Short Answer Type Questions – II

[3 marks]

Que 1. Which term of the AP: 3, 8, 13, 18, …, is 78?

Sol. let a_n be the required term and we have given AP

3, 8, 13, 18, …

Here, a = 3, d = 8 – 3 = 5 and a_n = 78

Now, $a_n = a + (n - 1)d \Rightarrow \quad 78 = 3 + (N - 1) \times 5$

$\Rightarrow \quad 78 - 3 = (N - 1) \times 5 \quad \Rightarrow \quad 75 = (n - 1) \times 5$

$\Rightarrow \quad \frac{75}{5} = n - 1 \quad \Rightarrow \quad 15 = n - 1 \quad \Rightarrow \quad n = 15 + 1 = 16$

Hence, 16th term of given AP is 78.

Que 2. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol. Let the first term be a and common difference be d.

Now, we have

$a_{11} = 38 \quad \Rightarrow \quad a = (11 - 1)d = 38$

$\Rightarrow \quad a + 10d = 38 \quad \ldots(i)$

and $\quad a_{16} = 73 \quad \Rightarrow \quad a = (16 - 1)d = 73$

$\Rightarrow \quad a = 15d = 73 \quad \ldots(ii)$

Now subtracting (ii) from (i), we have

Now,

$$\begin{array}{r} a + 10d = 38 \\ -a \pm 15d = -73 \\ \hline -5d = -35 \end{array} \quad \text{or} \quad 5d = 35$$

$$d = \frac{35}{5} = 7$$

Putting the value of d in equation (i), we have

$a + 10 \times 7 = 38 \quad \Rightarrow \quad a + 70 = 38$

$\Rightarrow \quad a = 38 - 70 \quad \Rightarrow \quad a = -32$

We have, a = - 32 and d = 7

Therefore, $a_{31} = a + (31 - 1)d$

$\Rightarrow \quad a_{31} = a + 30d = (-32) + 30 \times 7 = -32 + 210$

$\Rightarrow \quad a_{31} = 178$

Que 3. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. Let a be the first term and d be the common difference.

Since, give AP consist of 50 terms, so n = 50

$a_3 = 12 \quad \Rightarrow \quad a + 2d = 12 \quad \dots (i)$

$i.e., \quad a_{50} = 106 \quad \Rightarrow \quad a + 49d = 106 \quad \dots (ii)$

Subtracting (i) from (ii), we have

$47d = 94 \quad \Rightarrow \quad d = \frac{94}{47} = 2$

Putting the value of d in equation (i), we have

$a + 2 \times 2 = 12 \quad \Rightarrow \quad a = 12 - 4 = 8$

Here, $a = 8, d = 2$

$\therefore$ 29th term is given by

$a_{29} = a + (29 - 1) d = 8 + 28 \times 2$

$\Rightarrow \quad a_{29} = 8 + 56 \quad \Rightarrow \quad a_{29} = 64$

Que 4. If the 8th term of an AP is 31 and the 15th term is 16 more than the 11th term, find the AP.

Sol. Let a be the first term and d be the common difference of the AP.

We have, $a_8 = 31$ and $a_{15} = 16 + a_{11}$

$\Rightarrow \quad a + 7d = 31$ and $a + 14d = 16 + a + 10d$

$\Rightarrow \quad a + 7d = 31$ and $4d = 16$

$\Rightarrow \quad a + 7d = 31$ and $d = 4 \quad \Rightarrow \quad a + 7 \times 4 = 31$

$\Rightarrow \quad a + 28 = 31 \quad \Rightarrow \quad a = 3$

Hence, the AP is a, a + d, a + 2d, a + 3d,…

$i.e., \quad 3, 7, 11, 15, 19, \dots$

Que 5. Which term of the arithmetic progression 5, 15, 25, … will be 130 more than its 31st term?

Sol. We have, a = 5 and d = 10

$\therefore \; a_{31} = a + 30d = 5 + 30 \times 10 = 305$

Let nth term of the given AP be 130 more than its 31st term. Then,

$a_n = 130 + a_{31}$

$\therefore \; a + (n - 1) d = 130 + 305$

⇒ $5 + 10(n-1) = 435$ ⇒ $10(n-1) = 430$

⇒ $n - 1 = 43$ ⇒ $n = 44$

Hence, 44th term of the given AP is 130 more than its 31st term.

Que 6. Find the sum given below:

$$\mathbf{7 + 10\frac{1}{2} + 14 + \ldots + 84}$$

Sol. let a be the first term, d be the common difference and a_n be the last term of given AP.

Thus, $a = 7$, $d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2}$ and $a_n = 84$

Now, $a_n = a + (n-1)d$

⇒ $84 = 7 + (n-1) \times \frac{7}{2}$ ⇒ $84 - 7 = (n-1) \times \frac{7}{2}$

⇒ $77 = (n-1) \times \frac{7}{2}$ ⇒ $11 \times 2 = (n-1)$ ⇒ $22 = n - 1$

∴ $n = 22 + 1 = 23$

Now, $S_n = \frac{n}{2}[2a + (n-1).d]$

⇒ $S_{23} = \frac{23}{2}\left[2 \times 7 + (23-1) \times \frac{7}{2}\right]$ ⇒ $S_{23} = \frac{23}{2}\left[14 + 22 \times \frac{7}{2}\right]$

$= \frac{23}{2}[14 + 77] = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$

Que 7. In an AP: given l = 28, S = 144, and there are total 9 terms. Find a.

Sol. We have, $l = 28$, $S = 144$ and $n = 9$

Now, $l = a_n = 28$

$28 = a + (n-1)d$ ⇒ $28 = a + (9-1)d$

⇒ $28 = a + 8d$ … (i)

And $S = 144$

⇒ $144 = \frac{n}{2}[2a + (n-1)d]$ ⇒ $144 = \frac{9}{2}[2a + (9-1)d]$

⇒ $\frac{144 \times 2}{9} = 2a + 8d$ ⇒ $32 = 2a + 8d$

⇒ $16 = a + 4d$ … (ii)

Now, subtracting equation (ii) from (i), we get

$4d = 12$ or $d = 3$

Putting the value of d in equation (i), we have

$a + 8 \times 3 = 28$

$\Rightarrow$ $a + 24 = 28$ $\Rightarrow$ $a = 28 - 24$

$\therefore$ $a = 4$.

Que 8. How many terms of the AP : 9, 17, 25, … must be taken to give a sum of 636?

Sol. let sum of n term be 636.

$S_n = 636$, $a = 9$, $d = 7 - 9 = 8$

$\Rightarrow$ $\frac{n}{2}[2a + (n - 1) d] = 636$ $\Rightarrow$ $\frac{n}{2}[2 \times 9 + (n - 1) \times 8] = 636$

$\Rightarrow$ $\frac{n}{2} \times 2 [9 + (n - 1) \times 4] = 636$ $\Rightarrow$ $n [9 + 4n - 4] = 636$

$\Rightarrow$ $n [5 + 4n] = 636$ $\Rightarrow$ $5n + 4n^2 = 636$

$\Rightarrow$ $4n^2 + 5n - 636$

$$\therefore \quad n = \frac{-5 \pm \sqrt{(5)^2 - 4 \times (-636)}}{2 \times 4} = \frac{-5 \pm \sqrt{25 + 10176}}{8}$$

$$= \frac{-5 \pm \sqrt{10201}}{8} = \frac{-5 \pm 101}{8} = \frac{96}{8}, \frac{-106}{8} = 12, -\frac{53}{4}$$

But $n \neq \frac{-53}{4}$ So, $n = 12$

Thus, the sum of 12 terms of given AP is 636.

Que 9. How many terms of the series 54, 51, 48 …… be taken so that, their sum is 513? Explain the double answer.

Sol. Clearly, the given sequence is an AP with first term a = 54 and common difference d = - 3. Let the sum of n terms be 513. Then,

$S_n = 513$

$\Rightarrow$ $\frac{n}{2}\{2a + (n - 1) d\} = 513$ $\Rightarrow$ $\frac{n}{2}[108 + (n - 1) \times -3] = 513$

$\Rightarrow$ $n^2 - 37n + 342 = 0$ $\Rightarrow$ $(n - 18)(n - 19) = 0$

$\Rightarrow$ $n = 18$ or 19

Here, the common difference is negative. So, 19th term is given by

$a_{19} = 54 + (19 – 1) \times – 3 = 0$

Thus, the sum of 18 terms as well as that of 19 terms is 513.

Que 10. The first term, common difference and last term of an AP are 12, 6 and 252 respectively. Find the sum of all terms of this AP.

Sol. We have, a = 12, d = 6 and l = 252

Now, l = 252 ⇒ $a_n = 252$

⇒ l = a + (n – 1) d ⇒ 252 = 12 + (n – 1) x 6

⇒ 240 = (n – 1) x 6 ⇒ n – 1 = 40 or n = 41

Thus, $S_n = \frac{n}{2}(a + l)$

⇒ $S_{41} = \frac{41}{2}(12 + 252) = \frac{41}{2}(264) = 41 \times 132 = 5412$

Que 11. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol. We have, $S_7 = 49$

⇒ $49 = \frac{7}{2}[2a + (7 – 1) \times d]$ ⇒ $49 \times \frac{2}{7} = 2a + 6d$

⇒ 14 = 2a + 6d ⇒ a + 3d = 7 …(i)

And $S_{17} = 289$

⇒ $289 = \frac{17}{2}[2a + (17 – 1) d]$ ⇒ $2a + 16d = \frac{289 \times 2}{17} = 34$

⇒ a + 8d = 17 …(ii)

Now, subtracting equation (i) from (ii), we have

5d = 10 ⇒ d = 2

Putting the value of d in equation (i), we have

a + 3 x 2 = 7 ⇒ a = 7 – 6 = 1

Here, a = 1 and d = 2

Now, $S_n = \frac{n}{2}[2a + (n – 1) d]$

$= \frac{n}{2}[2 \times 1 + (n – 1) \times 2] = \frac{n}{2}[2 + 2n – 2] = \frac{n}{2} \times 2n = n^2$

Que 12. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. We have, $a = 5$ and $S_n = 400$

Now, $l = 45$

$\Rightarrow \quad a_n = 45 \quad \Rightarrow \quad a + (n-1)d = 45$

$\Rightarrow \quad 5 + (n-1) \times d = 45 \quad \Rightarrow \quad (n-1)d = 40$ …(i)

Again $S_n = 400 \quad \Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 400$

$\Rightarrow \frac{n}{2}[2 \times 5 + (n-1)d] = 400$ …(ii)

$\frac{n}{2}[10 + 40] = 400$ (Using equation (i)

$\Rightarrow \frac{n}{2} \times 50 = 400 \quad \Rightarrow \quad n = \frac{400}{25} = 16$

Now, putting the value of n in equation (i), we have

$(16-1)d = 40 \quad \Rightarrow \quad 15d = 40$

$\therefore \quad d = \frac{40}{15} = \frac{8}{3}$

Hence, number of terms is 16 and common difference is $\frac{8}{3}$.

Que 13. If the seventh terms of an AP is $\frac{1}{9}$ and its ninth terms is $\frac{1}{7}$, find its 63rd term.

Sol. Given, $a_7 = \frac{1}{9}$ and $a_9 = \frac{1}{7}$

$a_7 = a + (7-1)d = \frac{1}{9}$

$a + 6d = \frac{1}{9}$ …(i)

$a_9 = a + (9-1)d = \frac{1}{7}$

$a + 8d = \frac{1}{7}$ …(ii)

Subtracting (i) from (ii) we get

$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \quad \Rightarrow \quad d = \frac{1}{63}$

Putting the value of d in (i)

$a + \left(6 \times \frac{1}{63}\right) = \frac{1}{9}$ ⇒ $a = \frac{1}{9} - \frac{1}{63} = \frac{7-6}{63}$ ⇒ $a = \frac{1}{63}$

∴ $a_{63} = a + (63 - 1)\, d$

$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{63}{63} = 1$

Que 14. The sum of the 5th and the 9th terms of an AP is 30. If its 25th term is three times its 8th term, find the AP.

Sol. $a_5 + a_9 = 30$

⇒ $(a + 4d) + (a + 8d) = 30$

⇒ $2a + 12d = 30$ ⇒ $a + 6d = 15$

⇒ $a = 15 - 6d$ …(i)

$a_{25} = 3a_8$ ⇒ $a + 24d = 3(a + 7d)$

$a + 24d = 3a + 21d$ ⇒ $2a = 3d$

Putting the value of a form (i), we have

$2\,(15 - 6d) = 3d$ ⇒ $30 - 12d = 3d$

⇒ $15d = 30$ ⇒ $d = 2$

So, $a = 15 - 6 \times 2 = 15 - 12$ [From equation (i)]

⇒ $a = 3$

The AP will be 3, 5, 7, 9…

Que 15. The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161. Find the 28th terms of this AP.

Sol. Sum of first seven terms,

$S_n = \frac{n}{2}\,[2a + (n - 1)\, d]$

$S_7 = \frac{7}{2}\,[2a + (7 - 1)\, d] = \frac{7}{2}\,[2a + 6d]$

⇒ $63 = 7a + 21a$ ⇒ $a = \frac{63 - 21d}{7}$ …(i)

$S_{14} = \frac{14}{2}\,[2a + 13d]$ ⇒ $S_{14} = 7\,[2a + 13d] = 14a + 91d$

But ATQ, $S_{1-7} + S_{8-14} = S_{14}$

$63 + 161 = 14a + 91d$ ⇒ $224 = 14a + 91d$

$2a + 13d = 32$ ...(ii)

$2\left(\frac{63 - 21d}{7}\right) + 13d = 32 \Rightarrow 126 - 42d + 91d = 224$

$\Rightarrow 49d = 98 \Rightarrow d = 2$

$\Rightarrow a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = \frac{21}{7} = 3$

$\Rightarrow a_{28} = a + 27d = 3 + 27 \times 2 \Rightarrow a_{28} = 3 + 54 = 57$

Que 16. If the ratio of the sum of first in terms of two AP's is (7n + 1): (4n + 27), find the ratio of their mth terms.

Sol. $\frac{S_n}{S'_n} = \frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{7n+1}{4n+27} = \frac{a + \frac{n-1}{2}d}{a' + \frac{n-1}{2}d'} = \frac{7n+1}{4n+27}$ ...(i)

Since $\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)d'}$, = So replacing $\frac{n-1}{2}$ by m – 1 and n by 2m – 1 in (i)

$\frac{a+(m-1)d}{a'+(m-1)d'} = \frac{7(2m-1)+1}{4(2m-1)+27} \Rightarrow \frac{t_m}{t'_m} = \frac{14m-6}{8m+23}$