one function [2 Marks]

Onto/Into : Let $y \in R - \{1\}$ (co-domain)

Then one element $x \in R - \{3\}$ in domain is such that

$$f(x) = y \Rightarrow \frac{x - 2}{x - 3} = y \Rightarrow x - 2 = xy - 3y \Rightarrow x = \left(\frac{3y - 2}{y - 1}\right)$$

[2 Marks]

$\therefore$ The pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\} \Rightarrow$ f is onto [1 Mark]

33. $$\text{L.H.S.} = [aI + bA]^n = \left[a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]^n$$

$$= \left[\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}\right]^n = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}^n$$ [1 Mark]

$\text{R.H.S.} = a^nI + na^{n-1}bA$

$$= a^n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + na^{n-1}b\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a^n & 0 \\ 0 & a^n \end{pmatrix} + \begin{pmatrix} 0 & na^{n-1}b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{pmatrix}$$ [1 Mark]

We have to prove that

$$\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}^n = \begin{bmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{bmatrix}$$

Applying the principle of Mathematical Induction

$$\text{Put } P(n) : \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}^n = \begin{bmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{bmatrix} \quad \text{for } n = 1,$$

$$P(1) : \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$

$\therefore$ P (n) is true for $n = 1$, Let P (n) be true for $n = k$ [1 Mark]

$$\therefore \; P(k) : \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}^k = \begin{bmatrix} a^k & ka^{k-1}b \\ 0 & a^k \end{bmatrix}$$

Multiplying both sides by $\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$$\text{L.H.S.} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}^k \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}^{n+1}$$ [1 Mark]

$$\text{R.H.S.} = \begin{bmatrix} a^k & ka^{k-1}b \\ 0 & a^k \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & ba^k + ka^k b \\ 0 & a^{k+1} \end{bmatrix} = \begin{bmatrix} a^{k+1} & (k+1)a^k b \\ 0 & a^{k+1} \end{bmatrix}$$

This shows P (n) is true for n = k + 1 then by principle of mathematical induction, P (n) is true for all positive integral values of n. [1 Mark]

OR

$\therefore$ Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & -6 \end{bmatrix}$$

$$\because \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & -6 \end{bmatrix}$$ [1 Mark]

Equating the corresponding elements

$a + 4b = -7$ ...(i)

$2a + 5b = -8$ ...(ii)

$3a + 6b = -9$ ...(iii)

Multiplying (i) by 2

$2a + 8b = -14$ ...(iv)

and $2a + 5b = -8$

Subtracting (ii) from (iv) [1 Mark]

$3b = -6 \quad \therefore b = -2$

Putting the value of b in (i)

$a - 8 = -7, \quad \therefore a = 8 - 7 = 1$ [1 Mark]

a = 1, b = – 2 satisfy eqn. (iii) also equating the element of second row

$c + 4d = 2$ ...(v)

$2c + 5d = 4$ ...(vi)

$3c + 6d = 6$ ...(vii)

Multiplying equ. (v) by 2

$2c + 8d = 4$ ...(viii)

Subtracting (vi) from (viii)

$3d = 4 - 4 = 0 \quad \therefore \quad d = 0$ [1 Mark]

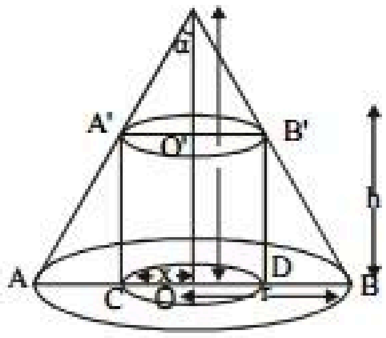
$\therefore$ from $c + 4d = 2 \quad \therefore \quad c = 2$

c = 2, d = 0, satisfy equ. (vii) also

Thus a = 1, b = – 2, c = 2, d = 0

Hence, $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ [1 Mark]

34. Let VAB be the cone of radius of base r and height 'h'. Let radius of base of the inscribed cylinder be x.



Noe, we observe that $\Delta VOB \sim \Delta B'DB$

$$\Rightarrow \quad \frac{VO}{B'D} = \frac{OB}{DB}$$

$$\Rightarrow \quad \frac{h}{B'D} = \frac{r}{r-x}$$

$$\Rightarrow \quad B'D = \frac{h(r-x)}{r}$$ [1 Mark]

Let C be the curved surface area of cylinder.
Then,

$$C = 2\pi(OC)(B'D)$$

$$\Rightarrow \quad C = \frac{2\pi \times h(r-x)}{r} = \frac{2\pi h}{r}(rx - x^2)$$ [1 Mark]

Differentiating wrt x, we get

$$\frac{dC}{dx} = \frac{2\pi h}{r}(r - 2x)$$ [1 Mark]

For maximina and minima, put $\frac{dC}{dx} = 0$

$$\therefore \quad \frac{2\pi h}{r}(r - 2x) = 0$$

$$\Rightarrow \quad r - 2x = 0$$

$$\Rightarrow \quad r = 2x \text{ or } x = \frac{r}{2}$$ [1 Mark]

Hence, radius of cylinder is half of that of cone.

Also, $\frac{d^2C}{dx^2} = \frac{d}{dx}\left(\frac{2\pi h(r-2x)}{r}\right)$

$$= \frac{2\pi h}{r}(-2) = \frac{-4\pi h}{r} < 0 \text{ as } h, r > 0$$

$$\because \frac{d^2C}{dx^2} < 0 \Rightarrow C \text{ is maximum or greatest.}$$

Hence, C is greatest at $x = \frac{r}{2}$ [1 Mark]

35. Equation of the line AB is

$$\frac{x-0}{0-(-3)}=\frac{y-6}{6-(-6)}=\frac{z-(-9)}{-9-3} \Rightarrow \frac{x}{1}=\frac{y-6}{4}=\frac{z+9}{-4},$$

[1 Mark]

Any point on it is given by (r, 4r + 6, – 4r – 9), let it be D. Now C is (7, 4, –1)

∴ Direction ratios of the line CD are r – 7, 4r + 6 – 4, 4r – 9(–1) i.e. r – 7, 4r + 2), –4r –8. [2 Marks]

As CD⊥AB, So $(r-7)\cdot 1+(4r+2)\cdot 4+(-4r-8)\cdot(-4)=0$

$\Rightarrow 33r+33=0$

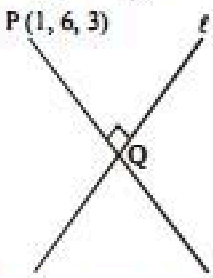
$\Rightarrow r=-1$, ∴ The point D is (–1, 2, –5). [2 Marks]

OR

For image of P (1, 6, 3) in L draw a line PR ⊥ ℓ then R is its image of Q is mid point of PR and PR ⊥ ℓ. Let λ, μ, ν be the d.r's of PR. PR ⊥ ℓ.

$\Rightarrow \lambda\times 1+\mu\times 2+\nu\times 3=0 \Rightarrow \lambda+2\mu+3\nu=0$ [1 Mark]

and equ. of PR is $\frac{x-1}{\lambda}=\frac{y-6}{\mu}=\frac{z-3}{\nu}$



Any point on it is (λk + 1, μk + 6, νk + 3) let it be θ. As θ lies on l, so. [1 Mark]

$$\frac{xk-1}{1}=\frac{\mu k+6-1}{2}=\frac{\nu k+3-2}{2} \Rightarrow \frac{\lambda k+1}{1}=\frac{\mu k+5}{3}$$

$$R(x',y',z')=\frac{1(\lambda k+1)+2(\mu k+5)+3(\nu k+1)}{1\times 1+2\times 2+3\times 3}$$

$$=\frac{14+(\lambda+2\mu+3\nu)k}{14}=1$$

$\Rightarrow \lambda k=0, \mu k=-3, \nu k=2$

$\Rightarrow Q(0+1,-3+6,2+3)=(1,3,5)$ [2 Marks]

As Q is the mid point of PR, so

$$\frac{1+x'}{2}=1, \frac{6+y'}{2}=3, \frac{3+z'}{2}=5 \quad \Rightarrow x'=1, y'=0, z'=7$$

⇒ R (1, 0, 7). Which is the image of P. [1 Mark]

36. (i) Income from rent = NP

Maintenance cost = 500 N

∴ Profit = NP – 500 N = N(P – 500) [1 Mark]

(ii) N = 50 – x

P = 1000 + 250 x

Profit = N(P – 500) = (50 – x) (10000 + 250 x – 500)

= (50 – x) (9500 + 250 x) = 250 (50 – x) (38 + x)

[1 Mark]

(iii) If P = 11000 then 11000 = 10000 + 250 x ⇒ x = 4

So, profit = 250 (50 – 4)(38 + 4) = 483000 [2 Marks]

OR

Let P(x) = 250(50 – x)(38 + x)

P′(x) = 250 (12 – 2x) = 0 ⇒ x = 6

P″(x) = – 500 < 0 (Profit Maximum)

∴ Rent = 10000 + 250 × 6 = ₹ 11500 [2 Marks]

37. (i) Centoid of ΔQRS = $G\left(\frac{3+4+2}{3},\frac{0+3+3}{3},\frac{1+6+2}{3}\right)$

= G(3, 2, 3) [1 Mark]

(ii) $\overline{PG}=(3-0)\hat{i}+(2-1)\hat{j}+(3-2)\hat{k}$

$=3\hat{i}+\hat{j}+\hat{k}$

$|\overline{PG}|=\sqrt{9+1+1}=\sqrt{11}$ [1 Mark]

(iii) Area of $\Delta PQR=\frac{1}{2}|\overline{PQ}\times\overline{PR}|$

$$\overline{PQ}\times\overline{PR}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ (3-0) & (0-1) & (1-2)\\ (4-0) & (3-1) & (6-2)\end{vmatrix}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & -1\\ 4 & 2 & 4\end{vmatrix}$$

$=\hat{i}(-4+2)-\hat{j}(12+4)+\hat{k}(6+4)$

$=-2\hat{i}-16\hat{j}+10\hat{k}$ [1 Mark]

$\therefore |\overline{PQ}\times\overline{PR}|=\sqrt{(-2)^2+(-16)^2+(10)^2}=6\sqrt{10}$

∴ Area of $\Delta PQR=\frac{1}{2}\times 6\sqrt{10}=3\sqrt{10}$ sq.unit

[1 Mark]

OR

$\overline{PQ}=3\hat{i}-\hat{j}-\hat{k} \Rightarrow |\overline{PQ}|=\sqrt{9+1+1}=\sqrt{11}$

$\overline{PR}=4\hat{i}+2\hat{j}+4\hat{k} \Rightarrow |\overline{PR}|=\sqrt{4^2+2^2+4^2}=\sqrt{36}=6$

[1 Mark]

$\therefore |\overline{PQ}|+|\overline{PR}|=\sqrt{11}+6=3.32+6=9.32$ units. [1 Mark]

38. Sample space = {1, 2, 3, 4, 5, 6}, A ∩ B = {5}

B ∩ C = {2, 5}, A ∩ C = {1, 5}, A ∩ B ∩ C = {5}

and {A ∪ B} ∩ C = {1,2,5}

$P(A)=\frac{2}{6},\ P(B)=\frac{2}{6},\ P(A\cap C)=\frac{2}{6}$

$P(A\cap B\cap C)=\frac{1}{6}$, and $P((A\cap B)\cap C)=\frac{3}{6}$

(i) $P(A\cap B/C)=\frac{P(A\cap B\cap C)}{P(C)}=\frac{1/6}{3/6}=\frac{1}{3}$ [2 Marks]

(ii) $P(A\cup B/C)=\frac{P(A\cup B\cap C)}{P(C)}=\frac{3/6}{3/6}=1.$ [2 Marks]