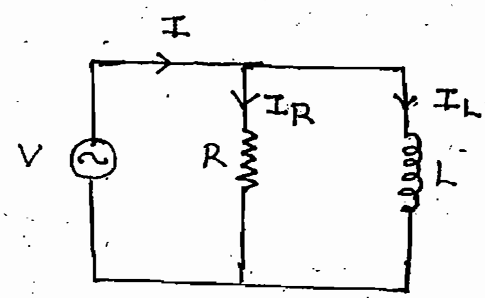
Lecture - 5

R-L Parallel circuit:-



By KCL

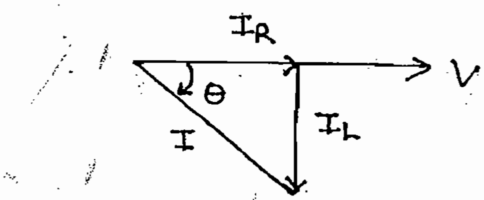
$I = I_R\angle 0 + I_L\angle -90^\circ$

$\Rightarrow \frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L}$

$\Rightarrow VY = VG - jVB_L$

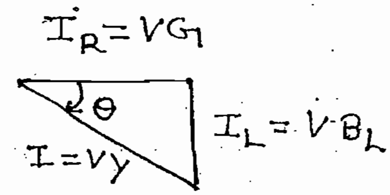
$\Rightarrow \boxed{Y = G - jB_L}$

mho mho mho



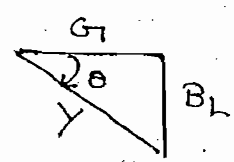
$\boxed{I = \sqrt{I_R^2 + I_L^2}}$

$\boxed{\theta = \tan^{-1}\left(\frac{-I_L}{I_R}\right)}$



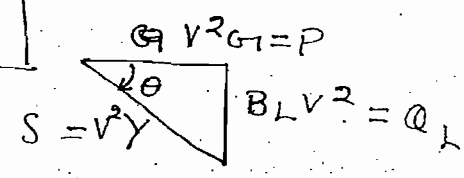
$\boxed{Y = \sqrt{G^2 + B_L^2}}$

$\boxed{\theta = \tan^{-1}\left(-\frac{B_L}{G}\right)}$



$\boxed{S = \sqrt{P^2 + Q_L^2}}$

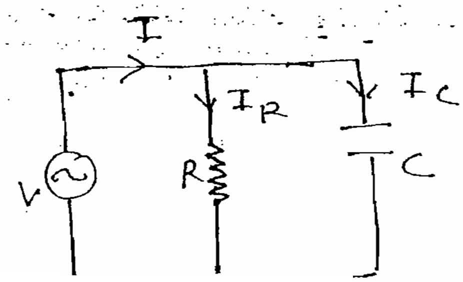
$\boxed{\theta = \tan^{-1}\left(\frac{-Q_L}{P}\right)}$



Power factor:-

$\rightarrow \boxed{\cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S} = \text{Lagging}}$

R-C Parallel circuit:-



By KCL

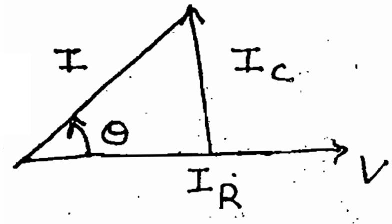
$I = I_R\angle 0 + I_C\angle +90$

$\Rightarrow \frac{V}{Z} = \frac{V}{R} + j\frac{V}{X_C}$

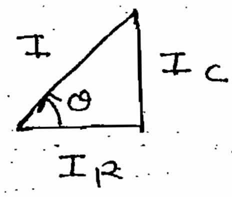
$\Rightarrow \quad V.Y = VG + jVB_c$

$\Rightarrow \quad \boxed{Y = G + jB_c}$

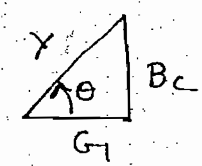
$\downarrow$ Mho $\quad \downarrow$ Mho $\quad \downarrow$ Mho



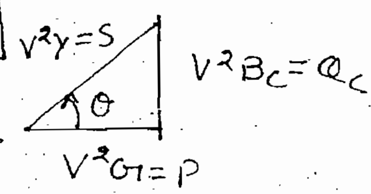
$$\boxed{I = \sqrt{I_R^2 + I_C^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{I_C}{I_R}\right)}$$



$$\boxed{Y = \sqrt{G^2 + B_C^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{B_C}{G}\right)}$$



$$\boxed{S = \sqrt{P^2 + Q_C^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{Q_C}{P}\right)}$$



Power Factor $\Rightarrow$

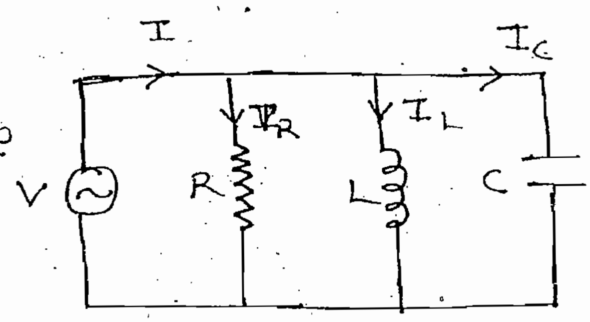
$$\boxed{\cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S} = \text{leading}}$$

RLC Parallel Circuit :-

By KCL

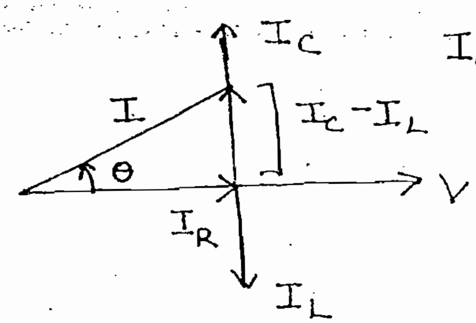
$$I = I_R\angle 0 + I_L\angle -90 + I_C\angle +90$$

$$\Rightarrow \frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_C}$$



$$\Rightarrow VY = V\left[G + j(B_C - B_L)\right]$$

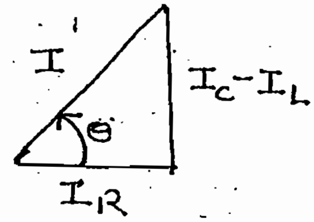
$$\Rightarrow \quad \boxed{Y = G + j(B_C - B_L)}$$



$I_C > I_L$

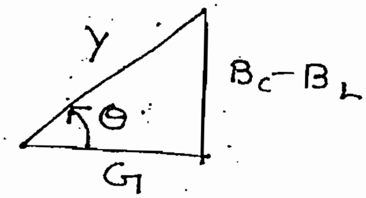
$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\theta = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$$



$$Y = \sqrt{G^2 + (B_C - B_L)^2}$$

$$\theta = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)$$



$$\text{Power Factor} = \cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S}$$

(I) $I_C > I_L \rightarrow$ leading

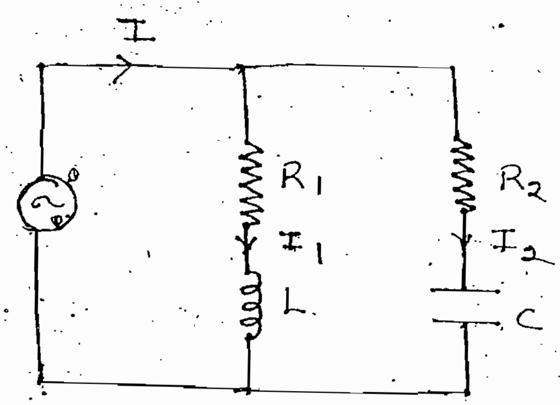
(II) $I_C < I_L \rightarrow$ lagging

(IV) $I_C = I_L \rightarrow$ Unity Power factor

<u>Note:-</u>

When combinational elements are present then we can't directly find conductance, admittance etc. Then following procedure is used

$$I_1 = \left(\frac{V}{R_1 + jX_L}\right)\left(\frac{R_1 - jX_L}{R_1 - jX_L}\right)$$



$$\Rightarrow \frac{V}{Z_1} = V\left[\frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}\right]V$$

$$\Rightarrow \boxed{Y_1 = G_1 - jB_L}$$

$$I_2 = \frac{V}{R_2 - jX_C} \; \frac{R_2 + jX_C}{R_2 + jX_C}$$

$$\Rightarrow \frac{V}{Z_2} = V\left[\frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}\right]$$

$$\boxed{Y_2 = G_2 + jB_C}$$

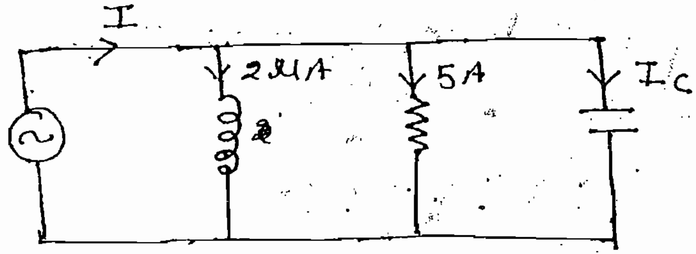
$$I = I_1 + I_2$$

$$\Rightarrow \quad V Y_{eq} = V Y_1 + V Y_2$$

$$\Rightarrow \quad Y_{eq} = Y_1 + Y_2$$

$$\Rightarrow \quad \boxed{Y_{eq} = (G_1 + G_2) + j(B_C - B_L)}$$

Ques:- Find I_C and I of the circuit shown



Soln:-

$$13 = \sqrt{5^2 + I_C^2} \quad \Rightarrow \quad I_C = 12A$$

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\Rightarrow \quad I = \sqrt{5^2 + (12 - 24A)^2} = 13\ A$$

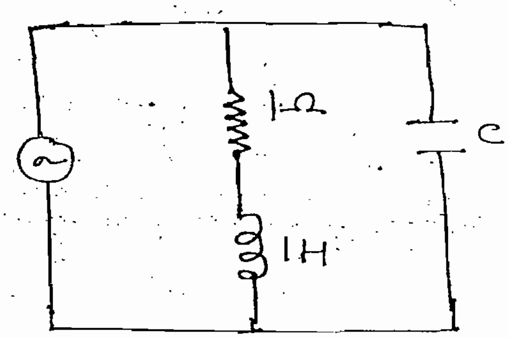
Note:-

A.C $\longrightarrow$ (KVL; KCL) $\longrightarrow$ Phasor sum

D.C $\longrightarrow$ (KVL; KCL) $\longrightarrow$ Arithmatic sum

Ques:- Find capacitance of the capacitor when power factor of the circuit is 0.8 lagging

$v(t) = \sin t$



Soln:- For Branch-1

$$X_L = \omega L = 1$$

$$G_1 = \frac{R_1}{R_1^2 + X_L^2} = \frac{1}{1^2 + 1^2} = \frac{1}{2}$$

$$B_L = \frac{X_L}{R_1^2 + X_L^2} = \frac{1}{1^2+1^2} = \frac{1}{2}$$

$$Y_1 = G_1 - jB_L$$

$$\Rightarrow Y = \frac{1}{2} - j\frac{1}{2}$$

For Branch-2

$$Y_2 = +jB_C$$

$$Y_2 = j\frac{1}{X_C} = j\omega C \quad \Rightarrow \quad Y_2 = \omega C$$

$$Y_{eq} = Y_1 + Y_2$$

$$\Rightarrow Y_{eq} = \frac{1}{2} + j\cdot\left(C - \frac{1}{2}\right)$$

$$\cos\theta = \frac{G_1}{\sqrt{G_1^2 + (B_C - B_L)^2}}$$

$$\Rightarrow 0.8 = \frac{1/2}{\sqrt{(1/2)^2 + (C - 1/2)^2}}$$

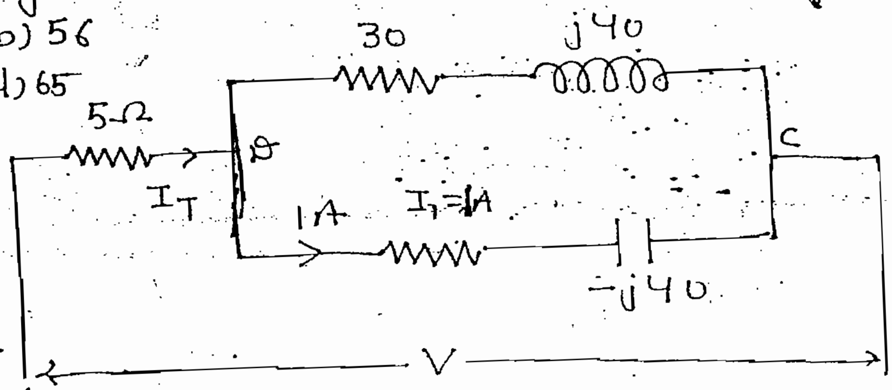
$$\Rightarrow C = \frac{7}{8} \text{ or } \frac{1}{8}$$

Since $B_L > B_C$

$\frac{1}{2} > C$ Hence, $\boxed{C = \frac{1}{8}}$ Ans

Ques:- Find voltage across A and B of the circuit shown (a) 55 (b) 56 (c) 60 (d) 65

Soln:-



Apply current division technique,

A B

$$I_1 = I_T \frac{30 + j40}{30 + j40 + 30 - j40} =$$

$$\Rightarrow \quad 1 = I_T \cdot \frac{50 \angle \tan^{-1}(4/3)}{60}$$

$$\Rightarrow \quad I_T = \frac{60}{50 \angle \tan^{-1}(4/3)} = 1.2 \angle \tan^{-1}(-4/3)$$

$$V_{AD} = I_T \times 5$$

$$\Rightarrow \quad V_{AD} = (1.2 \times 5) \angle \tan^{-1}(-4/3)$$

$$\Rightarrow \quad V_{AD} = 6 \angle \tan^{-1} 4/3$$

$$V_{DC} = (30 - j40)\, 1$$

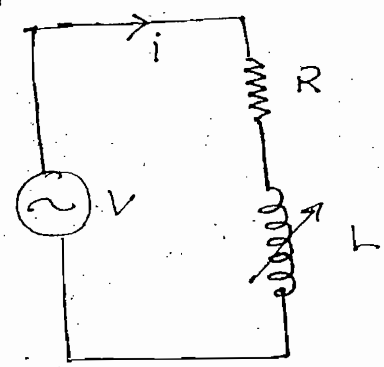
$$V_{DC} = 50 \angle \tan^{-1}(-4/3)$$

Angles are same hence they can be added

$$V_{AB} = V_{AC} = \cancel{V_D}\ V_{AD} + V_{DC}$$

$$\Rightarrow \quad \boxed{V_{AB} = 56 \angle \tan^{-1}(-4/3)} \quad \text{Ans}$$

Locus Diagram :—



$$X_L = 2\pi f L$$

$$X_L = 0$$

$$Z = R \quad \Rightarrow I = \frac{V}{R}$$

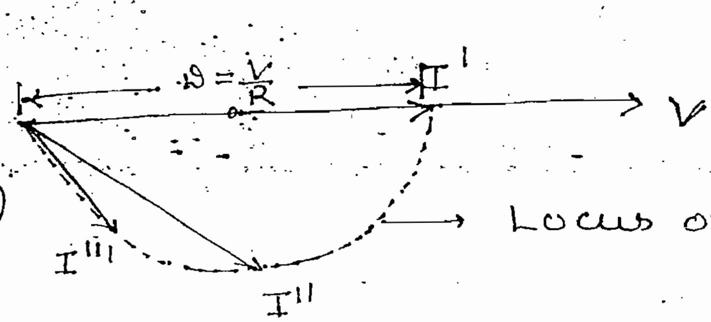
$$\Rightarrow \quad \theta = 0$$

$X_L \uparrow$ | $Z \uparrow$ $\quad I \downarrow$ $\qquad \boxed{\theta = \tan^{-1}\left(\frac{X_L}{R}\right)} \uparrow$

$X_L \simeq \infty \quad Z \simeq \infty \quad I = 0$

→ semicircle

→ centre $\left(\frac{V}{2R}, 0\right)$



→ Locus of I

→ Locus diagram are useful for analysis and designing of the circuit eg:- filters

→ With respect to practical application it is possible to develop the following locus diagram

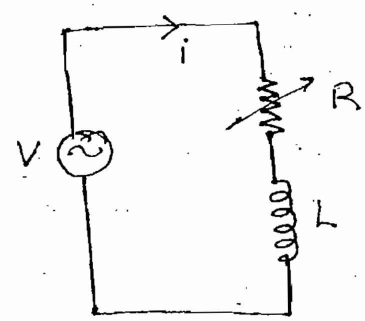
(I) current locus diagram
(II) Voltage locus diagram
(III) Impedance locus diagram
(IV) Admittance locus diagram

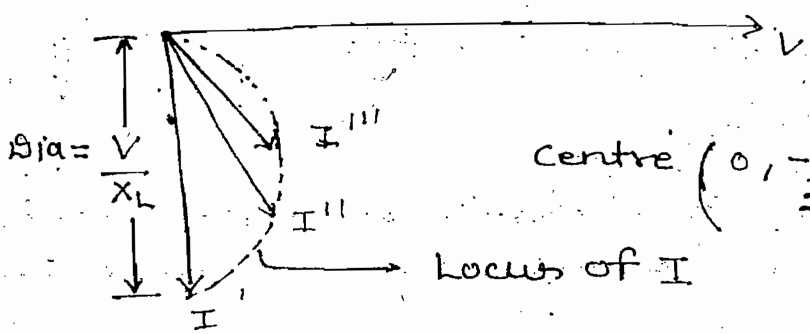
→ The path traced by terminous of current vectors by varying either any of the circuit elements or by varying source frequency is called as current locus

→ In the above circuit by keeping all the elements constant and by varying source frequency also same shape of the current locus diagram is obtained.

→ Develop current locus of the circuit shown

$R=0$	$R\uparrow$
$Z=X_L$	$Z\uparrow$
$I=\frac{V}{X_L}$	$I\downarrow$
$\theta=90°$	$\theta=\tan^{-1}\left(\frac{X_L}{R\uparrow}\right)\downarrow$

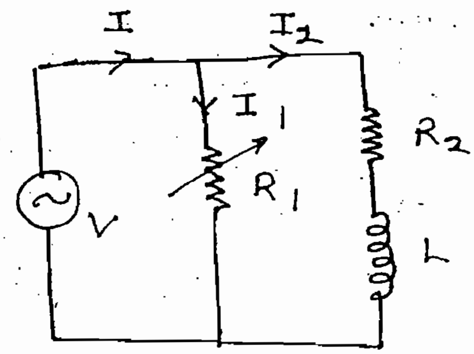




Centre $\left(0, \frac{-V}{2X_L}\right)$

→ Diameter is always due to constant elemen

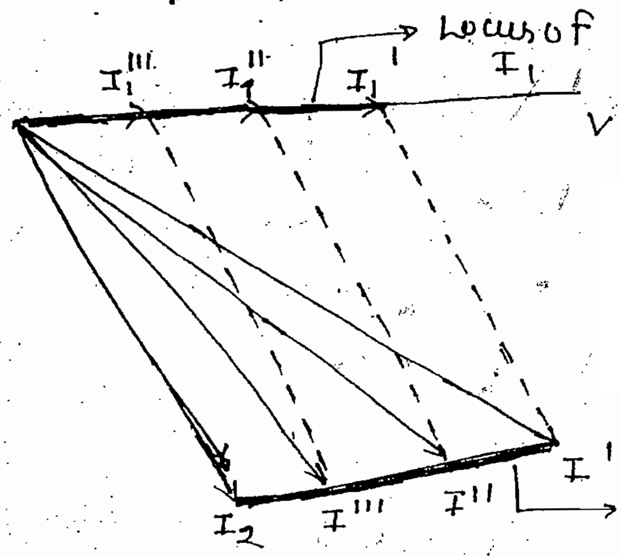
Ques:- Develop current locus of I_1 and I of the circuit shown:-



Soln:-

$R_1 \simeq 0.1\Omega$	$R_1 \uparrow$	$R_1 \simeq \infty$
$I_1 = \frac{V}{R_1}$	$I_1 \downarrow$	$I_1 = 0$
$\theta_1 = 0$	$\theta_1 = 0$	$I_2 = I$
$I = I_1 + I_2$	$I_2 =$ Constant	
	$I \downarrow$	

$$I_2 = \frac{V}{R_2 + jX_L}$$

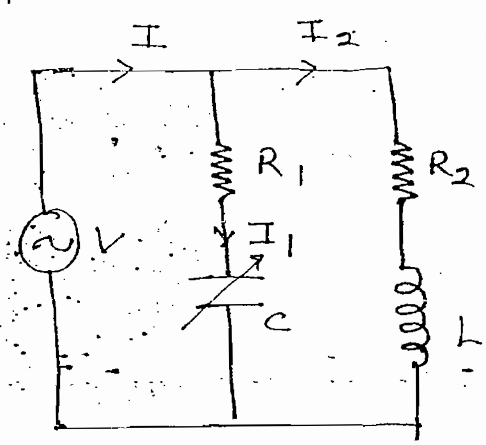


Note:-

→ When power factor angle is variable the shape of current locus is semi-circle

→ When power factor angle is constant the shape of the current locus diagram is straight line.

Ques:- Develop current locus of I_1 and I of the circuit shown

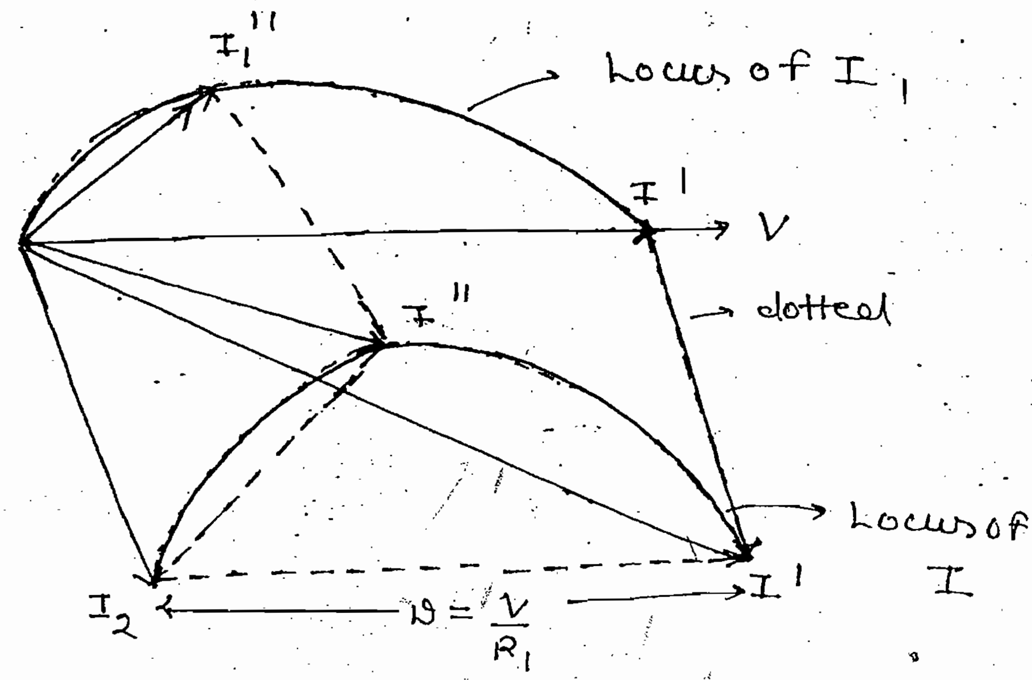


Soln:-

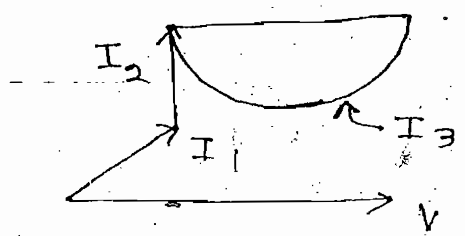
$X_C = \frac{1}{2\pi f C}$	$X_C \uparrow$	$X_C \simeq \infty$
$X_C \simeq 0$	$Z_1 = \sqrt{R_1^2 + X_C^2} \uparrow$	$I_1 = 0$
$I_1 = \frac{V}{R_1}$	$I_1 \downarrow$	$I = I_2$
$\theta_1 = 0$	$\theta_1 = \tan^{-1}\left(\frac{-X_C}{R}\right) \uparrow$	
$I = I_1 + I_2$	$I_2 =$ Constant	
	$I_1 \downarrow$	

$$I_2 = \frac{V}{R_2 + jX_L}$$

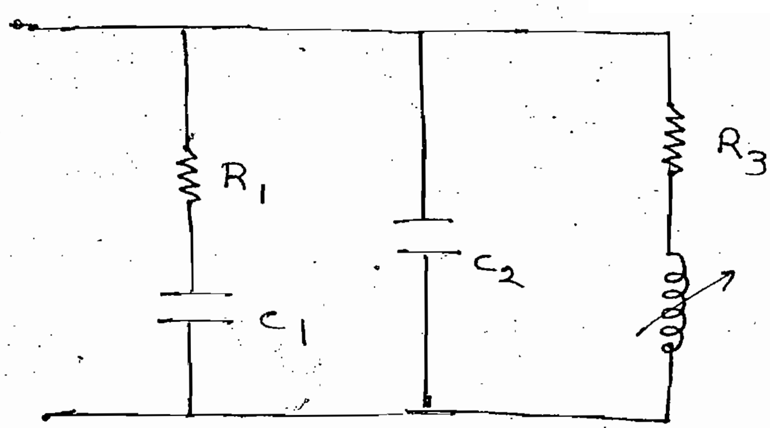
Focus only angles not sign but focus on magnitude



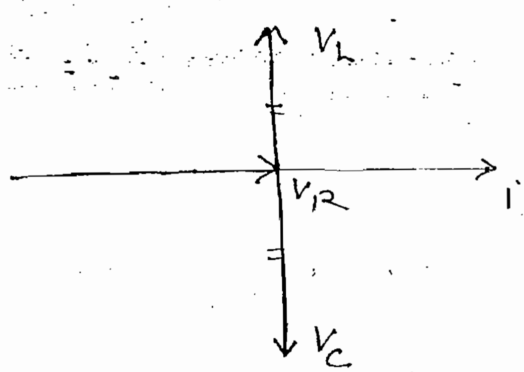
Ques:- Design a N/w for given current locus diagram

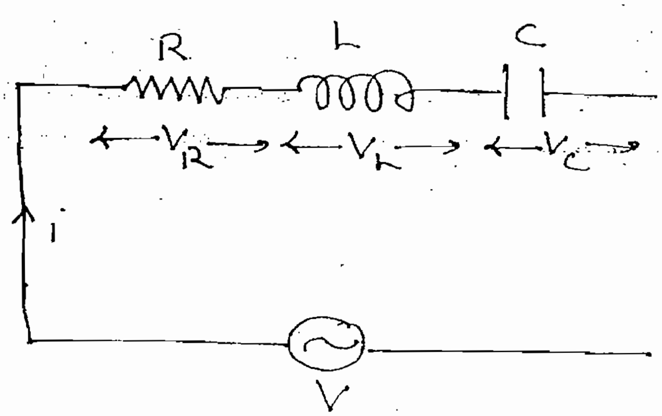


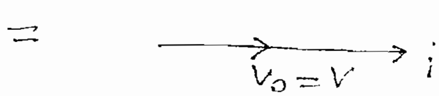
Soln:-



Resonance :-







By KVL, $V = V_R \angle 0 + V_L \angle 90 + V_C \angle -90$

At resonance,

$$V_L = V_C$$

$$I X_L = I X_C$$

$$\Rightarrow \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \text{ rad/sec}$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

→ For occurence of resonance in any circuit two energies are required. In RLC circuit inductor is having energy in the form of magnetic field capacitor is having energy in the form of electric field. When these two energies are present at particular frequency wide variations are present in the system is called as resonance

→ The circuit is said to be resonance when source current is ~~influence~~ phase with source voltage

→ The frequency at which $X_C = X_L$ is called as resonant frequency

→ The resonant frequency indicates rate at which energy transformation is done b/w inductor and capacitor

1 → $$Z = R + j\underbrace{(X_L - X_C)}_{=0}$$

$$Z_{min} = R$$

2 → $$I_{max} = \frac{V}{Z_{min}} = \frac{V}{R}$$

3 → $\cos\theta = 1$

4 → $V_R = V$

5. Net Reactive.

6. Voltage across inductor or voltage across ~~capacit~~ capacitor greater than source. This phenomenon is called as **voltage magnification**

Application:—

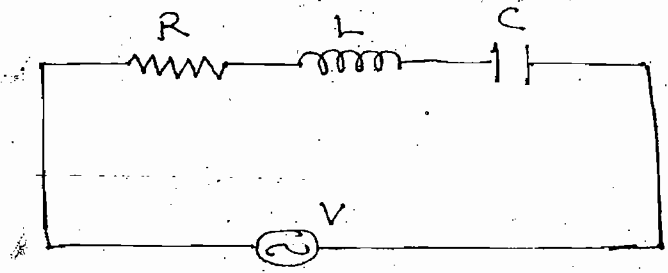
→ Oscillators

→ Filters (BP, BE)

→ Tuning circuits

→ Induction heating

Variation of voltage across inductor and voltage across capacitor w.r.t frequency:—



$V_C = I X_C$

$X_L = 2\pi f L$, $X_C = \frac{1}{2\pi f C}$

→ $f = 0$, $X_L = 0$, $X_C = \infty$

→ $X = |X_L - X_C| = \infty$

=> $Z = \infty$ & $I = 0$ => $\boxed{V_C = V}$

~~For inductor~~:—

→ $f\uparrow$ $X_L\uparrow$ $X_C\downarrow$ $X = |X_L - X_C|\downarrow\downarrow$ $Z\downarrow\downarrow$ $I\uparrow\uparrow$ $\boxed{V_C\uparrow}$

(10Ω) (100Ω) (90Ω)

→ $f\uparrow\uparrow$ $X_L\uparrow\uparrow$ $X_C\downarrow\downarrow$ $Z = R + j(X_L - X_C)\uparrow\uparrow\uparrow$ $I\downarrow\downarrow\downarrow$

(very low) $V_C\downarrow$

→ $$V_C = \frac{V X_C}{\sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow V_C = \frac{V\frac{1}{\omega C}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \quad \text{—(1)}$$

Differentiate eq-(1) w.r.t ω and equal to zero we get

$$\boxed{f_C = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \left(\frac{R^2 C}{2L}\right)^2}}$$

JL

For inductor :-

→ $V_L = I X_L$

→ $X_L = 2\pi f L$ & $X_C = \frac{1}{2\pi f C}$

→ $f = 0$, $X_L = 0$, $X_C = \infty$ $X = |X_L - X_C| = \infty$

$Z = \infty$, $I = 0$, $\boxed{V_L = 0}$

→ $f\uparrow$ $X_L\uparrow$ $X_C\downarrow$ $X = |X_L - X_C| \downarrow\downarrow$ $Z\downarrow\downarrow$

(10Ω) (100Ω)

$I\uparrow\uparrow$ $\boxed{V_L\uparrow}$

→ $f\uparrow\uparrow$ $X_L\uparrow\uparrow$ $X_C\downarrow\downarrow$

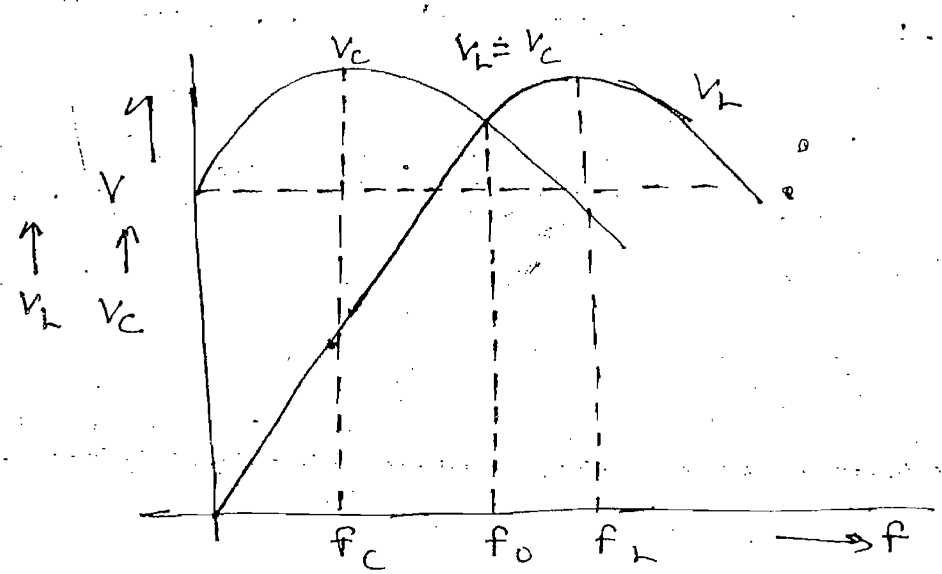
(Very low)

$Z = R + j(X_L - X_C)\uparrow\uparrow\uparrow$ $I_C\downarrow\downarrow\downarrow$ $\boxed{V_L\downarrow}$

→ $$V_L = \frac{V X_L}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V\omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})}} \quad -(11)$$

Differentiate eq-(11) w.r.t ω and equal to zero

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1 - \left(\frac{R^2 C}{2L}\right)}}}$$



Quality factor :–

→ Q-factor is a ratio of max. energy stored the circuit to power dissipation per cycle

$$Q =$$

$$i(t) = I_m \sin\omega t \quad —(1)$$

$$V_c(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int I_m \sin\omega t\, dt$$

$$\Rightarrow V_c(t) = -\frac{I_m}{\omega C}\cos\omega t \quad —(11)$$

$$W_g = \frac{1}{2}Li^2 + \frac{1}{2}CV_c^2$$

$$\Rightarrow W_g = \frac{1}{2}L(I_m \sin\omega t)^2 + \frac{1}{2}C\left(-\frac{I_m}{\omega C}\cos\omega t\right)^2$$

$$\Rightarrow W_g = \frac{1}{2}LI_m^2 \sin^2\omega t + \frac{1}{2}C\,\frac{I_m^2}{\omega^2 C^2}\cos^2\omega t$$

$$\Rightarrow W_g = \frac{1}{2}LI_m^2 \sin^2\omega t + \frac{1}{2}LI_m^2\cos^2\omega t \qquad \left[\omega^2 = \frac{1}{LC},\ L = \frac{1}{\omega^2 C}\right]$$

$$\Rightarrow \boxed{W_g = \frac{1}{2}LI_m^2} \qquad \boxed{W_g = \frac{1}{2}CV_{cm}^2}$$

$$\left(\because V_{cm} = \frac{I_m}{\omega C}\right)$$

$$Q = \frac{\frac{1}{2}LI_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R\,\frac{1}{\omega}} \qquad I^2R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$$\Rightarrow \boxed{Q = \frac{\omega L}{R}} \qquad \left(\omega = \frac{1}{\sqrt{C}}\right)$$

$$\Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

→ $$\boxed{Q = \frac{\omega L}{R} = \frac{IX_L}{IR} = \frac{V_L}{V_R} = \frac{V_L}{V} \quad (V_R = V)}$$

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$\rightarrow$ $\boxed{Q = \frac{V_L \text{ or } V_C}{V}}$ $\quad (\because V_L = V_C)$

$\rightarrow$ $\boxed{Q = \frac{I^2 X_L}{I^2 R} = \frac{Q_L}{P}}$

$\rightarrow$ $\boxed{Q = \frac{X_L \text{ or } X_C}{R}}$ $\quad (\because X_L = X_C)$

$\rightarrow$ $\boxed{Q = \frac{X_C}{R} = \frac{1}{\omega RC}}$

$\rightarrow$ $Q > 1$, $X_L > R$, $X_C > R$

$\rightarrow$ $Q \propto \frac{1}{\text{Power loss } (I^2R = P)}$

$\rightarrow$ $Q \propto \frac{1}{BW}$

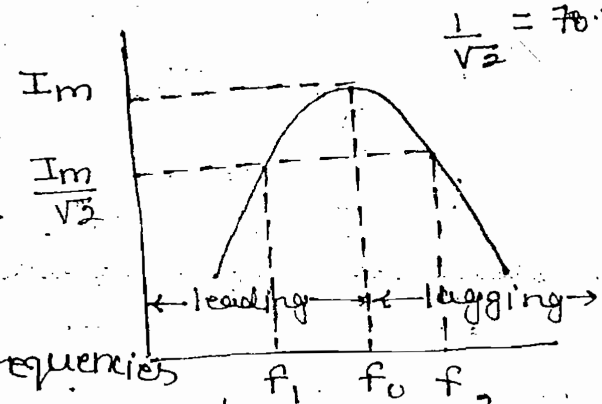
Note:-

$\rightarrow$ To obtain high efficiency circuit is designed with high Q-factor

$\rightarrow$ To obtain wide BW circuit is designed with low Q-factor

Bandwidth:-

$\frac{1}{\sqrt{2}} = 70.7\%$



$\rightarrow$ When the curve is developed b/w current and frequency then curve is called as resonance curve

$\rightarrow$ BW is the range of frequencies on either side of the resonant frequencies where the current falls from max. value to 70.7% of the max. value and it is given by

$$BW = f_2 - f_1$$

where f_2 = upper cut-off frequency

f_1 = lower " " "

f_1, f_2 = 3dB points or half power frequencies

1. $f_0 \rightarrow I_m$, $Z = R$

$f_1, f_2 \longrightarrow \frac{I_m}{\sqrt{2}}$ $\Rightarrow$ $\boxed{Z = \sqrt{2} R}$

2. $f_0 \longrightarrow \cos\theta = 1$

$f_1, f_2 \longrightarrow Z = R \pm jX$

$Z = \sqrt{R^2 + X^2} = \sqrt{2} R$

$\Rightarrow \boxed{X = R}$

$X = X_L - X_C$

$X_L = 2\pi f L$

$X_C = \frac{1}{2\pi f C}$

$f_1 \rightarrow X_C > X_L$

$X \longrightarrow -ve$

$f_2 \longrightarrow X_L > X_C$

$X \rightarrow +ve$

w.r.t

$\underline{f_1} \longrightarrow Z = R - jX$ $X = R$

Impedance Angle $= \tan^{-1}\left(\frac{-X}{R}\right)$

$= -45°$

$$I = \frac{V\angle 0}{Z\angle -45} = \frac{V}{Z} \angle +45$$

Power factor angle = 45

$\Rightarrow$ Power factor $= \cos 45 = \frac{1}{\sqrt{2}} \rightarrow$ leading

w.r.t f_2 :-

$f_2 \longrightarrow Z = R + jX$ $X = R$

Impedance angle $= \theta = \tan^{-1}(X/R) = +45°$

Power factor angle $= -45°$

Power factor $= \cos(-45°) = \frac{1}{\sqrt{2}} \rightarrow$ lagging

3. $f_1 \longrightarrow X_C > X_L$ $X = R$

$\frac{1}{\omega_1 C} - \omega_1 L = R$ —— (1)

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$f_2 \longrightarrow X_L > X_C \qquad X = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{— (II)}$$

From (I) & (II)

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{— (III)}$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC} \quad \text{— (IV)}$$

From (III) & (IV)

$$\omega_0^2 = \omega_1 \omega_2$$

$$\Rightarrow \boxed{\begin{aligned} \omega_0 &= \sqrt{\omega_1 \omega_2} \\ f_0 &= \sqrt{f_1 f_2} \end{aligned}}$$

Add eq-(I) & (II)

$$\frac{1}{C}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\Rightarrow \frac{1}{C}\left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\Rightarrow L\left[\omega_2 - \omega_1\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\boxed{\begin{aligned} BW &= \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec} \\ BW &= f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz} \end{aligned}}$$

4. $Q = \dfrac{\omega_0 L}{R} \Rightarrow Q = \dfrac{\omega_0}{R/L}$

$$\Rightarrow \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}}$$