CBSE Test Paper 05

CH-2 Polynomials

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1. Which of the following expression is a monomial
   a. $4x^3$
   b. $x^6 + 2x^2 + 2$
   c. None of these
   d. 3 + x
2. The value of $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$ is
   a. $3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$
   b. $3(a-b)(b-c)(c-a)$
   c. $3(a+b)(b+c)(c+a)$
   d. none of these
3. The value of $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right)(x+y)\left(x^2+y^2\right)$ is
   a. $(x^4 + y^4)$
   b. $(x^4 - y^4)$
   c. $(x+y)^4$
   d. $(x-y)^4$
4. A polynomial containing one nonzero term is called a ________.
   a. trinomial
   b. binomial
   c. none of these

d. monomial

5. The value of $(x-a)^3+(x-b)^3+(x-c)^3$ - 3 (x-a) (x-b) (x-c) when a + b + c= 3x, is

a. 1

b. 2

c. 3

d. 0

6. Fill in the blanks:

A polynomial containing three non-zero terms is called a ________.

7. Fill in the blanks:

The coefficient of x in the expansion of $(x + 3)^3$ is ________.

8. Write the degree of the following polynomial: 5t - $\sqrt{7}$

9. Whether the following are zero of the polynomial, indicated against them.p(x) = lx + m, x = - $\frac{m}{l}$

10. Expand: $\left(\frac{1}{x}+\frac{y}{3}\right)^3$

11. Find the following product: (2x - y + 3z) $(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$

12. Determine the remainder when the polynomial p(x) = $x^4 - 3x^2 + 2x + 1$ is divided by x - 1.

13. Simplify: $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

14. Factorize: $4x^2+9y^2+16z^2+12xy-24yz-16xz$

15. If x - 3 and x - $\frac{1}{3}$ are both factors of $px^2 + 5x + r$, then show that p = r

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Solution

1. (a) $4x^3$

   Explanation: $4x^3$ because monomial means only one term in an expression.

2. (a) $3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

   Explanation:

   $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$

   Here,

   $a^2-b^2+b^2-c^2+c^2-a^2=0$

   Therefore,

   $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$ = $3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)$ [Since $x^3+y^3+z^3=3xyz$ if $x+y+z=0$]

   $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$ = $3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

3. (b)(x^4-y^4)

   Explanation:

   $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$

   = $\left[\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right](x+y)\left(x^2+y^2\right)$

   = $(x-y)(x+y)\left(x^2+y^2\right)$

   = $\left[(x)^2-(y)^2\right]\left(x^2+y^2\right)$

   = $\left(x^2-y^2\right)\left(x^2+y^2\right)$

   = $\left(x^2\right)^2-\left(y^2\right)^2$

= $x^4 - y^4$

4. (d) monomial

   Explanation: A polynomial containing one nonzero term is called a monomial.

   Example: 3x, $5x^2$, y^3

5. (d) 0

   Explanation:

   $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$

   = [x - a + x - b + x - c]
   $\left[x-a^2\right)+\left(x-b^2\right)+\left(x-c^2\right)-(x-a)(x-b)-(x-b)(x-c)-(x-c)(x-$

   = [3x -({a + b + c]

   $\left[(x-a)^2+(x-b)^2+(x-c)^2-(x-a)(x-b)-(x-b)(x-c)-(x-c)(x-\right.$

   = 3x - 3x
   $\left[(x-a)^2+(x-b)^2+(x-c)^2-(x-a)(x-b)-(x-b)(x-c)-(x-c)(x-\right.$

   =

   $[0]\left[(x-a)^2+(x-b)^2+(x-c)^2-(x-a)(x-b)-(x-b)(x-c)-(x-c)(x\right.$

   = 0

6. trinomial

7. 27

8. Term with the highest power of t = 5t

   Exponent of t in this term = 1

   ∴ Degree of this polynomial = 1

9. p(-$\frac{m}{l}$) = l(-$\frac{m}{l}$) + m = - m + m = 0

   ∴ -$\frac{m}{l}$ is a zero of p(x).

10. $(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$

$$\left[\frac{1}{x}+\frac{y}{3}\right]^3=\left(\frac{1}{x}\right)^3+3\left(\frac{1}{x}\right)^2\frac{y}{3}+3\frac{1}{x}\left(\frac{y}{3}\right)^2+\left(\frac{y}{3}\right)^3$$
$$=\left(\frac{1}{x}\right)^3+3\cdot\left(\frac{1^2}{x^2}\right)\frac{y}{3}+3\cdot\frac{1}{x}\frac{y^2}{3^2}+\frac{y^3}{3^3}$$
$$=\frac{1}{x^3}+\frac{y}{x^2}+\frac{y^2}{3x}+\frac{y^3}{27}$$

11. $(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$

$= (2x + (-y) + 3z)\{(2x)^2 + (-y)^2 + (3z)^2 - 2x \times (-y) - (-y) \times (3z) - 2x \times 3z)\}$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$, where $a = 2x$, $b = -y$, $c = 3z$

$= a^3 + b^3 + c^3 - 3abc$

$= (2x)^3 + (-y)^3 + (3z)^3 - 3 \times 2x \times (-y) \times 3z$

$= 8x^3 - y^3 + 27z^3 + 18xyz$

12. By remainder theorem, the required remainder is equal to p(1).

Now, $p(x) = x^4 - 3x^2 + 2x + 1$

$\Rightarrow p(1) = (1)^4 - 3 \times 1^2 + 2 \times 1 + 1 = 1 - 3 + 2 + 1 = 1$

Hence, required remainder = p(1) = 1

13. We have,

$(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

$\therefore (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

$\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)$

Similarly, we have,

$(a - b) + (b - c) + (c - a) = 0$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

$$\therefore \quad \frac{\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$
$$=\frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)} = (a+b)(b+c)(c+a)$$

14. $4x^2+9y^2+16z^2+12xy-24yz-16xz$

The given expression can be re written as

$$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x$$

As we know, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x,$

$\Rightarrow (2x+3y-4z)^2 = (2x+3y-4z)(2x+3y-4z)$

$\Rightarrow 4x^2+9y^2+16z^2+12xy-24yz-16xz = (2x+3y-4z)(2x+3y-4z)$

15. $\because$ x - 3 and x - $\frac{1}{3}$ are factors of

$px^2 + 5x + r \therefore$ x = 3, x = $\frac{1}{3}$

zero of px^2+5x+r

Putting x = 3 in given polynomial,

$\therefore p(3)^2+5\times 3+r=0$

9p + 15 + r = 0

9p + r = - 15 - - - - - (1)

Again putting x = $\frac{1}{3}$ in given polynomial,

$p\left(\frac{1}{3}\right)^2+5\times\frac{1}{3}+r=0$

$\frac{p}{9}+\frac{5}{3}+r=0$

$\frac{p+15+9r}{9}=0$

$p+9r=-15------(2)$

Fron eq.(1) and eq.(2), we have,

9p + r = p + 9r

9p-p=9r-r

8p=8r

p=r

Hence proved