Long Answer Questions

Q. 1. Find an expression for the electric field strength at a distant point situated (i) on the axis and (ii) along the equatorial line of an electric dipole.
[CBSE (AI) 2013, (F) 2015]

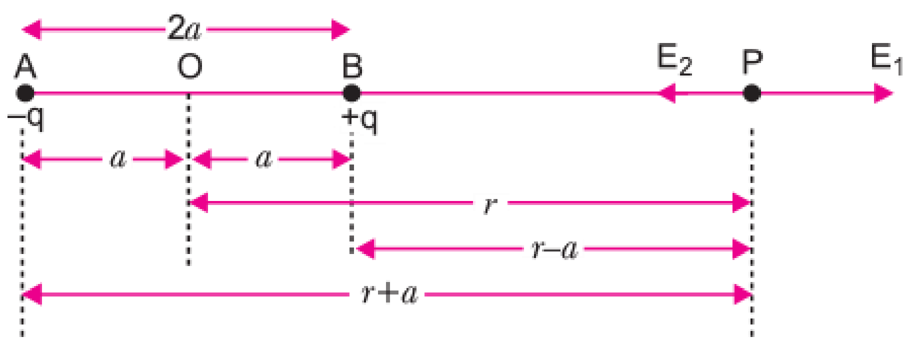
OR

Derive an expression for the electric field intensity at a point on the equatorial line of an electric dipole of dipole moment $\vec{p}$ and length 2a. What is the direction of this field?
[CBSE South 2016]

Ans. Consider an electric dipole AB. The charges –q and +q of dipole are situated at A and B respectively as shown in the figure. The separation between the charges is 2a.

Electric dipole moment, p = q.2a

The direction of dipole moment is from –q to +q.



(i) At axial or end-on position: Consider a point P on the axis of dipole at a distance r from mid-point O of electric dipole.

The distance of point P from charge +q at B is

BP = r – a

And distance of point P from charge –q at A is, AP = r + a.

Let E_1 and E_2 be the electric field strengths at point P due to charges +q and –q respectively.

We know that the direction of electric field due to a point charge is away from positive charge and towards the negative charge. Therefore,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ (from } B \text{ to } P\text{) and } E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2} \text{ (from } P \text{ to } A\text{)}$$

Clearly the directions of electric field strengths $\overrightarrow{E}_1$ are $\overrightarrow{E}_2$ along the same line but opposite to each other and $E_1 > E_2$ because positive charge is nearer.

∴ The resultant electric field due to electric dipole has magnitude equal to the difference of E_1 and E_2 direction from B to P i.e.

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4\,\mathrm{ra}}{(r^2-a^2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{2(q2a))r}{(r^2-a^2)^2}$$

But $q.2l = p$ (electric dipole moment)

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{2\,\mathrm{pr}}{(r^2-a^2)^2} \qquad \ldots(i)$$

If the dipole is infinitely small and point P is far away from the dipole, then r >> l, therefore equation (i) may be expressed as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\,\mathrm{pr}}{r^4} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \qquad ..(\mathrm{ii})$$

This is the expression for the electric field strength at axial position due to a short electric dipole.

(ii) At a point of equatorial line: Consider a point P on broad side on the position of dipole formed of charges +q and –q at separation 2a. The distance of point P from mid-point (O) of electric dipole is r. Let $\overrightarrow{E}_1$ and $\overrightarrow{E}_2$ be the electric field strengths due to charges +q and – q of electric dipole.

From fig. AP $=$ BP $= \sqrt{r^2 + a^2}$

$\therefore \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2+a^2}$ along B to P

$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2+a^2}$ along P to A

Clearly $\vec{E}_1$ and $\vec{E}_2$ are equal in magnitude *i.e.* $|\vec{E}_1| = |\vec{E}_2|$ or $E_1 = E_2$

To find the resultant of $\vec{E}_1$ and $\vec{E}_2$, , we resolve them into rectangular components.

Component of $\vec{E}_1$ parallel to $AB = E_1 \cos\theta$, in the direction to $\overrightarrow{\mathrm{BA}}$

Component of $\vec{E}_1$ perpendicular to $AB = E_1 \sin\theta$ along OP

Component of $\vec{E}_2$ parallel to $AB = E_2 \cos\theta$ in the direction $\overrightarrow{\mathrm{BA}}$

Component of $\vec{E}_2$ perpendicular to $AB = E_2 \sin\theta$ along PO

Clearly, components of $\vec{E}_1$ and $\vec{E}_2$ perpendicular to AB: $E_1 \sin\theta$ and $E_2 \sin\theta$ being equal and opposite cancel each other, while the components of $\vec{E}_1$ and $\vec{E}_2$ parallel to AB : $E_1 \cos\theta$ and $E_2 \cos\theta$, being in the same direction add up and give the resultant electric field whose direction is parallel to $\overrightarrow{\mathrm{BA}}$.

$\therefore$ Resultant electric field at P is $E = E_1 \cos\theta + E_2 \cos\theta$

But $E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2+a^2)}$

From the figure, $\cos\theta = \frac{\mathrm{OB}}{\mathrm{PB}} = \frac{l}{\sqrt{r^2+a^2}} = \frac{l}{(r^2+a^2)^{1/2}}$

$E = 2E_1 \cos\theta = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2+a^2)} \cdot \frac{l}{(r^2+a^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0} \frac{2\,\mathrm{ql}}{(r^2+a^2)^{3/2}}$

But $q.2l = p =$ electric dipole moment ...(iii)

$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2+a^2)^{3/2}}$

If dipole is infinitesimal and point P is far away, we have a << r, so l^2 may be neglected as compared to r^2 and so equation (iii) gives

$$E = \frac{1}{4\pi\,\varepsilon_0} \frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\,\varepsilon_0} \frac{p}{r^3}$$

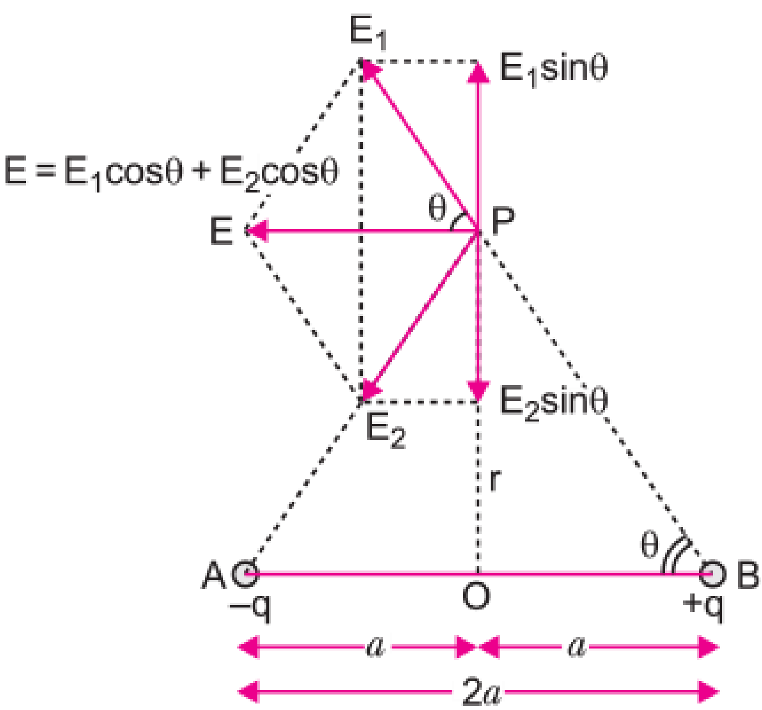
i.e., electric field strength due to a short dipole at broadside on position

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3} \text{ in the direction parallel to } \overrightarrow{\mathrm{BA}} \qquad ...(iv)$$

Its direction is parallel to the axis of dipole from positive to negative charge.

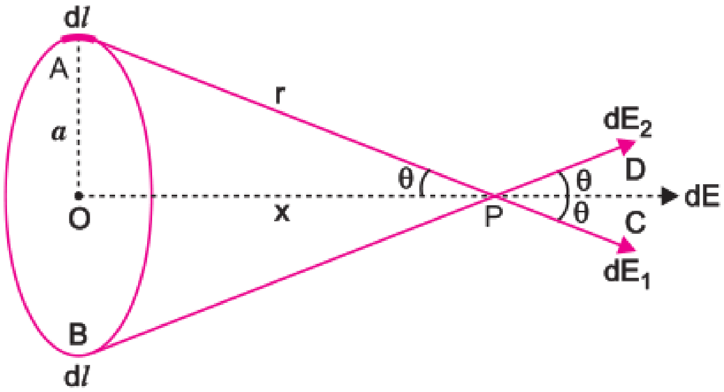
It may be noted clearly from equations (ii) and (iv) that electric field strength due to a short dipole at any point is inversely proportional to the cube of its distance from the dipole and the electric field strength at axial position is twice that at broad-side on position for the same distance.

Important: Note the important point that the electric field due to a dipole at large distances falls off as $\frac{1}{r^3}$ and not as $\frac{1}{r^2}$ as in the case of a point charge.



Q. 2. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for point's at large distances from the ring, it behaves like a point charge. [CBSE Delhi 2016]

Ans.



Consider a point P on the axis of uniformly charged ring at a distance x from its centre O. Point P is at distance r = $\sqrt{a^2 + x^2}$ from each element dl of ring. If q is total charge on ring, then, charge per metre length, $\lambda = \frac{q}{2\pi a}$.

The ring may be supposed to be formed of a large number of ring elements.

Consider an element of length dl situated at A.

The charge on element, dq = λ dl

∴ The electric field at P due to this element

$$\mathrm{dE_1} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{dq}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\ \mathrm{dl}}{r^2},\ \text{along } \overrightarrow{\mathrm{PC}}$$

The electric field strength due to opposite symmetrical element of length *dl* at *B* is

$$\overrightarrow{dE_2} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{dq}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\ \mathrm{dl}}{r^2},\ \text{along } \overrightarrow{\mathrm{PD}}$$

If we resolve $\overrightarrow{dE_1}$ and $\overrightarrow{dE_2}$ along the axis and perpendicular to axis, we note that the components perpendicular to axis are oppositely directed and so get cancelled, while those along the axis are added up. Hence, due to symmetry of the ring, the electric field strength is directed along the axis.

The electric field strength due to charge element of length dl, situated at A, along the axis will be

$$\mathrm{dE} = \mathrm{dE}_1 \cos\theta = \frac{1}{4\pi\varepsilon_0}\ \frac{\lambda\ \mathrm{dl}}{r^2}\cos\theta$$

But, $\cos\theta = \frac{x}{r}$

$$\therefore \qquad \mathrm{dE} = \frac{1}{4\pi\varepsilon_0}\ \frac{\lambda\ \mathrm{dl}\ x}{r^3} = \frac{1}{4\pi\varepsilon_0}\ \frac{\lambda x}{r^3}\ \mathrm{dl}$$

The resultant electric field along the axis will be obtained by adding fields due to all elements of the ring, i.e.,

$$\therefore \qquad E = \int \frac{1}{4\pi\,\varepsilon_0}\ \frac{\lambda x}{r^3}\,\mathrm{dl} = \frac{1}{4\pi\,\varepsilon_0}\ \frac{\lambda x}{r^3}\int \mathrm{dl}$$

But, $\int \mathrm{dl}$ = whole length of ring = $2\pi a$ and $r = \left(a^2 + x^2\right)^{1/2}$

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0}\ \frac{\lambda x}{(a^2+x^2)^{3/2}}2\pi a$$

As, $\lambda = \frac{q}{2\pi a}$, we have $E = \frac{1}{4\pi\varepsilon_0}\ \frac{\left(\frac{q}{2\pi a}\right)x}{(a^2+x^2)^{3/2}}2\pi a\ E = \frac{1}{4\pi\varepsilon_0}\ \frac{\mathrm{qx}}{(a^2+x^2)^{3/2}}$

or, $\qquad E = \frac{1}{4\pi\varepsilon_0}\ \frac{\mathrm{qx}}{(a^2+x^2)^{3/2}}$, along the axis

At large distances i.e., $x >> a$, $E = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{x^2}$

i.e., the electric field due to a point charge at a distance x.

For points on the axis at distances much larger than the radius of ring, the ring behaves like a point charge.

Q. 3. State and Prove Gauss theorem in electrostatics. [CBSE Ajmer 2015]

Ans. Statement: The net-outward normal electric flux through any closed surface of any shape is equal to $1/\varepsilon_0$ times the total charge contained within that surface, $1/\varepsilon_0$ i.e.,

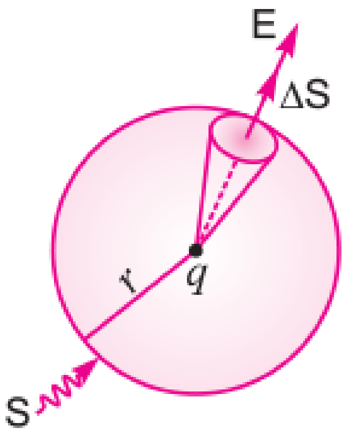
$$\oint S\ \vec{E} \bullet d\vec{S} = \frac{1}{\varepsilon_0}\sum q$$

Where $\oint_S$ indicates the surface integral over the whole of the closed surface, $\sum q$

Is the algebraic sum of all the charges (i.e., net charge in coulombs) enclosed by surface S and remain unchanged with the size and shape of the surface.

Proof: Let a point charge +q be placed at centre O of a sphere S. Then S is a Gaussian surface. Electric field at any point on S is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



The electric field and area element points radially outwards, so θ = 0°.

Flux through area $\overrightarrow{dS}$ is

$$d\varphi = \overrightarrow{E} \,.\, \mathrm{dS} = E \;\mathrm{dS}\cos 0^{\circ} = E \;\mathrm{dS}$$

Total flux through surface S is

$$\varphi = \oint_S d\varphi = \oint_S E\mathrm{dS} = E \oint_S dS = E \times \text{Area of Sphere}$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2} . 4\pi r^2$$

or, $\varphi = \frac{q}{\varepsilon_0}$ which proves Gauss's theorem.

Q. 4. (i) Using Gauss Theorem show mathematically that for any point outside the shell, the field due to a uniformly charged spherical shell is same as the entire charge on the shell, is concentrated at the centre.

(ii) Why do you expect the electric field inside the shell to be zero according to this theorem? [CBSE Allahabad 2015]

OR

A thin conducting spherical shell of radius R has charge Q spread uniformly over its surface. Using Gauss's theorem, derive an expression for the electric field at a point outside the shell. (CBSE Delhi 2009)

Draw a graph of electric field E(r) with distance r from the centre of the shell for $0 \leq r \leq \infty$.

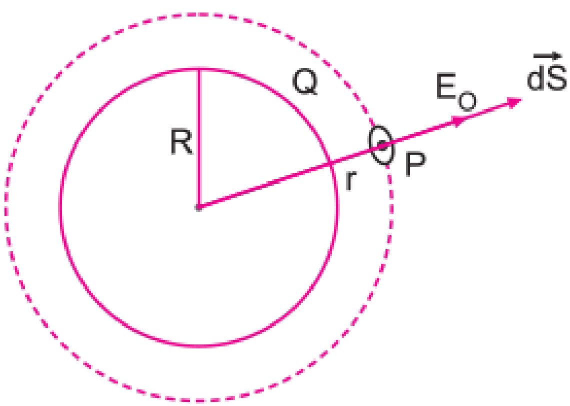
OR

Find the electric field intensity due to a uniformly charged spherical shell at a point (i) outside the shell and (ii) inside the shell. Plot the graph of electric field with distance from the centre of the shell. [CBSE North 2016]

OR

Using Gauss's law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for r > R and r < R.
[CBSE Delhi 2011; (AI) 2013]

Ans.



(i) Electric field intensity at a point outside a uniformly charged thin spherical shell: Consider a uniformly charged thin spherical shell of radius R carrying charge Q. To find the electric field outside the shell, we consider a spherical Gaussian surface of radius r (>R), concentric with given shell. If $\vec{E}$ is electric field outside the shell, then by symmetry electric field strength has same magnitude E0 on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E}_i$ and $\vec{dS}$ is zero at each point. Hence, electric flux through Gaussian surface.

$\oint S = \vec{E}_0 \bullet \vec{dS}.$

$\oint = E_0 \ \mathrm{dS} \cos 0 = E_0 \ . \ 4\pi r^2$

Now, Gaussian surface is outside the given charged shell, so charge enclosed by Gaussian surface is Q.

Hence, by Gauss's theorem

$\oint s = \vec{E}_0 \bullet \vec{dS} = \frac{1}{\varepsilon_0} \times$ charged enclosed

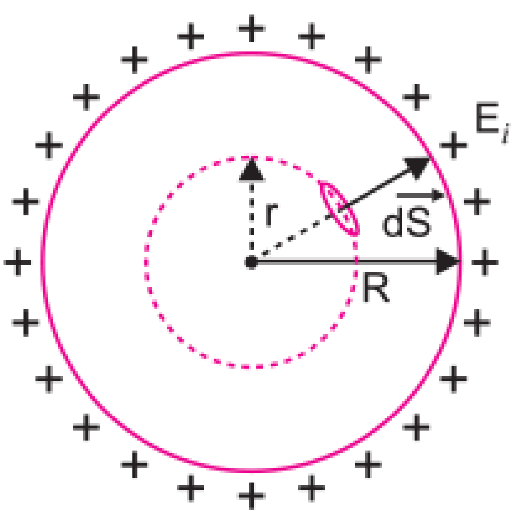
$\Rightarrow \qquad E_0 \ . \ 4\pi r^2 = \frac{1}{\varepsilon_0} \times Q \ \Rightarrow \ E_0 = \frac{1}{4\pi\varepsilon_0} \ \frac{Q}{r^2}$

Thus, electric field outside a charged thin spherical shell is the same as if the whole charge Q is concentrated at the centre.

If σ is the surface charge density of the spherical shell, then

$Q = 4\pi R^2 \sigma$ coulomb

$\therefore \qquad E_0 = \frac{1}{4\pi\varepsilon_0} \ \frac{4\pi R^2 \sigma}{r^2} = \frac{R^2 \sigma}{\varepsilon_0 r^2}$



(ii) Electric field inside the shell (hollow charged conducting sphere): The charge resides on the surface of a conductor. Thus a hollow charged conductor is equivalent to a charged spherical shell. To find the electric field inside the shell, we consider a spherical Gaussian surface of radius r (< R) concentric with the given shell. If $\vec{E}$ is the electric field inside the shell, then by symmetry electric field strength has the same magnitude E_i on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E}_i$ and $\vec{dS}$ is zero at each point.

Hence, electric flux through Gaussian surface

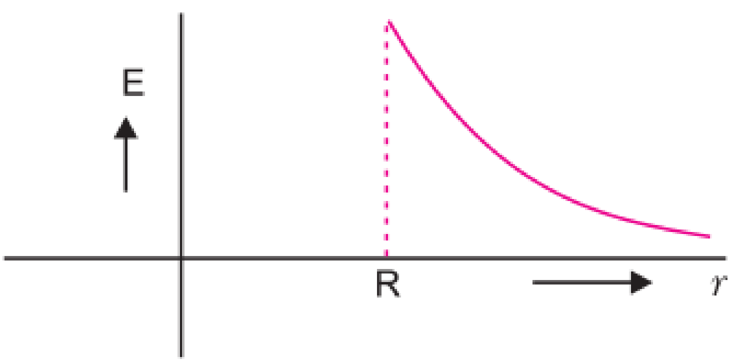
$$= \int_S \vec{E}_i \cdot \vec{dS} = \int E_i \, \mathrm{dS} \cos 0 = E_i \cdot 4\pi r^2$$

Now, Gaussian surface is inside the given charged shell, so charge enclosed by Gaussian surface is zero.

Hence, by Gauss's theorem

$$\int_S \vec{E}_i \cdot \vec{dS} = \frac{1}{\varepsilon_0} \times \text{ charge enclosed}$$

$$\Rightarrow \qquad E_i \, 4\pi r^2 = \frac{1}{\varepsilon_0} \times 0 \quad \Rightarrow \quad E_i = 0$$



Thus, electric field at each point inside a charged thin spherical shell is zero. The graph is shown in fig.

Q. 5. State Gauss theorem in electrostatics. Apply this theorem to obtain the expression for the electric field at a point due to an infinitely long, thin, uniformly charged straight wire of linear charge density λ C m^{-1}. [CBSE Delhi 2009]

Ans. Gauss Theorem: Refer to point 12 of Basic Concepts.

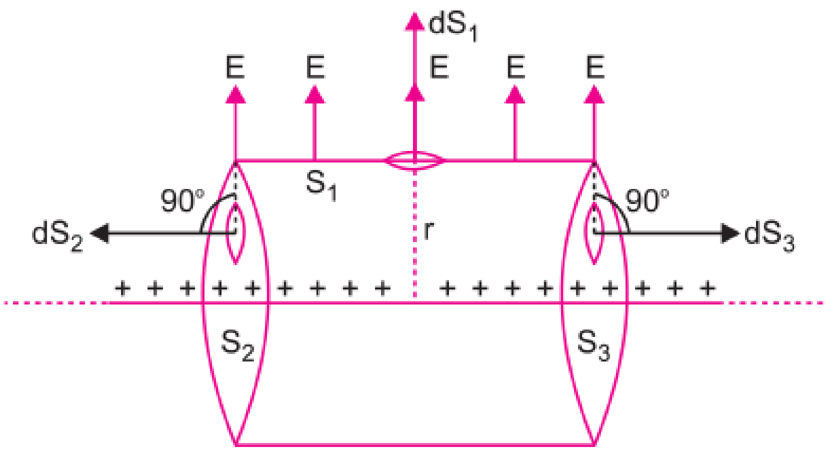
Electric field due to infinitely long, thin and uniformly charged straight wire: Consider an infinitely long line charge having linear charge density λ coulomb metre^{-1} (linear charge density means charge per unit length). To find the electric field strength at a distance r, we consider a cylindrical Gaussian surface of radius r and length l coaxial with line charge. The cylindrical Gaussian surface may be divided into three parts:

(i) Curved surface S_1 (ii) Flat surface S_2 and (iii) Flat surface S_3.

By symmetry, the electric field has the same magnitude E at each point of curved surface S_1 and is directed radially outward.

We consider small elements of surfaces S_1, S_2 and S_3 The surface element vector $\overrightarrow{dS_1}$ is directed along the direction of electric field (i.e., angle between $\overrightarrow{E}$ and $\overrightarrow{dS_1}$ is zero); the elements $\overrightarrow{dS_2}$ and $\overrightarrow{dS_3}$ are directed perpendicular to field vector $\overrightarrow{E}$ (i.e., angle between $\overrightarrow{dS_2}$ and $\overrightarrow{E}$ is 90° and so also angle between $\overrightarrow{dS_3}$ and $\overrightarrow{E}$).

Electric Flux through the cylindrical surface



$$\oint_S \vec{E} \bullet \vec{dS} = \oint_{S_1} \vec{E} \bullet \vec{dS_1} + \oint_{S_2} \vec{E} \bullet \vec{dS_2} + \oint_{S_3} \vec{E} \bullet \vec{dS_3}$$

$$= \oint_{S_1} E\ \mathrm{dS_1} \cos 0^\circ + \oint_{S_2} E\ \mathrm{dS_2} \cos\ 90^\circ\ +\ \oint_{S_3} E\ \mathrm{dS_3} \cos\ 90^\circ$$

$$=\ \int E\ \mathrm{dS_1} + 0 + 0$$

$= E \oint \mathrm{dS_1}$ (since electric field E is the same at each point of curved surface)

$= E\ 2\pi \mathrm{rl}$ (since area of curved surface = 2 π rl)

As λ is charge per unit length and length of cylinder is l therefore, charge enclosed by assumed surface = (λl)

∴ By Gauss's theorem

$$\int \vec{E} \bullet \vec{dS} = \frac{1}{\epsilon_0} \times \text{ charge enclosed}$$

$$\Rightarrow \qquad E.2\pi \mathrm{rl} = \frac{1}{\epsilon_0}(\lambda\, l\,) \qquad \Rightarrow \qquad E = \frac{\lambda}{2\pi \epsilon_0 r}$$

Thus, the electric field strength due to a line charge is inversely proportional to r.

Q. 6. Answer the following questions:

(i) Define electric flux. Write its SI unit.

(ii) Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

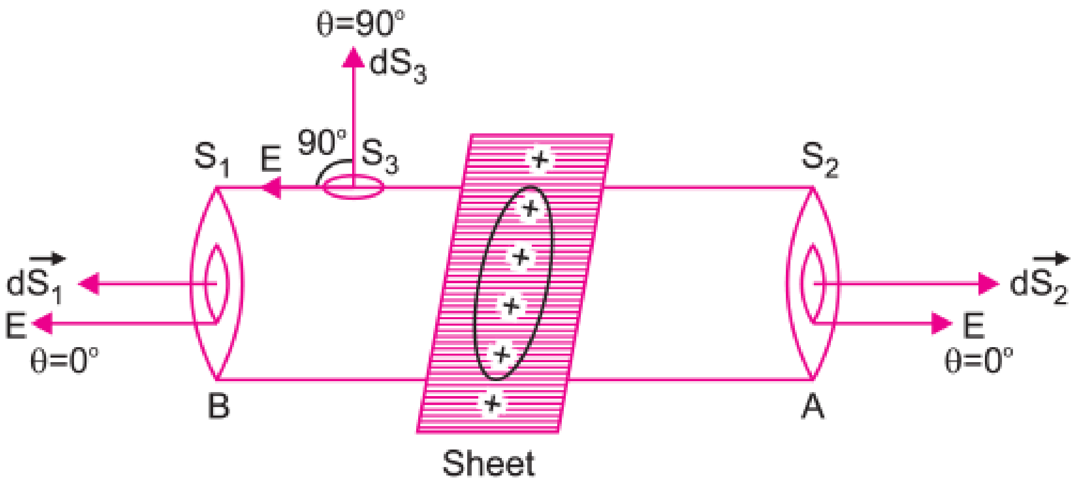
(iii) How is the field directed if (i) the sheet is positively charged, (ii) negatively charged? [CBSE Delhi 2012, Central 2016]

Ans. (i) Electric flux: It is defined as the total number of electric field lines passing through an area normal to its surface.

$$\varphi\ =\ \oint \vec{E}\ .\ \vec{dS}$$

The SI unit is Nm^2/C or volt-metre.

(ii)



Let electric charge be uniformly distributed over the surface of a thin, non-conducting infinite sheet. Let the surface charge density (i.e., charge per unit surface area) be σ. We need to calculate the electric field strength at any point distant r from the sheet of charge.

To calculate the electric field strength near the sheet, we now consider a cylindrical Gaussian surface bounded by two plane faces A and B lying on the opposite sides and parallel to the charged sheet and the cylindrical surface perpendicular to the sheet (fig). By symmetry the electric field strength at every point on the flat surface is the same and its direction is normal outwards at the points on the two plane surfaces and parallel to the curved surface.

Total electric flux

$$\text{or} \qquad \oint_S \overrightarrow{E} \ . \ \overrightarrow{dS} = \oint_{S_1} \overrightarrow{E} \ . \ \overrightarrow{dS}_1 + \oint_{S_2} \overrightarrow{E} \ . \ \overrightarrow{dS}_2 + \oint_{S_3} \overrightarrow{E} \ . \ \overrightarrow{dS}_3$$

$$\oint_S \overrightarrow{E} \ . \ \overrightarrow{dS} = \oint_{S_1} E \ \mathrm{dS_1} \cos 0^\circ + \oint_{S_2} E \ \mathrm{dS_2} \cos 0^\circ + \oint_{S_3} E \ \mathrm{dS_3} \cos \ 90 \ ^\circ$$

$$= E \oint \mathrm{dS_1} + E \oint \mathrm{dS_2} = \ \mathrm{Ea} \ + \ \mathrm{Ea} \ = \ 2 \ \mathrm{Ea}$$

∴ Total electric flux = 2Ea

As σ is charge per unit area of sheet and a is the intersecting area, the charge enclosed by Gaussian surface = σa

According to Gauss's theorem,

Total electric flux = $\frac{1}{e_0} \times$ (total charge enclosed by the surface)

i.e., $$2\mathrm{Ea} = \frac{1}{e_0} \ (\sigma a) \qquad \therefore \qquad E = \frac{p}{2 \, e_0}.$$

Thus electric field strength due to an infinite flat sheet of charge is independent of the distance of the point.

(iii) (i) If σ is positive, $\overrightarrow{E}$ points normally outwards/away from the sheet.

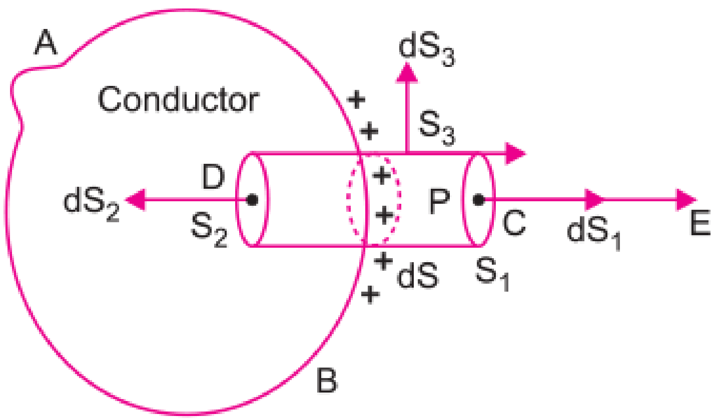
(ii) If σ is negative, $\overrightarrow{E}$ points normally inwards/towards the sheet.

Q. 7. Apply Gauss's Theorem to find the electric field near a charged conductor.

OR

Show that the electric field at the surface of a charged conductor is $\overrightarrow{E}=\frac{p}{e_0}\widehat{n}$ where σ is surface charge density and $\widehat{n}$ is a unit vector normal to the surface in the outward direction.

Ans.



Let a charge Q be given to a conductor, this charge under electrostatic equilibrium will redistribute and the electric field inside the conductor is zero (i.e., $E_{in} = 0$).

Let us consider a point P at which electric field strength is to be calculated, just outside the surface of the conductor. Let the surface charge density on the surface of the conductor in the neighbourhood of P be σ coulomb/metre^2. Now consider a small cylindrical box CD having one base C passing through P; the other base D lying inside the conductor and the curved surface being perpendicular to the surface of the conductor.

Let the area of each flat base be a. As the surface of the conductor is equipotential surface, the electric field strength **E** at P, just outside the surface of the conductor is perpendicular to the surface of the conductor in the neighbourhood of P.

The flux of electric field through the curved surface of the box is zero, since there is no component of electric field E normal to curved surface. Also the flux of electric field through the base D is zero, as electric field strength inside the conductor is zero. Therefore the resultant flux of electric field through the entire surface of the box is same as the flux through the face C. This may be analytically seen as:

If S_1 and S_2 are flat surfaces at C and D and S3 is curved surface, then

Total electric flux

$$\oint_S \overrightarrow{E} \ . \ \overrightarrow{dS} = \oint_{S_1} \overrightarrow{E} \ . \ \overrightarrow{dS}_1 + \oint_{S_2} \overrightarrow{E} \ . \ \overrightarrow{dS}_2 + \oint_{S_3} \overrightarrow{E} \ . \ \overrightarrow{dS}_3$$

$$= \oint_{S_1} E \ \mathrm{dS}_1 \ \cos 0 + \oint_{S_2} \overrightarrow{0} \ . \ \overrightarrow{dS}_2 + \oint_{S_3} E \ \mathrm{dS}_3 \ \cos \ 90^{\circ}$$

$$\oint_S E \ \mathrm{dS}_1 = \mathrm{She}$$

As the charge enclosed by the cylinder is (σa) coulomb, we have, using Gauss's theorem, = $\frac{1}{e_0} \times$ charge enclosed

$$\mathrm{Ea} = \frac{1}{e_0}(\sigma a) \qquad \text{or} \qquad E = \frac{p}{e_0} \qquad \ldots(i)$$

***Thus the electric field strength at any point close to the surface of a charged conductor of any shape is equal to 1/ε_0* times the surface charge density σ.** This is known as Coulomb's law. The electric field strength is directed radially away from the conductor if σ is positive and towards the conductor if σ is negative.

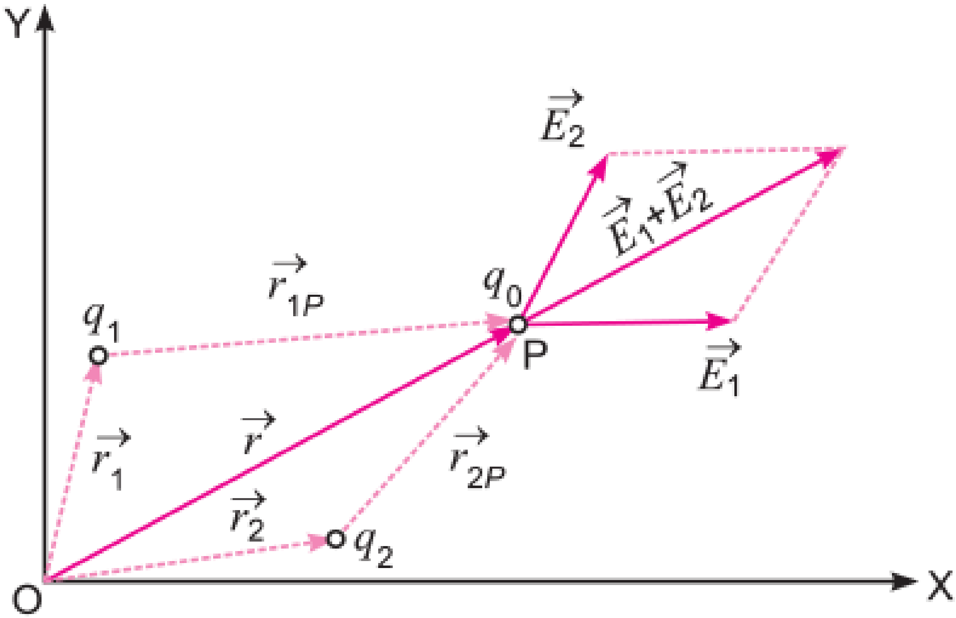
If $\widehat{n}$ is unit vector normal to surface in outward direction, then

$$\boxed{\overrightarrow{E} = \frac{p}{e_0}\widehat{n}}$$

Obviously electric field strength near a plane conductor is twice of the electric field strength near a non-conducting thin sheet of charge.

Q. 8. Consider a system of n charges q_1, q_2, ... q_n with position vectors $\overrightarrow{r}_1, \overrightarrow{r}_2, \overrightarrow{r}_3, \ldots \overrightarrow{r}_n$ relative to some origin 'O'. Deduce the expression for the net electric field $\overrightarrow{E}$ at a point P with position vector $\overrightarrow{r}_p$, due to this system of charges.

Ans.



Electric field due to a system of point charges.

Consider a system of *N* point charges $q_1, q_2, \ldots. q_N$, having position vectors $\vec{r_1}, \vec{r_2}, \ldots, \vec{r_N}$, with respect to origin O. We wish to determine the electric field at point P whose position vector is $\vec{r}$.

According to Coulomb's law, the force on charge q_0 due to charge q_1 is

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_0}{r_{2p}^2} \hat{r}_{1P}$$

Where $\hat{r}_{1P}$ is a unit vector in the direction from q_1 to P and r_1p is the distance between q_1 and P.

Hence the electric field at point P due to charge q_1 is

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{1P}^{\ 2}} \hat{r}_{1P}$$

Similarly, electric field at P due to charge q_2 is

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2P}^2} \hat{r}_{2P}$$

According to the principle of superposition of electric fields, the electric field at any point due to a group of point charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.

Hence, the electric field at point P due to the system of N charges is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_N$$

$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_{1P}^2} \hat{r}_{1P} + \frac{q_2}{r_{2P}^2} \hat{r}_{2P} + \ldots + \frac{q_N}{r_{NP}^2} \hat{r}_{\mathrm{NP}} \right] = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_{iP}^2} \hat{r}_{\mathrm{iP}}$$

Q. 9. A uniform electric field $\vec{E} = E_x \hat{i}$ N/C for x > 0 and $\vec{E} = -E_x \hat{i}$ N/C for x < 0 are given. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along the X-axis. Find out the net outward flux. Using Gauss's law, write the expression for the net charge within the cylinder. [HOTS]

Ans. Electric flux through flat surface S_1

$$\varphi_1 = \oint_{S_1} \overrightarrow{E}_1 . \overrightarrow{dS}_1 = \oint_{S_1} (E_x \hat{i}). (\mathrm{dS}_1 \hat{i}) = E_x S_1$$

Electric flux through flat surface S_2

$$\varphi_2 = \int_{S_2} \overrightarrow{E}_2 . \overrightarrow{dS}_2 = \int_{S_2} (-E_x \hat{i}). (-\mathrm{dS}_2 \hat{i}) = \int_{S_2} E_x \,\mathrm{dS}_2 = E_x S_2$$

Electric flux through curved surface S_3

$$\varphi_3 = \int_{S_3} (\overrightarrow{E}_3 . \overrightarrow{\mathrm{dS}_3}) = \int_{S_3} E_3 \,\mathrm{dS}_3 \cos 90^\circ = 0$$

$\therefore$ Net electric flux, $\varphi = \varphi_1 + \varphi_2 = E_x (S_1 + S_2)$

But $S_1 = S_2 = \pi \left(r \times 10^{-2}\right)^2 \mathrm{m}^2 = \pi r^2 \times 10^{-4} \mathrm{m}^2$

$\therefore \varphi = E_x . 2 \left(\pi r^2 \times 10^{-4}\right)$ units

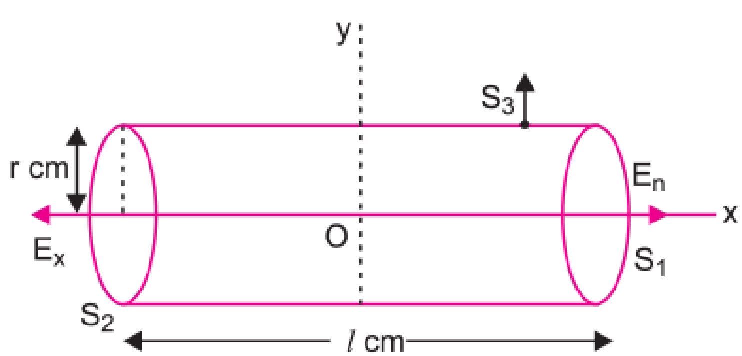
By Gauss's law, $\varphi = \frac{1}{\varepsilon_0} q$

$q = \varepsilon_0 \varphi = \varepsilon_0 E_x \left(2 \pi r^2 \times 10^{-4}\right)$

$$= 2\pi\varepsilon_0 E_x r^2 \times 10^{-4} = 4\pi\varepsilon_0 \left(\frac{E_x r^2 \times 10^{-4}}{2}\right)$$

$$= \frac{1}{9\times10^9} \left[\frac{E_x r^2 \times 10^{-4}}{2}\right]$$

$$= 5.56 E_x r^2 \times 10^{-11} \text{ coulomb}.$$



Q. 10. Answer the following Questions.

(a) Find expressions for the force and torque on an electric dipole kept in a uniform electric field. [CBSE (AI) 2014]

OR

An electric dipole is held in a uniform electric field. (i) Using suitable diagram show that it does not undergo any translatory motion, and (ii) derive an expression for torque acting on it and specify its direction.

(b) Derive an expression for the work done in rotating a dipole from the angle θ_0 to θ_1 in a uniform electric field E. [CBSE East 2016]

OR

(i) Define torque acting on a dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Express it in the vector form and point out the direction along which it acts.

(ii) What happens if the field is non-uniform?

(iii) What would happen if the external field $\vec{E}$ is increasing (i) parallel to $\vec{p}$ and (ii) anti-parallel to $\vec{p}$? [CBSE (F) 2016]

Ans.

Let an electric dipole be rotated in electric field from angle θ_0 to θ_1 in the direction of electric field. In this process the angle of orientation θ is changing continuously; hence the torque also changes continuously. Let at any time, the angle between dipole moment $\vec{p}$ and electric field $\vec{E}$ be θ then

Torque on dipole $\tau = pE \sin\theta$

The work done in rotating the dipole a further by small angle $d\theta$ is

dW = Torque × angular displacement= $pE \sin\theta \, d\theta$

Total work done in rotating the dipole from angle θ_0 to θ_1 is given by

$$W = \int_{\theta_0}^{\theta_1} \text{pE} \sin\theta \, d\theta = \text{pE} \left[-\cos\theta\right]_{\theta_0}^{\theta_1}$$

$$= -pE[\cos\theta_1 - \theta_0] = pE(\cos\theta_0 - \cos\theta_1) \qquad ..(i)$$

Special case : If electric dipole is initially in a stable equilibrium position $(\theta_0 = 0^\circ)$ and rotated through an angle $\theta(\theta_1 = \theta)$ then work done

$$W = pE[\cos 0^\circ - \cos\theta] = pE(1 - \cos\theta) \qquad ..(ii)$$