Introduction to Set

Exercise

Solution 1:

1. 6 ∈ {1, 2, 4, 6}6 lies in the set {1, 2, 4, 6}
2. {20}⊂{20, 30, 40}Since, 20 lies in {20, 30, 40}
3. 7 ∈ {x/x is a prime natural number}7 is a prime natural number.
4. 9 ∉ {x/x is multiple of 18}Multiple of 18 are 18, 36, 48 ..... Hence, 9 does not lie in that set.{1, 2, 3}⊂N
5. 1, 2, 3 are natural numbers.
6. {-1, 1, 0}⊄NIn the set {-1, 1, 0}, -1 is not a natural number.
7. If A = {a, b, c} and B = {1, 2, 3}, then A ~ B.n(A) = 3 and n(B) = 3.

Solution 2:

No.	Groups	The listing method	The property method
(1)	Positive multiples of 5	{5, 10, 15, 20, ...}	{x/x is a positive multiple of 5}
(2)	Prime numbers between 21 and 30	{23, 29}	{x/x is a prime between 21 and 30}
(3)	Positive integers smaller than 6	{1, 2, 3, 4, 5}	{x/x is a positive integer smaller than 6}
(4)	Factors of 21	{1, 3, 7, 21}	{x/x is a factor of 21}

Solution 3:

1. {x/x is a prime number less than 3}The only prime less than 3 is 2.

   ∴The given set is {2} which is a singleton set.
2. {5}Since the given set {5} has only one element, it is a singleton set.
3. {x/x + 1 = 1, x ∈ N}x + 1 = 1∴x = 1 − 1∴x = 0But, 0 ∈ N

   Hence, there is not a single element in the given set. So, the given set is an empty set.
4. {x/x is the additive identity} or {x/x is a neural element for addition}

   The additive identity is 0.So, the given set is {0} and has only one element.

   Hence, the given set is a singleton set.

Solution 4:

1. **The given set is a finite set.Reason:**
   The number of citizens of India is a finite positive integer at a given time.
2. **The given set is a finite set.Reason:**
   The number of three digit numbers greater than 100 is a finite positive integer, 899.
3. **The given set is an infinite set.Reason:**
   There are infinite numbers like 7, 17, 27, … having 7 at the units place.
4. **The given set is an infinite set.Reason:** There are infinite prime numbers.

Solution 5:

1. P ~ QHere, n(p) = 3 and n(Q) = 3, but P and Q do not have identical elements.
   Hence, they are not equal sets.
2. F = { } = ØThus, n(F) = 0G = {x/x is four digit number less than 1000}
   There is no four digit number less than 1000.
   ∴G = Ø
   i.e. n(G) = 0
   Thus, we have F = G and F ~ G
3. A = {1, 4, 9, 16} andB = {x/x is a perfect square number less than 25}
   ∴B = {1, 4, 9, 16}∴A = B
   Moreover, all the equal sets are equivalent sets.
   ∴A ~ B
4. Sets D and E have the same elements.
   ∴D = E and D ~ E
5. Here, n(A) = 3 and n(B) = 3.But those elements are different.
   ∴ A ~ B

Solution 6:
U = N = {1, 2, 3, …}
A = {1, 2, 3, …10}
Now, A' = {x/x ∈ U and x ∈ A}
∴ A' = {11, 12, 13, …}
Again, (A')' = {x/x ∈ U and x ∈ A'}
∴ (A')' = {1, 2, 3, …10} = A

Solution 7:
There are two possible one-to-one correspondences between A = {x, y} and B = {a, b}.
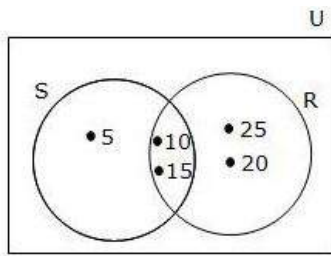
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(i)          (ii)
x ↔ a        x ↔ b
y ↔ b        y ↔ a
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Solution 8:

1. A ∪ B = {x, y, z, w, a, b, c} and A ∩ B = {x, y}Venn diagram is as follows:

2. S ∪ R = {5, 10, 15, 25, 20} and S ∩ R = {10, 15}
   Venn diagram is as follows:

Solution 9:

From the given Venn diagram, we have

A = {0, 1, 4, 5, 7}

B = {3, 4, 6, 7}

C = {2, 5, 6, 7}

U = {0, 1, 2, 3…9}.

1. A ∩ B= {4, 7}
2. = {0, 1, 4, 5, 7} ∩ {3, 4, 6, 7}
3. (A ∪ B) ∪ (B ∪ C)= {0, 1, 3, 4, 5, 6, 7}
   = {2, 3, 4, 5, 6, 7}= {0, 1, 3, 4, 5, 6, 7} ∪ {2, 3, 4, 5, 6, 7}
4. = {0, 1, 2, 3, 4, 5, 6, 7}
5. Now, (A ∪ B) ∪ (B ∪ C)
6. (B ∪ C) = {3, 4, 6, 7} ∪ {2, 5, 6, 7}
7. (A ∪ B) = {0, 1, 4, 5, 7} ∪ {3, 4, 6, 7}
8. A ∩ (B ∪ C)B ∪ C = {3, 4, 6, 7} ∪ {2, 5, 6, 7}Now, A ∩ (B ∪ C)= {4, 5, 7}
9. = {0, 1, 4, 5, 7} ∩ {2, 3, 4, 5, 6, 7}
10. = {2, 3, 4, 5, 6, 7}
11. A = {0, 1, 4, 5, 7}
12. (A ∪ C) ∩ B= {0, 1, 2, 4, 5, 6, 7}Now, (A ∪ C) ∩ B= {4, 6, 7}
13. = {0, 1, 2, 4, 5, 6, 7} ∩ {3, 4, 6, 7}
14. B = {3, 4, 6, 7}
15. A ∪ C = {0, 1, 4, 5, 7} ∪ {2, 5, 6, 7}
16. U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Practice 1

Solution 1:

1. 3 ∈ {1, 2, 3, 4} Since 3 lies in the set {1, 2, 3, 4}
2. 100 ∉ {1, 2, 3, ….., 99}Since 100 does not lie in the set {1, 2, 3, …., 99}
3. 5 ∉ {x/x is a multiple of 10}{x/x is a multiple of 10} = {…, -20, -10, 0, 10, 20, …}
   Hence, 5 is not an element of this set.
4. 2 ∉{x/x is a prime factor of 15}Prime factors of 15 are 3 and 5.
   Thus, {x/x is a prime factor of 15} = {3, 5}.Hence, 2 is not an element of this set.
5. 0 ∉ {x/x is a natural number}0 is not a natural number.

Solution 2:

No.	Groups	The listing method
1.	The district of Gujarat starting with 'A'	{Ahmedabad, Amrali, Anand}
2.	Such a district of Gujarat whose boundary touches Uttar Pradesh	φ or {}
3.	The bay lying in Gujarat	{Bay (gulf) of Khambhat, Bay (gulf) of Kachchh}
4.	The smallest district	{Dang}
5.	The largest district	{Kachchh}

Solution 3:

1. Singleton sets in the given table: {Dang}, {Kachchh}
2. All the sets in the given table are finite sets. An empty set is also a finite set.
3. The set corresponding to the group of "Such a district of Gujarat whose boundary touches Uttar Pradesh" is any empty set because the boundary of not a single district of Gujarat touches Uttar Pradesh.

Practice 2

Solution 1:

1. $N \subseteq Z$

   'N' denotes Natural numbers and 'Z' denotes Integers.
   Since natural numbers are positive integers, set N is subset of set Z.

2. $\{3, 1, -1\} \not\subseteq N$

   -1 is not a natural number.
   $\therefore -1 \notin N$
   $\therefore \{3, 1, -1\} \not\subseteq N$

3. $Z \subseteq Q$

   'Z' denotes Integers and 'Q' denotes Rational numbers.
   Since, group of integers are Rational numbers, $Z \subset Q$

4. $\left\{\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}\right\} \subseteq Q$

   'Q' denotes Rational Numbers.
   Since $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}$ are of the form $\dfrac{p}{q}$, where $q \neq 0$.

Solution 2:

(1) $3 \in A$

(2) $\dfrac{1}{2} \notin A$

(3) $A = \varnothing$

(4) $A = B$

(5) $C \subset D$

(6) $B \sim C$

(7) $A \not\subset B$

(8) $\{0\} \subset B$

Solution 3:

A = {x/x is an even natural number less than 10}

Thus, A = {2, 4, 6, 8}

B = {-2, -3, -4, -5}

Here, n(A) = 4 and n(B) = 4

Since n(A) = n(B) = 4, set A and set B are equivalent sets.

Symbolically, A ~ B.

Sets A and B do not have the same elements. Hence, set A and set B are not equal sets.

Practice 3

Solution 1:

1. U = {x/x is name of months of English calendar}Thus,

   U = {January, February, March…December}

   A = {March, May, July, June}Now, A' = {x/x ∈ U and x ∉ A}

   ∴A' = {January, February, April, August, September, October, November, December}

2. U = {x/x is main colour of rainbow}Thus,

   U = {Violet, Indigo, Blue, Green, Yellow, Orange, Red}

   R = {Violet, Red, Yellow}Now, R' = {x/x ∈ U and x ∉ R}

   ∴R' = {Indigo, Blue, Green, Orange}

3. U = {x ∈ N/x ≤ 9}Thus,

   U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

   A = {2, 3, 5}Now, A' = {x/x ∈ U and x ∉ A}

   ∴A' = {1, 4, 6, 7, 8, 9}Again, (A')' = {x/x ∈ U and x ∉ A'}

   ∴(A')' = {2, 3, 5} = AB = {4, 5, 7}Now, B' = {x/x ∈ U and x ∉ B}

   ∴B' = {1, 2, 3, 6, 8, 9}

   Again, (B')' = {x/x ∈ U and x ∉ B'}

   ∴ (B')' = {4, 5, 7} = B

Solution 2:

From the Venn diagram, we have

A = {0, 1, 2, 3, 5}

B = {2, 3, 4, 8, 9}

C = {3, 4, 5, 6, 7}

U = {0, 1, 2, …12}

1. A ⋃ B= {0, 1, 2, 3, 4, 5, 8, 9}
2. = {0, 1, 2, 3, 5} ⋃ {2, 3, 4, 8, 9}
3. A ⋂ B= {2, 3}
4. = {0, 1, 2, 3, 5} ⋂ {2, 3, 4, 8, 9}
5. For (A ⋂ C) ⋃ B,(A ⋂ C) ⋃ B = {3, 5} ⋃ {2, 3, 4, 8, 9}
6. ∴= {2, 3, 4, 5, 8, 9}
7. A ⋂ C = {0, 1, 2, 3, 5} ⋂ {3, 4, 5, 6, 7} = {3, 5}
8. For (A ⋃ C) ⋃ B,= {0, 1, 2, 3, 4, 5, 6, 7}∴= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
9. (A ⋃ C) ⋃ B = {0, 1, 2, 3, 4, 5, 6, 7} ⋃ {2, 3, 4, 8, 9}
10. A ⋃ C = {0, 1, 2, 3, 5} ⋃ {3, 4, 5, 6, 7}
11. The Universal set, U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

Solution 3:

A = {x/x is a natural number less than 5}

∴ A = {1, 2, 3, 4}

B = {x/3 < x < 7; x ∈ N}

∴ B = {4, 5, 6}

Now, A $\bigcup$ B = {1, 2, 3, 4} $\bigcup$ {4, 5, 6}

= {1, 2, 3, 4, 5, 6}

And A $\bigcap$ B = {1, 2, 3, 4} $\bigcap$ {4, 5, 6}

= {4}

A ∪ B   A ∩ B