

Inverse Trigonometric Functions

Functions	Domain (x)	Range (y)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1}x$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \operatorname{cosec}^{-1}x$	$\mathbb{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \cot^{-1}x$	$\mathbb{R}$	$(0, \pi)$

✓ Properties of inverse trigonometric functions

$$\sin^{-1}(\sin x) = x \quad : \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1}(\cos x) = x \quad : \quad x \in [0, \pi]$$

$$\tan^{-1}(\tan x) = x \quad : \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\cot^{-1}(\cot x) = x \quad : \quad x \in (0, \pi)$$

$$\sec^{-1}(\sec x) = x \quad : \quad x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad : \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$\sin^{-1}x \left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}x \quad : \quad x \geq 1 \text{ OR } x \leq -1$$

$$\cos^{-1}x \left(\frac{1}{x}\right) = \sec^{-1}x \quad : \quad x \geq 1 \text{ OR } x \leq -1$$

$$\tan^{-1}x \left(\frac{1}{x}\right) = \cot^{-1}x \quad : \quad x > 0$$

$$\tan^{-1}x \left(\frac{1}{x}\right) = -\pi + \cot^{-1}x \quad : \quad x < 0$$

$\mathbb{R}$ = Real Numbers

$$\sin x (\sin^{-1}x) : x \in [-1, 1]$$

$$\cos x (\cos^{-1}x) : x \in [-1, 1]$$

$$\tan x (\tan^{-1}x) : x \in \mathbb{R}$$

$$\cot x (\cot^{-1}x) : x \in \mathbb{R} - (-1, 1)$$

$$\sec x (\sec^{-1}x) : x \in \mathbb{R} - (-1, 1)$$

$$\operatorname{cosec} x (\operatorname{cosec}^{-1}x) : x \in \mathbb{R}$$

$$\sin^{-1}(-x) = -\sin^{-1}x \quad : \quad x \in [-1, 1]$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}x \quad : \quad x \in [-1, 1]$$

$$\tan^{-1}(-x) = -\tan^{-1}x \quad : \quad x \in \mathbb{R}$$

$$\cot^{-1}(-x) = \pi - \cot^{-1}x \quad : \quad x \in \mathbb{R}$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}x \quad : \quad |x| \geq 1$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \quad : \quad |x| \geq 1$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad : \quad x \in [-1, 1]$$

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad : \quad x \in \mathbb{R}$$

$$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \quad : \quad |x| \geq 1$$

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$$

$$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$$

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\{xy - y\sqrt{1-x^2}\sqrt{1-y^2}\}$$

$$\cos^{-1}x - \cos^{-1}y = \cos^{-1}\{xy + y\sqrt{1-x^2}\sqrt{1-y^2}\}$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) : xy < 1$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) : xy > -1$$

$$2 \sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}) : -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$2 \cos^{-1}x = \cos^{-1}(2x^2 - 1) : 0 \leq x \leq 1$$

$$2 \tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) : -1 < x < 1$$

$$2 \tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) : -1 \leq x \leq 1$$

$$2 \tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) : x \geq 0$$

$$2 \tan^{-1}x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) : \text{if } x > 1$$

$$3 \sin^{-1}x = \sin^{-1}(3x - 4x^3) : -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$3 \cos^{-1}x = \cos^{-1}(4x^3 - 3x) : \frac{1}{2} \leq x \leq 1$$

$$3 \tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) : -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \sin^{-1}x &= \cos^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) \\ &= \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right) \end{aligned}$$

$$\begin{aligned} \cos^{-1}x &= \sin^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) \\ &= \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) \end{aligned}$$

$$\begin{aligned} \tan^{-1}x &= \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) \\ &= \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(\sqrt{1+x^2}) = \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right) \end{aligned}$$

Expression	Substitution
$a^2 + x^2$ OR $\sqrt{a^2 + x^2}$	$x = a \tan \theta$ OR $x = a \cot \theta$
$a^2 - x^2$ OR $\sqrt{a^2 - x^2}$	$x = a \sin \theta$ OR $x = a \cos \theta$
$x^2 - a^2$ OR $\sqrt{x^2 - a^2}$	$x = a \sec \theta$ OR $x = a \operatorname{cosec} \theta$
$\sqrt{\frac{a-x}{a+x}}$ OR $\sqrt{\frac{a+x}{a-x}}$	$x = a \cos 2\theta$
$\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$ OR $\sqrt{\frac{a^2 + x^2}{a^2 - x^2}}$	$x^2 = a^2 \cos 2\theta$
$\sqrt{\frac{x}{a-x}}$ OR $\sqrt{\frac{a-x}{x}}$	$x = a \sin^2 \theta$ OR $x = a \cos^2 \theta$
$\sqrt{\frac{x}{a+x}}$ OR $\sqrt{\frac{a+x}{x}}$	$x = a \tan^2 \theta$ OR $x = a \cot^2 \theta$