7 State Space Analysis

Multiple Choice Questions

Q.1 A certain LTI system has state model

$$\dot{x} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$y = [1 \quad 1]x$ and the values of

$x_1(0) = 1$; $x_2(0) = -1$; $u(0) = 0$. Find $\left.\dfrac{dy}{dt}\right|_{t=0}$

(a) 1 (b) −1
(c) 0 (d) 2

Q.2 The transfer function $Y(s)/U(s)$ of a system described by the state equations $\dot{x}(t) = -2x(t) + 2u(t)$ and $y(t) = 0.5x(t)$ is

(a) $0.5/(s-2)$ (b) $1/(s-2)$
(c) $0.5/(s+2)$ (d) $1/(s+2)$

[GATE-2002]

Q.3 The state-space representation of a system is given by

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U \text{ and } Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T X$$

Then the transfer function of the system is

(a) $\dfrac{1}{s^2+3s+2}$ (b) $\dfrac{1}{s+2}$
(c) $\dfrac{s}{s^2+3s+2}$ (d) $\dfrac{1}{s+1}$ **[ESE-2003]**

Q.4 $$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

If the control signal is

$u = [-0.5 \quad -3 \quad -5]x + v$

Find the eigen values

(a) 0, −1, −2 (b) 0, −1, −3
(c) 0, −2, −3 (d) 0, .1, −2

Q.5 The system mode described by the state equations

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad Y = [1 \quad 1]\,x \text{ is}$$

(a) controllable and observable
(b) controllable but not observable
(c) observable but not controllable
(d) neither controllable nor observable

[GATE-1999]

Q.6 Consider the system $\dot{x} = Ax + BU$

where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $B = \begin{bmatrix} p \\ q \end{bmatrix}$

which of the following about controllability is true.

(a) Controllable for any non-zero value of p & q
(b) Controllable only if $P \neq q = 0$
(c) Un-Controllable for all values of p & q
(d) Cannot be concluded from the data given

Q.7 Let $\dot{X} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$

$y = [b \quad 0]x$

where b is an unknown constant

This system is

(a) observable for all values of b
(b) unobservable for all values of b
(c) observable for all non-zero values of b
(d) unobservable for all non-zero values of b

[ESE-2002]

Q.8 The transfer function of a system

$$\frac{Y(s)}{U(s)}=\frac{1}{s^4+5s^3+8s^2+6s+3}$$

Which one of the following will be 'A' matrix in state variable form?

(a) $\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -3 & -6 & -8 & -5\end{bmatrix}$

(b) $\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -5 & -8 & -6 & -3\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -3 & -6 & -8 & -5\end{bmatrix}$

(d) $\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 3 & 6 & 8 & 5\end{bmatrix}$

Q.9 The zero-input response of a system given by the state-space equation

$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ and $\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$ is

(a) $\begin{bmatrix}te^t\\ t\end{bmatrix}$ (b) $\begin{bmatrix}e^t\\ t\end{bmatrix}$

(c) $\begin{bmatrix}e^t\\ te^t\end{bmatrix}$ (d) $\begin{bmatrix}t\\ te^t\end{bmatrix}$ **[GATE-2003]**

Q.10 Given $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, the state transition matrix e^{At} is given by

(a) $\begin{bmatrix}0 & e^{-t}\\ e^{-t} & 0\end{bmatrix}$ (b) $\begin{bmatrix}e^t & 0\\ 0 & e^t\end{bmatrix}$

(c) $\begin{bmatrix}e^{-t} & 0\\ 1 & e^{-t}\end{bmatrix}$ (d) $\begin{bmatrix}0 & e^t\\ e^t & 0\end{bmatrix}$

[GATE-2004]

Q.11 The state-space representation in phase-variable form for the transfer function $G(s)=\frac{2s+1}{s^2+7s+9}$ is

(a) $\dot{x}=\begin{bmatrix}0 & 1\\ -9 & -7\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u; y[1\ \ 2]x$

(b) $\dot{x}=\begin{bmatrix}0 & 1\\ -9 & -7\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u; y[0\ \ 1]x$

(c) $\dot{x}=\begin{bmatrix}-9 & 0\\ 0 & -7\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u; y[2\ \ 0]x$

(d) $\dot{x}=\begin{bmatrix}-9 & -7\\ 1 & 0\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u; y[1\ \ 2]x$

[ESE-2002]

Q.12 Consider the system described by following state space equations:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u;\quad y=[1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

If u is unit step input, then the steady state error of the system is

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) 1

[GATE-2014]

Q.13 The state variable representation of a system is given as

$$x=\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}x,\quad x(0)=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$y=[0\ \ 1]x$

The response $y(t)$ is

(a) sin (t) (b) 1– e^{t}

(c) 1 – cos (t) (d) 0

[GATE-2015]

Q.14 A network is described by the state model as

$$\dot{x}_1 = 2x_1 - x_2 + 3u$$
$$\dot{x}_2 = -4x_2 - u$$
$$y = 3x_1 - 2x_2$$

The transfer function $H(s)\left(=\frac{Y(s)}{U(s)}\right)$ is

(a) $\frac{11s+35}{(s-2)(s+4)}$ (b) $\frac{11s-35}{(s-2)(s+4)}$

(c) $\frac{11s+38}{(s-2)(s+4)}$ (d) $\frac{11s-38}{(s-2)(s+4)}$

[GATE-2015]

Q.15 The state variable description of an LTI system is given by

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}u$$

$$y=(1\ \ 0\ \ 0)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

(a) $a_1 \neq 0,\ a_2 = 0,\ a_3 \neq 0$

(b) $a_1 = 0,\ a_2 \neq 0,\ a_3 \neq 0$

(c) $a_1 = 0,\ a_2 \neq 0,\ a_3 = 0$

(d) $a_1 \neq 0,\ a_2 \neq 0,\ a_3 = 0$ **[GATE-2012]**

Q.16 A second-order linear time-invariant system is described by the following state equations

$$\frac{d}{dt}x_1(t)+2x_1(t)=3u(t)$$
$$\frac{d}{dt}x_2(t)+x_2(t)=u(t)$$

where $x_1(t)$ and $x_2(t)$ are the two state variables and $u(t)$ denotes the input. If the output $c(t) = x_1(t)$, then the system is

(a) controllable but not observable

(b) observable but not controllable

(c) both controllable and observable

(d) neither controllable nor observable

[GATE-2016]

Q.17 Consider a linear time invariant system $\dot{x} = Ax$, with initial conditions $x(0)$ at $t = 0$. Suppose α and β are eigenvectors of (2 × 2) matrix A corresponding to distinct eigenvalues λ_1 and λ_2 respectively. Then the response $x(t)$ of the system due to initial condition $x(0) = \alpha$ is

(a) $\alpha\, e^{\lambda_1 t}$ (b) $e^{\lambda_2 t_\alpha}\beta$

(c) $e^{\lambda_2 t_\alpha}\alpha$ (d) $e^{\lambda_1 t_\alpha}+e^{\lambda_2 t\beta}$

[EE: GATE-2016]

Numerical Data Type Questions

Q.18 An unforced linear time invariant (LTI) system is represented by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

If the initial conditions are $x_1(0) = 1$ and $x_2(0) = -1$, the solution of the state equation is $x_1(t) = k_1\, e^{-t}$ and $x_2 = k_2\, e^{-2t}$.

Where k_1 and k_2 are respectively ________.

[GATE-2014]

Q.19 Consider the following state-space representation of a linear time-invariant system.

$$\dot{x}(t)=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}x(t),\quad y(t)=c^T x(t),$$

$$c=\begin{bmatrix}1\\ 1\end{bmatrix}\text{ and } x(0)=\begin{bmatrix}1\\ 1\end{bmatrix}$$

The value of $y(t)$ for $t = \log_e 2$ is _________.

[EE: GATE-2016]

Try Yourself

T1. The state equations in the phase variable canonical from can be obtained from the transfer function by

(a) Cascaded decomposition

(b) Direct decomposition

(c) Inverse decomposition

(d) Parallel decomposition

[Ans: (b)]

T2. The state equation of a second-order linear system is given by

$$\dot{x}(t) = Ax(t),\ x(0) = x_0$$

For $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ and for $x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

$$x(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}.$$

When $x_0 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $x(t)$ is

$$\left[\text{Ans: } \begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{-t} + 16e^{-2t} \end{bmatrix}\right]$$

T3. Find the state transition matrix $\phi(t)$ of given system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

$$\left[\text{Ans: } \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}\right]$$

T4. Consider the state space model of a system, as given below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} u;\ y = [1 \quad 1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

The system is

(a) controllable and observable

(b) uncontrollable and observable

(c) uncontrollable and unobservable

(d) controllable and unobservable

[Ans: (b)]

T5. System matrix of continuous time system is

$A = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}$ then characteristic equation is__________.

[Ans: $s^2 + 5s + 3 = 0$]