CHAPTER 4

Permutations and Combinations

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

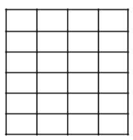
1. In a certain test, a_i students gave wrong answers to atleast i questions, where $i = 1, 2, \ldots, k$. No student gave more than k wrong answers. The total number of wrong answers given is .... *(1982 - 2 Marks)*
2. The side AB, BC and CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is .... *(1984 - 2 Marks)*
3. Total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together is .... *(1988 - 2 Marks)*
4. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is _________.

B True / False

1. The product of any r consecutive natural numbers is always divisible by $r!$. *(1985 - 1 Mark)*

C MCQs with One Correct Answer

1. ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is : *(1979)*
   (a) 1 (b) 2
   (c) 3 (d) None of these.
2. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated are *(1982 - 2 Marks)*
   (a) 69760 (b) 30240
   (c) 99748 (d) none of these
3. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to *(1982 - 2 Marks)*
   (a) ${}^{47}C_5$ (b) ${}^{52}C_5$
   (c) ${}^{52}C_4$ (d) none of these
4. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4 ; and then the men select the chairs from amongst the remaining. The number of possible arrangements is *(1982 - 2 Marks)*
   (a) ${}^6C_3 \times {}^4C_2$ (b) ${}^4P_2 \times {}^4P_3$
   (c) ${}^4C_2 + {}^4P_3$ (d) none of these
5. A five-digit numbers divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is *(1989 - 2 Marks)*
   (a) 216 (b) 240 (c) 600 (d) 3125
6. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ? *(2000S)*
   (a) 16 (b) 36 (c) 60 (d) 180
7. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals *(2001S)*
   (a) 5 (b) 7 (c) 6 (d) 4
8. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is *(2002S)*
   (a) 40 (b) 60 (c) 80 (d) 100
9. A rectangle with sides of length $(2m - 1)$ and $(2n - 1)$ units is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is *(2005S)*



   (a) $(m + n - 1)^2$ (b) 4^{m+n-1}
   (c) m^2n^2 (d) $m(m + 1)n(n + 1)$
10. If the LCM of p, q is $r^2t^4s^2$, where r, s, t are prime numbers and p, q are the positive integers then the number of ordered pair (p, q) is *(2006 - 3M, –1)*
   (a) 252 (b) 254 (c) 225 (d) 224
11. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is *(2007 -3 marks)*
   (a) 360 (b) 192 (c) 96 (d) 48
12. The number of seven digit i ntegers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is *(2009)*
   (a) 55 (b) 66 (c) 77 (d) 88

13. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is *(2012)*

(a) 75 (b) 150 (c) 210 (d) 243

14. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is *(JEE Adv. 2014)*

(a) 264 (b) 265 (c) 53 (d) 67

15. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 memoers) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is *(JEE Adv. 2016)*

(a) 380 (b) 320 (c) 260 (d) 95

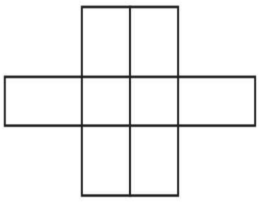
D MCQs with One or More than One Correct

1. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n- digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is *(1998 - 2 Marks)*

(a) 6 (b) 7 (c) 8 (d) 9

E Subjective Problems

1. Six X's have to be placed in the squares of figure below in such a way that each row contains at least one X. In how many different ways can this be done. *(1978)*



2. Five balls of different colours are to be placed in there boxes of different size. Each box can hold all five. In how many different ways can we place the balls so that no box remains empty ? *(1981 - 4 Marks)*

3. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated is $\dfrac{m!(m+1)!}{(m-n+1)!}$ *(1983 - 2 Marks)*

4. 7 relatives of a man comprises 4 ladies and 3 gentlemen; his wife has also 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of the wife's relatives? *(1985 - 5 Marks)*

5. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw? *(1986 - 2½ Marks)*

6. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Determine the number of ways in which the sitting arrangements can be made. *(1991 - 4 Marks)*

7. A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if at least five women have to be included in a committee? In how many of these committees *(1994 - 4 Marks)*

(a) The women are in majority?

(b) The men are in majority?

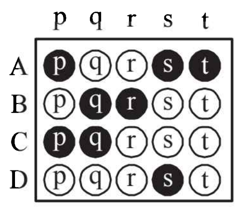
8. Prove by permutation or otherwise $\dfrac{(n^2)!}{(n!)^n}$ is an integer $(n \in I^+)$. *(2004 - 2 Marks)*

9. If total number of runs scored in n matches is $\left(\dfrac{n+1}{4}\right)(2^{n+1}-n-2)$ where $n > 1$, and the runs scored in the kth match are given by k. 2^{n+1-k}, where $1 \le k \le n$. Find n. *(2005 - 2 Marks)*

F Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Consider all possible permutations of the letters of the word ENDEANOEL. Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. *(2008)*

Column I	**Column II**
(A) The number of permutations containing the word ENDEA is	(p) 5!

(B) The number of permutations in which the letter E occurs in the first and the last positions is — (q) $2 \times 5!$

(C) The number of permutations in which none of the letters D, L, N occurs in the last five positions is — (r) $7 \times 5!$

(D) The number of permutations in which the letters A, E, O occur only in odd positions is — (s) $21 \times 5!$

G Comprehension Based Questions

PASSAGE - 1

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n–digit integers ending with digit 0. *(2012)*

1. The value of b_6 is
   (a) 7 (b) 8
   (c) 9 (d) 11
2. Which of the following is correct?
   (a) $a_{17} = a_{16} + a_{15}$ (b) $c_{17} \neq c_{16} + c_{15}$
   (c) $b_{17} \neq b_{16} + c_{16}$ (d) $a_{17} = c_{17} + b_{16}$

I Integer Value Correct Type

1. Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is *(JEE Adv. 2013)*
2. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is *(JEE Adv. 2014)*
3. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is *(JEE Adv. 2014)*
4. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are **[2002]**
   (a) 216 (b) 375 (c) 400 (d) 720
2. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4 (repetition allowed). Their number is **[2002]**
   (a) 125 (b) 105 (c) 375 (d) 625
3. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4 and 5 without repetition. Total number of such numbers are **[2002]**
   (a) 312 (b) 3125 (c) 120 (d) 216
4. The sum of integers from 1 to 100 that are divisible by 2 or 5 is **[2002]**
   (a) 3000 (b) 3050 (c) 3600 (d) 3250
5. If ${}^nC_r$ denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals **[2003]**
   (a) ${}^{n+1}C_{r+1}$ (b) ${}^{n+2}C_r$ (c) ${}^{n+2}C_{r+1}$ (d) ${}^{n+1}C_r$.
6. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is **[2003]**
   (a) 346 (b) 140 (c) 196 (d) 280
7. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[2003]**
   (a) $7! \times 5!$ (b) $6! \times 5!$ (c) $30!$ (d) $5! \times 4!$
8. How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order **[2004]**
   (a) 480 (b) 240 (c) 360 (d) 120
9. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[2004]**
   (a) ${}^8C_3$ (b) 21 (c) 3^8 (d) 5
10. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number **[2005]**
    (a) 601 (b) 600 (c) 603 (d) 602

11. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is **[2006]**
(a) 5040 (b) 6210 (c) 385 (d) 1110

12. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is **[2007]**
(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$ (c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$

13. In a shop there are five types of ice-creams available. A child buys six ice-creams.
Statement-1 : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.
Statement -2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. **[2008]**
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

14. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
(a) $8.{}^6C_4.{}^7C_4$ (b) $6.7.{}^8C_4$
(c) $6.8.{}^7C_4$. (d) $7.{}^6C_4.{}^8C_4$

15. From 6 different novels and 3 different dictionaries,4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is: **[2009]**
(a) at least 500 but less than 750
(b) at least 750 but less than 1000
(c) at least 1000
(d) less than 500

16. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[2010]**
(a) 36 (b) 66 (c) 108 (d) 3

17. **Statement-1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.
Statement-2: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$. **[2011]**
(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

18. These are 10 points in a plane, out of these 6 are collinear, if N is the number of triangles formed by joining these points. then: **[2012]**
(a) $n \leq 100$ (b) $100 < n \leq 140$
(c) $140 < n \leq 190$ (d) $n > 190$

19. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is : **[2012]**
(a) 880 (b) 629 (c) 630 (d) 879

20. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[JEE M 2013]**
(a) 7 (b) 5 (c) 10 (d) 8

21. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[JEE M 2015]**
(a) 120 (b) 72 (c) 216 (d) 192

22. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[JEE M 2016]**
(a) 52nd (b) 58th
(c) 46th (d) 59th

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4 Permutations and Combinations

Section-A : JEE Advanced/ IIT-JEE

A	**1.**	$a_1 + a_2 + ... + a_k$	**2.**	205	**3.**	35	**4.**	9				
B	**1.**	T										
C	**1.**	(c)	**2.**	(a)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(c)
	7.	(b)	**8.**	(a)	**9.**	(c)	**10.**	(c)	**11.**	(c)	**12.**	(c)
	13.	(b)	**14.**	(c)	**15.**	(a)						
D	**1.**	(b)										
E	**1.**	26	**2.**	300	**4.**	485	**5.**	64	**6.**	${}^{11}C_5 \times 9! \times 9!$		
	7.	6062, (a) 2702, (b) 1008			**9.**	7						
F	**1.**	(A) - p; (B) - s; (C) - q; (D) - q										
G	**1.**	(b)	**2.**	(a)								
I	**1.**	5	**2.**	7	**3.**	5	**4.**	5				

Section-B : JEE Main/ AIEEE

1.	(d)	**2.**	(c)	**3.**	(d)	**4.**	(b)	**5.**	(c)	**6.**	(c)
7.	(a)	**8.**	(c)	**9.**	(b)	**10.**	(a)	**11.**	(c)	**12.**	(a)
13.	(a)	**14.**	(d)	**15.**	(c)	**16.**	(c)	**17.**	(d)	**18.**	(a)
19.	(d)	**20.**	(b)	**21.**	(d)	**22.**	(b)				

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Number of students who gave wrong answers to exactly one question = $a_1 - a_2$, Two questions = $a_2 - a_3$
Three questions = $a_3 - a_4$, $k-1$ question = $a_{k-1} - a_k$, $k_{\text{question}} = a_k$

$\therefore$ Total number of wrong answers

$= 1\,(a_1 - a_2) + 2\,(a_2 - a_3) + 3\,(a_3 - a_4) + ...\,(k-1)\,(a_{k-1} - a_k) + k\,a_k$

$= \quad a_1 + a_2 + a_3 + ...a_k$

2. We have total $3 + 4 + 5 = 12$ points out of which 3 fall on one line, 4 on other line and 5 on still other line. So number of Δ's that can be formed using 12 such points are

$= \ {}^{12}C_3 - {}^3C_3 - {}^4C_3 - {}^5C_3$

$$= \frac{12 \times 11 \times 10}{6} - 1 - 4 - \frac{5 \times 4}{2 \times 1} = 220 - 15 = 205$$

3. '+' signs can be put in a row in 1 way, creating 7 ticked places to keep '–' sign so that no two '–' signs occur together

$\surd + \surd + \surd + \surd + \surd + \surd + \surd$

Out of these 7 places 4 can be chosen in ${}^7C_4$ ways.

$\therefore$ Required no. of arrangements are

$$= {}^7C_4 = {}^7C_3 = \frac{7.\,6.5}{3.2.1} = 35$$

4. **KEY CONCEPT :** We know that number of dearrangements of n objects

$$= n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - .....\frac{1}{n!}\right]$$

$\therefore$ No. of ways of putting all the 4 balls into boxes of different colour

$$= 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 4!\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)$$

$$= 24\left(\frac{12 - 4 + 1}{24}\right) = 9$$

B. True/False

1. Consider $\dfrac{(n+1)(n+2)...(n+r)}{r!}$

$$= \frac{1.2.3...(n-1)\,n\,(n+1)(n+2)...(n+r)}{1.2.3...n.r!}$$

$= \frac{(n+r)!}{n!r!} = {}^{n+r}C_r$ = some integral value

$\Rightarrow$ $(n+1)(n+2)...(n+r)$ is divisible by $r!$

Thus given statement is true.

C. MCQs with ONE Correct Answer

1. **(c)** ${}^nC_{r-1} = 36, {}^nC_r = 84, {}^nC_{r+1} = 126$

KEY CONCEPT : We know that

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{r}{n-r+1} \Rightarrow \frac{36}{84} = \frac{r}{n-r+1}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{3}{7} \Rightarrow 3n - 10r + 3 = 0 \quad ....(1)$$

$$\text{Also } \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{r+1}{n-r} = \frac{84}{126} = \frac{2}{3}$$

$$\Rightarrow 2n - 5r - 3 = 0 \quad ....(2)$$

Solving (1) and (2), we get $n = 9$ and $r = 3$.

2. **(a)** Total number of words that can be formed using 5 letters out of 10 given different letters

$= 10 \times 10 \times 10 \times 10 \times 10$ (as letters can repeat)

$= 1,00,000$

Number of words that can be formed using 5 different letters out of 10 different letters

$= {}^{10}P_5$ (none can repeat)

$= \frac{10!}{5!} = 30,240$

$\therefore$ Number of words in which at least one letter is repeated

= total words–words with none of the letters repeated

$= 1{,}00{,}000 - 30{,}240 = 69760$

3. **(c)** $${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3 = {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3$$

$$+ {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4)$$

[Using ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$]

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ({}^{48}C_3 + {}^{48}C_4)$$

$$= {}^{51}C_3 + {}^{50}C_3 + ({}^{49}C_3 + {}^{49}C_4)$$

$$= {}^{51}C_3 + ({}^{50}C_3 + {}^{50}C_4) = {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$$

4. **(d)** 1 2 3 4 5 6 7 8

Two women can choose two chairs out of 1, 2, 3, 4, in ${}^4C_2$ ways and can arrange themselves in 2! ways. Three men can choose 3 chairs out of 6 remaining chairs in ${}^6C_3$ ways and can arrange themselves in 3! ways

$\therefore$ Total number of possible arrangements are

${}^4C_2 \times 2! \times {}^6C_3 \times 3! = {}^4P_2 \times {}^6P_3$

5. **(a)** **KEY CONCEPT :** We know that a number is divisible by 3 if the sum of its digits is divisibly by 3.

Now out of 0, 1, 2, 3, 4, 5 if we take 1, 2, 3, 4, 5 or 0, 1, 2, 4, 5 then the 5 digit numbers will be divisible by 3.

Case I : Number of 5 digit numbers formed using the digits $1, 2, 3, 4, 5 = 5! = 120$

Case II : Taking 0, 1, 2, 4, 5 if we make 5 digit number then

I place can be filled in = 4 ways (0 can not come at I place)

II place can be filled in = 4 ways

III place can be filled in = 3 ways

IV place can be filled in = 2 ways

V place can be filled in = 1 ways

$\therefore$ Total numbers are $= 4 \times 4! = 96$

Thus total numbers divisible by 3 are $= 120 + 96 = 216$

6. **(c)** $X - X - X - X - X$. The four digits 3, 3, 5, 5 can be arranged at (–) places in $\frac{4!}{2!2!} = 6$ ways.

The five digits 2, 2, 8, 8, 8 can be arranged at

(X) places in $\frac{5!}{2!3!} = 10$ ways.

Total no. of arrangements $= 6 \times 10 = 60$ ways.

7. **(b)** $\therefore T_n = {}^nC_3 ; T_{n+1} = {}^{n+1}C_3$

As per question,

$$T_{n+1} - T_n = 21 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$$

$$\frac{(n+1)n(n-1)}{3.2.1} - \frac{n(n-1)(n-2)}{3.2.1} = 21$$

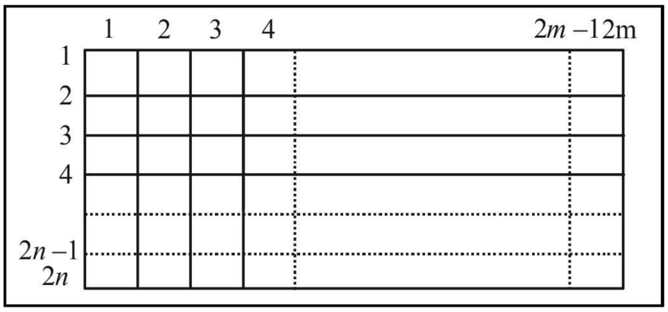
$$\Rightarrow n(n-1)(n+1-n+2) = 126$$

$$\Rightarrow n(n-1) = 42 \Rightarrow n(n-1) = 7 \times 6 \Rightarrow n = 7.$$

8. **(a)** Total number of ways of arranging the letters of the word BANANA is $\frac{6!}{2!3!} = 60$. Number of words in which 2 N's come together is $\frac{5!}{3!} = 20$.

Hence the required number $= 60 - 20 = 40$

9. **(c)**



If we see the blocks in terms of lines then there are $2m$ vertical lines and $2n$ horizontal lines. To form the required rectangle we must select two horizontal lines, one even numbered (out of 2, 4, ...2n) and one odd numbered (out of 1, 3....2n–1) and similarly two vertical lines. The number of rectangles is

$^mC_1 \cdot {}^mC_1 \cdot {}^nC_1 \cdot {}^nC_1 = m^2n^2$

10. (c) $\because$ r, s, t are prime numbers,

$\therefore$ Section of (p, q) can be done as follows

p	q
r^0	r^2
r^1	r^2
r^2	r^0, r^1, r^2

$\therefore$ r can be selected 1 + 1 + 3 = 5 ways

Similarly s and t can be selected in 9 and 5 ways respectivley.

$\therefore$ Total ways = 5 × 9 × 5 = 225

11. (c) The letter of word *COCHIN* in alphabetic order are *C, C, H, I, N, O*.

Fixing first letter *C* and keeping *C* at second place, rest 4 can be arranged in 4! ways.

Similarly the words starting with *CH, CI, CN* are 4! in each case.

Then fixing first two letters as *CO* next four places when filled in alphabetic order give the word *COCHIN*.

$\therefore$ Numbers of words coming before *COCHIN* are

$4 \times 4! = 4 \times 24 = 96$

12. (c) We have to form 7 digit numbers, using the digits 1, 2 and 3 only, such that the sum of the digits in a number = 10.

This can be done by taking 2, 2, 2, 1, 1, 1, 1, or by taking 2, 3, 1, 1, 1, 1, 1.

$\therefore$ Number of ways $= \frac{7!}{3!4!} + \frac{7!}{5!} = 77.$

13. (b) $\because$ Each person gets at least one ball.

$\therefore$ 3 Persons can have 5 balls in the following systems

Person	I	II	III
No. of balls	1	1	3

or

Person	I	II	III
No. of balls	1	2	2

The number of ways to distribute the balls in first system

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3$

Also 3, persons having 1, 1 and 3 balls can be arranged in $\frac{3!}{2!}$ ways.

$\therefore$ No. of ways to distribute 1, 1, 3 balls to the three persons

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times \frac{3!}{2!} = 60$

Similarly the total no. of ways to distribute 1, 2, 2 balls to the three persons $= {}^5C_1 \times {}^4C_2 \times {}^2C_2 \times \frac{3!}{2!} = 90$

$\therefore$ The required number of ways = 60 + 90 = 150

14. (c) $\because$ Card numbered 1 is always placed in envelope numbered 2, we can consider two cases.

Case I: Card numbered 2 is placed in envelope numbered 1. Then it is dearrangement of 4 objects, which can be done in $4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) = 9$ ways

Case II: Card numbered 2 is not placed in envelope numbered 1.
Then it is dearrangement of 5 objects, which can be done in $5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) = 44$ ways

$\therefore$ Total ways = 44 + 9 = 53

15. (a) Either one boy will be selected or no boy will be selected. Also out of four members one captain is to be selected.

$\therefore$ Required number of ways $= ({}^4C_1 \times {}^6C_3 + {}^6C_4) \times {}^4C_1$
$= (80 + 15) \times 4 = 380$

D. MCQs with ONE or MORE THAN ONE Correct

1. (b) Distinct n digit numbers which can be formed using digits 2, 5 and 8 are 3^n.

We have to find n so that $3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$

$\Rightarrow n - 2 \geq 5 \Rightarrow n \geq 7$. So the least value of n is 7.

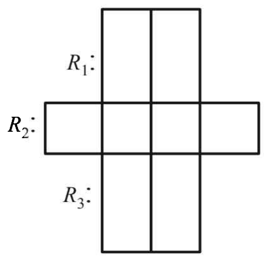
E. Subjective Problems

1. As all the X's are identical, the question is of selection of 6 squares from 8 squares, so that no row remains empty. Here R_1 has 2 squares, R_2 has 4 squares and R_3 has 2 squares. The selection scheme is as follows :

	R_1	R_2	R_3
	1	4	1
or	1	3	2
or	2	3	1
or	2	2	2

$\therefore$ Number of selections are

$^2C_1 \times {}^4C_4 \times {}^2C_1 + {}^2C_1 \times {}^4C_3 \times {}^2C_2$
$+ {}^2C_2 \times {}^4C_3 \times {}^2C_1 + {}^2C_2 \times {}^4C_2 \times {}^2C_2$
$= 4 + 8 + 8 + 6 = 26$



2. The various possibilities to put 5 different balls in 3 different size boxes, when no box remains empty : The balls can be 1, 1 and 3 in different boxes or 2, 2, 1.

Case I : To put 1, 1 and 3 balls in different boxes. Selection of 1, 1 and 3 balls out of 5 balls can be done in ${}^5C_1 \times {}^4C_1 \times {}^3C_3$ ways and then 1, 1, 3 can permute (as defferent size boxes) in 3 ! ways.

$\therefore$ No. of ways $= {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times 3! = 5 \times 4 \times 1 \times 6 = 120$

Case II : To put 2, 2 and 1 ball in different boxes. Selection of 2, 2 and 1 balls out of 5 balls can be done in ${}^5C_2 \times {}^3C_2 \times {}^1C_1$ ways

and then 2, 2, 1 can permute (different boxes) in 3 ! ways

$\therefore$ No. of ways $= {}^5C_1 \times {}^3C_2 \times {}^1C_1 \times 3! = 10 \times 3 \times 1 \times 6 = 180$

Combining case I and II, total number of required ways are $= 120 + 180 = 300$.

3. m men can be seated in m! ways creating $(m+1)$ places for ladies to sit.

n ladies out of $(m+1)$ places (as $n < m$) can be seated in ${}^{m+1}P_n$ ways

$\therefore$ Total ways $= m! \times {}^{m+1}P_n$

$$= m! \times \frac{(m+1)!}{(m+1-n)!} = \frac{(m+1)!\,m!}{(m-n+1)!}$$

4. There are four possibilities:

(i) 3 ladies from husband's side and 3 gentlemen from wife's side.

No. of ways in this case $= {}^4C_3 \times {}^4C_3 = 4 \times 4 = 16$

(ii) 3 gentlemen from husband's side and 3 ladies from wife's side.

No. of ways in this case $= {}^3C_3 \times {}^3C_3 = 1 \times 1 = 1$

(iii) 2 ladies and one gentlemen from husband's side and one lady and 2 gentlemen from wife's side.

No. of ways in this case

$= ({}^4C_2 \times {}^3C_1) \times ({}^3C_1 \times {}^4C_2) = 6 \times 3 \times 3 \times 6 = 324$

(iv) One lady and 2 gentlemen from husband's side and 2 ladies and one gentlemen from wife's side.

No. of ways in this case

$= ({}^4C_1 \times {}^3C_2) \times ({}^3C_2 \times {}^4C_1) = 4 \times 3 \times 3 \times 4 = 144$

Hence the total no. of ways are

$= 16 + 1 + 324 + 144 = 485$

5. Number of ways of drawing at least one black ball

= 1 black and 2 other or 2 black and 1 other or 3 black

$= {}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3 = 3 \times 15 + 3 \times 6 + 1$

$= 45 + 18 + 1 = 64$

6. Out of 18 guests half i.e. 9 to be seated on side A and rest 9 on side B. Now out of 18 guests, 4 particular guests desire to sit on one particular side say side A and other 3 on other side B. Out of rest $18 - 4 - 3 = 11$ guests we can select 5 more for side A and rest 6 can be seated on side B. Selection of 5 out of 11 can be done in ${}^{11}C_5$ ways and 9 guests on each sides of table can be seated in $9! \times 9!$ ways. Thus there are total ${}^{11}C_5 \times 9! \times 9!$ arrangements.

7. Given that there are 9 women and 8 men. A committee of 12 is to be formed including at least 5 women.

This can be doen in the following ways.

$=$ $5W$ and $7M$ or $6W$ and $6M$

or $7W$ and $5M$

or $8W$ and $4M$

or $9W$ and $3M$

No. of ways of forming committee is

$= {}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$

$$= \frac{9.8.7.6}{4.3.2.1} \times 8 + \frac{9.8.7}{3.2.1} \times \frac{8.7}{2.1} + \frac{9.8}{2.1} \times \frac{8.7.6}{3.2.1}$$

$$+ 9 \times \frac{8.7.6.5}{4.3.2.1} + 1 \times \frac{8.7.6}{3.2.1}$$

$= 126 \times 8 + 84 \times 28 + 36 \times 56 + 9 \times 70 + 56 = 6062$ ways.

(a) The women are in majority in

$2016 + 630 + 56 = 2702$ ways.

(b) The men are in majority in 1008 ways.

8. Let there be n sets of different objects each set containing n identical objects [eg (1, 1, 1 ... 1 (n times)), (2, 2, 2 ..., 2 (n times) ... (n, n, n ... n (n times))]

Then the no. of ways in which these $n \times n = n^2$ objects can be arranged in a row $= \dfrac{(n^2)!}{n!n!...n!} = \dfrac{(n^2)!}{(n!)^n}$

But these number of ways should be a natural number.

Hence $\dfrac{(n^2)!}{(n!)^n}$ is an integer. $(n \in I^+)$

9. Given that

runs scored in kth match $= k.2^{n+1-k}$, $1 \le k \le n$

and runs scored in n matches $= \dfrac{n+1}{4}(2^{n+1} - n - 2)$

$$\therefore \sum_{k=1}^{n} k.2^{n+1-k} = \frac{n+1}{4}(2^{n+1} - n - 2)$$

$$\Rightarrow 2^{n+1}\left[\sum_{k=1}^{n} \frac{k}{2^k}\right] = \frac{n+1}{4}(2^{n+1} - n - 2)$$

$$\Rightarrow 2^{n+1}\left[\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + ... + \frac{n}{2^n}\right]$$

$$= \frac{n+1}{4}(2^{n+1} - n - 2) \quad ....(i)$$

Let $S = \dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + ... + \dfrac{n}{2^n}$

$$\frac{1}{2}S = \frac{1}{2^2} + \frac{2}{2^3} + ... + \frac{n-1}{2^n} + \frac{n}{2^{n+1}}$$

Subtracting the above two, we get

$$\frac{1}{2}S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^n} - \frac{n}{2^{n+1}}$$

$$\Rightarrow \frac{1}{2}S = \frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} - \frac{n}{2^{n+1}} \Rightarrow S = 2\left[1-\frac{1}{2^n}-\frac{n}{2^{n+1}}\right]$$

$\therefore$ Equation (i) becomes

$$2.2^{n+1}\left[1-\frac{1}{2^n}-\frac{n}{2^{n+1}}\right] = \frac{n+1}{4}\left[2^{n+1}-n-2\right]$$

$$\Rightarrow 2.\left[2^{n+1}-2-n\right] = \frac{n+1}{4}\left[2^{n+1}-2-n\right]$$

$$\Rightarrow \frac{n+1}{4} = 2 \Rightarrow n = 7$$

F. Match the Following

1. **(A)-p; (B)-s; (C)-q; (D)-q**

(A) For the permutations containing the word ENDEA we consider 'ENDEA' as single letter. Then we have total ENDEA, N, O, E, L i.e. 5 letters which can be arranged in 5! ways.

$\therefore$ (A) $\to$ (p)

(B) If E occupies the first and last position, the middle 7 positions can be filled by N, D, E, A, N, O, L.

These can be arranged in $\frac{7!}{2!} = 21 \times 5!$ ways.

$\therefore$ (B) $\to$ (s)

(C) If none of the letters D, L, N occur in the last five positions then we should arrange D, D, L, N at first four positions and rest five i.e. E, E, E, A,O at last five positions. This can be done in

$\frac{4!}{2!} \times \frac{5!}{3!}$ ways. (C) $\to$ (q)

(D) As per question A, E, E, E, O can be arranged at 1st, 3rd, 5th, 7th and 9th positions and rest N, D, N, L at rest 4 positions. This can be done in

$\frac{5!}{3!} \times \frac{4!}{2!}$ ways $= 2 \times 5!$ways (D) $\to$ (q)

G. Comprehension Based Questions

1. **(b)** $\because$ a_n = number of all n digit +ve integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0.

and b_n = number of such n digit integers ending with 1

c_n = number of such n digit integers ending with 0

Clearly, $a_n = b_n + c_n$ ($\because$ a_n can end with 0 or 1)

Also $b_n = a_{n-1}$

and $c_n = a_{n-2}$ [$\because$ if last digit is 0, second last has to be 1]

$\therefore$ We get $a_n = a_{n-1} + a_{n-2}, n \geq 3$

Also $a_1 = 1, a_2 = 2,$

By this recurring formula

$a_3 = a_2 + a_1 = 3$

$a_4 = a_3 + a_2 = 3 + 2 = 5$

$a_5 = a_4 + a_3 = 5 + 3 = 8$

Also $b_6 = a_5 = 8$

2. **(a)** By recurring formula, $a_{17} = a_{16} + a_{15}$ is correct

Also $c_{17} \neq c_{16} + c_{15}$

$\Rightarrow a_{15} \neq a_{14} + a_{13}$ ($\because$ $c_n = a_{n-2}$)

$\therefore$ Incorrect

Similarly, other parts are also incorrect.

I. Integer Value Correct Type

1. **(5)** Given 8 vectors are

(1, 1, 1), (–1, –1, –1); (–1, 1, 1), (1, –1, –1); (1, –1, 1), (–1, 1, –1); (1, 1, –1), (–1, –1, 1)

These are 4 diagonals of a cube and their opposites.

For 3 non coplanar vectors first we select 3 groups of diagonals and its opposite in ${}^4C_3$ ways. Then one vector from each group can be selected in $2 \times 2 \times 2$ ways.

$\therefore$ Total ways $= {}^4C_3 \times 2 \times 2 \times 2 = 32 = 2^5$ $\therefore p = 5$

2. **(7)** $\because n_1, n_2, n_3, n_4$ and n_5 are positive integers such that $n_1 < n_2 < n_3 < n_4 < n_5$

Then for $n_1 + n_2 + n_3 + n_4 + n_5 = 20$

If n_1, n_2, n_3, n_4 take minimum values 1, 2, 3, 4 respectively then n_5 will be maximum 10.

$\therefore$ Corresponding to $n_5 = 10$, there is only one solution

$n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 4.$

Corresponding to $n_5 = 9$, we can have, only solution

$n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 5$ i.e., one solution

Corresponding to $n_5 = 8$, we can have, only solution

$n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 6$

or $n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 4$

i.e., 2 solution

For $n_5 = 7$, we can have

$n_1 = 1, n_2 = 1, n_3 = 4, n_4 = 6$

or $n_1 = 1, n_2 = 3, n_3 = 4, n_4 = 5$ *i.e.* 2 solutions

For $n_5 = 6$, we can have

$n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5$

i.e., one solution

Thus there can be 7 solutions.

3. **(5)** Number of adjacent lines $= n$

Number of non adjacent lines $= {}^nC_2 - n$

$\therefore {}^nC_2 - n = n \Rightarrow \frac{n(n-1)}{2} - 2n = 0$

$\Rightarrow n^2 - 5n = 0 \Rightarrow n = 0$ or 5

But $n \geq 2 \Rightarrow n = 5$

4. **(5)** $n = 5! \times 6!$

For second arrangement,

5 boys can be made to stand in a row in 5! ways, creating 6 alternate space for girls. A group of 4 girls can be selected in ${}^5C_4$ ways. A group of 4 and single girl can be arranged at 2 places out of 6 in ${}^6P_2$ ways. Also 4 girls can arrange themselves in 4! ways.

$\therefore$ $m = 5! \times {}^6P_2 \times {}^5C_4 \times 4!$

$$\frac{m}{n} = \frac{5! \times 6 \times 5 \times 5 \times 4!}{5! \times 6!} = 5$$

Section-B JEE Main/ AIEEE

1. **(d)** Required number of numbers
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

2. **(c)** Required number of numbers $= 3 \times 5 \times 5 \times 5 = 375$

3. **(d)** We know that a number is divisible by 3 only when the sum of the digits is divisible by 3. The given digits are 0, 1, 2, 3, 4, 5.
Here the possible number of combinations of 5 digits out of 6 are ${}^5C_4 = 5$, which are as follows–
$1+2+3+4+5 = 15 = 3 \times 5$
$0+2+3+4+5 = 14$ (not divisible by 3)
$0+1+3+4+5 = 13$ (not divisible by 3)
$0+1+2+4+5 = 12 = 3 \times 4$
$0+1+2+3+5 = 11$ (not divisible by 3)
$0+1+2+3+4 = 10$ (not divisible by 3)
Thus the number should contain the digits 1, 2, 3, 4, 5 or the digits 0, 1, 2, 4, 5.
Taking 1, 2, 3, 4, 5, the 5 digit numbers are $= 5! = 120$
Taking 0, 1, 2, 4, 5, the 5 digit numbers are $= 5! - 4! = 96$
$\therefore$ Total number of numbers $= 120 + 96 = 216$

4. **(b)** Required sum
$= (2+4+6+...+100) + (5+10+15+...+100)$
$-(10+20+...+100)$
$= 2550 + 1050 - 530 = 3050.$

5. **(c)** ${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r = {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$
$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

6. **(c)** As for given question two cases are possible.
(i) Selecting 4 out of first five question and 6 out of remaining 8 question $= {}^5C_4 \times {}^8C_6 = 140$ choices.
(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions $= {}^5C_5 \times {}^8C_5 = 56$ choices.
$\therefore$ total number of choices $= 140 + 56 = 196.$

7. **(a)** No. of ways in which 6 men can be arranged at a round table $= (6 - 1)! = 5!$
Now women can be arranged in ${}^6P_5 = 6!$ Ways.
Total Number of ways $= 6! \times 5!$

8. **(c)** Total number of arrangements of letters in the word GARDEN $= 6! = 720$ there are two vowels A and E, in half of the arrangements A preceeds E and other half A follows E.
So, vowels in alphabetical order in $\frac{1}{2} \times 720 = 360$

9. **(b)** We know that the number of ways of distributing n identical items among r persons, when each one of them receives at least one item is ${}^{n-1}C_{r-1}$
$\therefore$ The required number of ways
$= {}^{8-1}C_{3-1} = {}^7C_2 = \frac{7!}{2!5!} = \frac{7 \times 6}{2 \times 1} = 21$

10. **(a)** Alphabetical order is
A, C, H, I, N, S
No. of words starting with A – 5!
No. of words starting with C – 5!
No. of words starting with H – 5!
No. of words starting with I – 5!
No. of words starting with N – 5!
SACHIN – 1
$\therefore$ sachin appears at serial no 601

11. **(c)** ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$
$= 10 + 45 + 120 + 210 = 385$

12. **(a)** Set S = {1, 2, 3, ... 12}
$A \cup B \cup C = S,\ A \cap B = B \cap C = A \cap C = \phi$
$\therefore$ The number of ways to partition
$= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4 = \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!} = \frac{12!}{(4!)^3}$

13. **(a)** The given situation in statement 1 is equivalent to find the non negative integral solutions of the equation
$x_1 + x_2 + x_3 + x_4 + x_5 = 6$
which is coeff. of x^6 in the expansion of
$(1 + x + x^2 + x^3 + ...\infty)^5 =$ coeff. of x^6 in $(1-x)^{-5}$
$=$ coeff. of x^6 in $1 + 5x + \frac{5.6}{2!}x^2$..........
$= \frac{5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{6!} = \frac{10!}{6!4!} = {}^{10}C_6$
$\therefore$ Statement 1 is wrong.
Number of ways of arranging 6A's and 4B's in a row
$= \frac{10!}{6!4!} = {}^{10}C_6$ which is same as the number of ways the child can buy six icecreams.
$\therefore$ Statement 2 is true.

14. **(d)** First let us arrange M, I, I, I, I, P, P
Which can be done in $\frac{7!}{4!2!}$ ways
√ M √ I √ I √ I √ I √ P √ P √

Now 4 S can be kept at any of the ticked places in $^8C_4$ ways so that no two S are adjacent.
Total required ways

$$=\frac{7!}{4!2!}\,{}^8C_4=\frac{7!}{4!2!}\,{}^8C_4=7\times{}^6C_4\times{}^8C_4$$

15. (c) 4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in ${}^6C_4\times{}^3C_1$ ways

Then 4 novels with one dictionary in the middle can be arranged in 4! ways.

$\therefore$ Total ways of arrangement $={}^6C_4\times{}^3C_1\times4!=1080$

16. (c) Total number of ways $={}^3C_2\times{}^9C_2$

$$=3\times\frac{9\times8}{2}=3\times36=108$$

17. (d) The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box empty is same as the number of ways of selecting $(r-1)$ places out of $(n-1)$ different places, that is ${}^{n-1}C_{r-1}$.

Hence required number of ways $={}^{10-1}C_{4-1}={}^9C_3$

$\therefore$ Both statements are correct and second statement is a correct explanation of statement -1.

18. (a) Number of required triangles

$$={}^{10}C_3-{}^6C_3$$

$$=\frac{10\times9\times8}{6}-\frac{6\times5\times4}{6}=120-20=100$$

19. (d) Number of white balls = 10

Number of green balls = 9

and Number of black balls = 7

$\therefore$ Required probability $=(10+1)(9+1)(7+1)-1=11.10.8-1=879$

[$\because$ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind is $(p+1)(q+1)(r+1)-1$]

20. (b) We know,

$T_n={}^nC_3,\ T_{n+1}={}^{n+1}C_3$

ATQ, $T_{n+1}-T_n={}^{n+1}C_3-{}^nC_3=10$

$\Rightarrow\ {}^nC_2=10$

$\Rightarrow\ n=5.$

21. (d) Four digits number can be arranged in $3\times4!$ ways.

Five digits number can be arranged in 5! ways.

Number of integers $=3\times4!+5!=192.$

22. (b) ALLMS

No. of words starting with

A : A _ _ _ _ $\frac{4!}{2!}=12$

L : L _ _ _ _ $4!=24$

M : M _ _ _ _ $\frac{4!}{2!}=12$

S : S A _ _ _ $\frac{3!}{2!}=3$

: S L _ _ _ $3!=6$

SMALL $\rightarrow$ 58th word