1 Real Numbers

Fastrack« Revision

- **Fundamental Theorem of Arithmetic:** Every composite number can be uniquely expressed as a product of primes, except for the order in which these prime factors occur. This theorem is also called as **Unique Factorisation Theorem.**
- **For any Two Positive Integers a and b**
  1. HCF (a, b): Product of the smallest power of each common prime factor in the numbers.
  2. LCM (a, b): Product of the greatest power of each common and uncommon prime factors in the numbers.
  3. HCF (a, b) × LCM (a, b) = Product of numbers $(a \times b)$

COMMON ERR(!)R

If p, q and r are positive integers, then HCF $(p, q, r) \times$ LCM $(p, q, r) \neq p \times q \times r$.

- **LCM and HCF of Rational Numbers**

LCM of rational numbers $= \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

HCF of rational numbers $= \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

- **Rational and Irrational Numbers:** A number of the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$, is called rational number. *e.g.,* $\dfrac{3}{7}, \dfrac{4}{9}, \ldots\ldots$

A number that cannot be expressed in the form $\dfrac{p}{q}$ is called irrational number. *e.g.,* $\sqrt{2}, \sqrt{5}, \pi, \ldots$

- If p is a prime number and p divides a^2, then p divides a, where a is a positive integer.

Knowledge BOOSTER

1. *A number is said to be a composite number, if it has at least one factor other than 1 and the number itself.*
   e.g., 4, 6, 9, 24, ... *are composite numbers.*
2. *The smallest even composite number is 4 and the smallest odd composite number is 9.*
3. *If there is no common prime factor, then HCF of given number is 1.*
4. *When we find the greatest or maximum numbers between two or more numbers, we use HCF.*
5. *When we find the least or minimum numbers between two or more numbers, we use LCM.*
6. *If p, q, r are positive integers, then*

$$\text{LCM}(p, q, r) = \frac{p \cdot q \cdot r \cdot \text{HCF}(p, q, r)}{\text{HCF}(p, q) \cdot \text{HCF}(q, r) \cdot \text{HCF}(p, r)}$$

and

$$\text{HCF}(p, q, r) = \frac{p \cdot q \cdot r \cdot \text{LCM}(p, q, r)}{\text{LCM}(p, q) \cdot \text{LCM}(q, r) \cdot \text{LCM}(p, r)}$$

Practice Exercise

Multiple Choice Questions

Q 1. The prime factorisation of natural number 288 is: [CBSE 2023]

a. $2^4 \times 3^3$ b. $2^4 \times 3^2$
c. $2^5 \times 3^2$ d. $2^5 \times 3^1$

Q 2. The exponent of 5 in the prime factorisation of 3750 is: [CBSE 2021 Term-I]

a. 3 b. 4 c. 5 d. 6

Q 3. If a and b are two co-prime numbers, then a^3 and b^3 are: [CBSE 2021 Term-I]

a. co-prime b. not co-prime
c. even d. odd

Q 4. If the HCF of 360 and 64 is 8, then their LCM is: [CBSE 2023]

a. 2480 b. 2780 c. 512 d. 2880

Q 5. $7 \times 11 \times 17 + 17$ is:

a. a prime number
b. a composite number
c. an odd number
d. divisible by 5

Q 6. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a, b) is: [CBSE SQP 2023-24]

a. xy b. xy^2
c. x^3y^3 d. x^2y^2

Q 7. Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF $(a, b) = p^mq^n$ and LCM $(a, b) = p^rq^s$, then $(m+n)(r+s) =$ [CBSE SQP 2022-23]

a. 15 b. 30 c. 35 d. 72

Q 8. What is the greatest possible speed at which a girl can walk 95 m and 171 m in an exact number of minutes? [CBSE 2021 Term-I]

a. 17 m/min b. 19 m/min
c. 23 m/min d. 13 m/min

Q 9. The LCM of smallest two-digit composite number and smallest composite number is: [CBSE SQP 2023-24]

a. 12 b. 4 c. 20 d. 44

Q 10. If LCM $(x, 18) = 36$ and HCF $(x, 18) = 2$, then x is: [CBSE SQP 2021 Term-I]

a. 2 b. 3 c. 4 d. 5

Q 11. If the LCM of 12 and 42 is $10m + 4$, then the value of m is:

a. 50 b. 8 c. 1/5 d. 1

Q 12. The LCM and HCF of two non-zero positive numbers are equal, then the numbers must be:

a. prime b. co-prime
c. composite d. equal

Q 13. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is: [NCERT EXEMPLAR]

a. 10 b. 100 c. 504 d. 2520

Q 14. If 'p' and 'q' are natural numbers and 'p' is the multiple of 'q', then what is the HCF of 'p' and 'q'? [CBSE 2023]

a. pq b. p
c. q d. $p+q$

Q 15. The least number which when divided by 18, 24, 30 and 42 will leave in each case the same remainder 1, would be:

a. 2520 b. 2519
c. 2521 d. None of these

Q 16. Three alarm clocks ring their alarms at regular intervals of 20 min, 25 min and 30 min, respectively. If they first beep together at 12 noon, at what time will they beep again for the first time? [CBSE 2021 Term-I]

a. 4:00 pm b. 4:30 pm
c. 5:00 pm d. 5:30 pm

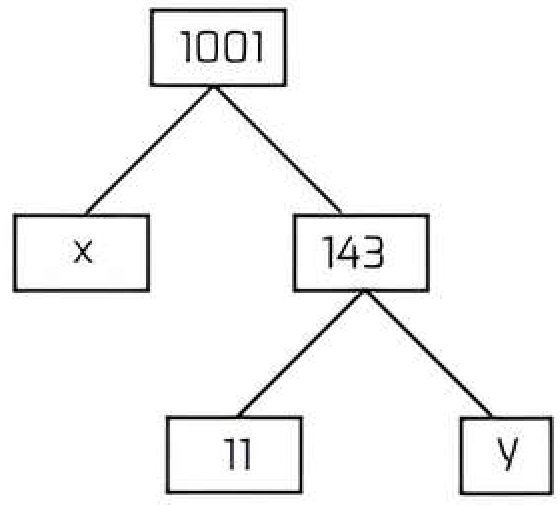
Q 17. A forester wants to plant 66 apple trees, 88 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also he wants to make distinct rows of trees (*i.e.*, only one type of trees in one row). The number of minimum rows required are:

a. 2 b. 3
c. 10 d. 12

Q 18. The greatest number which when divides 1251, 9377 and 15628 and leaves remainders 1, 2 and 3 respectively, is: [CBSE 2021 Term-I]

a. 575 b. 450 c. 750 d. 625

Q 19. The values of x and y in the given figure are:



a. 7, 13 b. 13, 7 c. 9, 12 d. 12, 9

Q 20. If $p^2 = \frac{32}{50}$, then p is a/an: [CBSE 2023]

a. whole number b. integer
c. rational number d. irrational number

Assertion & Reason Type Questions

Directions (Q. Nos. 21-25): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 21. Assertion (A): $11 \times 4 \times 3 \times 2 + 4$ is a composite number.
Reason (R): Every composite number can be expressed as product of primes.

Q 22. Assertion (A): For no value of n, where n is a natural number, the number 8^n ends with the digit zero.
Reason (R): The prime factorisation of a natural number is not unique, except for the order of its factors.

Q 23. Assertion (A): $\sqrt{a}$ is an irrational number, where a is a prime number.
Reason (R): Square root of any prime number is an irrational number.

Q 24. Assertion (A): If HCF of two numbers is 5 and their product is 150. Then their LCM is 40.
Reason (R): For any two positive integers a and b, HCF $(a, b) \times$ LCM $(a, b) = a \times b$. [CBSE SQP 2023-24]

Q 25. Assertion (A): If product of two numbers is 5780 and their HCF is 17, then their LCM is 340.
Reason (R): HCF is always a factor of LCM. [CBSE SQP 2022-23]

Fill in the Blanks Type Questions

Q 26. Two numbers are in the ratio 21 : 17. If their HCF is 5, the numbers are .... and ....

Q 27. The LCM of the smallest prime number and the smallest odd composite number is ....

Q 28. The greatest possible speed at which a man can walk 52 km and 91 km in an exact number of minutes is ....

Q 29. If n is a natural number, then the number of consecutive zeroes in 7^n is ....

Q 30. $\sqrt{3}+2$ is a.................number (rational/ irrational).

True/False Type Questions

Q 31. If there is no common prime factor, then HCF of given number is 1.

Q 32. For some integer q, every odd integer is of the form $q+1$.

Q 33. The LCM of the smallest prime number and the smallest composite number is 4.

Q 34. The product of a non-zero rational and an irrational numbers is always irrational.

Q 35. The sum of $(3+\sqrt{3})$ and $(5-\sqrt{3})$ is an irrational number.

Solutions

1. (c) The prime factorisation of given number is

$$288 = 2\times2\times2\times2\times2\times3\times3 = 2^5\times3^2$$

2. (b) The prime factorisation of given number is

$$3750 = 2\times3\times5^4$$

So, the exponent of 5 in the given prime factorisation number is 4.

3. (a) Since, a and b are two co-prime numbers. So, their cubes a^3 and b^3 will also have no common factor *i.e.*, they are also co-prime.

4. (d) Let the required LCM be x. Then,

Product of two numbers = HCF × LCM of two numbers

$\therefore \quad 360\times64 = 8\times x$

$\Rightarrow \quad x = \frac{360\times64}{8} = 360\times8 = 2880$

So, LCM (360, 64) = 2880.

5. (b) $7\times11\times17+17 = (7\times11+1)17 = 78\times17$, which is a composite number as it has a factors of 17 and 78.

6. (b) Given, $a = x^3y^2$ and $b = xy^3$

$\therefore$ HCF (a, b) = Product of the smallest power of each common prime factors in the numbers

$= xy^2$

7. (c) Given, $a = p^3q^4$ and $b = p^2q^3$

Also, given

HCF $(a, b) = p^mq^n$ and LCM $(a, b) = p^rq^s$

$\Rightarrow \quad p^2q^3 = p^mq^n$ and $p^3q^4 = p^rq^s$

Equating the exponents both sides, we have

$m = 2, n = 3$ and $r = 3, s = 4$

$\therefore (m+n)(r+s) = (2+3)(3+4) = 5\times7 = 35.$

8. (b)

TR!CK

The greatest possible speed is the HCF of the distances 95m and 171m.

The prime factors of 95 and 171 are

$95 = 5\times19$

and $171 = 3\times3\times19 = 3^2\times19$

$\therefore$ The greatest possible speed = HCF (95, 171)

= 19 m/min

9. (c) As we know that,

Smallest composite number $= 4 = 2^2$

and smallest two-digit composite number $= 10 = 2\times5$

$\therefore$ LCM = Product of the greatest power of each common and uncommon prime factors in the numbers

$= 2^2\times5 = 4\times5 = 20$

10. (c) Given, LCM $(x, 18) = 36$ and HCF $(x, 18) = 2$

Here, first number $= x$

and second number $= 18$

$\because$ First number × second number = LCM × HCF

$\therefore \quad x\times18 = 36\times2$

$\Rightarrow \quad x = \frac{2\times36}{18} = 4$

11. (b) We have, $12 = 2\times2\times3 = 2^2\times3$

and $42 = 2\times3\times7$

$\therefore \quad$ LCM $(12, 42) = 2^2\times3\times7 = 84$ (given)

$\Rightarrow \quad 10m+4 = 84$

$\Rightarrow \quad 10m = 84-4 = 80$

$\Rightarrow \quad m = \frac{80}{10} = 8.$

12. (d) Let two non-zero positive numbers be a and b.

Given, HCF (a, b) = LCM $(a, b) = k$ (say)

Since, HCF $(a, b) = k \Rightarrow a = km$ and $b = kn$, for some natural numbers m, n.

We know that HCF × LCM = Product of two numbers

$\therefore \quad k\times k = km\times kn$

$\Rightarrow \quad 1 = m\cdot n$

$\Rightarrow m = n = 1$, since, m, n are natural numbers.

Therefore, $a = km = k$ and $b = kn = k$

$\Rightarrow a = b = k$ *i.e.*, the numbers must be equal.

13. (d) Prime factorisation of numbers from 1 to 10 are

$1 = 1;\ 2 = 1\times2;\ 3 = 1\times3;\ 4 = 1\times2\times2;$

$5 = 1\times5;\ 6 = 1\times2\times3;\ 7 = 1\times7;$

$8 = 1\times2\times2\times2;\ 9 = 1\times3\times3;$

$10 = 1\times2\times5$

∴ Required least number

$= \text{LCM of numbers from 1 to 10}$

$= \text{LCM}(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$

$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2520.$

14. (c) We know that HCF of two natural numbers is the largest positive integer that divides both of them without leaving any remainder.

Given, p is a multiple of q. This means that there exists a natural number k such that $p = kq$.

To find the HCF of p and q, we need to find the largest positive integer that divides both p and q without leaving any remainder. Since, q is a factor of p (p is a multiple of q). We know that q is a common factor of p and q.

Moreover, since p is a multiple of q, any common factor of p and q must also be a factor of q. Therefore, the HCF of p and q is simply q.

15. (c) We have,

$$18 = 2 \times 3^2, \quad 24 = 2^3 \times 3$$
$$30 = 2 \times 3 \times 5, \quad 42 = 2 \times 3 \times 7$$

$\therefore \text{LCM}(18, 24, 30, 42) = 2^3 \times 3^2 \times 5 \times 7 = 2520$

So, required least number $= 2520 + 1 = 2521$.

COMMON ERR(!)R

Sometimes students make mistake of adding 1 instead of subtracting 1. So, please be careful of these types of questions.

16. (c) The prime factors of 20, 25 and 30 are

$$20 = 2 \times 2 \times 5 = 2^2 \times 5$$
$$25 = 5 \times 5 = 5^2$$
$$30 = 2 \times 3 \times 5$$

∴ LCM of (20, 25, 30) = Product of the greatest power of each prime factor in the numbers

$= 2^2 \times 3 \times 5^2 = 300 \text{ min}$

$= 5 \times 60 \text{ min} = 5 \text{ hours}$

Since, first beep start at 12 noon. Therefore, they will beep, again at 12 noon + 5 hours = 5 : 00 pm.

17. (d) Prime factors of given numbers are

$$66 = 2 \times 3 \times 11$$
$$88 = 2 \times 2 \times 2 \times 11 = 2^3 \times 11$$
$$110 = 2 \times 5 \times 11$$

HCF of 66, 88 and 110 $= 2 \times 11 = 22$

$\therefore \text{Number of rows} = \frac{66}{22} + \frac{88}{22} + \frac{110}{22}$

$= 3 + 4 + 5 = 12.$

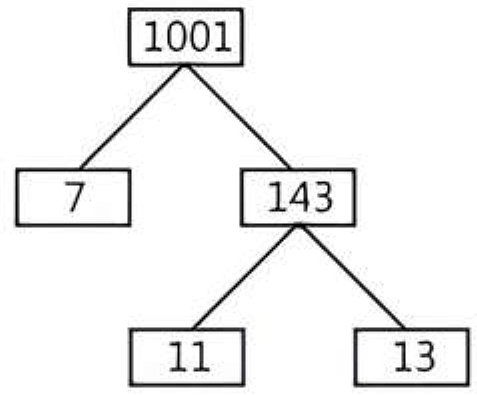
18. (d) The required number is (1251 – 1), (9377 – 2) and (15628 –3) *i.e.*, 1250, 9375, 15625.

Now, prime factors of these numbers are

$$1250 = 2 \times 5 \times 5 \times 5 \times 5 = 2^1 \times 5^4$$
$$9375 = 3 \times 5 \times 5 \times 5 \times 5 \times 5 = 3^1 \times 5^5$$
$$15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$$

∴ HCF of 1250, 9375 and 15625 $= 5^4 = 625$

19. (a) Given number is 1001. Then, the factor tree of 1001 is given as below:



$\because \quad 1001 = 7 \times 11 \times 13$

By comparing with given factor tree, we get

$x = 7, y = 13$

20. (c) Given, $p^2 = \frac{32}{50} \Rightarrow p^2 = \frac{16}{25}$

$\Rightarrow \quad p = \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

$\Rightarrow \quad p = \frac{\text{rational}}{\text{rational}} = \text{rational}$

21. (a) **Assertion (A):** We have, $11 \times 4 \times 3 \times 2 + 4$

$= (11 \times 3 \times 2 + 1)\, 4 = 67 \times 4 = 67 \times 2^2$

The given number can be expressed as product of primes. So, it is a composite number.

∴ Assertion (A) is true.

Reason (R): It is true to say that every composite number can be expressed as product of primes.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

22. (c) **Assertion (A):** We have, $8^n = (2^3)^n = 2^{3n}$, so the only prime in the factorisation of 8^n is 2. So, from the uniqueness of the Fundamental Theorem of Arithmetic, we can say that there are no other prime factorisation of 8^n. So, there is no natural number n for which 8^n ends with the digit zero.

So, Assertion (A) is true.

Reason (R): It is not true.

Hence, Assertion (A) is true but Reason (R) is false.

23. (a) **Assertion (A):** As we know that square root of every prime number is an irrational number. So, it is a true statement.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

24. (d) **Assertion (A):** Product of two numbers

$= \text{HCF} \times \text{LCM of two numbers}$

$\therefore \quad 150 = 5 \times \text{LCM}$

$\Rightarrow \text{LCM} = \frac{150}{5} = 30$

So, Assertion (A) is false.

Reason (R): It is a true statement.

Hence, Assertion (A) is false but Reason (R) is true.

25. (b) **Assertion (A):** Given, product of numbers = 5780
and HCF = 17

TR!CK

Use formula,

HCF × LCM = Product of two numbers

$\therefore \quad 17 \times LCM = 5780$

$\Rightarrow \quad LCM = 340$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

26. Given numbers are in the ratio 21 : 17

Let two numbers be $21x$ and $17x$

$\therefore \quad HCF(21x, 17x) = x$

But it is given that, $HCF(21x, 17x) = 5$

$\therefore \quad x = 5$

Hence, the numbers are $21 \times 5 = 105$ and $17 \times 5 = 85$.

27.

TR!CK

The smallest prime number is 2 and smallest odd composite number is 9.

$LCM(2, 9) = 2 \times 9 = 18$

28. The greatest possible speed at which a man can walk of 52 km and 91 km in exact number of minutes is

$HCF(52, 91) = HCF(13 \times 2 \times 2, 13 \times 7)$

$= 13$ km/min

COMMON ERR!R

Some of students make mistake instead of finding HCF, they find LCM. So, adequate practice is required.

29. We have 7^n.

Now $7^1 = 7$,

$7^2 = 49$,

$7^3 = 343$

$7^4 = 2401$,

$7^5 = 16807, ...$

Here, we see that there is no power of 7, in which we get 0 in the last digit.

Hence, consecutive zeroes in 7^n is nil.

30. $\because \sqrt{3}$ is an irrational number and 2 is a rational number.

$\therefore \sqrt{3} + 2$ will be an irrational number.

31. True

32. False

33.

TR!CK

The smallest prime number is 2 and smallest composite number is 4.

$LCM(2, 4) = 4$

Hence, given statement is true.

34. True

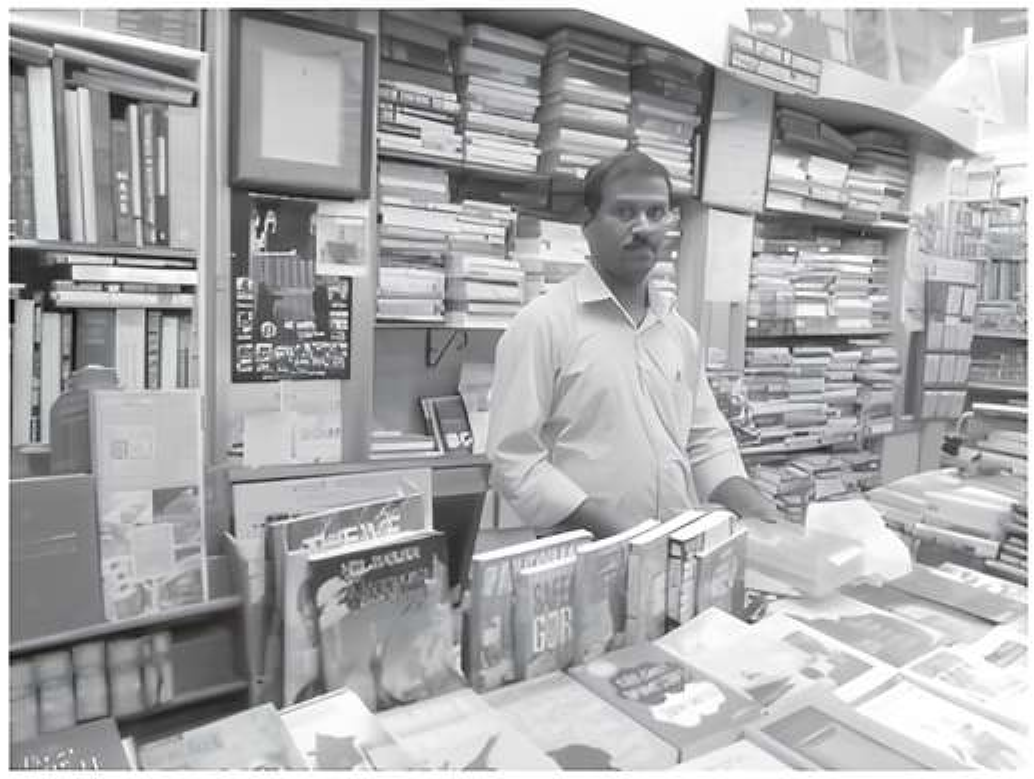
35. $\because (3 + \sqrt{3}) + (5 - \sqrt{3}) = 8$, which is a rational number.

Hence, given statement is false.

Case Study Based Questions

Case Study 1

A shopkeeper has 420 science stream books and 130 arts stream books. He wants to stack them in such a way that each stack has the same number and they take up the least area of the surface.



Based on the above information, solve the following questions:

Q 1. A number has no factor other than 1 and number itself is:

a. composite b. prime
c. do not say anything d. None of these

Q 2. What is the maximum number of books that can be placed in each stack for this purpose?

a. 10 b. 14
c. 12 d. 15

Q 3. Which mathematical concept is used to solve the problem?

a. Prime factorisation method
b. Area of triangle
c. Arithmetic progression
d. None of the above

Q 4. If the shopkeeper double the quantity, then the maximum number of books that can be placed in each stack:

a. remains same b. double
c. triple d. None of these

Q 5. Find the LCM of the given book streams:

a. 5450 b. 5460
c. 2730 d. None of these

Solutions

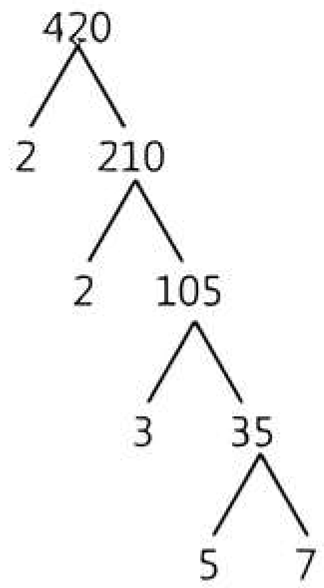
1. A number has no factor other than 1 and number itself is a prime number.

So, option (b) is correct.

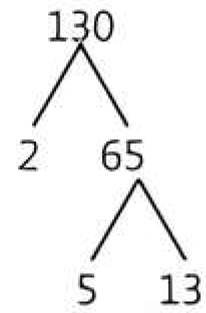
2. Given number of science books = 420

and number of arts books = 130

$420 = 2 \times 2 \times 3 \times 5 \times 7$



$130 = 2 \times 5 \times 13$



Maximum number of books that can be placed in each stack for the given purpose

$= \text{HCF }(420, 130) = 2^1 \times 5^1 = 10$

So, option (a) is correct.

3. Prime factorsation method is used to solve the problem.

So, option (a) is correct.

4. If the shopkeeper double the quantity, then the maximum number of books that can be placed in each stack is also doubled.

So, option (b) is correct.

5. LCM of $(420, 130) = 2^2 \times 3 \times 5 \times 7 \times 13$

$= 4 \times 15 \times 91 = 5460$

So, option (b) is correct.

Case Study 2

A seminar is being conducted by an educational organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

Based on the above information, solve the following questions:

Q 1. The sum of the powers of each prime factor of 108 is:

a. 2 b. 3

c. 4 d. 5

Q 2. In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number of participants that can accommodated in each room are:

a. 14 b. 12 c. 16 d. 18

Q 3. What is the minimum number of rooms required during the event?

a. 11 b. 31 c. 41 d. 21

Q 4. The LCM of 60, 84 and 108 is:

a. 3780 b. 3680 c. 4780 d. 4680

Q 5. The product of HCF and LCM of 60, 84 and 108 is:

a. 55360 b. 35360

c. 45500 d. 45360

Solutions

1. Prime factorisation of 108

$= 2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^3$

$\therefore$ Required sum of the powers $= 2 + 3 = 5$

So, option (d) is correct.

2. Using prime factorisation,

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

and $108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

$\therefore$ Maximum number of participants that can be accommodated in each room = HCF (60, 84, 108)

= Product of the smallest power of each common prime factor in the numbers

$= 2^2 \times 3 = 4 \times 3 = 12$

So, option (b) is correct.

COMMON ERR(!)R

Some students take LCM (60, 84, 108) for finding maximum number of participants that can be accommodated in each room.

3. Given,

The number of participants in Hindi = 60

The number of participants in English = 84

and the number of participants in Mathematics = 108

$\therefore$ Total number of participants $= 60 + 84 + 108$

$= 252$

Hence, minimum number of rooms required during

$$\text{event} = \frac{\text{Total number of participants}}{\text{Maximum number of participants that can be accommodated in each room}}$$

$$= \frac{252}{12} = 21$$

So, option (d) is correct.

4. LCM (60, 84, 108) = Product of the greatest power of each prime factor in the numbers

$= 2^2 \times 2^3 \times 5 \times 7$

$= 4 \times 27 \times 5 \times 7 = 3780$

So, option (a) is correct.

5. HCF (60, 84, 108) = $2^2 \times 3 = 12$

and LCM (60, 84, 108) = 3780

$\therefore$ HCF × LCM = 12 × 3780

= 45360

So, option (d) is correct.

Case Study 3

Old age homes mean for senior citizens who are unable to stay with their families or destitute. These old age homes have special medical facilities for senior citizens such as mobile health care systems, ambulances, nurses and provision of well balanced meals.

Himanshu, Gaurav and Gagan start preparing greeting cards for each person of an old age home on new year. In order to complete one card, they take 10, 16 and 20 min respectively.

Based on the above information, solve the following questions:

Q 1. Co-prime numbers are those numbers which do not have any common factor other than 1. Is this statement true?

Q 2. Find the sum of the powers of all different prime factors of the numbers 10, 16 and 20.

Q 3. If all of them started together, then what time will they start preparing a new card together?

OR

What is the common time to make one card?

Solutions

1. True

2. By prime factorisation,

$10 = 2^1 \times 5^1$

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

$\therefore$ Required sum = sum of the power of 2 + sum of the power of 5 = (1 + 4 + 2) + (1 + 1) = 7 + 2 = 9

3. The required number of minutes after which they start preparing a new card together is the LCM of 10, 16 and 20 min.

Now, $10 = 2 \times 5$

$16 = 2 \times 2 \times 2 \times 2$

$20 = 2 \times 2 \times 5 = 2^2 \times 5$

$\therefore$ LCM (10, 16, 20) = $2^4 \times 5^1 = 16 \times 5 = 80$ min

So, they will start preparing a new card together after 80 min *i.e.*, 1 h 20 min.

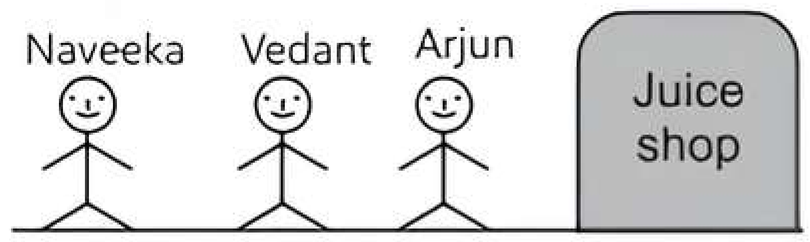
OR

$\because$ The common time to make one card

= HCF of (10, 16, 20) = 2 min

Case Study 4

In a morning walk, Naveeka, Arjun and Vedant step off together, their steps measuring 240 cm, 90 cm, 120 cm respectively. They want to go for a juice shop for a health issue, which is situated near by them.



Based on the above information, solve the following questions:

Q 1. Factor tree is a chain of factors, which is represented in the form of a tree. Is this statement true?

Q 2. Find the sum of the powers of all common prime factors of the numbers 240, 90 and 120.

Q 3. Find the minimum distance of shop from where they start to walk together, so that one can cover the distance in complete steps.

Or

Find the number of common steps covered by all of them to reach the juice shop.

Solutions

1. True

2. By prime factorisation,

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 \times 3^1 \times 5^1$

$90 = 2 \times 3 \times 3 \times 5 = 2^1 \times 3^2 \times 5^1$

$120 = 2 \times 2 \times 3 \times 2 \times 5 = 2^3 \times 3^1 \times 5^1$

$\therefore$ Required sum = sum of the power of 2 + sum of the power of 3 + sum of the power of 5 = 1 + 1 + 1 = 3.

3. Minimum required distance to reach the juice shop

= LCM (240, 90, 120)

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 \times 3 \times 5$

$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

and $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

Now, LCM = $2^4 \times 3^2 \times 5 = 16 \times 9 \times 5 = 720$

Hence, required minimum distance is 720 cm.

OR

The number of common steps covered by all of them = HCF (240, 90, 120) = $2 \times 3 \times 5 = 30$

Very Short Answer Type Questions

Q 1. If HCF (26, 169) = 13, then find LCM (26, 169).

Q 2. What is the HCF of smallest composite number and the smallest prime number? [CBSE 2018]

Q 3. Two positive integers a and b can be written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers. Find the LCM of (a, b). [CBSE 2019]

Q 4. Write the number of zeroes in the end of a number whose prime factorisation is $2^2 \times 5^3 \times 3^2 \times 17$. [CBSE 2019]

Q 5. The HCF of two numbers a and b is 5 and their LCM is 200. Find the product of ab. [CBSE 2019]

Q 6. Check whether $\frac{5\sqrt{125}-3\sqrt{5}}{\sqrt{5}}$ is a rational or irrational number.

Q 7. Find the LCM and HCF of 92 and 510, using prime factorisation. [CBSE 2023]

Q 8. Prove that 4^n can never end with digit 0, where n is a natural number. [CBSE 2023]

Short Answer Type-I Questions

Q 1. Find the HCF of 1260 and 7344. [CBSE 2019]

Q 2. Check whether the pair of numbers 50 and 20 are co-prime or not.

Q 3. Show that $(15)^n$ cannot end with the digit 0 for any natural number 'n'. [CBSE 2023]

Q 4. Two tankers contain 850 L and 680 L of petrol, respectively. Find the maximum capacity of a container which can measure the petrol of either tanker, in exact number of times.

Q 5. Find the largest number which divides 70 and 125 leaving remainder 5 and 8 respectively. [CBSE 2015]

Q 6. Is $7 \times 5 \times 3 \times 2 + 3$ a composite number? Justify your answer. [CBSE 2016, 15]

Q 7. The product of two consecutive positive integers is divisible by 2. Is this statement true or false? Give reason. [NCERT EXERCISE]

Q 8. Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers? [CBSE 2023]

Short Answer Type-II Questions

Q 1. Find the HCF and LCM of 26, 65 and 117, using prime factorisation. [CBSE 2023]

Q 2. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, find the other number.

Q 3. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point? [NCERT EXERCISE]

Q 4. Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 am, when will they ring together again? [CBSE 2023]

Q 5. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 8 am, at what time will they change together again?

Q 6. Prove that $\sqrt{3}$ is an irrational number. [CBSE SQP 2023-24]

Q 7. Prove that $\sqrt{2}$ is an irrational number. [CBSE SQP 2023-24]

Q 8. Prove that $3+7\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number. [CBSE 2023]

Q 9. Prove that $5-\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. [CBSE 2023]

Q 10. Prove that $4+2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. [CBSE 2023]

Long Answer Type Questions

Q 1. A fruit vendor has 990 apples and 945 oranges. He packs them into baskets. Each basket contains only one of the two fruits but in equal number. Find the number of fruits to be put in each basket in order to have minimum number of baskets. [CBSE 2016]

Q 2. Find by prime factorisation, the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers. [CBSE 2023]

Q 3. The HCF of 65 and 117 is expressible in the form $65m - 117$. Find the value of m. Also, find the LCM of 65 and 117, using prime factorisation method.

Q 4. Prove that $2-3\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number. [CBSE 2023]

Q 5. National Art convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same art-form and the number of students in each group is the same. Find the number of students in each group. Find the number of groups in each art-form. How many rooms are required if each group will be allotted a room? [CBSE SQP 2023-24]

Solutions

Very Short Answer Type Questions

1. Given, HCF (26, 169) = 13

$\because$ Product of two numbers = HCF × LCM of two numbers

$\therefore \quad 26 \times 169 = 13 \times \text{LCM}$

$\Rightarrow \quad \text{LCM} = \dfrac{26 \times 169}{13} = 2 \times 169 = 338$

2. Smallest composite number = 4 = $2 \times 2 = 2^2$

and smallest prime number = 2

> **TiP**
>
> *HCF (a, b) = Product of the smallest power of each common prime factor in the numbers.*

$\therefore \quad \text{HCF}(4, 2) = 2$

> **COMMON ERR(!)R**
>
> *Sometimes students assume smallest composite number as 2 as they get confused in prime number and composite number.*

3. Given, $a = x^3y^2$ and $b = xy^3$

$\therefore$ LCM (a, b) = Greatest power of x and y from a and b

$= x^3y^3$

4. Given expression $= 2^2 \times 5^3 \times 3^2 \times 17$

We know that, zeroes in an expression are a result of number of 10's in it.

> **TR!CK**
>
> *The only way to make a 10 is the product of 2 and 5. Hence, the number of zeroes in an expression will be the number of 2 or 5 whichever is minimum.*

$\therefore \quad 2^2 \times 5^3 \times 3^2 \times 17$

$= 2 \times 2 \times 5 \times 5 \times 5 \times 3^2 \times 17$

$= (2 \times 5) \times (2 \times 5) \times 5 \times 3^2 \times 17$

$= 10 \times 10 \times 5 \times 3^2 \times 17$

Hence, from the expression, we can see that there will be 2 zeroes in the given expression.

> **COMMON ERR(!)R**
>
> *Some students do multiplication of the expression directly in haste which can make calculations wrong. So first we should convert given factors in the form of 10 as many as possible.*

5. We know that,

HCF × LCM = Product of two numbers

$\therefore \quad 5 \times 200 = a \times b \quad \Rightarrow \quad ab = 1000$

6. $\dfrac{5\sqrt{125} - 3\sqrt{5}}{\sqrt{5}} = \dfrac{5 \times 5\sqrt{5} - 3\sqrt{5}}{\sqrt{5}}$

$= \dfrac{\sqrt{5}(25-3)}{\sqrt{5}} = 22$

Hence, given expression is a rational number.

7. Prime factorisation of 92 and 510:

$92 = 2 \times 2 \times 23$

$= 2^2 \times 23$

and $\quad 510 = 2 \times 3 \times 5 \times 17$

$\therefore$ LCM (92, 510) = Product of the greatest power of each common and uncommon prime factors in the numbers

$= 2^2 \times 3 \times 5 \times 17 \times 23$

$= 60 \times 17 \times 23$

$= 23460$

and HCF (92, 510) = Product of the smallest power of each common prime factor in the numbers,

$= 2$

8. Prime factorisation of $4^n = (2 \times 2)^n = (2)2^n$

Here, 5 is not in the prime factorisation of 4^n.

Hence, for any value of n, 4^n will not be divisible by 5.

> **TR!CK**
>
> *If any number ends with the digit 0, it should be divisible by 2 and 5 as 10 = 2 × 5.*

Therefore, 4^n can never end with the digit 0 for any natural number n. **Hence proved.**

Short Answer Type-I Questions

1. $\because \quad 1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2^2 \times 3^2 \times 5 \times 7$

and $\quad 7344 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 17$

$= 2^4 \times 3^3 \times 17$

2	1260
2	630
3	315
3	105
5	35
7	7
	1

2	7344
2	3672
2	1836
2	918
3	459
3	153
3	51
17	17
	1

$\therefore$ HCF (1260, 7344)

$= 2^2 \times 3^2$

$= 4 \times 9$

$= 36$

Hence, the HCF of 1260 and 7344 is 36.

2. By prime factorisation,

$50 = 2 \times 5 \times 5$

$= 2 \times 5^2$

and $\quad 20 = 2 \times 2 \times 5$

$= 2^2 \times 5$

2	50
5	25
5	5
	1

2	20
2	10
5	5
	1

$\therefore \quad \text{HCF}(50, 20) = 2 \times 5$

Here, we see that HCF has more than two factors.

Hence, the pair of numbers are not co-prime.

3.

> **TR!CK**
>
> *If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as 10 = 2 × 5.*

If $(15)^n$ end with digit zero, then the number should be divisible by 2 and 5.

As $\quad 2 \times 5 = 10$.

This means the prime factorisation of $(15)^n$ should contain prime factors 2 and 5.

$\Rightarrow \quad (15)^n = (3 \times 5)^n$

It does not have the prime factor 2 but have 3 and 5. Since, 2 is not present in the prime factorisation, there is no natural number for which $(15)^n$ ends with digit zero.

So, $(15)^n$ cannot end with digit zero.

Hence proved.

4. Given capacities of two tankers are 850 L and 680 L. Maximum capacity of required container is the HCF of 850 L and 680 L.

$\therefore \quad 850 = 2 \times 5 \times 5 \times 17$
$= 2 \times 5^2 \times 17$

2	850
5	425
5	85
17	17
	1

and $\quad 680 = 2 \times 2 \times 2 \times 5 \times 17$
$= 2^3 \times 5 \times 17$

2	680
2	340
2	170
5	85
17	17
	1

$\therefore \quad \text{HCF}(850, 680) = 2 \times 5 \times 17 = 170$

Hence, the maximum capacity of the required container is 170 L.

5.

TR!CK

The required largest number is the HCF of (70 − 5 and 125 − 8) i.e., 65 and 117.

It is given that on dividing 70 by the required number, there is a remainder 5. This means that $70 - 5 = 65$ is exactly divisible by the required number.

Similarly, $125 - 8 = 117$ is also divisible by the required number.

Now, using prime factorisation

$65 = 5 \times 13$

and $\quad 117 = 3 \times 3 \times 13$
$= 3^2 \times 13$

5	65
13	13
	1

3	117
3	39
13	13
	1

$\therefore \quad \text{HCF}(65, 117) = 13$

Hence, the required number is 13.

TiP

HCF (a, b) = Product of the smallest power of each prime factor in the numbers.

6. $7 \times 5 \times 3 \times 2 + 3 = 3(7 \times 5 \times 2 + 1) = 3(70 + 1) = 3 \times 71$

By Fundamental theorem of Arithmetic, every composite number can be expressed as product of primes in a unique way, apart from the order of factors.

The given number has more than two factors.

Hence, $7 \times 5 \times 3 \times 2 + 3$ is a composite number.

7. True.

Because product of two consecutive numbers (even and odd or odd and even) in the form of $n(n+1)$ will always be even.

e.g., Let $n = 5$, then $n + 1 = 6$, so 5×6 is divisible by 2.

8. Let the numbers be $2x$ and $3x$.

Given, LCM $(2x, 3x) = 180$

Clearly HCF $(2x, 3x) = x$

$\because$ Product of two numbers,

$a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b)$

$\therefore \quad 2x \times 3x = \text{HCF}(2x, 3x) \times \text{LCM}(2x, 3x)$

$\Rightarrow \quad 6x^2 = x \times 180$

$\Rightarrow \quad x^2 = 30x = 0$

$\Rightarrow \quad x(x - 30) = 0$

$\Rightarrow \quad x = 0 \text{ or } x = 30$

Since, $x = 0$ is not possible as HCF can't be 0.

So, HCF $(2x, 3x) = x = 30$.

Short Answer Type-II Questions

1.

TiP

Every composite number can be uniquely expressed as a product of primes, except for the order in which these prime factors occurs.

By prime factorisation,

$26 = 2 \times 13$

$65 = 5 \times 13$

and $\quad 117 = 3 \times 3 \times 13 = 3^2 \times 13$

$\therefore$ HCF (26, 65, 117) = Product of the smallest power of each common prime factor in the numbers
$= 13$

and LCM (26, 65, 117) = Product of the greatest power of each common and uncommon prime factors in the numbers
$= 13 \times 3^2 \times 2 = 26 \times 9 = 234$

2. Let HCF of two numbers be x, then LCM will be $14x$.

Now, $\quad$ LCM + HCF = 600 $\quad$ (given)

$\Rightarrow \quad 14x + x = 600$

$\Rightarrow \quad 15x = 600$

$\Rightarrow \quad x = \frac{600}{15}$

$\Rightarrow \quad x = 40$

So, $\quad$ HCF = 40

and $\quad$ LCM = $14 \times 40 = 560$

Since, $\quad$ HCF × LCM = One number × Other number

$\therefore \quad 40 \times 560 = 280 \times$ Other number

$\Rightarrow$ Other number $= \frac{40 \times 560}{280} = 80$

Hence, the other number is 80.

3.

TR!CK

The total time taken for completing 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round respectively, i.e., LCM of 18 minutes and 12 minutes.

Using prime factorisation,

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

and $12 = 2 \times 2 \times 3 = 2^2 \times 3$

TiP

Every composite number can be uniquely expressed as a product of primes, except for the order in which these prime factors occur.

LCM of 12 and 18 = Product of the greatest power of each prime factor in the numbers

$= 2^2 \times 3^2 = 36$

Hence, Ravi and Sonia will meet together at the starting point after 36 minutes.

4. Given that, at intervals of 6, 12 and 18 minutes, three chimes ring.

We have to find when will the three bells sound simultaneously once more if they rang at 6 am.

Let us find the LCM of 6, 12 and 18.

By prime factorisation,

$6 = 2 \times 3$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

and $18 = 2 \times 3 \times 3 = 2 \times 3^2$

$\therefore$ LCM (6, 12, 18) = Product of the greatest power of each common and uncommon prime factors in the numbers

$= 2^2 \times 3^2 = 4 \times 9 = 36$

Therefore, all the three bells will rang together again at 6 : 36 am.

5.

TR!CK

If the traffic light changes simultaneously at 8 am, then they will change again simultaneously by the LCM value of the respective times.

Using prime factorisation,

$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

and $108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

So, LCM = Product of the greatest power of each prime factor in the numbers

$= 2^4 \times 3^3 = 16 \times 27$

$= 432$ seconds

$= \frac{432}{60}$ minutes

$= 7\frac{12}{60}$ minutes

$= 7$ minutes $+ \frac{12}{60}$ minutes

$= 7$ minutes $+ \frac{12}{60} \times 60$ seconds

($\because$ 1 minute = 60 s)

= 7 minutes 12 seconds

So, the required time = 8 am + 7 minutes 12 seconds

= 8 : 07 : 12 am

Hence, the traffic lights will change together again at 8 : 07 : 12 am.

6. Let if possible $\sqrt{3}$ is a rational number instead of an irrational number.

Then, $\sqrt{3} = \frac{p}{q}$, where $q \neq 0$ and p and q are positive integers.

Let p and q have no common factor other than 1.

Now, $\sqrt{3} = \frac{p}{q}$

$\therefore$ $p = \sqrt{3}q$

Squaring both sides, $p^2 = 3q^2$

$\therefore$ p^2 is divisible by 3.

$\therefore$ p is also divisible by number 3.

Now, p is divisible by 3, then let $p = 3r$

Squaring both sides, $p^2 = 9r^2$

But it is known that $p^2 = 3q^2$

$\therefore$ $3q^2 = 9r^2$ or $q^2 = 3r^2$

Then q^2 is divisible by 3.

Then, q is also divisible by 3.

$\because$ p and q, both are divisible by 3.

$\therefore$ 3 is a common prime factor of p and q (except 1).

It is a contradiction because according to our assumption, p and q have no common prime factor (except 1).

$\therefore$ It indicates that our assumption "$\sqrt{3}$ is a rational number" is wrong.

Therefore, $\sqrt{3}$ is an irrational number.

Hence proved.

7. Do same as solution-6.

8. Let us assume $3 + 7\sqrt{2}$ be rational, then it must be in the form of p/q, where p and q are co-prime integers and $q \neq 0$.

i.e., $3 + 7\sqrt{2} = \frac{p}{q}$

$\Rightarrow$ $7\sqrt{2} = \frac{p}{q} - 3$

$\Rightarrow$ $\sqrt{2} = \frac{p - 3q}{7q}$

Since, p, q, 7 and 3 are integers and $q \neq 0$, so RHS of eq. (1) is rational. But LHS of eq. (1) is $\sqrt{2}$ which is irrational. This is not possible.

This contradiction has arisen due to our wrong assumption that $3 + 7\sqrt{2}$ is rational.

So, $3 + 7\sqrt{2}$ is irrational. **Hence proved.**

9. Since, $\sqrt{3}$ is an irrational number and 5 is a rational number.

Therefore, difference of rational and irrational numbers is irrational number.

Hence, $5 - \sqrt{3}$ is an irrational number. **Hence proved.**

10. Let us assume $4+2\sqrt{3}$ be rational, then it must be in the form of p/q, where p and q are co-prime integers and $q \neq 0$.

i.e., $4+2\sqrt{3}=\frac{p}{q} \Rightarrow 2\sqrt{3}=\frac{p}{q}-4$

$$\Rightarrow \sqrt{3}=\frac{p-4q}{2q} \quad ...(1)$$

Since p, q, 4 and 2 are integers and $q \neq 0$, so RHS of eq. (1) is rational. But LHS of eq. (1) is $\sqrt{3}$ which is irrational. This is not possible.

This contradiction has arisen due to our wrong assumption that $4+2\sqrt{3}$ is rational.

So, $4+2\sqrt{3}$ is irrational. **Hence proved.**

Long Answer Type Questions

1. Number of fruits in each basket = HCF of 990 and 945

$990 = 2\times3\times3\times5\times11 = 2\times3^2\times5\times11$

and $945 = 3\times3\times3\times5\times7 = 3^3\times5\times7$

$\therefore$ HCF (990, 945) = Product of the smallest power of each common prime factor in the numbers

$= 3^2\times5 = 9\times5 = 45$

So, maximum number of fruits that can be packed in each basket = 45

Now, total number of fruits = 990 + 945 = 1935

Hence, minimum number of baskets required

$$= \frac{\text{Total number of fruits}}{\text{Max. number of fruits that can be packed in each basket}}$$

$$= \frac{1935}{45} = 43.$$

2. By prime factorisation,

$18180 = 2\times2\times3\times3\times5\times101 = 2^2\times3^2\times5\times101$

and $7575 = 3\times5\times5\times101 = 3\times5^2\times101$

$\therefore$ LCM (18180, 7575) = Product of the greatest power of each common and uncommon prime factors in the numbers

$= 2^2\times3^2\times5^2\times101$

$= 4\times9\times25\times101$

$= 90900$

$\because$ Product of two numbers,

$a\times b = \text{HCF}(a, b)\times\text{LCM}(a, b)$

$\therefore 18180\times7575 = \text{HCF}(18180, 7575)\times90900$

$$\Rightarrow \text{HCF}(18180, 7575) = \frac{18180\times7575}{90900} = 1515.$$

3. By prime factorisation, $65 = 5\times13$

and $117 = 3\times3\times13 = 3^2\times13$

$\therefore$ HCF of (65, 117) = 13

5	65
13	13
	1

3	117
3	39
13	13
	1

Hence, HCF of 65 and 117 is 13.

According to the question,

$65m - 117 = 13$

$\Rightarrow 65m = 13 + 117$

$= 130$

$\Rightarrow m = \frac{130}{65} = 2$ or $m = 2$

Now, we know that,

LCM × HCF = One number × Other number

LCM × 13 = 65 × 117

$\Rightarrow \text{LCM} = \frac{65\times117}{13} = 5\times117$

$\therefore$ LCM = 585

Hence, $m = 2$ and LCM of 65 and 117 is 585.

4. Let us assume $2-3\sqrt{5}$ be rational, then it must be in the form of p/q, where p and q are co-prime integers and $q \neq 0$.

i.e., $2-3\sqrt{5}=\frac{p}{q}$

$\Rightarrow 3\sqrt{5}=2-\frac{p}{q}$

$$\Rightarrow \sqrt{5}=\frac{2q-p}{3q} \quad ...(1)$$

Since, p, q, 3 and 2 are integers and $q \neq 0$, so RHS of eq. (1) is rational. But LHS of eq. (1) is $\sqrt{5}$ which is irrational. This is not possible.

This contradiction has arisen due to our wrong assumption that $2-3\sqrt{5}$ is rational.

So, $2-3\sqrt{5}$ is irrational. **Hence proved.**

COMMON ERR(!)R

Sometimes students fail to prove the solution. They commit errors in explaining the steps properly.

5.

TR!CK

Number of students in each group is the HCF of students which are interested in different art-forms.

By prime factorisation,

$60 = 2\times2\times3\times5 = 2^2\times3\times5.$

$84 = 2\times2\times3\times7 = 2^2\times3\times7$

and $108 = 2\times2\times3\times3\times3 = 2^2\times3^3$

$\therefore$ HCF (60, 84, 108) = $2^2\times3 = 4\times3 = 12$

So, number of students in each group subject to the given condition = HCF (60, 84, 108) = 12.

Now, number of groups in music $= \frac{60}{12} = 5$

Number of groups in dance $= \frac{84}{12} = 7$

Number of groups in handicrafts $= \frac{108}{12} = 9$

Hence, total number of rooms required

$= 5 + 7 + 9 = 21.$

Chapter Test

Multiple Choice Questions

1. LCM of first two consecutive odd number is:

a. 2 b. 3 c. 1 d. 4

2. HCF of 99 and 186 is:

a. 6 b. 9 c. 3 d. 15

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): $n^2 - n$ is divisible by 2 for every positive integer n.

Reason (R): $\sqrt{2}$ is not a rational number.

Q 4. Assertion (A): If LCM = 182, product of integers is 26×91, then HCF = 13.

Reason (R): LCM × product of integers = HCF.

Fill in the Blanks

Q 5. $\sqrt{2}, \sqrt{5}, \sqrt{11}$ etc., are .... numbers.

Q 6. The sum of the exponents of prime factors in the prime factorisation of 250 is ....

True/False

Q 7. HCF of two numbers is always a factor of their LCM.

Q 8. Two numbers have 12 as their HCF and 350 as their LCM.

Case Study Based Question

Q 9. A sweetseller has 420 kaju barfis and 130 gola barfis. He wants to stack them in such a way that each stack has same number and they take up the least area of the tray.

Based on the given information, solve the following questions:

(i) Find the product of exponents of the prime factors of total number of sweets.

(ii) What is the number of sweets that can be placed in each stack for this purpose?

(iii) Find the sum of exponents of the prime factors of the number of sweets that can be placed in each stack for this purpose.

Or

What is the total number of rows in which they can be placed?

Very Short Answer Type Questions

Q 10. If the product of two numbers is 1080 and their HCF is 30, find their LCM.

Q 11. Two positive integers a and b can be written as $a = x^2y^3$ and $b = xy^2$, where x and y are prime numbers. Find LCM (a, b).

Short Answer Type-I Questions

Q 12. Prove that $\sqrt{11}$ is an irrational number.

Q 13. In a school, there are two sections — section A and section B of class X. There are 32 students in section A and 36 students in section B. Find the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B.

Short Answer Type-II Questions

Q 14. Factorise 612 and 1314 by using tree method and find HCF and LCM.

Q 15. Six bells commence tolling together and toll at intervals 2, 4, 6, 8, 10 and 12 minutes, respectively. After how many minutes they will toll together?

Long Answer Type Question

Q 16. There are 156, 208 and 260 students in groups A, B and C respectively. Buses are to be hired to take them for a field trip. Find the minimum number of buses to be hired, if the same number of students should be accommodated in each bus.