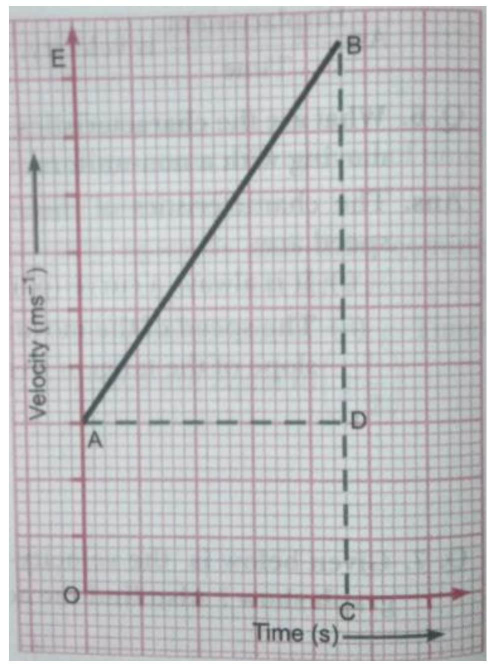
Long Answer Type Questions

[5 marks]

Q. 1. with the help of a graph, derive the relation *v = u + at.*

Ans. Consider the velocity-time graph of an object that moves under uniform acceleration as shown in the figure ($u \neq O$).

From this graph, we can see that initial velocity of the object (at point A) is u and then it increases to v (at point B) in time t. The velocity changes at a uniform rate a. As shown in the figure, the lines BC and BE are drawn from point B on the time and the velocity axes respectively, so that the initial velocity is represented by OA, the final velocity is represented by BC and the time interval t is represented by OC. BD = BC - CD, represents the change in velocity in time interval t. If we draw AD parallel to OC, we observe that



BC = BD + DC = BD + OA

Substituting, BC with v and OA with u, we get

v = BD + u

or BD = $v - u$

Thus, from the given velocity-time graph, the acceleration of the object is given by

$$A = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$= \frac{BD}{AD} = \frac{BD}{OC}$$

Substituting, OC with t, we get

$$a = \frac{BD}{t} \text{ or } BD = at$$

From equations (1) and (2), we have

$v - \boldsymbol{u} = \boldsymbol{at}$ **or** $v = \boldsymbol{u + at}$

Q. 2. Deduce the following equations of motion:

(i) $s = ut + \left(\frac{1}{2}\right)at^2$ **(ii)** $v^2 = u^2 + 2as$

Ans. (i) Consider a body which starts with initial velocity u and due to uniform acceleration a, its final velocity becomes v after time t. Then, its average velocity is given by

$$\text{Average velocity} = \frac{\text{Initial velocity + Final velocity}}{2} = \frac{u+v}{2}$$

$\therefore$ The distance covered by the body in time t is given by

Distance, s = Average velocity x Time

or $s = \frac{u+v}{2} \times t$ or $s = \frac{u+(u+\text{at})}{2} \times t$

$\therefore$ $s = \frac{2ut+at^2}{2}$ $or\ s = ut + \frac{1}{2}at^2$

(ii) We know that

$$s = ut + \frac{1}{2}at^2$$

Also, $a = \frac{v-u}{t}$

$\Longrightarrow$ $t = \frac{v-u}{a}$

Putting the value of t in (1), we have

$$s = u\left(\frac{\boldsymbol{v-u}}{\boldsymbol{a}}\right) + \frac{1}{2}a\left(\frac{\boldsymbol{v-u}}{\boldsymbol{a}}\right)^2$$

or $s = \frac{uv-u^2}{a} + \frac{v^2+u^2-2uv}{2a}$

or $2as = 2uv - 2u^2 + v^2 + u^2 - 2uv$

or $v^2 - u^2 = 2as$

Q.3. Obtain a relation for the distance travelled by an object moving with a uniform acceleration in the interval between 4th and 5th seconds.

Ans. Using the equation of motion $s = ut + \frac{1}{2}at^2$

Distance travelled in 5 seconds, $s = u \times 5 + \frac{1}{2}a \times 5^2$

Or $s = 5u + \frac{25}{2}a$

…(1)

Similarly, distance travelled in 4 seconds, $s' = 4u + \frac{16}{2}a$ …(2)

Distance travelled in the interval between 4th and 5th seconds

$= (s - s') = (u + \frac{9}{2}a)$ m

Q. 4. Two stones are thrown vertically upwards simultaneously with their initial velocities u_1 and u_2 respectively. Prove that the heights reached by them would be in the ratio of $\boldsymbol{u_1^2 : u_2^2}$.
(Assume upward acceleration is -g and downward acceleration to be + g).

Ans. We know the upward motion,$u^2 = u^2$-2gh or h = $\frac{u^2+v^2}{2g}$

But at highest point $v = 0$

Therefore, $h = \frac{u^2}{2g}$

For first ball, $h_1 = \frac{u_1^2}{2g}$

and for second ball, $h_2 = \frac{u_2^2}{2g}$

Thus, $\frac{h_1}{h_2} = \frac{\frac{u_1^2}{2g}}{\frac{u_2^2}{2g}} = \frac{u_1^2}{u_2^2}$

Or $h_1 : h_2 = u_1^2 : u_2^2$

Q. 5. The driver of train A travelling at a speed of 54 kmh^{-1} applies brakes and retards the train uniformly. The train stops in 5 seconds. Another train B is travelling on the parallel with a speed of 36 kmh^{-1}. Its driver applies the brakes and the train retards uniformly; train B stops in 10 seconds. Plot speed-time graphs for both the trains on the same axis. Which of the trains travelled farther after the brakes were applied?

Ans. For train A, the initial velocity,

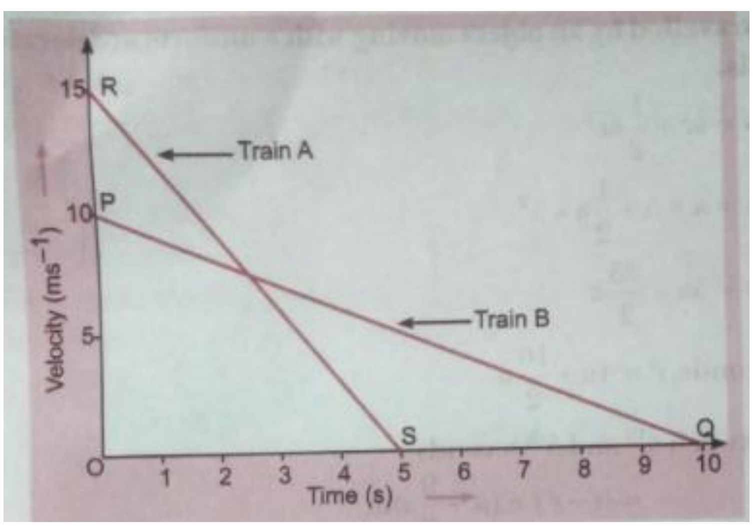
u = 54 kmh^{-1} = 54 × $\frac{5}{18}$ = 15 ms^{-1}

Final velocity, v = 0 and time, t = 5s

For train B, u = 36 kmh^{-1} = 36 x $\frac{5}{18}$ = 10 ms^{-1}

v = 0; t =10 s

Speed-time graph for train A and B are shown in the figure.



Distance travelled by train A

= Area under straight line graph RS

= Area of ΔORS

$= \frac{1}{2} \times OR \times OS = \frac{1}{2} \times 15\ ms^{-1} \times 5\ s = 37.5\ m$

Distance travelled by train B = Area under PQ = Area of ΔOPQ

$= \frac{1}{2} \times OP \times OQ = \frac{1}{2} \times 10\ ms^{-1} \times 10\ s = 50\ m$

Thus, train B travelled farther after the brakes were applied.