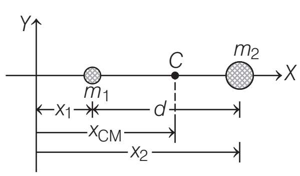
System of Particles and Rotational Motion

Quick Revision

1. **Rigid Body** A body is said to be a rigid body, when it has a perfectly definite shape and size. e.g. A wheel can be considered as rigid body by ignoring a little change in its shape.
2. **Rotational Motion (Fixed Axis of Rotation)** In pure rotational motion, every particle of the rigid body moves in circles of different radii about a fixed line, which is known as **axis of rotation**.

   e.g. A potter's wheel, a merry-go-round, etc.
3. **Centre of Mass** A point at which the entire mass of the body or system of bodies is supposed to be concentrated is known as the centre of mass.
   - **For a System of two Particles** The centre of mass of the system at a point which is at distance x_{CM} from origin is given by



$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

   - **For a System of *n*-Particles** Suppose a system having masses $m_1, m_2, m_3, \ldots, m_n$ occupying x-coordinates $x_1, x_2, x_3, \ldots, x_n$, then

x_{CM} is x-coordinates of centre of mass of system is expressed as,

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

Centre of mass, $x_{CM} = \dfrac{\sum_{i=1}^{n} m_i x_i}{\Sigma m_i}$

   - If particles are distributed in three-dimensional space, then the centre of mass has 3-coordinates, which are

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i\, x_i,\; y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

$$z_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

where, $M = m_1 + m_2 + m_3 + \ldots = \sum_{i=1}^{n} m_i$ is the total mass of the system. The index i runs from 1 to n, m_i is the mass of the ith particle and the position of the ith particle is given by (x_i, y_i, z_i).

   - **Relation between position vectors of particles and centre of mass,**

$$\mathbf{R} = \frac{\sum_{i=1}^{n} m_i \mathbf{r}_i}{m}$$

where, $\mathbf{r}_i = (x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}})$ is the position vector of the ith particle and $\mathbf{R} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}})$ is the position vector of the centre of mass.

4. **Centre of Mass of Rigid Continuous Bodies** For a real body which is a continuous distribution of matter, point masses are differential mass elements dm and centre of mass is given as

$$x_{CM} = \frac{1}{M}\int x\,dm,\quad y_{CM} = \frac{1}{M}\int y\,dm$$

and $$z_{CM} = \frac{1}{M}\int z\,dm$$

where, M is total mass of that real body.

If we choose the origin of coordinates axes at centre of mass, then

$$\int x\,dm = \int y\,dm = \int z\,dm = 0$$

5. **Motion of Centre of Mass**
   - **Velocity** about centre of mass,

$$\mathbf{v}_{CM} = \frac{\sum_{i=1}^{n} m_i \mathbf{v}_i}{M}$$

where, $\mathbf{v} = \frac{d\mathbf{r}}{dt}$, i.e. rate of change of position vector is velocity.

   - **Acceleration** about centre of mass,

$$\mathbf{a}_{CM} = \frac{\sum_{i=1}^{n} m_i \mathbf{a}_i}{M},$$

But $m_i \mathbf{a}_i$ is the resultant force on the ith particle, so

$$M\mathbf{a}_{CM} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots + \mathbf{F}_n$$
$$M\mathbf{a}_{CM} = \mathbf{F}_{net}$$

6. **Linear Momentum of a System of a Particle** The total momentum of a system of particles is equal to the product of the total mass and velocity of its centre of mass.

$\therefore$ Total linear momentum, $\mathbf{p} = M\mathbf{v}_{CM}$

7. **Moment of Force (Torque)** Torque is also known as moment of force. We can define the torque for a particle about a point as the vector product of position vector of the point, where the force acts and with the force itself. Let us consider a particle P and force $\mathbf{F}$ acting on it.

Torque, $\tau = \mathbf{r} \times \mathbf{F}$

The magnitude of torque $|\tau|$ is

$$= Fr\sin\theta$$

$$\therefore \quad \tau = Fr_{\perp}$$

Here, $r_{\perp}$ is the perpendicular distance of the line of action of $\mathbf{F}$ from the origin.

8. **Angular Momentum of a Particle** The angular momentum of a particle of mass m moving with velocity $\mathbf{v}$ (having a linear momentum, $\mathbf{p} = m\mathbf{v}$) about a point O is defined by the following vector product,

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$

or Angular momentum, $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

Angular momentum will be zero $(L = 0)$, if

$p = 0$ or $r = 0$ or $\theta = 0°, 180°$

It is a vector quantity and its direction could be found out with the help of cross-product.

The SI unit of angular momentum is kg-m^{2}s^{-1}.

9. **Relation between Torque (τ) and Angular Momentum (L)**

$$\frac{d\mathbf{L}}{dt} = \tau$$

Above equation gives Newton's second law of motion in angular form, i.e. the rate of change of angular momentum is equal to the torque applied.

10. **Couple** A pair of equal and opposite forces with parallel lines of action are known as a couple. It produces rotation without translation.

11. **Principle of Moments** When an object is in rotational equilibrium, then algebraic sum of all torques acting on it is zero. Clockwise torques are taken as negative and anti-clockwise torques are taken as positive.

12. **Centre of Gravity** If a body is supported on a point such that total gravitational torque about this point is zero, then this point is called centre of gravity of the body.

13. **Moment of Inertia** For a rotating body, its moment of inertia is $I = \sum_{i=1}^{n} m_i r_i^2$

or Moment of inertia, $I = mR^2$

The SI unit of moment of inertia is kg-m^2 and its dimensional formula is $[ML^2]$.

14. **Relation between Angular Momentum and Moment of Inertia**

For a rigid body (about an fixed axis),

$$L = \text{sum of angular momenta of all particles}$$

$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots\ldots)\,\omega$$

$$L = I\omega$$

where, I = moment of inertia and ω = angular velocity of rigid body.

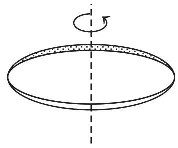
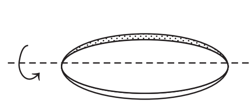
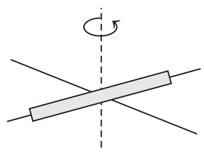
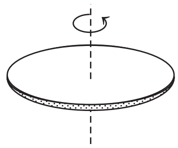
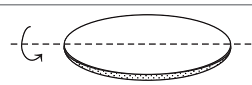
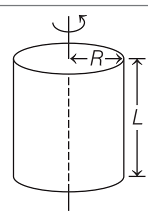
15. **Radius of Gyration** The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of whole body and moment of inertia is equal to the moment of inertia of the body about the axis. It is given as, $K = \sqrt{\dfrac{I}{M}}$

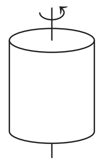
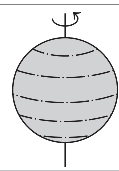
where, K is radius of gyration of the body.

For rotating body, $K = \sqrt{\dfrac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}}$

Hence, radius of gyration of a rotating body about a given axis is equal to root mean square distance of constituent particles from the given axis.

16. **Moment of Inertia in Some Standard Cases**

Body	**Axis of Rotation**	**Figure**	**Moment of Inertia**	***K***	***K*²/*R*²**
Thin circular ring, radius R	About an axis passing through CG and perpendicular to its plane		MR^2	R	1
Thin circular ring, radius R	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Thin rod, length L	Perpendicular to rod at mid-point		$\frac{1}{12}ML^2$	$\frac{L}{\sqrt{12}}$	
Circular disc, radius R	Perpendicular to plane of disc at centre		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Circular disc, radius R	About its diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Hollow cylinder, radius R	About its own axis		MR^2	R	1

Body	Axis of Rotation	Figure	Moment of Inertia	K	K^2/R^2
Solid cylinder, radius R	About its own axis		$\frac{MR^2}{2}$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Solid sphere, radius R	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$

17. From the given table below, we can compare translational motion and rotational motion about a fixed axis, i.e. Z-axis.

Pure Translational	Pure Rotational
Linear position, x	Angular position, θ
Linear velocity, $v = \frac{dx}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$
Linear acceleration, $a = \frac{dv}{dt}$	Angular acceleration, $\alpha = \frac{d\omega}{dt}$
Mass, m	Rotational inertia, I
Newton's second law, $F = ma$	Newton's second law, $\tau = I\alpha$
Work done, $W = \int F\,dx$	Work done, $W = \int \tau\,d\theta$
Kinetic energy, $K = \frac{1}{2}mv^2$	Kinetic energy, $K = \frac{1}{2}I\omega^2$
Power, $P = Fv$	Power, $P = \tau\omega$
Linear momentum, $p = mv$	Angular momentum, $L = I\omega$

18. **Rolling Motion** The rolling motion can be regarded as the combination of pure rotation and pure translation. It is also one of the most common motions observed in daily life.

19. **Kinetic Energy of a Rolling Body** The kinetic energy of a body rolling without slipping is the sum of kinetic energies of translational and rotational motions.

$$\therefore \quad (\text{KE})_{\text{rolling}} = (\text{KE})_{\text{rotation}} + (\text{KE})_{\text{translation}}$$
$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{\text{CM}}^2$$
$$= \frac{1}{2}mv_{\text{CM}}^2\left[1 + \frac{K^2}{R^2}\right]$$
$$(\because v_{\text{CM}} = R\omega \text{ and } I = mK^2)$$

Objective Questions

Multiple Choice Questions

1. A system of particles is called a rigid body, when

(a) any two particles of system may have displacements in opposite directions under action of a force
(b) any two particles of system may have velocities on opposite directions under action of a force
(c) any two particles of system may have a zero relative velocity
(d) any two particles of system may have displacements in same direction under action of a force

2. The centre of mass of a system of particles does not depend on

(a) masses of the particles
(b) internal forces of the particles
(c) position of the particles
(d) relative distance between two particles

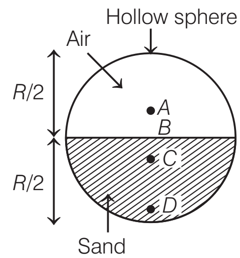
3. In pure rotation, all particles of the body

(a) move in a straight line
(b) move in concentric circles
(c) move in non-concentric circles
(d) have same speed

4. For n particles in a space, the suitable expression for the x-coordinate of the centre of mass of a system is

(a) $\frac{\Sigma m_i x_i}{m_i}$ (b) $\frac{\Sigma m_i x_i}{M}$
(c) $\frac{\Sigma m_i y_i}{M}$ (d) $\frac{\Sigma m_i z_i}{M}$

5. Which of the following points is the likely position of the centre of mass of the system shown in figure?
(NCERT Exemplar)



(a) A (b) B (c) C (d) D

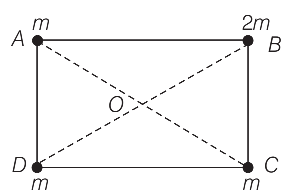
6. Two bodies of masses 1 kg and 2 kg are lying on x-y plane at (1, 2) and (– 1, 3) respectively. What are the coordinates of centre of mass?

(a) (2, –1) (b) $\left(\frac{8}{3}, -\frac{1}{3}\right)$
(c) $\left(-\frac{1}{3}, \frac{8}{3}\right)$ (d) None of these

7. Three identical spheres of mass M each are placed at the corners of an equilateral triangle of side 2 m. Taking one of the corner as the origin, the position vector of the centre of mass is

(a) $\sqrt{3}(\hat{i} - \hat{j})$ (b) $\frac{\hat{i}}{\sqrt{3}} + \hat{j}$
(c) $\frac{\hat{i} + \hat{j}}{3}$ (d) $\hat{i} + \frac{\hat{j}}{\sqrt{3}}$

8. Centre of mass of the given system of particles will be at



(a) OD (b) OC (c) OB (d) AO

9. Two particles of equal masses have velocities $\mathbf{v}_1 = 4\,\hat{\mathbf{i}}\ \text{ms}^{-1}$ and $\mathbf{v}_2 = 4\,\hat{\mathbf{j}}\ \text{ms}^{-1}$. First particle has an acceleration $\mathbf{a}_1 = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{ms}^{-2}$, while the acceleration of the other particle is zero. The centre of mass of the two particles moves in a path of

(a) straight line
(b) parabola
(c) circle
(d) ellipse

10. The centre of mass of three particles of masses 1 kg, 2 kg and 3 kg is at (3, 3, 3) with reference to a fixed coordinate system. Where should a fourth particle of mass 4 kg be placed, so that the centre of mass of the system of all particles shifts to a point (1, 1, 1)?

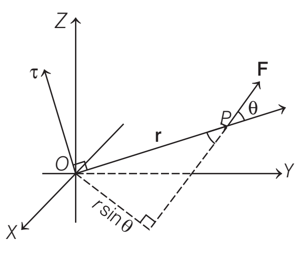
(a) (−1, −1, −1) (b) (−2, −2, −2)
(c) (2, 2, 2) (d) (1, 1, 1)

11. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The velocity of the centre of mass

(a) of the box remains constant
(b) of the box and the ball system remains constant
(c) of the ball remains constant
(d) of the ball relative to the box remains constant

12. A force **F** is applied on a single particle *P* as shown in the figure. Here, **r** is the position vector of the particle.

The value of torque τ is

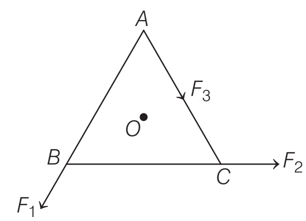


(a) $\mathbf{F} \times \mathbf{r}$ (b) $\mathbf{r} \times \mathbf{F}$ (c) $\mathbf{r} \cdot \mathbf{F}$ (d) $\mathbf{F} \cdot \mathbf{r}$

13. A force $\mathbf{F} = 5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ acts on a particle whose position vector is $\mathbf{r} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$. What is the torque about the origin ?

(a) $8\hat{i} + 10\hat{j} + 12\hat{k}$ (b) $8\hat{i} + 10\hat{j} - 12\hat{k}$
(c) $8\hat{i} - 10\hat{j} - 8\hat{k}$ (d) $10\hat{i} - 10\hat{j} - \hat{k}$

14. *ABC* is an equilateral triangle with *O* as its centre. $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ represent three forces acting along the sides *AB*, *BC* and *AC*, respectively. If the total torque about *O* is zero, then the magnitude of $\mathbf{F}_3$ is



(a) $F_1 + F_2$ (b) $F_1 - F_2$
(c) $\dfrac{F_1 + F_2}{2}$ (d) $2(F_1 + F_2)$

15. The angular momentum **L** of a single particle can be represented as

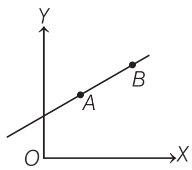
(a) $\mathbf{r} \times \mathbf{p}$ (b) $rp\sin\theta\,\hat{n}$
(c) $rp \perp \hat{n}$ (d) Both (a) and (b)

($\hat{\mathbf{n}}$ = unit vector perpendicular to plane of *r*, so that *r*, *p* and $\hat{\mathbf{n}}$ make right handed system)

16. Newton's second law for rotational motion of a system of particle can be represented as (**L** for a system of particles)

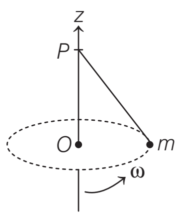
(a) $\dfrac{d\mathbf{p}}{dt} = \tau_{ext}$ (b) $\dfrac{d\mathbf{L}}{dt} = \tau_{int}$
(c) $\dfrac{d\mathbf{L}}{dt} = \tau_{ext}$ (d) $\dfrac{d\mathbf{L}}{dt} = \tau_{int} + \tau_{ext}$

17. A particle of mass *m* moves in the *xy*-plane with a velocity *v* along the straight line *AB*. If the angular momentum of the particle with respect to origin *O* is L_A, when it is at *A* and L_B when it is at *B*, then



(a) $L_A > L_B$
(b) $L_A = L_B$
(c) the relationship between L_A and L_B depends upon the slope of the line AB
(d) $L_A < L_B$

18. A point mass m is attached to a massless string whose other end is fixed at P as shown in figure. The mass is undergoing circular motion in xy-plane with centre O and constant angular speed ω. If the angular momentum of the system, calculated about O and P be $\mathbf{L}_O$ and $\mathbf{L}_P$ respectively, then



(a) $\mathbf{L}_O$ and $\mathbf{L}_P$ do not vary with time
(b) $\mathbf{L}_O$ varies with time while $\mathbf{L}_P$ remains constant
(c) $\mathbf{L}_O$ remains constant while $\mathbf{L}_P$ varies with time
(d) $\mathbf{L}_O$ and $\mathbf{L}_P$ both vary with time

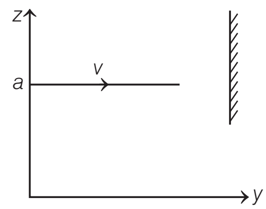
19. A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev min^{-1}. How much is the angular speed of the child, if he folds his hands back and thereby reduces his moment of inertia to $(2/5)$ times the initial value? Assume that the turntable rotates without friction. *(NCERT Exemplar)*

(a) 40 rpm (b) 45 rpm (c) 55 rpm (d) 100 rpm

20. If the torque of the rotational motion will be zero, then the constant quantity will be

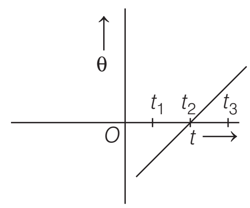
(a) angular momentum
(b) linear momentum
(c) angular acceleration
(d) centripetal acceleration

21. A particle of mass m is moving in yz-plane with a uniform velocity v with its trajectory running parallel to +ve y-axis and intersecting z-axis at $z = a$ in figure. The change in its angular momentum about the origin as it bounces elastically from a wall at $y =$ constant is *(NCERT Exemplar)*



(a) $mva\,\hat{\mathbf{e}}_x$ (b) $2\,mva\,\hat{\mathbf{e}}_x$
(c) $ymv\,\hat{\mathbf{e}}_x$ (d) $2\,ymv\,\hat{\mathbf{e}}_x$

22. The variation of angular position θ of a point on a rotating rigid body with time t is shown in figure.



In which direction, the body is rotating? *(NCERT Exemplar)*

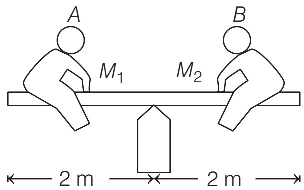
(a) Clockwise
(b) Anti-clockwise
(c) May be clockwise or anti-clockwise
(d) None of the above

23. A rigid body is said to be in partial equilibrium only, if

(a) it is in rotational equilibrium
(b) it is in translational equilibrium
(c) Either (a) or (b)
(d) None of the above

24. In the game of see-saw, what should be the displacement of boy B from right edge to keep the see-saw in equilibrium?

(Given, $M_1 = 40$ kg and $M_2 = 60$ kg)



(a) $\frac{4}{3}$ m
(b) 1m
(c) $\frac{2}{3}$ m
(d) Zero

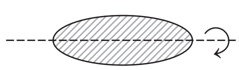
25. The centre of gravity of a homogeneous body is the point at which the whole

(a) volume of the body is assumed to be concentrated
(b) area of the surface of the body is assumed to be concentrated
(c) weight of the body is assumed to be concentrated
(d) All of the above

26. One solid sphere A and another hollow sphere B are of same mass and same outer radius. Their moments of inertia about their diameters are I_A and I_B respectively, such that

(a) $I_A = I_B$
(b) $I_A > I_B$
(c) $I_A < I_B$
(d) None of these

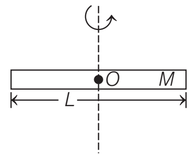
27. A disc of mass M and radius R is rotating about one of its diameter. The value of radius of gyration for the disc is



(a) $R/4$
(b) $R/2$
(c) $R/6$
(d) None of these

28. A rod is rotating about an axis passing through its centre and perpendicular to its length.

The radius of gyration for the rod is



(a) $L/12$
(b) $L/\sqrt{12}$
(c) $L/6$
(d) $L/\sqrt{6}$

29. A wheel is rotating at 900 rpm about its axis. When the power is cut-off, it comes to rest in 1 min. The angular retardation (in rad s^{-2}) is

(a) $-\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{2}$

30. If object starts from rest and covers angle of 60 rad in 10 s in circular motion, then magnitude of angular acceleration will be

(a) 1.2 rad s^{-2}
(b) 1.5 rad s^{-2}
(c) 2 rad s^{-2}
(d) 2.5 rad s^{-2}

31. When a ceiling fan is switched OFF, its angular velocity fall to half while it makes 36 rotations. How many more rotations will it make before coming to rest? (Assume uniform angular retardation)

(a) 36
(b) 24
(c) 18
(d) 12

32. A disc is rotating with angular velocity ω. A force **F** acts at a point whose position vector with respect to the axis of rotation is **r**. The power associated with torque due to the force is given by

(a) $(\mathrm{r} \times \mathrm{F}) \cdot \omega$
(b) $(\mathrm{r} \times \mathrm{F}) \times \omega$
(c) $\mathrm{r} \times (\mathrm{F} \cdot \omega)$
(d) $\mathrm{r} \cdot (\mathrm{F} \times \omega)$

33. A flywheel of moment of inertia 0.4 kg-m^2 and radius 0.2 m is free to rotate about a central axis. If a string is wrapped around it and it is pulled with a force of 10N, then its angular velocity after 4 s will be

(a) 10 rads^{-1}
(b) 5 rads^{-1}
(c) 20 rads^{-1}
(d) None of these

34. Two discs having mass ratio (1/2) and diameter ratio (2/1), then find ratio of moment of inertia.

(a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 2 : 3

35. A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere?

(a) Rotational kinetic energy
(b) Moment of inertia
(c) Angular velocity
(d) Angular momentum

36. A body having a moment of inertia about its axis of rotation equal to 3 kg-m^2 is rotating with angular velocity of 3 rad s^{-1}. Kinetic energy of this rotating body is same as that of a body of mass 27 kg moving with a velocity v. The value of v is

(a) 1 ms^{-1} (b) 0.5 ms^{-1} (c) 2 ms^{-1} (d) 1.5 ms^{-1}

37. A disc of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_k. What was the velocity of its centre of mass before being brought in contact with the table? *(NCERT Exemplar)*

(a) $\omega_0 R$ (b) Zero (c) $\frac{\omega_0 R}{2}$ (d) $2\omega_0 R$

38. Two bodies have their moments of inertia I and $2I$ respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

(a) 1 : 2 (b) $\sqrt{2}$: 1 (c) 2 : 1 (d) 1 : $\sqrt{2}$

39. By keeping moment of inertia of a body constant, if we double the time period, then angular momentum of body

(a) remains constant (b) becomes half
(c) doubles (d) quadruples

40. If frictional force is neglected and girl bends her hand, then (initially girl is rotating on chair)

(a) I_{girl} will reduce
(b) I_{girl} will increase
(c) ω_{girl} will reduce
(d) None of the above

41. A merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round of afterwards is *(NCERT Exemplar)*

(a) 2ω (b) ω
(c) $\frac{\omega}{2}$ (d) zero

42. A wheel of radius R rolls on the ground with a uniform velocity v. The velocity of topmost point relative to the bottommost point is

(a) v (b) $2v$
(c) $v/2$ (d) zero

43. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor, so that its centre of mass has a speed of 20 cms^{-1}. How much work has to be done to stop it? *(NCERT Exemplar)*

(a) 10 J (b) 12 J
(c) 4 J (d) 3 J

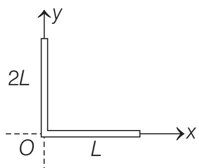
44. A drum of radius R and mass M rolls down without slipping along an inclined plane of angle θ. The frictional force

(a) converts translational energy into rotational energy
(b) dissipates energy as heat
(c) decreases the rotational motion
(d) decreases the rotational and translational motion

45. The centre of mass lie outside the body of a .... *(NCERT Exemplar)*

(a) pencil (b) shotput
(c) dice (d) bangle

46. Figure shows a composite system of two uniform rods of lengths as indicated. Then the coordinates of the centre of mass of the system of rods are......... .



(a) $\left(\frac{L}{2}, \frac{2L}{3}\right)$ (b) $\left(\frac{L}{4}, \frac{2L}{3}\right)$
(c) $\left(\frac{L}{6}, \frac{2L}{3}\right)$ (d) $\left(\frac{L}{6}, \frac{L}{3}\right)$

47. Analogue of mass in rotational motion is ....

(a) moment of inertia
(b) angular momentum
(c) gyration
(d) None of the above

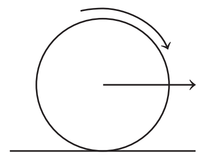
48. The angular acceleration of a flywheel of mass 5 kg and radius of gyration 0.5 m is ...., if a torque of 10N-m is applied on it.

(a) 2 rad s^{-2} (b) 4 rad s^{-2}
(c) 8 rad s^{-2} (d) zero

49. When a disc rotates with uniform angular velocity, which of the following statemnts is incorrect. *(NCERT Exemplar)*

(a) The sense of rotation remains same.
(b) The orientation of the axis of rotation remains same.
(c) The speed of rotation is non-zero and remains same.
(d) The angular acceleration is non-zero and remains same.

50. A bicycle wheel rolls without slipping on a horizontal floor. Which one of the following statements is true about the motion of points on the rim of the wheel, relative to the axis at the wheel's centre?



(a) Points near the top move faster than points near the bottom.
(b) Points near the bottom move faster than points near the top.
(c) All points on the rim move with the same speed.
(d) All points have the velocity vectors that are pointing in the radial direction towards the centre of the wheel.

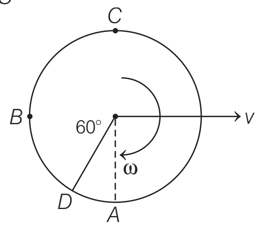
51. If radius of earth is reduced to half without changing its mass, then match the following columns and choose the correct option from the codes given below.

	Column I		Column II
A.	Angular momentum of earth	p.	Will become one fourth
B.	Time period of rotation of earth	q.	Will become four times
C.	Rotational kinetic energy of earth	r.	No change

Codes

	A	B	C		A	B	C
(a)	p	q	r	(b)	p	q	p
(c)	r	p	q	(d)	p	r	p

52. A rigid body is rolling without slipping on the horizontal surface, then match the Column I with Column II and choose the correct option from the codes given below.



	Column I		Column II
A.	Velocity at point A, i.e. v_A	p.	$v\sqrt{2}$
B.	Velocity at point B, i.e. v_B	q.	zero
C.	Velocity at point C, i.e. v_C	r.	v
D.	Velocity at point D, i.e. v_D	s.	$2v$

Codes

	A	B	C	D
(a)	q	p	s	r
(b)	p	r	s	q
(c)	s	r	q	p
(d)	q	r	s	p

Assertion-Reasoning MCQs

For question numbers 53 to 64, two statements are given-one labelled ***Assertion*** *(A) and the other labelled* ***Reason*** *(R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) are as given below*

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

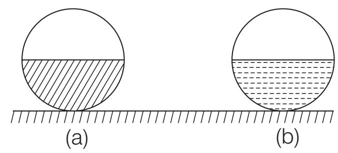
53. **Assertion** The motion of the centre of mass describes the translational part of the motion.

Reason Translational motion always means straight line motion.

54. **Assertion** The centre of mass of a body must lie on the body.

Reason The centre of mass of a body does not lie at the geometric centre of body.

55. **Assertion** Two identical spherical spheres are half filled with two liquids of densities ρ_1 and ρ_2 $(>\rho_1)$. The centre of mass of both the spheres lie at same level.



Reason The centre of mass does not lie at centre of the sphere.

56. **Assertion** If a particle moves with a constant velocity, then angular momentum of this particle about any point remains constant.

Reason Angular momentum does not have the units of Planck's constant.

57. **Assertion** When a particle is moving in a straight line with a uniform velocity, its angular momentum is constant.

Reason The angular momentum is non-zero, when particle moves with a uniform velocity.

58. **Assertion** For a system of particles under central force field, the total angular momentum is conserved.

Reason The torque acting on such a system is zero.

59. **Assertion** Inertia and moment of inertia are not same quantities.

Reason Inertia represents the capacity of a body that does not oppose its state of motion or rest.

60. **Assertion** Moment of inertia of a particle is different whatever be the axis of rotation.
Reason Moment of inertia does not depends on mass and distance of the particle from the axis of rotation.

61. **Assertion** The angular velocity of a rigid body in motion is defined for the whole body.

Reason All points on a rigid body performing pure rotational motion are having same angular velocity.

62. **Assertion** If bodies slide down an inclined plane without rolling, then all bodies reach the bottom simultaneously is not necessary.

Reason Acceleration of all bodies are equal and independent of the shape.

63. **Assertion** A solid sphere cannot roll without slipping on smooth horizontal surface.

Reason If the sphere is left free on smooth inclined surface, it can roll without slipping.

64. **Assertion** The work done against force of friction in the case of a disc rolling without slipping down an inclined plane is zero.

Reason When the disc rolls without slipping, friction is required because for rolling condition velocity of point of contact is zero.

Case Based MCQs

Direction *Answer the questions from 65-69 on the following case.*

Centre of Mass

The centre of mass of a body or a system of bodies is the point which moves as though all of the mass were concentrated there and all external forces were applied to it. Hence, a point at which the entire mass of the body or system of bodies is supposed to be concentrated is known as the centre of mass.

If a system consists of more than one particles (or bodies) and net external force on the system in a particular direction is zero with centre of mass at rest. Then, the centre of mass will not move along that direction. Even though some particles of the system may move along that direction.

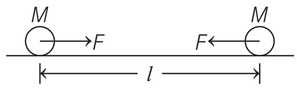
65. The centre of mass of a system of two particles divides, the distance between them

(a) in inverse ratio of square of masses of particles
(b) in direct ratio of square of masses of particles
(c) in inverse ratio of masses of particles
(d) in direct ratio of masses of particles

66. Two bodies of masses 1 kg and 2 kg are lying in xy-plane at $(-1, 2)$ and $(2, 4)$, respectively. What are the coordinates of the centre of mass?

(a) $\left(1, \frac{10}{3}\right)$ (b) $(1, 0)$
(c) $(0, 1)$ (d) None of these

67. Two balls of same masses start moving towards each other due to gravitational attraction, if the initial distance between them is l. Then, they meet at



(a) $\frac{l}{2}$ (b) l (c) $\frac{l}{3}$ (d) $\frac{l}{4}$

68. All the particles of a body are situated at a distance R from the origin. The distance of centre of mass of the body from the origin is

(a) $= R$ (b) $\leq R$ (c) $> R$ (d) $\geq R$

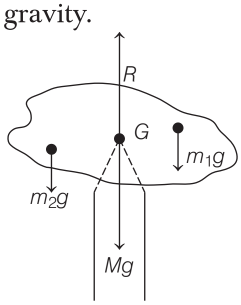
69. Two particles A and B initially at rest move towards each other under a mutual force of attraction. At the instant, when the speed of A is v and the speed of B is $2v$, the speed of centre of mass of the system is

(a) zero (b) v
(c) $1.5v$ (d) $3v$

Direction *Answer the questions from 70-74 on the following case.*

Torque and Centre of Gravity

Torque is also known as moment of force or couple. When a force acts on a particle, the particle does not merely move in the direction of the force but it also turns about some point. So, we can define the torque for a particle about a point as the vector product of position vector of the point where the force acts and with the force itself. In the given figure, balancing of a cardboard on the tip of a pencil is done. The point of support, G is the centre of gravity.



70. If the $\mathbf{F}_{\text{net, ext}}$ is zero on the cardboard, it means

(a) $R = Mg$ (b) $m_1 g = Mg$
(c) $m_2 g = Mg$ (d) $R = m_1 / g$

71. Choose the correct option.

(a) τ_{Mg} about CG $= 0$
(b) τ_R about CG $= 0$
(c) Net τ due to $m_1g, m_2g, \ldots, m_ng$ about CG $= 0$
(d) All of the above

72. The centre of gravity and the centre of mass of a body coincide, when

(a) g is negligible
(b) g is variable
(c) g is constant
(d) g is zero

73. If value of g varies, the centre of gravity and the centre of mass will

(a) coincide
(b) not coincide
(c) become same physical quantities
(d) None of the above

74. A body lying in a gravitational field is in stable equilibrium, if

(a) vertical line through CG passes from top
(b) horizontal line through CG passes from top
(c) vertical line through CG passes from base
(d) horizontal line through CG passes from base

Direction *Answer the questions from 75-79 on the following case.*

Moment of Inertia

A heavy wheel called flywheel is attached to the shaft of steam engine, automobile engine etc., because of its large moment of inertia, the flywheel opposes the sudden increase or decrease of the speed of the vehicle. It allows a gradual change in the speed and prevents jerky motion and hence ensure smooth ride of passengers.

75. Moment of inertia of a body depends upon

(a) axis of rotation (b) torque

(c) angular momentum (d) angular velocity

76. A particle of mass 1 kg is kept at (1m, 1m, 1m). The moment of inertia of this particle about Z-axis would be

(a) 1 kg-m^2

(b) 2 kg-m^2

(c) 3 kg-m^2

(d) None of the above

77. Moment of inertia of a rod of mass m and length l about its one end is I. If one-fourth of its length is cut away, then moment of inertia of the remaining rod about its one end will be

(a) $\frac{3}{4}I$ (b) $\frac{9}{16}I$

(c) $\frac{27}{64}I$ (d) $\frac{I}{16}$

78. A circular disc is to be made by using iron and aluminium, so that it acquires maximum moment of inertia about its geometrical axis. It is possible with

(a) iron and aluminium layers in alternate order

(b) aluminium at interior and iron surrounding it

(c) iron at interior and aluminium surrounding it

(d) Either (a) or (c)

79. Three thin rods each of length L and mass M are placed along X, Y and Z -axes such that one end of each rod is at origin. The moment of inertia of this system about Z-axis is

(a) $\frac{2}{3}ML^2$

(b) $\frac{4ML^2}{3}$

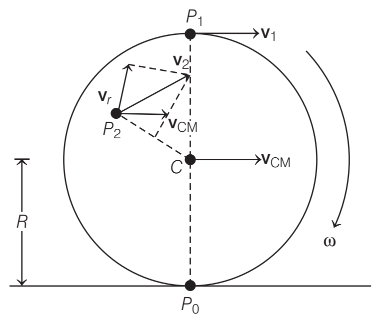
(c) $\frac{5ML^2}{3}$

(d) $\frac{ML^2}{3}$

Direction *Answer the questions from 80-84 on the following case.*

Rolling Motion

The rolling motion can be regarded as the combination of pure rotation and pure translation. It is also one of the most common motions observed in daily life.



Suppose the rolling motion (without slipping) of a circular disc on a level surface. At any instant, the point of contact P_0 of the disc with the surface is at rest (as there is no slipping). If $\mathbf{v}_{CM}$ is the velocity of centre of mass which is the geometric centre C of the disc, then the translational velocity of disc is $\mathbf{v}_{CM}$, which is parallel to the level surface.

Velocity of centre of mass, $\mathbf{v}_{CM} = R\omega$

80. A solid cylinder is sliding on a smooth horizontal surface with velocity v_0 without rotation. It enters on the rough surface. After that it has travelled some distance, the friction force increases its

(a) translational kinetic energy

(b) rotational kinetic energy

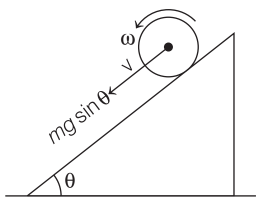
(c) total mechanical energy

(d) angular momentum about an axis passing through point of contact of the cylinder and the surface

81. A cylinder rolls down an inclined plane of inclination 30°, the acceleration of cylinder is

(a) $\frac{g}{3}$ (b) g (c) $\frac{g}{2}$ (d) $\frac{2g}{3}$

82. Sphere is in pure accelerated rolling motion in the figure shown,



Choose the correct option.

(a) The direction of f_s is upwards
(b) The direction of f_s is downwards
(c) The direction of gravitational force is upwards
(d) The direction of normal reaction is downwards

83. Kinetic energy of a rolling body will be

(a) $\frac{1}{2}mv_{CM}^2(1+k^2/R^2)$
(b) $\frac{1}{2}I\omega^2$
(c) $\frac{1}{2}mv_{CM}^2$
(d) None of the above

84. A body is rolling down an inclined plane. Its translational and rotational kinetic energies are equal. The body is a

(a) solid sphere
(b) hollow sphere
(c) solid cylinder
(d) hollow cylinder

ANSWERS

Multiple Choice Questions

1. (c)	2. (b)	3. (b)	4. (b)	5. (c)	6. (c)	7. (d)	8. (c)	9. (a)	10. (b)
11. (b)	12. (b)	13. (a)	14. (a)	15. (d)	16. (c)	17. (b)	18. (c)	19. (d)	20. (a)
21. (b)	22. (b)	23. (c)	24. (c)	25. (c)	26. (c)	27. (b)	28. (b)	29. (a)	30. (a)
31. (d)	32. (a)	33. (c)	34. (a)	35. (d)	36. (a)	37. (b)	38. (d)	39. (b)	40. (a)
41. (a)	42. (b)	43. (c)	44. (a)	45. (d)	46. (c)	47. (a)	48. (c)	49. (d)	50. (a)
51. (c)	52. (a)								

Assertion-Reasoning MCQs

53. (c)	54. (d)	55. (c)	56. (c)	57. (b)	58. (a)	59. (c)	60. (c)	61. (b)	62. (c)
63. (d)	64. (a)								

Case Based MCQs

65. (c)	66. (a)	67. (a)	68. (b)	69. (a)	70. (a)	71. (d)	72. (c)	73. (b)	74. (c)
75. (a)	76. (b)	77. (c)	78. (b)	79. (a)	80. (b)	81. (a)	82. (a)	83. (a)	84. (d)

SOLUTIONS

1. A rigid body does not deform under action of applied force and there is no relative motion of any two particles constituting that rigid body. So, it means that a system of particles is called a rigid body, when any two particles of system has a zero relative velocity.

2. A point at which the entire mass of the body or system of bodies. This is supposed to be concentrated is known as centre of mass.
It does not depend on the internal forces acting on the particle.

3. In pure rotational motion, all the particles of body moves in concentric circles without doing any translational motion.

4. For system of n-particles in space, the centre of mass of such a system is at (x, y, z), where

$$X = \frac{\Sigma m_i x_i}{M}, Y = \frac{\Sigma m_i y_i}{M}$$

and $Z = \dfrac{\Sigma m_i z_i}{M}$

Here, $M = \Sigma m_i$ is the total mass of the system. The index i runs from 1 to n. m_i is the mass of ith particle and position of ith particle is (x_i, y_i, z_i).

5. Centre of mass of a system lies towards the part of the system, having bigger mass. In the above diagram, lower part is heavier, hence CM of the system lies below the horizontal diameter.

Hence, option (c) is correct.

6. Let the coordinates of the centre of mass be (x, y) which are calculated as,

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1 \times 1 + 2 \times (-1)}{3}$$

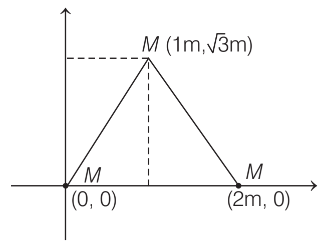
$$= \frac{1 - 2}{3} = -\frac{1}{3}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 3}{3}$$

$$= \frac{2 + 6}{3} = \frac{8}{3}$$

Therefore, the coordinates of centre of mass be $\left(-\dfrac{1}{3}, \dfrac{8}{3}\right)$.

7. The given system of spheres is as shown below



The x and y -coordinates of centre of mass is

$$x = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{M \times 0 + M \times 1 + M \times 2}{M + M + M} = 1$$

$$y = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{M \times 0 + M(\sqrt{3}) + M \times 0}{M + M + M}$$

$$\Rightarrow \quad y = \frac{\sqrt{3}\, M}{3M} = \frac{1}{\sqrt{3}}$$

So, position vector of the centre of mass is $\left(\hat{\mathbf{i}} + \dfrac{\hat{\mathbf{j}}}{\sqrt{3}}\right)$.

8. If all the masses were same, the centre of mass was at O. But as the mass at B is 2m, so the centre of mass of the system will shift towards B. So, centre of mass will be on the line OB.

9. Given, $\mathbf{v}_1 = 4\hat{\mathbf{i}}\ \text{ms}^{-1}$, $\mathbf{v}_2 = 4\hat{\mathbf{j}}\ \text{ms}^{-1}$

$\mathbf{a}_1 = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{ms}^{-2}$, $\mathbf{a}_2 = 0\ \text{ms}^{-2}$

$\therefore$ Velocity of centre of mass,

$$\mathbf{v}_{\text{CM}} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} = \frac{(\mathbf{v}_1 + \mathbf{v}_2)m}{2m}$$

$[\because m_1 = m_2 = m]$

$$= \frac{4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}}{2} = 2(\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-1}$$

Similarly, acceleration of centre of mass,

$$\mathbf{a}_{\text{CM}} = \frac{\mathbf{a}_1 + \mathbf{a}_2}{2}$$

$$= \frac{2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 0}{2} = (\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-2}$$

Since, from above values, it can be seen that $\mathbf{v}_{\text{CM}}$ is parallel to $\mathbf{a}_{\text{CM}}$, so the path will be a straight line.

10. Centre of mass of a system of particles is given by

$$x_{CM} = \frac{1 \times x_1 + 2 \times x_2 + 3 \times x_3}{1+2+3} = 3$$

$[\because x_{CM} = y_{CM} = z_{CM} = 3]$

$\Rightarrow \quad x_1 + 2x_2 + 3x_3 = (1+2+3)3 = 18 \quad \ldots$(i)

When fourth particle is placed, then

$x_{CM} = y_{CM} = z_{CM} = 1$ (given)

$$\Rightarrow \quad x_{CM} = \frac{1 \times x_1 + 2 \times x_2 + 3 \times x_3 + 4 \times x_4}{(1+2+3+4)}$$

$\Rightarrow$

$x_1 + 2\,x_2 + 3\,x_3 + 4\,x_4 = 1\,(1+2+3+4) = 10 \quad \ldots$(ii)

On solving Eqs. (i) and (ii), we get

$4x_4 = 10 - 18 \Rightarrow x_4 = -2$

Similarly, $\quad y_4 = -2,\ z_4 = -2$

$\therefore$ The fourth particle must be placed at the point $(-2, -2, -2)$.

11. Net external force on the system is zero. Hence, velocity of centre of mass of the box and ball system will remain constant.

12. If a force acts on a single particle at a point P whose position with respect to origin O is given by the position vector **r** as shown in given figure, the moment of the force acting on the particle with respect to the origin O is defined as the vector product.

$$\tau = \mathbf{r} \times \mathbf{F}$$

$$\Rightarrow \quad |\tau| = r\,F \sin\theta$$

13. Given, $\mathbf{F} = 5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $\mathbf{r} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

We know that, $\tau = \mathbf{r} \times \mathbf{F}$

So, torque about the origin will be given by

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -2 & 1 \\ 5 & +2 & -5 \end{vmatrix}$$

$$= \hat{\mathbf{i}}\,(10-2) - \hat{\mathbf{j}}\,(-5-5) + \hat{\mathbf{k}}\,(2+10)$$

$$= 8\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$$

14. If we take clockwise torque, then magnitude of total torque is

$$\tau_{net} = \tau_{F_1} + \tau_{F_2} + \tau_{F_3}$$

$$0 = -F_1 r - F_2 r + F_3 r$$

$$\Rightarrow \quad F_3 = F_1 + F_2$$

15. Angular momentum (L) can be defined as moment of linear momentum about a point. It is given by,

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

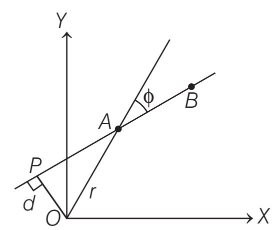
L can also be represented as, $L = rp \sin\theta\hat{n}$.

16. According to Newton's second law of rotational motion, the rate of the total angular momentum of a system of particles about a point is equal to the sum of the external torques acting on the system taken about the same point.

i.e. $\tau_{ext} = \dfrac{d\mathrm{L}}{dt}$

17. From the definition of angular momentum,

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = rmv\ \sin\phi\,(-\hat{\mathbf{k}})$

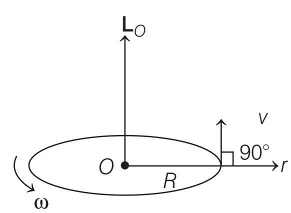


Therefore, the magnitude of L is $L = mvr,\ \sin\phi = mvd$, where $d = r\sin\phi$ is the distance of closest approach of the particle to the origin. As d is same for both the particles, hence $L_A = L_B$.

18. Angular momentum of a particle about a point is given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\,(\mathbf{r} \times \mathbf{v})$$

For $\mathbf{L}_O$,

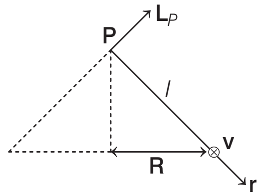


$|\mathbf{L}| = (mvr\sin\theta) = m\,(R\omega)(R)\sin 90° = mR^2\omega$
$= \text{constant}$

Direction of $\mathbf{L}_O$ is always upwards, therefore $\mathbf{L}_O$ is constant, both in magnitude as well as direction.

For $\mathbf{L}_P$, $|\mathbf{L}_P| = (mvr \sin\theta)$

$$= (m)(R\omega)(l)\sin 90^\circ = mRl\omega$$



Magnitude of $\mathbf{L}_P$ will remains constant but direction of $\mathbf{L}_P$ keeps on changing, i.e. it varies with time.

19. From law of conservation of angular momentum,

$$L_1 = L_2$$

$$\Rightarrow \quad I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2}$$

$$\Rightarrow \quad \omega_2 = \frac{I_1 \times 40}{\frac{2}{5}I_1} = \frac{200}{2} = 100 \text{ rpm}$$

20. As, torque, $\tau = \dfrac{dL}{dt}$

If $\tau = 0$, then L = constant.

Hence, option (a) is correct.

21. The initial velocity is $\mathbf{v}_i = v\hat{\mathbf{e}}_y$ and after reflection from the wall, the final velocity is $\mathbf{v}_f = -v\hat{\mathbf{e}}_y$. The trajectory is described as position vector $\mathbf{r} = y\hat{\mathbf{e}}_y + a\hat{\mathbf{e}}_z$.

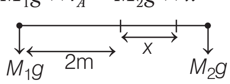
Hence, the change in angular momentum is $\mathbf{r} \times m(\mathbf{v}_f - \mathbf{v}_j) = 2mva\hat{\mathbf{e}}_x$.

22. As the slope of θ-t graph is positive and positive slope indicates anti-clockwise rotation.

23. A body may remain in partial equilibrium means that body may remain only in translational equilibrium or only in rotational equilibrium.

24. Let x be the distance from centre, then for rotational equilibrium,

$$M_1 g \times r_A = M_2 g \times x$$



$$(40 \times 10) \times 2 = (60 \times 10)\, x$$

$$\Rightarrow \quad x = \frac{8}{6} = \frac{4}{3} \text{ m}$$

So, 60 kg boy has to be displaced to

$$= 2 - \frac{4}{3} = \frac{2}{3} \text{ m}$$

25. The centre of gravity of a homogeneous body is the point at which the whole weight of the body is assumed to be concentrated.

26. Let mass and outer radii of solid sphere and hollow sphere be M and R, respectively. The moment of inertia of solid sphere A about its diameter,

$$I_A = \frac{2}{5}MR^2 \quad \ldots\text{(i)}$$

The moment of inertia of hollow sphere (spherical shell) B about its diameter,

$$I_B = \frac{2}{3}MR^2 \quad \ldots\text{(ii)}$$

It is clear from Eqs. (i) and (ii), that

$$I_A < I_B$$

27. I_{disc} about the axis along its diameter

$$= \frac{MR^2}{4} \quad \ldots\text{(i)}$$

Using radius of gyration, $I = Mk^2$ ...(ii)

Comparing Eqs. (i) and (ii), we get $k = \dfrac{R}{2}$.

28. As, moment of inertia of rod,

$$I_{\text{rod}} = \frac{ML^2}{12} \quad \ldots\text{(i)}$$

Using radius of gyration, $I = Mk^2$ ...(ii)

Comparing Eqs. (i) and (ii), we get

Radius of gyration, $k = L/\sqrt{12}$

29. Angular retardation,

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 900 \times \frac{2\pi}{60}}{60} \text{ rad s}^{-2}$$

$$= -\frac{900 \times 2 \times \pi}{3600} = -\frac{\pi}{2} \text{ rad s}^{-2}$$

30. Given, initial angular velocity of object, $\omega_0 = 0$

Angular displacement, $\theta = 60$ rad

and $\Delta t = 10$ s

From equation of rotational motion,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$60 = 0 \times t + \frac{1}{2} \times \alpha \times 10^2$$

$$\Rightarrow \quad \alpha = \frac{60}{50}$$

$$\Rightarrow \quad \alpha = 1.2 \text{ rads}^{-2}$$

31. Total angular displacement in 36 rotation,

$$\theta = 36 \times 2\pi$$

Using $\omega_2^2 - \omega_1^2 = 2\alpha\theta$, we get

$$(\omega/2)^2 - \omega^2 = 2\alpha(36 \times 2\pi) \quad \ldots(\text{i})$$

Similarly, $0^2 - (\omega/2)^2 = 2\alpha\,(n \times 2\pi)$ $\quad \ldots(\text{ii})$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{-\frac{3}{4}\omega^2}{-\omega^2/4} = \frac{36}{n} \Rightarrow n = 12$$

Hence, ceiling fan will make 12 more rotations before coming to rest.

32. Power, $P = \tau\omega$

$$P = (\mathbf{r} \times \mathbf{F}) \cdot \omega$$

33. Given, moment of inertia of flywheel,

$$I = 0.4 \text{ kg-m}^2$$

Radius, $r = 0.2$ m

Force, $F = 10$ N

$$\because \; F \times r = I\alpha = I\,\frac{(\omega_2 - \omega_1)}{t}$$

$$\Rightarrow \quad \omega_2 - \omega_1 = \frac{F \times r \times t}{I}$$

(from $\tau = F \times r$ and $\tau = I\alpha$)

$$= \frac{10 \times 0.2 \times 4}{0.4}$$

$$= 20 \text{ rads}^{-1}$$

34. Given, mass ratio of two discs,

$m_1 : m_2 = 1 : 2$, i.e. $\frac{m_1}{m_2} = \frac{1}{2}$

and diameter ratio, $\frac{d_1}{d_2} = \frac{2}{1}$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{d_1/2}{d_2/2} = \frac{d_1}{d_2} = \frac{2}{1}$$

$\therefore$ Ratio of their moment of inertia,

$$\frac{I_1}{I_2} = \frac{\frac{m_1 r_1^2}{2}}{\frac{m_2 r_2^2}{2}} = \frac{m_1}{m_2} \cdot \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{2}\left(\frac{2}{1}\right)^2 = \frac{2}{1}$$

$$\therefore \quad I_1 : I_2 = 2 : 1$$

35. As, we know that external torque, $\tau_{\text{ext}} = \frac{dL}{dt}$

where, L is the angular momentum.

Since, in the given condition,

$$\tau_{\text{ext}} = 0 \;\Rightarrow\; \frac{dL}{dt} = 0$$

or $\quad L =$ constant

Hence, when the radius of the sphere is increased keeping its mass same, only the angular momentum remains constant. But other quantities like moment of inertia, rotational kinetic energy and angular velocity changes.

36. We know that, kinetic energy,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$$

Given, $m = 27$ kg (mass of the body),

$\omega = 3 \text{ rads}^{-1}$ (angular velocity)

and $\quad I = 3 \text{ kg-m}^2$ (moment of inertia)

$$\Rightarrow \quad mv^2 = I\omega^2 \quad \Rightarrow v^2 = \frac{I\omega^2}{m}$$

$$v^2 = \frac{3 \times 3^2}{27} \;\Rightarrow\; v^2 = \frac{27}{27} = 1$$

$$\Rightarrow \quad v = \sqrt{1} = 1 \text{ ms}^{-1}$$

37. Before being brought in contact with the table, the disc was in pure rotational motion, hence $v_{\text{CM}} = 0$.

38. Rotational kinetic energy remains same.

i.e. $\quad \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}I_2\,\omega_2^2$

or $\frac{1}{2I_1}(I_1\omega_1)^2 = \frac{1}{2I_2}(I_2\,\omega_2)^2$

$$\Rightarrow \quad \frac{L_1^2}{I_1} = \frac{L_2^2}{I_2} \quad \text{or} \quad \frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}}$$

But $\quad I_1 = I,\; I_2 = 2I$

$$\Rightarrow \quad \frac{L_1}{L_2} = \sqrt{\frac{I}{2I}} = \frac{1}{\sqrt{2}} \;\Rightarrow\; L_1 : L_2 = 1 : \sqrt{2}$$

39. We know that, angular momentum of the body is given by

$$L = I\omega \;\text{ or }\; L = I \times \frac{2\pi}{T} \;\text{ or } L \propto \frac{1}{T} \;\Rightarrow\; \frac{L_1}{L_2} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{L}{L_2} = \frac{2T}{T} \qquad (\text{as, } T_2 = 2T \text{ and } L_1 = L)$$

So, $L_2 = \frac{L}{2}$. Thus, on doubling the time period, angular momentum of body becomes half.

40. As there is no external torque, so if the girl bends her hands, her moment of inertia about the rotational axis will decrease. By conservation of angular momentum,

$L = I\omega =$ constant. So, in order to keep L constant, if I is decreasing, then ω will increase.

41. As no external torque acts on the system, angular momentum should be conserved.

Hence, $I\,\omega =$ constant. ...(i)

where, I is moment of inertia of the system and ω is angular velocity of the system.

From Eq. (i) $I_1\omega_1 = I_2\,\omega_2$

where, ω_1 and ω_2 are angular velocities before and after jumping)

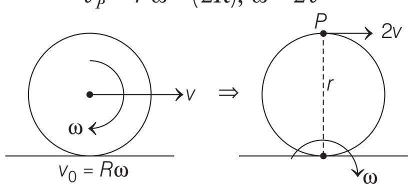
$\Rightarrow \quad I\omega = \frac{I}{2} \times \omega_2$

(as mass reduced to half, hence moment of inertia also reduced to half)

$\Rightarrow \omega_2 = 2\omega$

42. Velocity of the particle,

$v_P = r\,\omega = (2R),\ \omega = 2v$



43. Work done $= \Delta K =$ Change in rotational kinetic energy + Change in linear kinetic energy

$$= \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I\omega^2$$

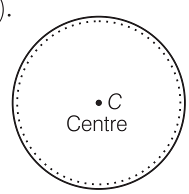
$(\because I = mr^2 \text{ and } v_{CM} = r\omega)$

$$= m v_{CM}^2 = 100 \times (20 \times 10^{-2})^2 = 4\text{ J}$$

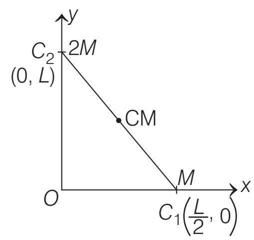
44. When a body rolls down without slipping along an inclined plane of inclination θ, it rotates about a horizontal axis through its centre of mass and its centre of mass also moves.

As it rolls down, it suffers loss in gravitational potential energy which provides translational energy and due to frictional force, it gets converted into rotational energy.

45. A bangle is in the form of a ring as shown in the adjacent diagram. The centre of mass lies at the centre, which is outside the body (boundary).



46. As rods are uniform, therefore centre of mass of both rods will be at their geometrical centres. The coordinates of CM of first rod C_1 are $\left(\frac{L}{2}, 0\right)$ and second rod C_2 are $(0, L)$.



$$\therefore\ x_{CM} = \frac{M\left(\frac{L}{2}\right) + 2M(0)}{M + 2M} = \frac{L}{6}$$

$$y_{CM} = \frac{M(0) + 2M(L)}{M + 2M} = \frac{2L}{3}$$

Hence, coordinates of CM are $\left(\frac{L}{6}, \frac{2L}{3}\right)$.

47. The role of moment of inertia in the study of rotational motion is analogous to that of mass in study of linear motion.

48. As, $\tau = I\alpha = Mk^2\alpha$

$\Rightarrow \alpha = \frac{\tau}{Mk^2}$

$\Rightarrow \alpha = \frac{10}{5 \times 0.5 \times 0.5} = 8 \text{ rad s}^{-2}$

49. We know that, angular acceleration,

$\alpha = \frac{d\omega}{dt}$, given $\omega =$ constant

where, ω is angular velocity of the disc.

$\Rightarrow \alpha = \frac{d\omega}{dt} = \frac{0}{dt} = 0$

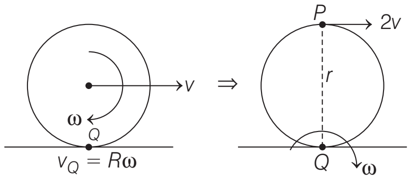
Hence, angular acceleration is zero.

50. Velocity of the particle at Q,

$$v_Q = r\omega = R\omega$$

Velocity of the particle at P,

$$v_P = r\,\omega = (2R)\,\omega = 2v_Q$$



Hence, points near the top move faster than points near the bottom.

51. $L_1 = I\omega = \frac{2}{5}MR^2\omega$

$$L_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2\omega'$$

As $L_1 = L_2 \Rightarrow \omega' = 4\omega$

$$\therefore \quad \frac{2\pi}{T'} = 4\left(\frac{2\pi}{T}\right)$$

$$\Rightarrow \quad T' = \frac{T}{4}$$

Time period will become $\left(\frac{1}{4}\right)$th.

Further, $K = \frac{L^2}{2I}$

Since, angular momentum is constant and I has become (1/4)th.

Therefore, kinetic energy will become 4 times.

Hence, A → r, B → p and C → q.

52. If v is the velocity of centre of mass of the body of radius r, then

velocity at point A, $v_A = 0$

velocity at point B, $v_B = v\sqrt{2}$

velocity at point C, $v_C = v + r\omega = 2v$

velocity of point D, $v_D = r\omega = v$

Hence, A → q, B → p, C → s and D → r.

53. The motion of centre of mass describes the translational part of the motion.

In translational motion, all points of a moving body move along a straight line, i.e. the relative velocities between any two particles, must be zero.

But it is not necessary that, translational motion of body is always in straight line. A parabolic motion of an object without rotation is also translational motion.

Therefore, A is true but R is false.

54. The centre of mass of a body may lie on or outside the body.

(a) (b)

Hence, in Fig. (a), centre of mass is on the body and in Fig. (b), centre of mass does not lie on the body.

The centre of mass of an object is the average position of all the parts of the system, weighted according to their masses. Therefore, centre of mass of a body lie at the geometric centre of body.

Therefore, A is false and R is also false.

55. We know that, centre of mass of half disc depends only on radius and not only the density of the material of disc similarly in this case centre of mass of half filled sphere will depends only on radius and not on density of liquid inside. Since, both sphere are of same radius so both have CM at the same level.

Therefore, A is true and R is false.

56. $L = mvr\sin\theta$ or $mvr_\perp$

In case of constant velocity m, v and $r_\perp$ all are constant.

Therefore, angular momentum is constant.

Further, $L = n\frac{h}{2\pi}$ (in Bohr's theory)

Hence, L and h have same units.

Therefore, A is true but R is false.

57. Angular momentum remains constant as particle is moving in a straight line. The angular momentum is constant, when particle moves with a uniform velocity.

Therefore, both A and R are true but R is not the correct explanation of A.

58. When $\tau_{ext} = 0$, then $\mathbf{L}$ = constant.

So, for a system of particles under central force field, the total angular momentum on the system is conserved because torque acting on such a system is zero.

Therefore, both A and R are true and R is the correct explanation of A.

59. There is a difference between inertia and moment of inertia of a body. The inertia of a body depends only upon the mass of the body but the moment of inertia of a body about an axis not only depend upon the mass of the body but also upon the distribution of mass about the axis of rotation.
Inertia represents the capacity (ability) of a body to oppose its state of motion or rest.

Therefore, A is true but R is false.

60. Moment of inertia changes with axis chosen. It is because moment of inertia of a particle depends on its mass and its distance from axis of rotation.

Therefore, A is true but R is false.

61. Angular velocity for a rigid body can be described as the rate of change at which the object rotates about an axis. It is defined for the whole body.

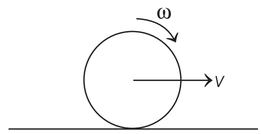
Angular velocity of particle of rigid body is same in rotational motion.

Therefore, both A and R are true but R is not the correct explanation of A.

62. Friction force between sliding body and inclined plane depends upon the nature of surfaces of both the body and inclined plane, hence if bodies slide down an inclined plane without rolling, then it is not necessary that all bodies reach the bottom simultaneously. Acceleration of all bodies are also not equal due to different values of friction between the surfaces of body and inclined plane.

Therefore, A is true but R is false.

63. Sphere can roll without slipping on surface, if $v = r\,\omega$ on an inclined plane, it is friction which creates rotation on sphere. So, smooth surface cannot create rotation.



Therefore, A is false and R is also false.

64. The work done on a body is given by

$$W = \int F \cdot v\,dt, \text{ where } F \text{ is force of friction.}$$

For the rolling disc without slipping down an inclined plane, the velocity of the particle on which the friction force is acting, is zero. Hence, work done is zero, i.e. when the disc rolls without slipping, the friction force is required because for rolling condition, velocity of point of contact is zero.

Therefore, both A and R are true and R is the correct explanation of A.

65. Centre of mass of a system of two particles is

Then, $r_{CM} = \dfrac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

If $m_1 + m_2 = M$ = total mass of the particles,

then $r_{CM} = \dfrac{m_1 r_1 + m_2 r_2}{M}$

$\therefore$ $r_{CM} \propto 1/M$

So, the above relation clearly shows that the centre of mass of a system of two particles divide the distance between them in inverse ratio of masses of particles.

66. Let the coordinates of the centre of mass be (x, y).

$$\therefore \quad x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1 \times (-1) + 2 \times 2}{3} = \frac{-1 + 4}{3} = 1$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 4}{3} = \frac{2 + 8}{3} = \frac{10}{3}$$

Therefore, the coordinates of centre of mass be $\left(1, \dfrac{10}{3}\right)$.

67. As the balls were initially at rest and the forces of attraction are internal, then their centre of mass (CM) will always remain at rest.

So, $v_{CM} = 0$

As CM is at rest, they will meet at CM. Hence, they will meet at $l/2$ from any initial positions.

68. For a single particle, distance of centre of mass from origin is R. For more than one particles, distance $\leq R$.

69. As per the question, two particles A and B are initially at rest, move towards each other under a mutual force of attraction. It means that, no external force is applied on the system. Therefore, $F_{ext} = 0$.

So, there is no acceleration of CM. This means velocity of the CM remain constant.

As, initial velocity of CM, $v_i = 0$ and final velocity of CM, $v_f = 0$.

So, the speed of centre of mass of the system will be zero.

70. The tip of the pencil provides a vertically upward force due to which the cardboard is in equilibrium. As shown in given figure, the reaction of the tip is equal and opposite to Mg, the total weight of the cardboard, i.e. $R = Mg$.

71. Net τ due to all the forces of gravity $m_1g, m_2g, \cdots, m_ng$ about CG is zero.

τ of reaction $\mathbf{R}$ about CG is also zero as it is at CG.

Point G is the centre of gravity of the cardboard and it is so located that the total torque on it due to forces $m_1\mathbf{g}, m_2\mathbf{g}, .., m_n\mathbf{g}$ is zero.

It means, $\tau_g = \Sigma \tau_i$

$$= \Sigma \mathbf{r}_i \times m_i \mathbf{g} = 0.$$

72. As, $\tau_g = \Sigma \mathbf{r}_i \times m_i \mathbf{g}$

(τ_g = total gravitational torque)

$\Sigma \mathbf{r}_i \times m_i \mathbf{g} = 0$

If g is constant,

$$(\Sigma m_i \mathbf{r}_i) \times \mathbf{g} = \mathbf{g} \Sigma m_i \mathbf{r}_i$$

As $\mathbf{g} \neq 0$, so $\Sigma m_i \mathbf{r}_i = 0$

It is the condition where the centre of mass (CM) of the body lies at origin and here origin is considered at centre of gravity (CG), when g is constant.

73. If the value of g varies, then CM and CG will not coincide. Keep in mind that, CG and CM both are two different concepts. CM has nothing to do with CG.

74. A body in a gravitational field will be in stable equilibrium, if the vertical line through CG passes from the base of the body.

75. Moment of inertia of a body depends on position and orientation of the axis of rotation with respect to the body.

76. Perpendicular distance from Z-axis would be $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$ m

$\therefore \; I = Mr^2 = (1)(\sqrt{2})^2 = 2$ kg-m^2

77. Initial moment of inertia, $I = \dfrac{ml^2}{3}$

New moment of inertia,

$$I' = \frac{(3m/4)(3l/4)^2}{3} = \frac{27}{64}\left(\frac{ml^2}{3}\right) = \frac{27}{64} I$$

78. A circular disc is made up of larger number of circular rings.

Moment of inertia of a circular ring in given by

$$I = MR^2$$

$$\Rightarrow \quad I \propto M$$

Since, mass is proportional to the density of material. The density of iron is more than that of aluminium. Hence to get maximum value of I, the less dense material should be used at interior and denser at the surrounding.

Therefore, using aluminium at the interior and iron at its surrounding will maximise the moment of inertia.

79. Moment of inertia of the rod lying along Z-axis will be zero. Moment of inertia of the rods along X and Y-axes will be $\dfrac{ML^2}{3}$ each.

Hence, total moment of inertia is $\dfrac{2}{3}ML^2$.

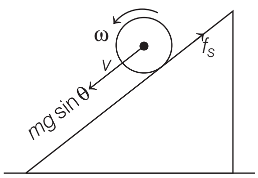
80. The frictional force will reduce v_0, hence translational KE will also decrease.

It will increases ω, which increases its rotational kinetic energy.

There is no torque about the line of contact, angular momentum will remain constant. The frictional force will decrease the mechanical energy.

81. $a = \dfrac{g \sin \theta}{1 + \dfrac{k^2}{R^2}} = \dfrac{g \sin 30°}{1 + \dfrac{1}{2}} \Rightarrow a = \dfrac{g/2}{3/2} = g/3$

82. As we know that,



The direction of f_s will be upwards to provide torque for rolling of sphere.

83. KE of a rolling body = Rotational KE + Translational KE

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv_{CM}^2 \qquad \left(\because I = mk^2 \text{ and } v_{CM} = R\omega\right)$$

$$= \frac{1}{2} \frac{m\,k^2 v_{CM}^2}{R^2} + \frac{1}{2} mv_{CM}^2$$

where, k is the corresponding radius of gyration of the body.

$$= \frac{1}{2} mv_{CM}^2 \left(1 + \frac{k^2}{R^2}\right)$$

It applies for any rolling body.

84. When a body rolls down on inclined plane, it is accompanied by rotational and translational kinetic energies.

Rotational kinetic energy $= \dfrac{1}{2} I\omega^2 = K_R$

where, I is the moment of inertia and ω is the angular velocity.

Translational kinetic energy for pure rolling,

$$v_{CM} = r\omega$$

$$= \frac{1}{2} mv_{CM}^2 = K_T = \frac{1}{2} m(r\omega)^2$$

where, m is mass of the body, v_{CM} is the velocity and ω is the angular velocity.

Given,

Translational KE = Rotational KE

$$\therefore \quad \frac{1}{2} m(r^2\omega^2) = \frac{1}{2} I\omega^2$$

$$\Rightarrow \quad I = mr^2$$

We know that, mr^2 is the moment of inertia of hollow cylinder about its axis, where m is the mass of hollow cylindrical body and r is the radius of the cylinder.