

## Chapter 3

# Current Electricity

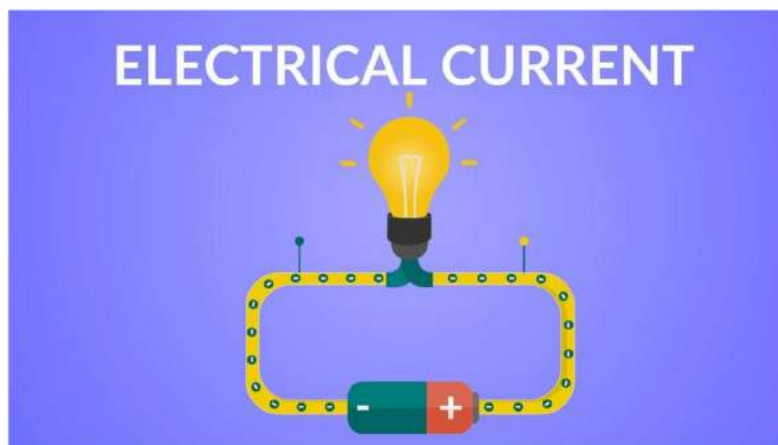
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### Electric Current, Ohm's Law & Drift of Electrons

#### What is Electric Current?

**Electric Current** is the rate of flow of electrons in a conductor. The SI Unit of electric current is the **Ampere**.

- Our **ancestors** relied on fire for light, warmth and cooking. Today at the flick of a switch, turn of a knob or the push of a button we have instant power. This is possible because of the electric current.
- From the basic bread toaster, baking oven to the commonly used television all require an electric current to operate. The most common device, mobile phones use the **electric current** to charge the battery for the operation. Besides playing a major part at home, electricity also plays an important role in industries, transportation and communication



- Electrons are minute particles that exist within the **molecular structure** of a substance. Sometimes, these electrons are tightly held, and other times they are loosely held. When electrons are loosely held by the nucleus, they are able to travel freely within the limits of the body.
- Electrons are negatively charged particles hence when they move a number of charges moves and we call this **movement of electrons** as

electric current. It should be noted that the number of electrons that are able to move governs the ability of a particular substance to conduct electricity.

- Some materials allow current to move better than others.

Based on the ability of the material to conduct electricity, materials are classified into Conductors, Insulators & Semiconductor.

### 1. Conductor

- In some materials, the outer electrons of each atom or molecules are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called **free electrons**.
- They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called **conductors**.
- **Examples of conductors:** Human body, aqueous solutions of salts and metals like iron, silver and gold.



### Did you know?

Silver is the best conductor of electricity.

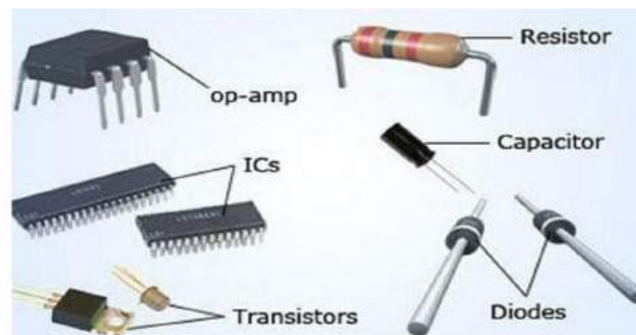
### 2. Insulator

- Another class of materials is called insulators in which all the electrons are tightly bound to their respective **atoms or molecules**.
- Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their **parent atoms or molecules** and hence can't move through long distances.
- Such materials are also called **dielectrics**.



### 3. Semiconductor

- In **semiconductors**, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field.
- As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a **conductor and an insulator** and hence, the name semiconductor.
- A freed electron in a semiconductor leaves a vacancy in its **normal bound position**. These vacancies also help in conduction.



### Semiconductors

#### Unit of Electric Current

- The magnitude of electric current is measured in coulombs per second.
- The SI unit of electric current is **Ampere** and is denoted by the letter A.
- Ampere is defined as one coulomb of charge moving past a point in one second. If there are  $6.241 \times 10^{18}$  electrons flowing through our frame in one second then the electrical current flowing through it is 'One Ampere.'
- The unit Ampere is widely used within electrical and electronic technology along with the multipliers like **milliamp (0.001A)**, **microamp (0.000001A)**, and so forth.

#### Properties of Electric Current

Electric current is an important quantity in **electronic circuits**. We have adapted electricity in our lives so much that it becomes impossible to imagine life without it. Therefore, it is important to know what is current and the properties of the electric current.

- We know that electric current is the result of the flow of electrons. The work done in moving the electron stream is known as **electrical energy**. Electrical energy can be converted into other forms of energy such as heat energy, light energy, etc. For example, in an iron box, electric energy is converted to heat energy. Likewise, the electric energy in a bulb is converted into light energy.
- There are two types of electric current known as **alternating current (AC)** and **direct current (DC)**.
- The direct current can flow only in one direction, whereas the alternating direction flows in **two directions**.
- Direct current is seldom used as a primary energy source in industries. It is mostly used in **low voltage applications** such as charging batteries, aircraft applications, etc. Alternating current is used to operate appliances for both household and industrial and commercial use.
- The electric current is measured in ampere. One ampere of current represents one coulomb of electric charge moving past a specific point in one second.

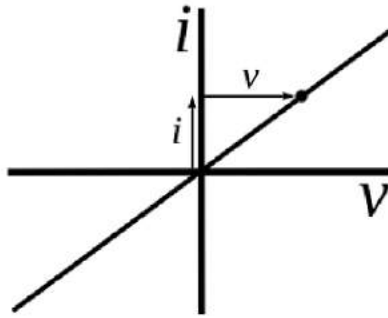
1 ampere = 1 coulomb / 1 second

- The **conventional direction** of an electric current is the direction in which a positive charge would move. Henceforth, the current flowing in the external circuit is directed away from the positive terminal and toward the negative terminal of the battery.

### What is Ohm's Law?

**Ohm's law** states that the voltage across a conductor is directly proportional to the current flowing through it, provided all physical conditions and temperature remain constant.

- Ohm's Law of Current Electricity is named after the scientist "Ohm". Most basic components of current electricity are voltage, current, and resistance. Ohm's law shows a simple relation between these three quantities.

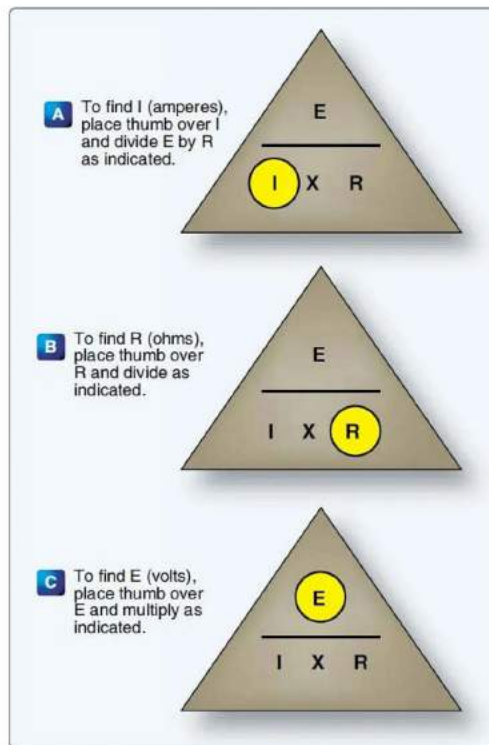


- Voltage = Current  $\times$  Resistance  
 $V = I \times R$   
 where  $V$  = voltage,  $I$  = current and  $R$  = resistance. The SI unit of resistance is **ohms** and is denoted by  $\Omega$ .
- In order to establish the current-voltage relationship, the ratio  $V / I$  remains constant for a given resistance, therefore a graph between the potential difference ( $V$ ) and the current ( $I$ ) must be a straight line.
- This law helps us in determining either voltage, current or impedance or resistance of a linear electric circuit when the other two quantities are known to us. It also makes power calculation simpler.

**Ohm's Law Equation:**  $V = IR$ , where  $V$  is the voltage across the conductor,  $I$  is the current flowing through the conductor and  $R$  is the resistance provided by the conductor to the flow of current.

### Ohm's Law Magic Triangle

You can make use of the **Ohm's law magic triangle** to remember the different equations for Ohm's law used to solve for different variables ( $V$ ,  $I$ ,  $R$ ).



If the value of voltage is asked and the values of the current and resistance are given, then to calculate voltage simply cover **V** at the top. So, we are left with the **I** and **R** or  $I \times R$ . So, the equation for Voltage is Current multiplied by Resistance. Examples of how the magic triangle is employed to determine the voltage using Ohm's law is given below.

**Example 1: If the resistance of an electric iron is  $50 \Omega$  and a current of  $3.2 \text{ A}$  flows through the resistance. Find the voltage between two points.**

If we are asked to calculate the value of voltage with the value of current and resistance given to us, then cover **V** in the triangle. Now, we are left with **I** and **R** or more precisely  $I \times R$ .

Therefore, we use the following formula to calculate the value of **V**:

$$V = I \times R$$

Substituting the values in the equation, we get

$$V = 3.2 \text{ A} \times 50 \div = 160 \text{ V}$$

**Example 2: An EMF source of  $8.0 \text{ V}$  is connected to a purely resistive electrical appliance (a light bulb). An electric current of  $2.0 \text{ A}$  flows through it. Consider the conducting wires to be resistance-free. Calculate the resistance offered by the electrical appliance.**

When we are asked to find out the value of resistance when the values of voltage and current are given, then we cover R in the triangle. This leaves us with only V and I, more precisely  $V \div I$ .

Substituting the values in the equation, we get

$$R = V \div I$$

$$R = 8 \text{ V} \div 2 \text{ A} = 4 \Omega$$

### Applications of Ohm's Law

The main applications of Ohm's law are:

- To determine the **voltage, resistance or current** of an electric circuit.
- Ohm's law is used to maintain the desired voltage drop across the electronic components.
- Ohm's law is also used in **DC ammeter** and other DC shunts to divert the current.

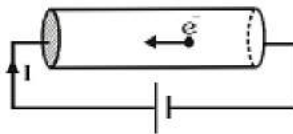
### Limitations of Ohm's Law

Following are the limitations of Ohm's law:

- Ohm's law is not applicable for **unilateral electrical elements** like diodes and transistors as they allow the current to flow through in one direction only.
- For **non-linear electrical elements** with parameters like capacitance, resistance etc the voltage and current won't be constant with respect to time making it difficult to use Ohm's law.

### Electric Current and Current Density

- Electric charges in motion constitute an **electric current**. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from a higher potential energy state to a lower potential energy state.



- Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors.
- When charge flows in a conductor from one place to the other, then the rate of flow of charge is called **electric current (I)**.

- When there is a **transfer of charge** from one point to other point in a conductor, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion of charge.
- If they are negative the current is opposite to the direction of motion. If a charge  $\Delta Q$  crosses an area in time  $\Delta t$  then the average electric current through the area, during this time as
  - (i) Average current  $I_{av} = \Delta Q / \Delta t$
  - (ii) Instantaneous current

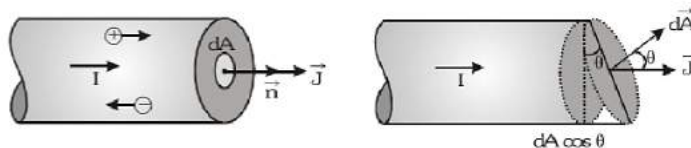
$$I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

**Example 3:** If  $q = 2t^2$ , find current at  $t = 2$  sec ?

$$i = dq/dt, i = 4t$$

$$\therefore i \text{ at } 2 \text{ sec} = 4 \times 2 = 8 \text{ A}$$

- Current is a **macroscopic quantity** and deals with the overall rate of flow of charge through a section. To specify the current with direction in the microscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.
- Current density at point P is given by



- If the cross-sectional area is not normal to the current but makes an angle  $\theta$  with the direction of current
 

then  $J = \frac{dI}{dA \cos \theta} \Rightarrow dI = J dA \cos \theta = \vec{J} \cdot d\vec{A} \Rightarrow I = \int \vec{J} \cdot d\vec{A}$
- Current density  $\vec{J}$  is a vector quantity. Its direction is same as that of  $I$ . Its S.I. unit is ampere/m<sup>2</sup> and dimension [L<sup>-2</sup>A].

**Example 4:** An electron beam has an aperture 1.0 mm<sup>2</sup>. A total of  $6.0 \times 10^{10}$  electrons go through any perpendicular cross-section per second. Find (a) the current and (b) the current density in the beam.

The total charge crossing a perpendicular cross-section in one second is

$$\begin{aligned} q &= ne \\ &= 6.0 \times 10^{16} \times 1.6 \times 10^{-19} \text{ C} \\ &= 9.6 \times 10^{-3} \text{ C} \end{aligned}$$

The current is

$$i = \frac{q}{t} = \frac{9.6 \times 10^{-3} \text{ C}}{1 \text{ s}} = 9.6 \times 10^{-3} \text{ A}$$

As the charge is negative, the current is opposite to the direction of motion of the beam.

(b) The current density is

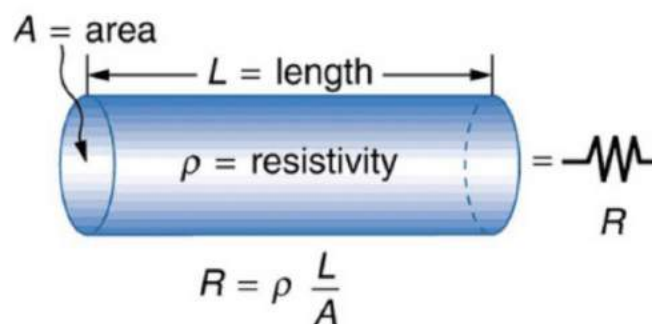
$$j = \frac{i}{S} = \frac{9.6 \times 10^{-3} \text{ A}}{(1.0 \text{ mm})^2} = \frac{9.6 \times 10^{-3} \text{ A}}{1.0 \times 10^{-8} \text{ m}^2} = 9.6 \times 10^3 \text{ A/m}^2$$

### Drift of Electrons & the Origin of Resistivity

- The **net velocity** of the circuit is zero when electrons move randomly in the circuit and the electric field is not applied to the circuit.
- Drift force is the force driving the electrons through a conductor and the force opposing the drift force is resistivity.

### What is Resistance or Resistivity?

The tendency of a material/device towards resistance is the resistivity of the device/circuit. The SI unit of resistivity is ohm-meter. The unit length across the cross-sectional area of the device is also resistivity. Therefore, the nature and temperature of the material also define resistivity ( $\sigma$ ).



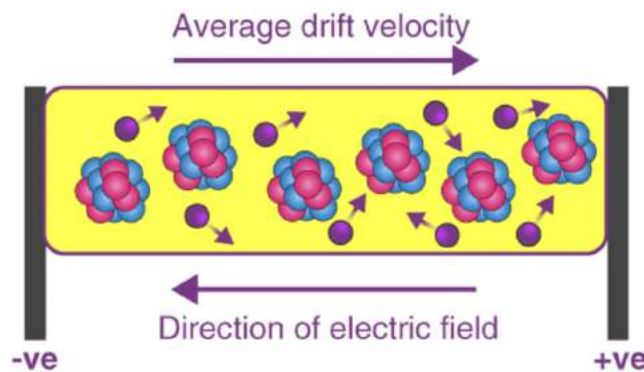
The graph of resistivity as follows. The graphs depict current (I) to voltage (V) ratio, whereas, dotted line A, B, C shows the idealized graph. After a certain amount of current, the device starts resisting to the current flowing in the system and the resistivity becomes constant.

## Drift of Electrons

- The free electrons in a conductor have random velocities and move in random directions. When current is applied across the conductor the randomly moving electrons are subjected to electrical forces along the direction of the electric field.
- Due to this electric field, free electrons still have their random moving nature, but they will move through the conductor with a certain along with force. The net velocity in a conductor due to the moving of electrons is referred to as the drift of electrons.

## Drift Velocity

- Drift velocity is defined as the velocity with which the **free electrons** get drifted towards the positive terminal under the effect of the applied external electric field.
- In addition to its **thermal velocity**, due to acceleration given by the applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



At any given time, an electron has a velocity,

$\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$ , where  $\vec{u}_1$  = the thermal velocity and  $\vec{a}\tau_1$  = the velocity acquired by the electron under the influence of the applied electric field.

$\tau_1$  = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, \dots, \vec{v}_N = \vec{u}_N + \vec{a}\tau_N$$

The average velocity of all the free electrons in the conductor is equal to the drift velocity  $\vec{v}_d$  of the free electrons

$$\begin{aligned}\vec{v}_d &= \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \dots + \vec{v}_N}{N} = \frac{(u_1 + \vec{a}\tau_1) + (u_2 + \vec{a}\tau_2) + \dots + (u_N + \vec{a}\tau_N)}{N} \\ &= \frac{(\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N)}{N} + \vec{a} \left( \frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right) \\ \therefore \frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N}{N} &= 0 \quad \therefore \vec{v}_d = \vec{a} \left( \frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right) \Rightarrow \vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau\end{aligned}$$

**Note:** Order of drift velocity is  $10^{-4}$  m/s.

### ➤ Relation between current and drift velocity

Let  $n$  = number density of free electrons and  $A$  = area of cross-section of conductor.  
Number of free electrons in conductor of length  $L$  =  $nAL$ , Total charge on these free electrons  $\Delta q = neAv_d$

Time taken by drifting electrons to cross conductor

$$\Delta t = \frac{L}{v_d} \quad \therefore \text{current } I = \frac{\Delta q}{\Delta t} = neAL \left( \frac{v_d}{L} \right) = neAv_d$$

### Mobility

- **Conductivity** arises from mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionised gas, they are electrons and positive charged ions; in an electrolyte, these can be both positive and negative ions.
- An important quantity is the **mobility**  $\mu$  defined as the magnitude of the drift velocity per unit electric field:

$$\mu = \frac{|v_d|}{E}$$

**Example 5:** Calculate the drift speed of the electrons when 1 A of current exists in a copper wire of cross-section  $2 \text{ mm}^2$ . The number of free electrons in  $1 \text{ cm}^3$  of copper is  $8.5 \times 10^{22}$ .

We have

$$j = nev_d$$

or,

$$v_d = -\frac{j}{ne} = \frac{i}{A_{ne}}$$

$$= \frac{1A}{(2 \times 10^{-8} m^2)(8.5 \times 10^{22} \times 10^8 m^{-3})(1.6 \times 10^{-19} C)} = 0.036 mm/s$$

We see that the drift speed is indeed small.

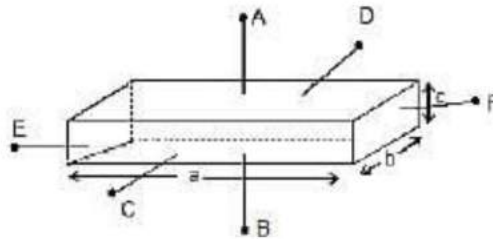
**Example 6:** Calculate the resistance of an aluminium wire of length 50 cm and cross-sectional area  $2.0 \text{ mm}^2$ . The resistivity of aluminium is  $\rho = 2.6 \times 10^{-8} \Omega m$ . The resistance is

$$R = \frac{\rho \ell}{A}$$

$$= \frac{(2.6 \times 10^{-8} \Omega - m) \times (0.50 m)}{2 \times 10^{-6} m^2} = 0.0065 \Omega$$

We arrived at Ohm's law by making several assumptions about the existence and behaviour of the free electrons. These assumption are not valid for semiconductors, insulators, solutions etc. Ohm's law cannot be applied in such cases.

**Example 7:** The dimensions of a conductor of specific resistance  $r$  are shown below. Find the resistance of the conductor across AB, CD and EF.

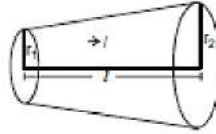


$$R_{AB} = \frac{\rho c}{ab}, \quad R_{CD} = \frac{\rho b}{ac}, \quad R_{EF} = \frac{\rho a}{bc}$$

**Example 8:** A portion of length  $L$  is cut out of a conical solid wire. The two ends of this portion have circular cross-sections of radii  $r_1$  and  $r_2$  ( $r_2 > r_1$ ). It is connected lengthwise to a circuit and a current  $i$  is flowing in it. The resistivity of the material of the wire is  $\rho$ . Calculate the resistance of the considered portion and the voltage developed across it.

If follows from the figure, that

$$\tan = \frac{r_2 - r_1}{L}$$



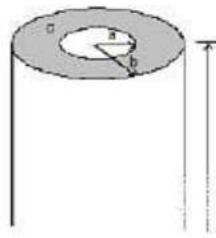
Therefore,

$$r = r_1 \times \tan q = r_1 \times \left( \frac{r_2 - r_1}{L} \right) = \frac{r_1 L + x(r_2 - r_1)}{L}$$

Therefore,

$$\begin{aligned} A &= \pi r^2 = \frac{\pi}{L^2} [r_1 L + (r_2 - r_1)x]^2 \\ dR &= \frac{\rho dx}{\pi r^2} = \frac{\rho dx L^2}{\pi [r_1 L + (r_2 - r_1)x]^2} \\ \Rightarrow R &= \int dR = \frac{\rho L^2}{\pi} \int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2} \\ &= \frac{\rho L^2}{\pi} \left[ \{r_1 L + \{r_2 - r_1\}x\}^{-1} \right]_0^L \left( -\frac{1}{(r_2 - r_1)} \right) \\ &= \frac{-\rho L}{\pi(r_2 - r_1)} \left( \frac{1}{r_2} - \frac{1}{r_1} \right) = \frac{\rho L}{\pi(r_1 r_2)} \\ \text{Therefore, } V = R &= \frac{\rho L}{\pi r_1 r_2} \end{aligned}$$

**Example 9:** The space between two coaxial cylinders, whose radii are  $a$  and  $b$  (where  $a < b$  as in (figure shown)) is filled with a conducting medium. The specific conductivity of the medium is  $\sigma$ .



(a) Compute the resistance along the length of the cylinder.

(b) Compute the resistance between the cylinders in the radial direction.

Assume that the cylinders are very long as compared to their radii, i.e.,  $L \gg b$ , where  $L$  is the length of the cylinders.

$$(a) \quad R = \frac{\rho l}{A} = \frac{l}{\sigma A} = \frac{l}{\sigma(\pi b^2 - \pi a^2)} = \frac{l}{\pi \sigma (b^2 - a^2)}$$

(b) From Ohm's law, we have

$$\vec{J} = \sigma \vec{E}$$

Assuming radial current density,  $\vec{J}$  becomes

$$\vec{J} = \frac{1}{2\pi r L} \hat{r} \text{ for } a < r < b$$

and, therefore,

$$\vec{E} = \frac{1}{2\pi r L} \hat{r}$$

Here we have used the assumption that  $L \gg b$  so that  $\vec{E}$  and are in cylindrically symmetric form. The potential drop across the medium is thus

$$V_{ab} = - \int_b^a \vec{E}(r) \cdot d\vec{r} = - \frac{1}{2\pi \sigma L} \int_b^a \frac{dr}{r} = \frac{1}{2\pi \sigma L} \ln \left( \frac{b}{a} \right)$$

The resistance

$$R_{ab} = \frac{V_{ab}}{I} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma L}$$

**Method 2:** We split the medium into differential cylindrical shell elements of width  $dr$ , in series. The current flow is cylindrically symmetric ( $L \gg b$ ). The area through which the current flows across a shell of radius  $r$  is  $A(r) = 2\pi rL$ . The length the current flows, passing through a shell of radius  $r$  is  $dr$ . Therefore, the resistance of the shell of radius  $r$  is:

$$dR = \frac{1}{\sigma} \frac{dr}{2\pi rL}$$

Since the shells are connected in a series, we have

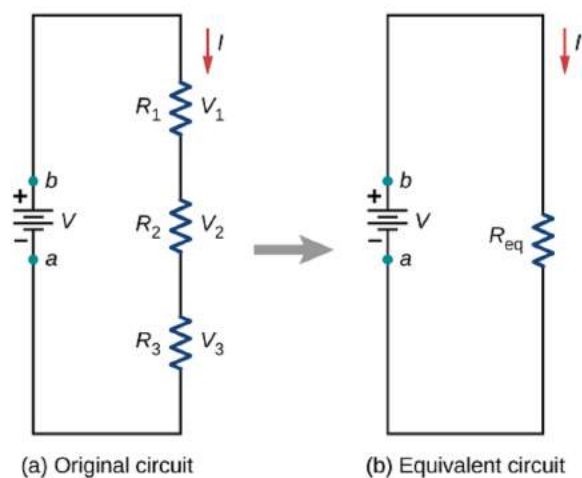
$$R_{ab} = \int_a^b dR = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma L}$$

## Resistors in Series & Parallel Combinations

### Combination of Resistance

A number of resistance can be connected in a circuit and any complicated combination can be, in general, reduced essentially to two different types, namely series and parallel combinations.

#### Resistance in Series

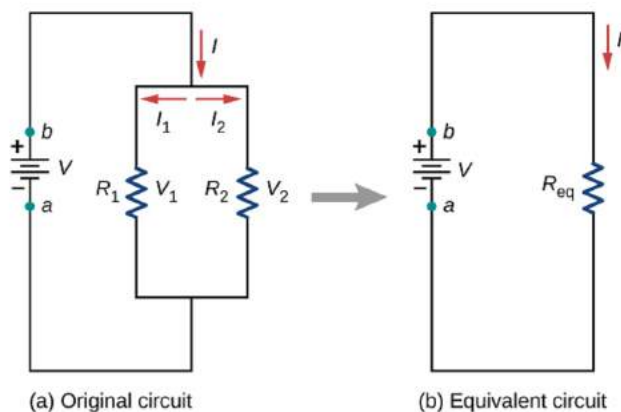


- In this combination the resistance are joined end to end. The second end of each resistance is joined to first end of the next resistance and so on.

A cell is connected between the first end of first resistance and second end of last resistance. Figure shows three resistances  $R_1$ ,  $R_2$  and  $R_3$  connected in this way. Let  $V_1$ ,  $V_2$  and  $V_3$  are the potential differences across these resistances.

- In this combination current flowing through each resistance will be same and will be equal to current supplied by the battery.
  - As resistances are different and current flowing through them is same, hence potential differences across them will be different. Applied potential difference will be distributed among three resistances directly in their ratio.
  - As  $i$  is constant, hence  $V$  is directly proportional to  $R$   
i.e.,  $V_1 = iR_1$ ,  $V_2 = iR_2$ ,  $V_3 = iR_3$
  - If the potential difference between the points A and D is  $V$ , then  
 $V = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$
  - If the combination of resistances between two points is replaced by a single resistance  $R$  such that there is no change in the current of the circuit in the potential difference between those two points, then the single resistance  $R$  will be equivalent to combination and  $V = iR$  i.e.,  
 $IR = I(R_1 + R_2 + R_3)$   
 $R = R_1 + R_2 + R_3$
  - **Thus in series combination of resistances, important conclusion are:**
    - (i) Equivalent Resistance > highest individual resistance
    - (ii) Current supplied by source = Current in each resistance
- or  $\frac{V}{R_1 + R_2 + R_3} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3}$
- (iii) The total potential difference  $V$  between points A and B is shared among the three resistances directly in their ratio  
 $V_1 : V_2 : V_3 = R_1 : R_2 : R_3$

### Resistance in Parallel



- When two or more resistance are combined in such a way that their first ends are connected to one terminal of the battery while other ends are connected to other terminal, then they are said to be connected in parallel. Figure shows three resistances  $R_1$ ,  $R_2$  and  $R_3$  joined in parallel between two points A and B. Suppose the current flowing from the battery is  $i$ . This current gets divided into three parts at the junction A. Let the currents in three resistance  $R_1$ ,  $R_2$  and  $R_3$ , are  $i_1$ ,  $i_2$ ,  $i_3$  respectively.

- Suppose potential difference between points A and B is  $V$ . Because each resistance is connected between same two points A and B, hence potential difference across each resistance will be same and will be equal to applied potential difference  $V$ .

- Since potential difference across each resistance is same, hence current approaching the junction A is divided among three resistances reciprocally in their ratio.

As  $V$  is constant, hence  $i \propto (1/R)$  i.e.,

$$i_1 = \frac{V}{R_1}, i_2 = \frac{V}{R_2} \text{ and } i_3 = \frac{V}{R_3}$$

- Because  $i$  the main current which is divided into three parts  $i_1$ ,  $i_2$  and  $i_3$  at the junction A.

$$\text{hence, } i = i_1 + i_2 + i_3 = V \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right]$$

- If the equivalent resistance between the points A and B is  $R$ , then  $i = \frac{V}{R}$

$$\text{Thus, } \frac{V}{R} = V \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] \text{ or } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- Thus in parallel combination of resistance important conclusion are :

(i) Equivalent resistance < lowest individual resistance

(ii) Applied potential difference = Potential difference across each resistance.

$$\text{or } iR = i_1R_1 = i_2R_2 = i_3R_3$$

(iii) Current approaching the junction A = Current leaving the junction B and current is shared among the three resistances in the inverse ratio of resistances

$$i_1 : i_2 : i_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

**Note:** (i) If two or more resistance are joined in parallel then  $i_1R = i_1R_2 = i_3R_3$ .....

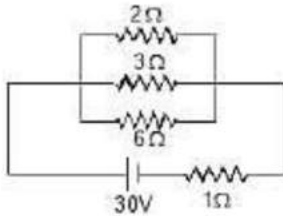
i.e.,  $iR = \text{constant}$  i.e., a low resistance joined in parallel always draws a higher current.

(ii) When two resistance  $R_1$  and  $R_2$  are joined in parallel, then

$$\frac{i_1R_1}{i_2R_2} = 1 \text{ or } \frac{i_1^2R_1^2}{i_2^2R_2^2} = 1 \text{ or } \frac{i_1^2R_1^2}{i_2^2R_2^2} = \frac{R_2}{R_1} \text{ or } \frac{H_1}{H_2} = \frac{R_2}{R_1}$$

i.e., heat produced will be maximum in the lowest resistance.

**Example1.** Find current which is passing through battery.

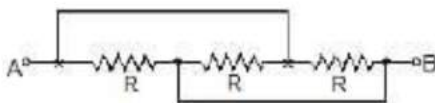


**Sol.** Here potential difference across each resistor is not 30 V  
Q battery has internal resistance here the concept of combination of resistors is useful.

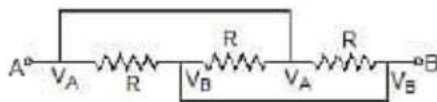
$$R_{eq} = 1 + 2 = 3\Omega$$

$$i = \frac{30}{3} = 10A$$

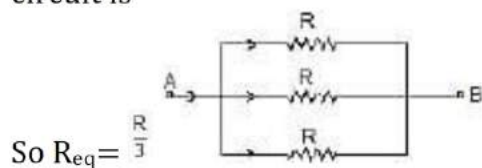
**Example 2.** Find equivalent Resistance



**Sol.**

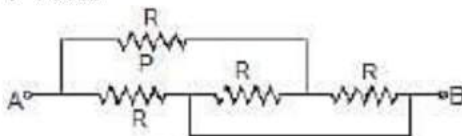


Here all the Resistance are connected between the terminals A and B. So, Modified circuit is

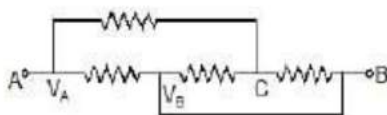


$$\text{So } R_{eq} = \frac{R}{3}$$

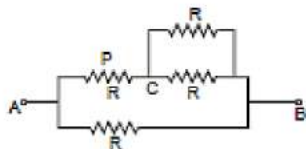
**Example 3.** Find the current in Resistance P if voltage supply between A and B is V volts



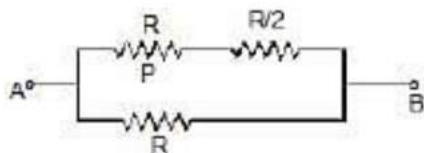
$$\text{Sol. } R_{eq} = \frac{3R}{5}$$



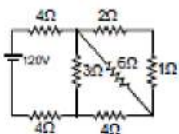
$$I = \frac{5V}{3R} \text{ Modified circuit}$$



$$\text{Current in } P = \frac{R \times \frac{5V}{3R}}{1.5R + R} = \frac{2V}{3R}$$

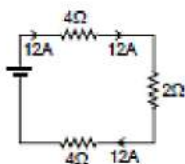


**Example 4. Find the current in 2 W resistance.**



**Sol.** 2Ω, 1Ω in series = 3Ω

3Ω, 6Ω in parallel =  $\frac{18}{9} = 2\Omega$



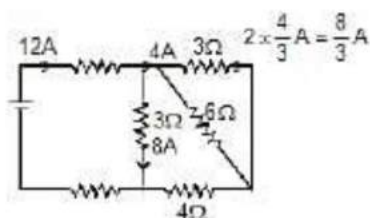
2Ω, 4Ω in series = 6Ω

6Ω, 3Ω in parallel = 2Ω

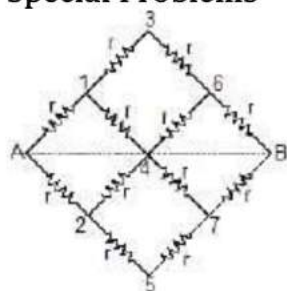
$R_{eq} = 4 + 4 + 2 = 10 \Omega$

$$i = \frac{120}{10} = 12A$$

So current in 2? Resistance =  $\frac{8}{3} A$



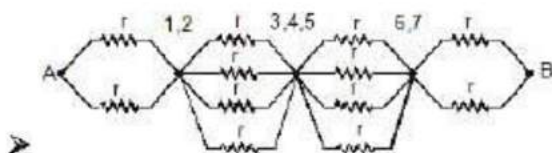
### Special Problems



We wish to determine equivalent resistance between A and B. In figure shown points (1,2) (3, 4, 5) and (6, 7) are at same potential. Equivalent circuit can be redrawn as in figure shown.

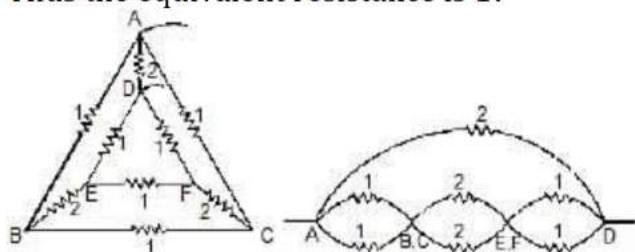
The equivalent resistance of this series combination is

$$R_{eq} = \frac{r}{2} + \frac{r}{4} + \frac{r}{4} + \frac{r}{2} = \frac{3r}{2}$$



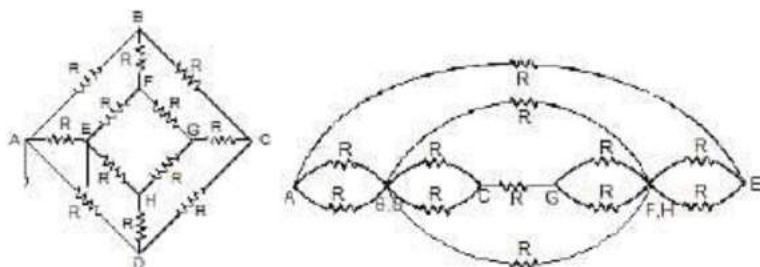
In the figure shown, the resistances specified are in ohms. We wish to determine the equivalent resistance between point A and D. Point B and C, E and F are the same potential so the circuit can be redrawn as in figure shown.

Thus the equivalent resistance is 1?



➤ In the network shown in figure shown all the resistances are equal, we wish to determine equivalent resistance between A and E. Point B and D have same

potential, similarly F and H have same potential. The equivalent circuit is shown in figure shown. The equivalent resistance of network is  $7R/2$ .



### Temperature Dependence of Resistivity

Resistivity is the nature of a material that allows or resists the flow of electric current through a given element or material. What is surprising about resistivity is the temperature dependence of electrical resistance! It is hard to comprehend how the temperature of an element can affect the degree of conductance of such material but believe it or not, this is the world of science and it happens almost every day, all around us!

### The Concept of Electrical Resistance

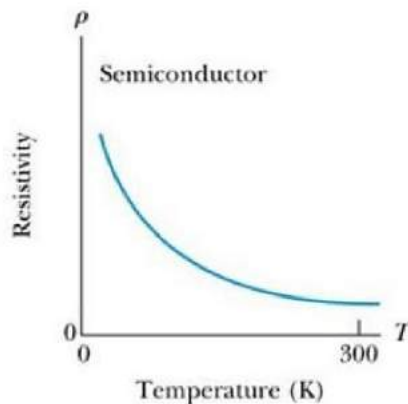
Resistivity is the phenomenon of specific electrical resistance of a material or volume resistivity of a material. It can also be defined as the intrinsic property of a material that displays how the material resists the flow of current in the material. The concept can also be defined as the resistance that is displayed by a conductor which has unit length and unit area of the given cross section.

So resistivity is not dependent upon the length and area of a cross-section of a given material. However, the resistance of a material depends upon the length and area of the cross-section of the material in question. The resistivity manifests as:

$$\rho = RA/L,$$

where R is the resistance in ohms, A is the area of cross-section in square meters and L is the length in meters. The unit of resistivity is universally accepted as ohm-meter.

### The Concept of Temperature Resistivity



The resistivity of materials is dependent upon the temperature of the material.

$$\rho_t = \rho_0 [1 + \alpha (T - T_0)]$$

is the equation that defines the connection between the temperature and the resistivity of a given material. In this equation  $\rho_0$  is the resistivity at an equilibrium temperature,  $\rho_t$  is the resistivity at  $t_0$  C,  $T_0$  is referred to as the reference temperature and  $\alpha$  is the temperature coefficient of resistivity.

### Understanding the Equation

It is known that an electric current is the movement of free electrons from one atom to the other when there is a potential difference between the two. In the case of conductors, no gap is present between the conduction band and valence band of the electrons. In most cases, these bands overlap each other.

The valence electrons in a given atom are loosely bound to the nucleus in a conducting material. Quite often, metals or conductors have a low ionization energy and therefore, they tend to lose electrons very fluidly. When an electric current is applied, the electrons are free to move within the structure on their own. This happens in the case of the normal temperature of a material.

However, when the temperature increases gradually, the vibrations in the metal ions in the lattice structure also undergo an increase. In this case, the atoms begin to vibrate with a higher amplitude. Such vibrations, in turn, cause frequent collisions between the free electrons and the remaining electrons.

Each such collision drains out some degree of energy of the free moving electrons and renders them in a condition in which they are not able to move. Thus, it causes a restriction in the movement of the delocalized electrons.

In the case of metals or conductors, it is rightly said that they hold a positive temperature coefficient. The value  $\alpha$  is positive. For most of the metals, the resistivity increases in a linear pattern with an increase in the temperature in a range of 500K.

## What happens in Insulators?

In the case of insulators, the forbidden energy gap between the conduction band and the valence band is very high. The valence band is filled with the electrons of the atoms. Diamond is a unique example of an insulator. Here, all the valence electrons are involved in the covalent bond formation and as a result, conduction does not take place. The electrons are too tightly bound to the nucleus of the atom.

## Resistance of Pure Metals

(i) We know that  $R = \left[ \frac{2m}{ne^2\tau} \right] \frac{l}{A}$

For a given conductor,  $l$ ,  $A$  and  $n$  are constant, hence  $R$  is directly proportional to  $(1/\tau)$

If  $\lambda$  represents the mean free path (Average distance covered between two successive collisions) of the electron and  $v_{rms}$ , the root-mean-square speed, then

$\tau = \frac{\lambda}{v_{rms}}$ . Hence  $R$  is directly proportional to  $\frac{v_{rms}}{\lambda}$

Now,

(a)  $\lambda$  decreases with rise in temperature because the amplitude of vibrations of the +ve ions of the metal increases and they create more hindrance in the movement of electrons and,

(b) (i)  $v_{rms}$  increases because  $v_{rms}$  is directly proportional to under root  $T$ .

Therefore, **Resistance of the metallic wire increases with rise in temperature**. As  $\rho$  is directly proportional to  $R$  and  $\sigma$  is directly proportional to  $(1/\rho)$ , hence resistivity increases and conductivity decreases with rise in temperature of the metallic of the metallic wires.

(ii) If  $R_0$  and  $R_t$  represent the resistances of metallic wire at  $0^\circ\text{C}$  and  $t^\circ\text{C}$  respectively then  $R_t$  is given by the following formula:

$$R_t - R_0 = R_0 \alpha t$$

where  $\alpha$  is called as the **Temperature coefficient of resistance** of the material of the wire.

$\alpha$  depends on material and temperature but generally it is taken as a constant for a particular material for small change.

$$R_t - R_0 = R_0 \alpha t$$

for very small change in temperature  $dR = R_0 \alpha dt$

## (c) Resistance of semiconductors

(i) There are certain substances whose conductivity lies in between that of insulators and conductors, higher than that of insulators but lower than that of conductors. These are called as semiconductors, e.g., silicon, germanium, carbon etc.

(ii) The resistivity of semiconductors decreases with increase in temperature i.e., **a for semiconductors is -ve and high**.

(iii) Though at ordinary temperature the value of  $n$  (no. of free electrons per unit volume) for these materials is very small as compared to metals, but increases very

rapidly with rise in temperature (this happens due to breaking of covalent bonds). Though  $\tau$  decreases but factor of  $n$  dominates. Therefore, the resistance

$R = \frac{m'l}{ne^2 \tau A}$  goes on decreasing with increase in temperature.

### Specific Resistance or Resistivity

- R of a conductor is directly proportional to its length (l).
- R of a conductor is inversely proportional to the area of cross-section (A).

$$R = \frac{\rho L}{A}$$

$\rho$  = resistivity  
 $L$  = length  
 $A$  = cross sectional area

Resistivity depends only upon the material of which the conductor is made. It is defined as the resistance of the conductor made of a given material having length of 1 meter and area of cross-section 1 m<sup>2</sup>

If  $\rho_1$  and  $\rho_2$  are resistivities of a material at temperatures  $T_1$  and  $T_2$  respectively, then:

$$\rho_2 = \rho_1 (1 + \alpha_\rho \theta)$$

where  $\theta = T_2 - T_1$  is the temperature difference and  $\alpha_\rho$  is known as temperature coefficient of resistivity.

### Solved Examples for You

**Question:** State the properties and features of temperature resistivity in conductors and insulators.

**Solution:** The resistivity of a material is defined as the resistance offered by a conductor having a given unit length and unit area of cross-section. The unit of resistivity is ohm meter. The formula for deriving resistivity is  $\rho = RA/L$ . Here, R is the resistance in ohms, A is the area of cross-section in square meters and L is the length in meters.

- In the case of metals or conductors, when the temperature increases, the resistivity of the metal increases as a result. Thus, the flow of current in the metal decreases. This phenomenon reflects a positive temperature coefficient. The value  $\alpha$  is positive in this case.
- In the case of insulators, the conductivity of the material generally increases, when the temperature is made to increase. When the conductivity of the material undergoes an increase, it is easy to decipher

that the resistivity of the material decreases and the current flow of the material increases.

## Electrical Energy & Power

### Electrical Power



- The energy liberated per second in a device is called its power, the electrical power  $P$  delivered by an electrical device is given by

$$P = \frac{dq}{dt} V = VI$$

- Power consumed by a resistor.

$$\frac{V^2}{R} \text{ watt}$$

$$P = VI = I^2R =$$

- The power  $P$  is in watts when  $I$  is in amperes,  $R$  is in ohms and  $V$  is in volts.
- The practical unit of power is  $1 \text{ kW} = 1000 \text{ W}$ .
- The formula for power  $P = I^2R = VI = \frac{V^2}{R}$  is true only when all the electrical power is dissipated as heat and not converted into mechanical work, etc. simultaneously.
- If the current enters the higher potential point of the device then electric power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source.)

### Joule's law of electrical heating

- When an electric current flows through a conductor electrical energy is used in overcoming the resistance of the wire. If the potential difference across a conductor of resistance  $R$  is  $V$  volt and if a current of  $I$  ampere

flows the energy expended in time  $t$  seconds is given by

$$\frac{V^2}{R} t$$

$$W = VIt \text{ joule} = I^2 R t \text{ joule} =$$

- The electrical energy so expended is converted into heat energy and this conversion is called the heating effect of electric current.
- The heat generated in joules when a current of  $I$  amperes flows through a resistance of  $R$  ohm for  $t$  seconds is given by

$$H = I^2 R t \text{ joule} = \frac{I^2 R t}{4.2} \text{ cal.}$$

This relation is known as Joule's law of electrical heating.

**Example 1.** If bulb rating is 100 watt and 220 V then determine

- Resistance of filament
- Current through filament
- If bulb operate at 110volt power supply then find power consume by bulb.

**Sol.** Bulb rating in 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consume is 100 W

Here  $V = 220$

$P = 100$

$$\frac{V^2}{R} = 100$$

So  $R = 484 \text{ W}$

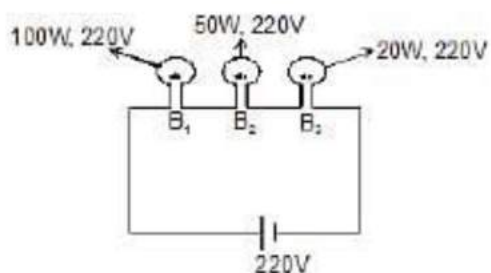
Since Resistance depends only on material hence it is constant for bulb

$$I = \frac{V}{R} = \frac{220}{22 \times 22} = \frac{5}{11} \text{ Amp}$$

power consumed at 110 V

$$\text{Therefore, power consumed} = \frac{110 \times 110}{484} = 25 \text{ W}$$

**Example 3.** In the following figure, grade the bulb in order of their brightness:



Sol.  $P_{\text{rated}} = \frac{V_{\text{rated}}^2}{R}$

$$R = \frac{V_{\text{rated}}^2}{P_{\text{rated}}}$$

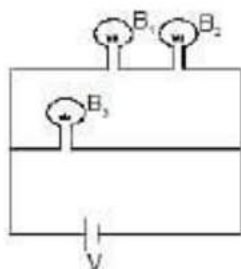
$$\therefore R_3 > R_2 > R_1$$

Power =  $i^2R$

As current passing through every bulb is same

Therefore, Brightness order is  $B_3 > B_2 > B_1$

**Example 4.**



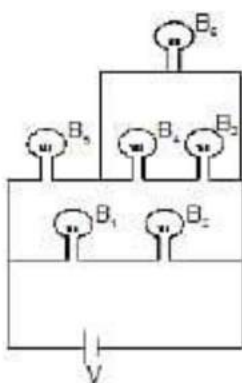
The above configuration shows three identical bulbs, Grade them in order of their brightness.

Sol. B<sub>1</sub> & B<sub>2</sub> withdraw less current as compared to B<sub>3</sub> because in series they give 2R resistance where as R is the resistance due to B<sub>3</sub>.

Power =  $i^2R$

Therefore, Brightness order:  $B_3 > B_2 = B_1$ .

**Example 5.**



Grade the bulbs in order of their brightness (All bulbs are identical)  
Sol.

$$\text{As } I \propto \frac{1}{R}$$

$$I_1 : I_2 = \frac{3}{5R} : \frac{1}{2R} = 6 : 5$$

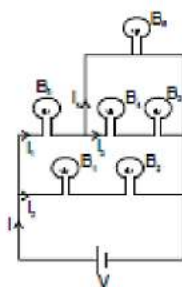
$$\therefore I_1 = \frac{6I}{11}, \quad I_2 = \frac{5I}{11}$$

$$I_3 : I_4 = \frac{1}{2R} : \frac{1}{R} = 1 : 2$$

$$\text{As } I_3 + I_4 = I_2$$

$$\therefore I_3 = \frac{2I}{11}, \quad I_4 = \frac{4I}{11}$$

$$\text{power} = I^2 R$$



Therefore, Order of Brightness:  $B_5 > B_1 = B_2 > B_6 > B_4 = B_3$

### (B) Maximum power transfer theorem

Let  $E$  be emf and  $r$  internal resistance of the battery. It is supplying current to an external resistance  $R$

$$\text{current in circuit } I = \frac{E}{(R+r)}$$

The power absorbed by load resistor  $R$  is

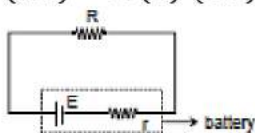
$$P = I^2 R = \left( \frac{E}{R+r} \right)^2 R$$

For maximum power transfer we take the derivative of  $P$  w.r.t  $R$ , set it equal to zero and solve the equation for  $R$ .

$$\frac{dP}{dR} = E^2 \frac{(R+r)^2 - R[2(R+r)]}{(R+r)^2} = 0$$

Solving for  $R$ , we have

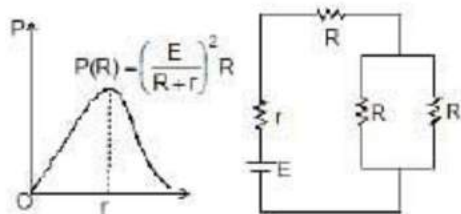
$$(Rr)^2 - R(2)(Rr) = 0$$



$$(Rr) - 2R = 0$$

$$R = r$$

For a given real battery the load resistance maximizes the power if it is equal to the internal resistance of the battery.



The maximum power transfer theorem in general, holds for any real voltage source. The resistance  $R$  may be a single resistor or  $R$  may be the equivalent resistance of a collection of resistors.

## Instruments

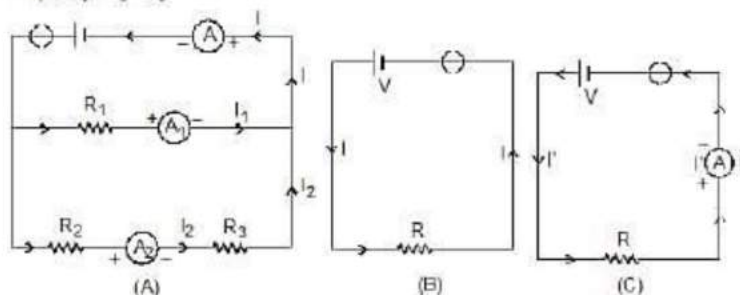
### Ammeter

- It is a device used to measure current and its always connected in series with the 'element' through which current is to be measured, e.g., in figure (A) ammeter  $A_1$  will measure the current ( $I_1$ ) through resistance  $R_1$ ,  $A_2$  measures current ( $I_2$ ) through  $R_2$  and  $R_3$  while  $A$ , measures current  $I$  ( $I_1 + I_2$ ).

Regarding an ammeter it is worth noting that :

- The reading of an ammeter is always lesser than actual current in the circuit, e.g., true current in the resistance  $R$  in the circuit shown in figure (B) is  $I = \frac{V}{R}$
- However, when an ammeter of resistance  $r$  is used to measure current as shown in figure (C), the reading will be

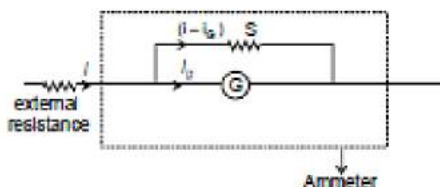
$$r = \frac{V}{(R+r)} < I \left( \frac{V}{R} \right)$$



- Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be **ideal** if its resistance ( $r$ ) is zero. However, as practically  $r \neq 0$ , ideal ammeter cannot be realised in practice.
- To convert a galvanometer into an ammeter of a certain range say  $I$ , a small resistance  $S$  (called shunt) is connected in parallel with the galvanometer so that the current passing through the galvanometer of resistance  $G$  becomes equal to its full scale deflection value  $I_g$ . This is possible only if

$$I_g G = (I - I_g) S$$

$$S = \frac{I_g}{(I - I_g)} G$$



i.e.,

**Example 6.** What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm?

**Sol.** As in figure  $R_g I_g = (I - I_g) S$

$$\Rightarrow 99 \times \frac{I}{10} = \left( I - \frac{I}{10} \right) \times S$$

$$\Rightarrow S = 11 \text{ ?}$$

For calculation it is simply a resistance

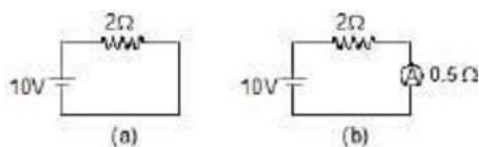


Resistance of ammeter

$$R_A = \frac{R_g \cdot S}{R_g + S}$$

$$\text{for } S \ll R_g \quad \Rightarrow \quad R_A = S$$

**Example 7.** Find the current in the circuit also determine percentage error in measuring in current through an ammeter (a) and (b).



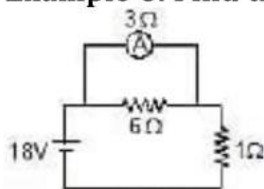
Sol. In A  $I = \frac{10}{2} = 5\text{A}$

In B  $I = \frac{10}{2.5} = 4\text{A}$

Percentage error is  $= \frac{i - I}{i} \times 100 = 20\%$  **Ans.**

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

**Example 8.** Find the reading of ammeter. Is this the current through 6 W ?



Sol.  $R_{eq} = \frac{3 \times 6}{3 + 6} + 1 = 3\Omega$

Current through battery

$$I = \frac{18}{3} = 6\text{A}$$

So, current through ammeter

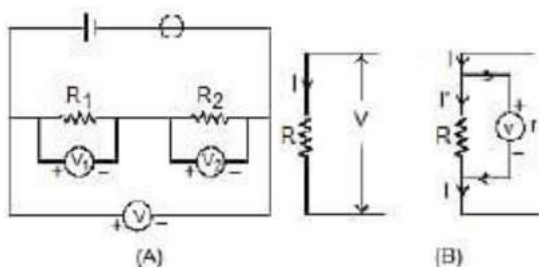
$$= 6 \times \frac{6}{9} = 4\text{A}$$

No, it is not the current through the 6W resistor.

Note: Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.

### Voltmeter

It is a device used to measure potential difference and is always put in parallel with the 'circuit element' across which potential difference is to be measured e.g., in Figure (A) voltmeter  $V_1$  will measure potential difference across resistance  $R_1$ ,  $V_2$  across resistance  $R_2$  and  $V$  across  $(R_1 R_2)$  with  $V = V_1 V_2$



Regarding a voltmeter it is worth noting that:

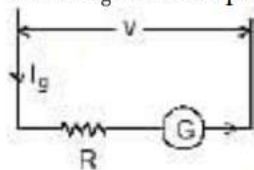
(1) The reading of a voltmeter is always lesser than true value, e.g., if a current  $I$  is passing through a resistance  $R$  [Fig. (B)], the true value  $V = IR$ . However, when a voltmeter having resistance  $r$  is connected across  $R$ , the current through  $R$  will become

$$I' = \frac{I}{(R+r)} \quad \text{and so} \quad V' = I'R = \frac{V}{[1 + (R/r)]}$$

and as voltmeter is connected across  $R$  its reading  $V'$  is lesser than  $V$ .

(2) Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be **ideal** if its resistance  $r$  is infinite, i.e., it draws no current from the circuit element for its operation. Ideal voltmeter has been realised in practice in the form of potentiometer.

(3) To convert a galvanometer into a voltmeter of certain range say  $V$ , a high resistance  $R$  is connected in series with the galvanometer so that current passing through the galvanometer of resistance  $G$  becomes equal to its full scale deflection value  $I_g$ . This is possible only if



$$V = I_g (G + R) \quad \text{i.e.,} \quad R = \frac{V}{I_g} - G$$

**Example 9.** A voltmeter has a resistance of  $G$  ohm and range of  $V$  volt. Calculate the resistance to be used in series with it to extend its range to  $nV$  volt.

**Sol.** Full scale current  $i_g = \frac{V}{G}$   
to change its range

$$V_1 = (G + R_s) i_g$$

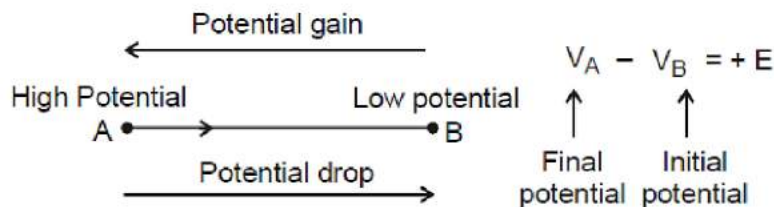
$$\Rightarrow nV = (G + R_s) \frac{V}{G}$$

## Kirchhoff's Laws

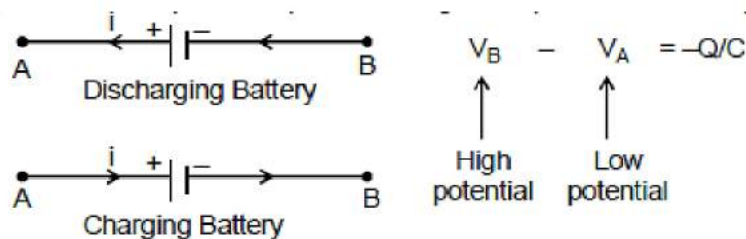
### Kirchhoff's Laws for Circuit Analysis

- Before moving on to the statement of Kirchhoff's law, we state some conventions to be followed in circuit analysis:

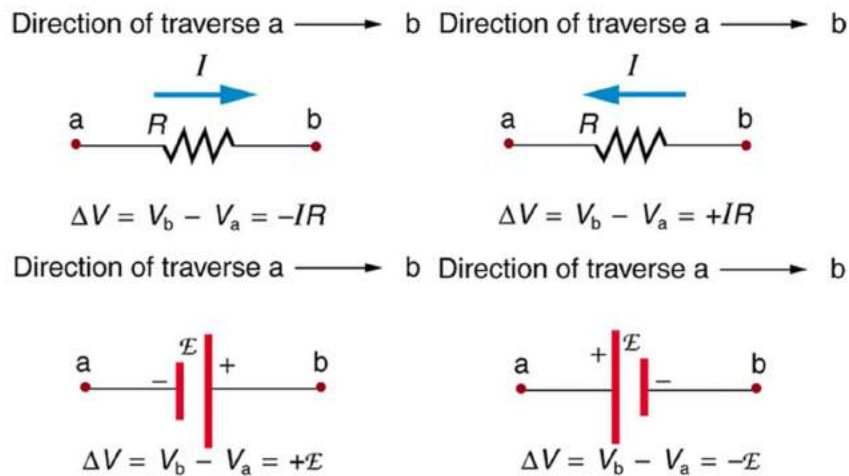
- (i) Direction of conventional current is from high potential to low potential terminal.
- (ii) Current flows from high potential node A to low potential node B. if we traverse from point A to B, there is drop of potential; similarly from B to A, there is gain of potential.
- If we traverse from point A to B, there is drop of potential; similarly from B to A, there is gain of potential. If a source of emf is traversed from negative to positive terminal, the change in potential is  $+E$ .



- While discharging, current is drawn from the battery, the current comes out from positive terminal and enters negative terminal, while charging of battery current is forced from positive terminal of the battery to negative terminal. Irrespective of direction of current through a battery the sign convention mentioned above holds.
- The positive plate of a capacitor is at high potential and negative plate at low potential. If we traverse a capacitor from positive plate to negative plate, the change in potential is  $-Q/C$



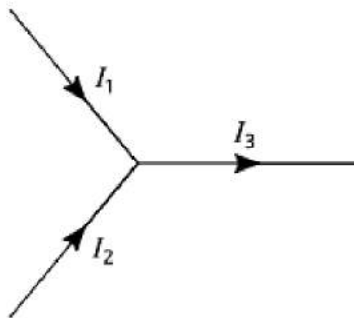
- If we traverse a resistor in the direction of current, the change in potential is  $-IR$ .
- If we traverse a resistor in the direction opposite to the direction of current, the change in potential is  $+IR$ .



- Positive terminal of source of emf is at high potential and negative terminal at low potential. If we traverse a source of emf from the positive terminal to negative terminal, the change in potential is  $-\mathcal{E}$ .
- If a capacitor is traversed from negative plate to positive plate, the change in potential is  $Q/C$ .

### The Kirchhoff's Current Law

- Current flow in circuits is produced when charge carriers travel through conductors. Current is defined as the rate at which this charge is carried through the circuit. A fundamental concept in physics is that charge will always be conserved.
- In the context of circuits this means that, since current is the rate of flow of charge, the current flowing into a point must be the same as current flowing out of that point.

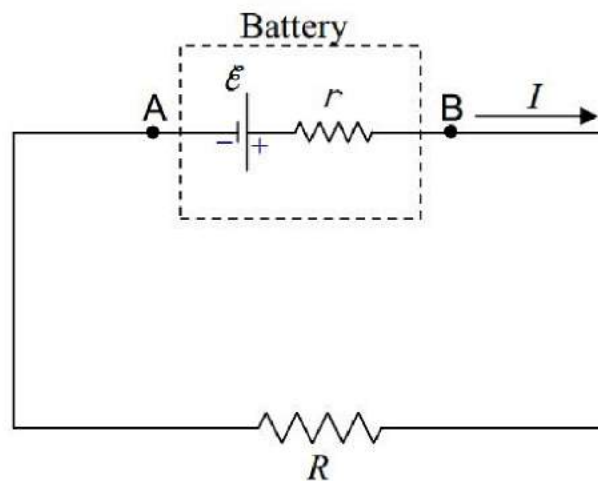


- Kirchhoff's current law states that for the diagram above, the currents in the three wires must be related by:  
 $I_1 + I_2 = I_3$

- It is important to note what is meant by the signs of the current in the diagram - a positive current means that the currents are flowing in the directions indicated on the diagram.

### The Kirchhoff's Voltage Law

- The Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential difference around any closed loop of an electric circuit is zero. **The KVL is a statement of conservation of energy.**
- The KVL reflects that electric force is conservative, the work done by a conservative force on a charge taken around a closed path is zero.
- We can move clockwise or anticlockwise, it will make no difference because the overall sum of the potential difference is zero.
- We can start from any point on the loop, we just have to finish at the same point.
- An ideal battery is modelled by an independent voltage source of emf  $\mathcal{E}$  and an internal resistance  $r$  as shown in figure A real battery always absorbs power when there is a current through it, thereby offering resistance to flow of current.

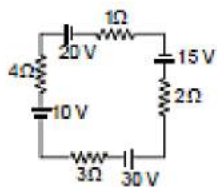


- Applying KVL around the single loop in anticlockwise direction, starting from point A, we have

$$+ IR \quad + \quad Ir \quad - \quad \mathcal{E} \quad = \quad 0$$

In the opposite direction to current
In the opposite direction to current
From positive to negative terminal

**Example 1. Find current in the circuit**

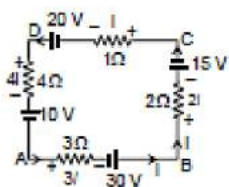


**Sol.** Therefore, all the elements are connected in series

Therefore, current in all of them will be same

let current =  $I$

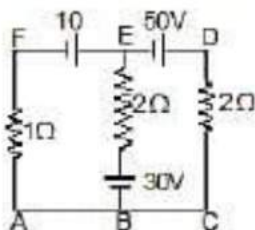
Applying kirchhoff's voltage law in ABCDA loop



$$10 + 4i - 20 + i + 15 + 2i - 30 + 3i = 0$$

$$10i = 25 \Rightarrow i = 2.5 \text{ A}$$

**Example 1.** Find the current in each wire applying only kirchhoff voltage law



**Sol.** Applying kirchhoff voltage law in loop ABEFA

$$i_1 + 30 + 2(i_1 + i_2) - 10 = 0$$

$$3i_1 + 2i_2 + 20 = 0 \dots(i)$$

Applying kirchhoff voltage law in BCDEB

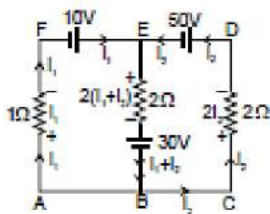
$$+30 + 2(i_1 + i_2) + 50 + 2i_2 = 0$$

$$4i_2 + 2i_1 + 80 = 0$$

$$2i_2 + i_1 + 40 = 0 \dots(ii)$$

Solving (i) and (ii)

$$3[-40 - 2i_2] + 2i_2 + 20 = 0$$



$$-120 - 4i_2 + 20 = 0$$

$$i_2 = -25 \text{ A}$$

$$\text{and } i_1 = 10 \text{ A}$$

$$\text{Therefore, } i_1 i_2 = -15 \text{ A}$$

current in wire AF = 10 A from A to E

current in wire EB = 15 A from B to E

current in wire DE = 25 A from D to C

### Potentiometer & Its Applications

#### What is Potentiometer?

Potentiometer working can be explained when the potentiometer is understood. It is defined as a three-terminal resistor having either sliding or rotating contact that forms an adjustable voltage divider. In order to use the potentiometer as a rheostat or variable resistor, it should have only two terminals with one end and the wiper. Following are the terms used to describe types of potentiometers:

1. **Slider pot or slide pot:** This can be adjusted by sliding the wiper right or left with a finger or thumb.
2. **Thumb wheel pot or thumb pot:** This can be adjusted infrequently with the help of small thumb wheel which is a small rotating potentiometer.
3. **Trimmer pot or trim pot:** This can be adjusted once for fine-tuning of an electric signal.

#### Necessity of Potentiometer

- Practically voltmeter has a finite resistance (ideally it should be  $\infty$ ). In other words, it draws certain current from the circuit. To overcome this problem a potentiometer is employed because at the instant of measurement, it draws no current from the circuit.

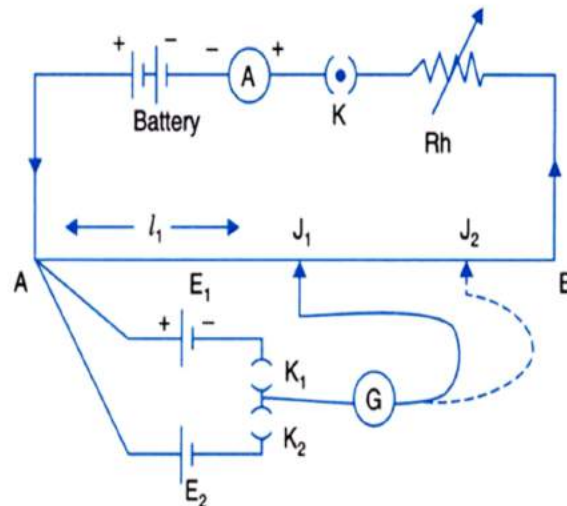
#### Working Principle of Potentiometer

- Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over the entire length of a potentiometer wire. This process is termed as zero deflection or null deflection method.

**Note:**

- (i) Potentiometer wire : Made up of alloys of manganin, constantan, eureka.
- (ii) Special properties of these alloys are high specific resistance, negligible temperature co-efficient of resistance ( $\alpha$ ). This results in invariability of resistance of potentiometer wire over a long period.

**Circuits of Potentiometer**



- Primary circuit contains source of constant voltage & rheostat or Resistance Box.
- Secondary circuit contains battery & galvanometer.

➤ **Potential gradient ( $x$ ) (V/m)**

- Potential difference corresponding to unit length of potentiometer wire is called potential gradient.
- Rate of growth/fall of potential per unit length of potentiometer wire is equal to potential gradient.
- Let  $r = 0$  and  $R_1 = 0$  then  $V_{AB} = E$  (max. in the ideal case) then  $x = E/L$   
Unit and dimensions : (V/m ;  $MLT^{-3}A^{-1}$ )
- Always  $V_{AB} < E$  ; ( $\because r + R_1 \neq 0$ )  
 $x = V_{AB}/L$
- Now  $V_{AB} = I R_P$  ( $R_P$  = resistance of potentiometer wire)
- Let  $\rho$  = Resistance per unit length of potentiometer wire

$$\text{So } x = \frac{I R_P}{L} = I \rho \quad \rho = \frac{R_P}{L}$$

current in primary circuit  $I$

$$= \frac{E}{R_1 + r + R_P} ; \quad x = \frac{E}{R_1 + R_P + r} \left( \frac{R_P}{L} \right)$$

- If cross-sectional radius is uniform  $\Rightarrow x$  is uniform over the entire length of potentiometer wire.
- If  $l$  constant, then
$$x \propto \frac{1}{(\text{radius})^2}$$
- ' $x$ ' depends on  $\rightarrow \rho, r, \sigma$  etc.

#### ➤ Factors affecting ' $x$ '

- If  $V_{AB} = \text{constant}$  and  $L = \text{constant}$  then for any change  $\rightarrow x$  remains unchanged.
- If there is no information about  $V_{AB}$  then always take  $V_{AB}$  as constant so ( $x \propto 1/L$ )
- If  $V_{AB}$  and  $L$  are constant :
- For any change like radius of wire, substance of wire ( $\sigma$ ) there is no change in  $x$ .
- Any change in the secondary circuit results in no change in  $x$  because  $x$  is an element of primary circuit.

**Note:**

$$x = \frac{E}{R_p + r + R_1} \left( \frac{R_p}{L} \right)$$

$x_{\max}$  or  $x_{\min}$  on the basis of range of rheostat or resistance box (R.B.)

$$\text{If } R_1 = 0 \Rightarrow x_{\max} = \frac{E}{R_p} \times \frac{R_p}{L} \quad (r \simeq 0)$$

$$\text{If } R_1 = R \Rightarrow x_{\min} = \frac{E}{R_p + R} \left( \frac{R_p}{L} \right)$$

$$\text{then } \frac{x_{\max}}{x_{\min}} = \frac{R_p + R}{R_p}$$

#### Applications of Potentiometer

##### Comparison of EMF's of two cells using potentiometer

- Consider the circuit arrangement of potentiometer given below used for comparison of emfs of two cells

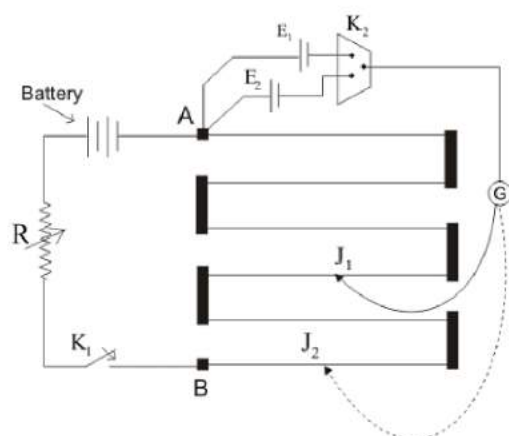


Figure 14. Potentiometer arrangement for comparison of emf's of two cells

- Positive terminals of two cells of emfs  $E_1$  and  $E_2$  (whose emf are to be compared) are connected to the terminals A and negative terminals are connected to jockey through a two way key  $K_2$  and a galvanometer
- Now first key  $K_1$  is closed to establish a potential difference between the terminals A and B then by closing key  $K_2$  introduce cell of EMF  $E_1$  in the circuit and null point junction  $J_1$  is determined with the help of jockey. If the null point on wire is at length

$l_1$  from A then

$$E_1 = Kl_1$$

Where  $K \rightarrow$  Potential gradient along the length of wire

- Similarly cell having emf  $E_2$  is introduced in the circuit and again null point  $J_2$  is determined. If length of this null point from A is  $l_2$  then

$$E_2 = Kl_2$$

Therefore

$$E_1/E_2 = l_1/l_2$$

This simple relation allows us to find the ratio of  $E_1/E_2$

- if the EMF of one cell is known then the EMF of other cell can be known easily

#### Determination of internal resistance of the cell

- Potentiometer can also be used to determine the internal resistance of a cell

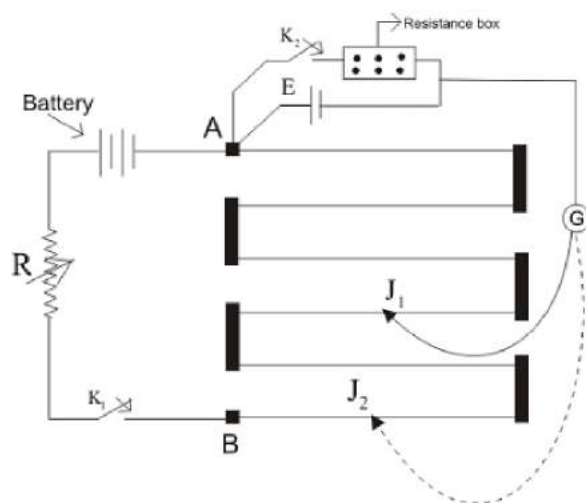


Figure 15. Potentiometer arrangement for determining internal resistance of a cell

- For this a cell whose internal resistance is to be determined is connected to terminal A of the potentiometer across a resistance box through a key  $K_2$
- First close the key  $K_1$  and obtain the null point. Let  $l_1$  be the length of this null point from terminal A then  

$$E = Kl_1$$
- When key  $K_2$  is closed, the cell sends current through resistance Box (R). If  $E_2$  is the terminal Potential difference and null point is obtained at length  $l_2(AJ_2)$  then  

$$V = Kl_2$$
  
 Thus  

$$E/V = l_1/l_2$$
  
 But  $E = I(R + r)$  and  $V = IR$   
 This gives  

$$E/V = (r + R)/R$$
  
 So  $(r + R)/R = l_1/l_2$   
 giving  

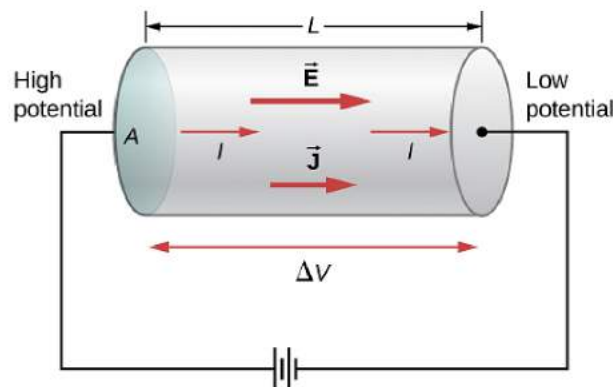
$$r = R(l_1/l_2 - 1)$$
- Using above equation we can find internal resistance of any given cell

## Cells, EMF, & Internal Resistance

### Battery and EMF

- A battery is a device which maintains a potential difference between its two terminals A and B.

- Figure shows a schematic diagram of a battery. Some internal mechanism exerts forces on the charges of the battery material.
- This force drives the positive charges of the battery material towards A and the negative charges of the battery material towards B. We show the force on a positive charge  $q$  as  $F_b$ . As positive charge accumulates on A and negative charge on B, a potential difference develops and grows between A and B. An electric field is developed in the battery material from A to B and exerts a force  $F_e = qE$  on a charge  $q$ . The direction of this force is opposite to that of  $F_b$ . In steady state, the charge accumulation on A and B is such that  $F_b = F_e$ . No further accumulation takes place.



- If a charge  $q$  is taken from the terminal B to the terminal A, the work done by the battery force  $F_b$  is  
 $W = F_b d$  where  $d$  is the distance between A and B. The work done by the battery force per unit charge is

$$\mathcal{E} = \frac{W}{q} = \frac{F_b d}{q}$$

- This quantity is called the emf of the battery. The full form of emf is electromotive force. The name is misleading in the sense that emf is not a force, it is work done/charge. We shall continue to denote this quantity by the short name emf. If nothing is connected externally between A and B,

$$F_b = F_e = qE$$

$$\text{or, } F_b d = qEd = qV$$

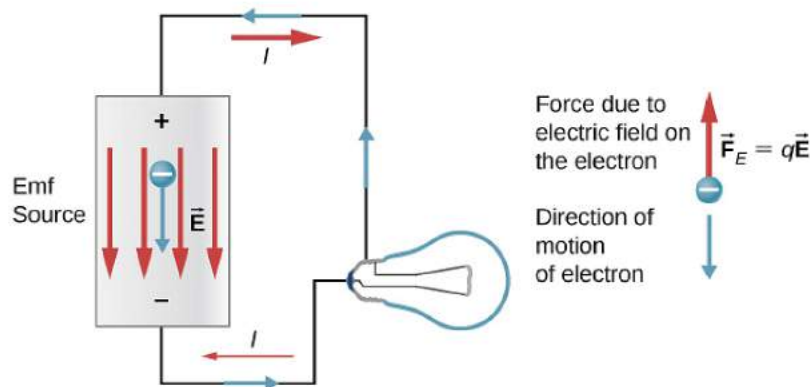
where  $V = Ed$  is the potential difference between the terminals. Thus,

$$\mathcal{E} = \frac{F_b d}{q} = V$$

- Thus, the emf of a battery equals the potential difference between its terminals when the terminals are not connected externally.
- Potential difference and emf are two different quantities whose magnitudes may be equal in certain conditions. The emf is the work done per unit charge by the battery force  $F_b$  which is non-electrostatic in

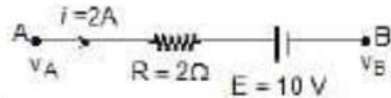
nature. The potential difference originates from the electrostatic field created by the charges accumulated on the terminals of the battery.

- A battery is often prepared by putting two rods or plates of different metals in a chemical solution. Such a battery, using chemical reactions to generate emf, is often called a cell.



- Now suppose the terminals of a battery are connected by a conducting wire as shown in above figure. As the terminal A is at a higher potential than B, there is an electric field in the wire in the direction shown in the figure.
- The free electrons in the wire move in the opposite direction and enter the battery at the terminal A. Some electrons are withdrawn from the terminal B which enter the wire through the right end. Thus, the potential difference between A and B tends to decrease. If this potential difference decreases, the electrostatic force  $F_e$  inside the battery also decreases. The force  $F_b$  due to the battery mechanism remains the same.
- Thus, there is a net force on the positive charges of the battery material from B to A. The positive charges rush towards A and neutralize the effect of the electrons coming at A from the wire. Similarly, the negative charges rush towards B. Thus, the potential difference between A and B is maintained.
- For calculation of current, motion of a positive charge in one direction is equivalent to the motion of a negative charge in opposite direction. Using this fact, We can describe the above situation by a simpler model. The positive terminal of the battery supplies positive charges to the wire. These charges are pushed through the wire by the electric field and they reach the negative terminal of the battery. The battery mechanism drives these charges back to the positive terminal against the electric field existing in the battery and the process continues. This maintains a steady current in the circuit

- Current can also be driven into a battery in the reverse direction. In such a case, positive charge enters the battery at the positive terminal, moves inside the battery to the negative terminal and leaves the battery from the negative terminal. Such a process is called charging of the battery. The more common process in which the positive charge comes out of the battery from the positive terminal is called discharging of the battery.



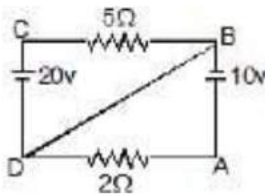
**Example 1.**

Find  $v_A - v_B$

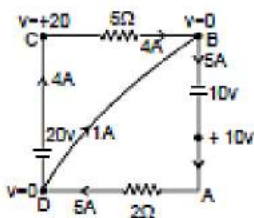
**Sol.**  $v_A - iR - E = v_B$

$$v_A - v_B = iR + E = 4 + 10 = 14 \text{ volt}$$

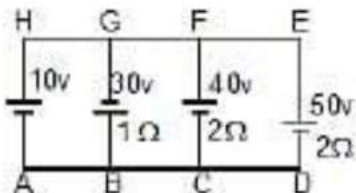
**Example 2. Shown in the figure. Find out the current in the wire BD**



**Sol.** Let at point D potential = 0 and write the potential of other points then current in wire AD =  $\frac{10}{2} = 5A$  from A to D current in wire CB =  $\frac{20}{5} = 4A$  from C to B. Therefore, current in wire BD = 1 A from D to B.

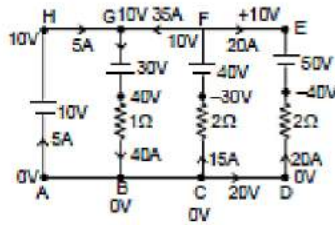


**Example 3. Find the current in each wire**



**Sol.** Let potential at point A is 0 volt then potential of other points is shown in figure.

Current in BG =  $\frac{40 - 0}{1} = 40$  A from G to B



Current in FC =  $\frac{0 - (-30)}{2} = 15$  A from C to K

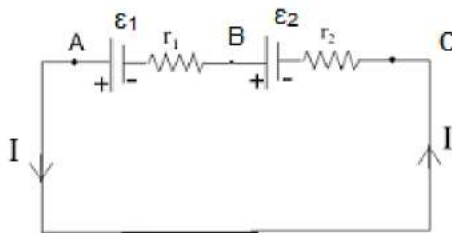
current in DE =  $\frac{0 - (-40)}{2} = 20$  A from D to E

current in wire AH =  $40 - 35 = 5$  A from A to H

### Combinations of cells

- A cell is used to maintain current in an electric circuit. We cannot obtain a strong current from a single cell.
- Hence need arises to combine two or more cells to obtain a strong current.
- Cells can be combined in three possible ways:
  - (i) In series
  - (ii) In parallel
  - (iii) In mixed grouping.

### Cells in Series



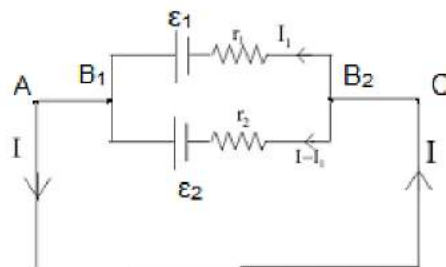
- In this combination, cells are so connected that -ve terminal of each cell is connected with the +ve terminal of next and so on. Suppose n cells are connected in this way. Let e.m.f and internal resistance of each cell are E and r respectively.
- Net e.m.f of the cells = nE. Total internal resistance = nr. Hence total resistance of the circuit = nr + R.

If total current in the circuit is  $I$ , then  $I = \frac{\text{net e.m.f}}{\text{Total Resistance}} = \frac{nE}{nr + R} \dots (1)$

**Case (i) :** If  $nr \ll R$ , then  $I \approx nE/R$  i.e., if total internal resistance of the cells is far less than external resistance, then current obtained from the cells is approximately equal to  $n$  times the current obtained from a single cell. Hence cells, whose total internal resistance is less than external resistance, just be joined in series to obtain strong current.

**Case (ii) :** If  $nr \gg R$ , then  $I \approx \frac{nE}{nr} = \frac{E}{r}$  i.e., if total internal resistance of the cells is much greater than the external resistance, then current obtained from the combination of  $n$  cells is nearly the same as obtained from a single cell. Hence there is no use of joining such cells in series.

### Cells in Parallel



#### ➤ When E.M.F's and internal resistance of all the cells are equal

- In this combination, positive terminals of all the cells are connected at one point and negative terminals at other point. Figure shown such cells connected in parallel across some external resistance  $R$ . Let e.m.f and internal resistance of each cell are  $E$  and  $r$  respectively.
- Because all the cells are connected in parallel between two points, hence e.m.f of battery =  $E$ .
- Total internal resistance of the combination of  $n$  cells =  $r/n$
- Because external resistance  $R$  is connected in series with internal resistance, hence total resistance of the circuit =  $(r/n) + R$
- If current in external resistance is  $I$ , then

$$I = \frac{\text{net E.M.F}}{\text{Total resistance}} = \frac{E}{(r/n) + R} = \frac{nE}{r + nR}$$

**Case (I) :** If  $r \ll R$ , the  $I \approx \frac{nE}{nR} = \frac{E}{R}$  i.e., if internal resistance of the cells is much less than external resistance, then total current obtained from combination is nearly equal to

current given by one cell only. Hence there is no use of joining cells of low internal resistance in parallel.

**Case (II):** If  $r \gg R$ , then  $I \approx \frac{nE}{r}$  i.e., if the internal resistance of the cells is much higher than the external resistance, then total current is nearly equal to  $n$  times the current given by one cell. Hence cells of high internal resistance must be joined in parallel to get a strong current.

**(II) When emf's and internal resistance of all the cells connected in parallel are different.**

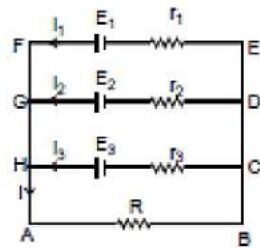
In this case, total current in external resistance is obtained with the help of Kirchhoff's laws. Figure shows three cells of e.m.f  $E_1, E_2$  and  $E_3$  and internal resistances  $r_1, r_2$  and  $r_3$  connected in parallel across some external resistance  $R$ . Suppose currents given by three cells are  $i_1, i_2$  and  $i_3$ . Hence according to Kirchhoff's first law, total current  $I$  in external resistance  $R$ , is given by

$$I = i_1 + i_2 + i_3 \dots (1)$$

Applying Kirchhoff's 2nd law to closed mesh ABEF we get

$$IR + i_1 r_1 = E_1 \text{ or } i_1 = \frac{E_1 - IR}{r_1} \dots (2)$$

Similarly, for closed meshes ABDG and ABCH, we get



$$i_2 = \frac{E_2 - IR}{r_2} \dots (3)$$

$$\text{and } i_3 = \frac{E_3 - IR}{r_3} \dots (4)$$

Substituting eq. (2), (3) and (4) in eq. (1), we have

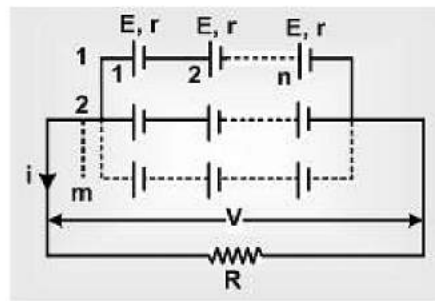
$$I = \frac{E_1 - IR}{r_1} + \frac{E_2 - IR}{r_2} + \frac{E_3 - IR}{r_3} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} - IR \left( \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \right)$$

$$\text{or } I \left[ 1 + R \left( \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \right) \right] = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} \text{ or } I = \frac{(E_1/r_1) + (E_2/r_2) + (E_3/r_3)}{1 + R(1/r_1 + 1/r_2 + 1/r_3)}$$

$$I = \frac{\sum \frac{E_i}{r_i}}{1 + R \sum \frac{1}{r_i}} \text{ and } E_{eq} = \frac{\sum \frac{E_i}{r_i}}{\sum \frac{1}{r_i}}, r_{eq} = \frac{1}{\sum \frac{1}{r_i}}$$

If  $n$  cells are joined in parallel, then

**Cells in Mixed Grouping**



In this combination, a certain number of cells are joined in series in various rows, and all such rows are then connected in parallel with each other.

Suppose  $n$  cells, each of e.m.f  $E$  and internal resistance  $r$ , are connected in series in every row and  $m$  such rows are connected in parallel across some external resistance  $R$ , as shown in figure.

Total number of cells in the combination =  $mn$ . As e.m.f. of each row =  $nE$  and all the rows are connected in parallel, hence net e.m.f of battery =  $nE$ .

Internal resistance of each row =  $nr$ . As  $m$  such rows are connected in parallel,

hence total internal resistance of battery =  $\left(\frac{nr}{m}\right)$

Hence total resistance of the circuit =  $\left[\left(\frac{nr}{m}\right) + R\right]$

If the current in external resistance is  $I$ , then

$$I = \frac{\text{net e.m.f}}{\text{Total resistance}} = \frac{nE}{\left(\frac{nr}{m}\right) + R} = \frac{mnE}{nr + mR}$$

$$= \frac{mnE}{\left(\sqrt{nr} - \sqrt{nr}\right)^2 + 2\sqrt{nmrR}}$$

It is clear from above equation that  $I$  will be maximum when

$$[(\sqrt{nr} - \sqrt{nr})^2 + 2\sqrt{nmrR}] \text{ is minimum.}$$

This will be possible when the quantity  $[\sqrt{nr} - \sqrt{nr}]^2$  is minimum. Because this quantity is in square, it can not be negative, hence its minimum value will be equal to zero, i.e.,

$$mR = nr \text{ or } R = \frac{nr}{m}$$

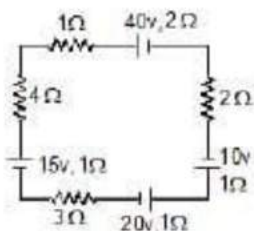
**Note:** In mixed grouping of cells, current in external resistance will be maximum when total internal resistance of battery is equal to external resistance.

Because power consumed in the external resistance or load =  $I^2R$ , hence when current in load is maximum, consumed power in it is also maximum, Hence

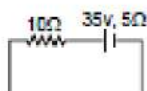
consumed power in the load will also be maximum when  $R = \frac{nr}{m}$ .

$$I_{\text{max}} = \frac{mnE}{2mR} \text{ or } \frac{mnE}{2nr} = \frac{nE}{2r} \text{ or } \frac{mE}{2r}$$

**Example 4.** Find the current in the loop.



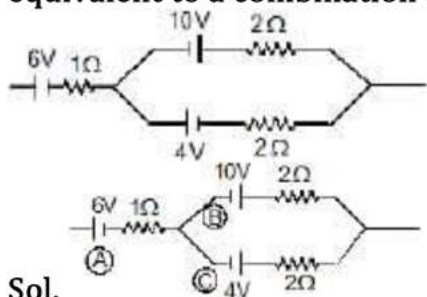
Sol. The given circuit can be simplified as



$$i = \frac{35}{10 + 5} = \frac{35}{15}$$

$$= \frac{7}{3} \text{ A} \Rightarrow i = \frac{7}{3} \text{ A}$$

**Example 5.** Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



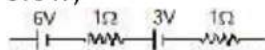
Sol.

Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent

$$\varepsilon_{BC} = \frac{\frac{10}{2} + \frac{-4}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{5 - 2}{1} = 3\text{V}$$

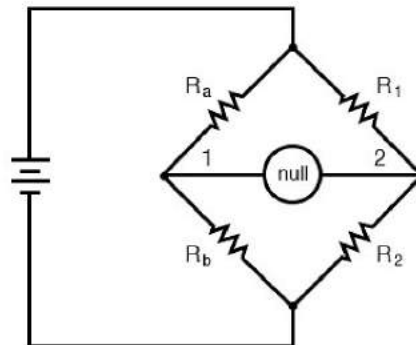
$$r_{BC} = 1 \Omega$$

$$\text{Now, } \varepsilon_{ABC} = 6 - 3 = 3\text{V}$$



$$r_{ABC} = 2\Omega$$

## Wheatstone Bridge & Meter Bridge



(i) Wheatstone designed a network of four resistances with the help of which the resistance of a given conductor can be measured. Such a network of resistances is known as Wheatstone's bridge.

(ii) In this bridge, four resistance P, Q, R and S are so connected so as to form a quadrilateral ABCD. A sensitive galvanometer and key  $K_2$  are connected between diagonally opposite corners B and D, and a cell and key  $K_1$  are connected between two other corners A and C (figure shown)

(iii) When key  $K_1$  is pressed, a current  $i$  flows from the cell. On reaching the junction A, the current  $i$  gets divided into two parts  $i_1$  and  $i_2$ . Current  $i_1$  flows in the arm AB while  $i_2$  in arm AD. Current  $i_1$ , on reaching the junction B gets further divided into two parts  $(i_1 - i_g)$  and  $i_g$ , along branches BC and BD respectively. At junction D, currents  $i_2$  and  $i_g$  are added to give a current  $(i_2 + i_g)$ , along branch DC.  $(i_1 - i_g)$  and  $(i_2 + i_g)$  add up at junction C to give a current  $(i_1 + i_2)$  or  $i$  along branch CE. In this way, currents are distributed in the different branches of bridge. In this position, we get a deflection in the galvanometer.

(iv) Now the resistance P, Q, R and S are so adjusted that on pressing the key  $K_2$ , deflection in the galvanometer becomes zero or current  $i_g$  in the branch BD becomes zero. In this situation, the bridge is said to be balanced.

(v) In this balanced position of bridge, same current  $i_1$  flows in arms AB and BC and similarly same current  $i_2$  in arms AD and DC. In other words, resistances P and Q and similarly R and S, will now be joined in series.

(vi) **Condition of balance:** Applying Kirchhoff's 2nd law to mesh ABDA,  $i_1P + i_gG - i_2R = 0 \dots(1)$

Similarly, for the closed mesh BCDB, we get,  $(i_1 - i_g)Q - (i_2 + i_g)S - i_gG = 0 \dots(2)$  When bridge is balanced,  $i_g = 0$ . Hence eq. (1) & (2) reduce to

$$i_1P - i_2R = 0 \text{ or } i_1P = i_2R \dots(3)$$

$$i_1Q - i_2S = 0 \text{ or } i_1Q = i_2S \dots(4)$$

Dividing (3) by (4), we have,  $\frac{P}{Q} = \frac{R}{S}$  ....(5)

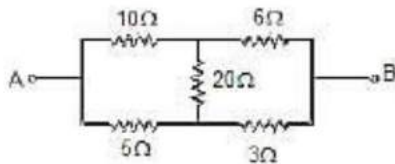
This is called as condition of balanced for Wheatstone's Bridge.

(vii) It is clear from above equation that if ratio of the resistance P and Q, and the resistance R are known, then unknown resistance S can be determined. This is the reason that arms P and Q are called as ratio arms, arm AD as known arm and arm CD as unknown arm.

(viii) When the bridge is balanced then on inter-changing the positions of the galvanometer and the cell there is no effect on the balance of the bridge. Hence the arms BD and AC are called as conjugate arms of the bridge.

(ix) The sensitivity of the bridge depends upon the value of the resistances. **The sensitivity of bridge is maximum when all the four resistances are of the same order.**

**Ex.22 Find equivalent resistance of the circuit between the terminals A and B.**

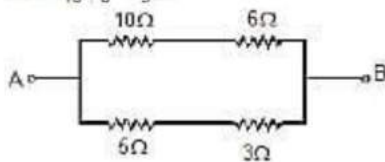


**Sol.** Since the given circuit is wheat stone bridge and it is in balance condition.

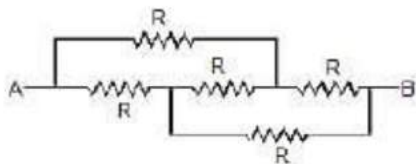
Because,  $10 \times 3 = 30 = 6 \times 5$

hence this is equivalent to

$$R_{eq} = \frac{16 \times 8}{16 + 8} = \frac{16}{3} \Omega$$

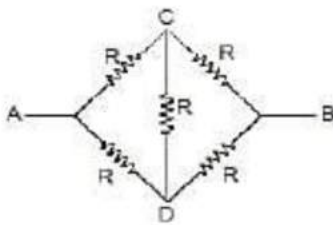


**Ex.23 Find the equivalent resistance between A and B**



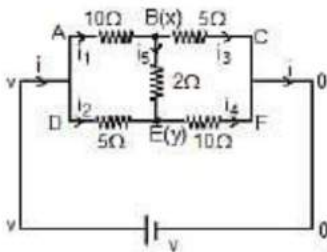
**Sol.** This arrangement can be modified as shown in figure since it is balanced wheat stone bridge.

$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$



## 8.1 Unbalanced Wheatstone Bridge

### Ex.24



Find equivalent resistance?

Sol. Let potential at point B is  $x$  and E is  $Y$

$$R_{eq} = \frac{v}{i}$$

Applying KCL at point B

$$\frac{x-v}{10} + \frac{x-y}{2} + \frac{x-0}{5} = 0$$

$$8x - 5y = v \dots (1)$$

Applying KCL at point E

$$\frac{y-v}{5} + \frac{y-x}{2} + \frac{y-0}{10} = 0$$

$$\Rightarrow 8y - 5x = 2v \dots (2)$$

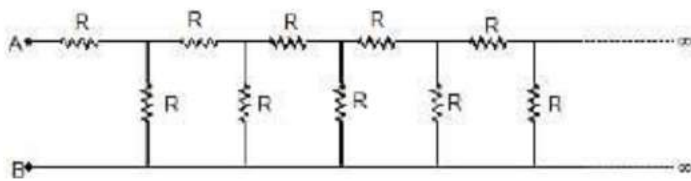
$$\text{solving } x \text{ \& } y \quad x = \frac{6v}{13}, \quad y = \frac{7v}{13}$$

current from branches BC & EF adds up to give total current ( $i$ ) flowing in the circuit.

$$i = i_3 + i_4 = \frac{x-0}{5} + \frac{y-0}{10} = \frac{19v}{130}$$

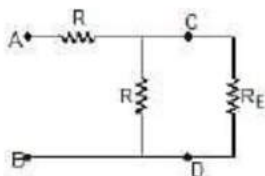
$$\text{Because, } i = \frac{v}{R_{eq}} \quad \text{Therefore, } R_{eq} = \frac{130}{19}$$

**Ladder Problem:**



Find the effective resistance between A & B?

**Sol.** Let the effective resistance between A & B be  $R_E$  since the network is infinite long, removal of one cell from the chain will not change the network. The effective resistance between points C & D would also be  $R_E$ . The equivalent network will be as shown below



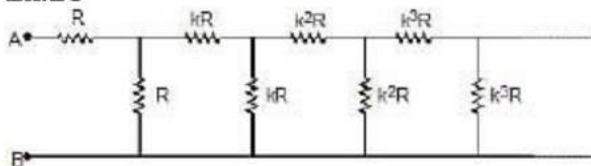
The original infinite chain is equivalent to  $R$  in series with  $R$  &  $R_E$  in parallel.

$$R_E = R + \frac{RR_E}{R + R_E}$$

$$R_E R + R_E^2 = R^2 + 2RR_E \Rightarrow R_E^2 - RR_E - R^2 = 0$$

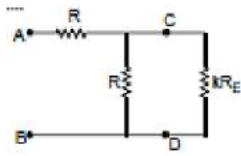
$$R_E = \frac{R(1 + \sqrt{5})}{2}$$

**Ex.25**



Find the equivalent resistance between A & B?

**Sol.** As moving from one section to next one, resistance is increasing by  $k$  times. Since the network is infinitely long, removal of one section from the chain will bring a little change in the network. The effective resistance between points C & D would be  $kR_E$  (where  $R_E$  is the effective resistance)



Therefore, Effective R between A & B.

$$R_E = R + \frac{R(kR_E)}{kR_E + R}$$

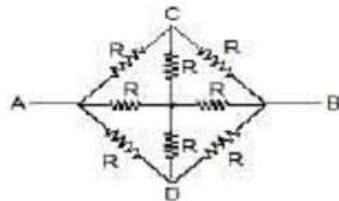
On solving we get

$$R_E = \frac{2kR - R + \sqrt{(R - 2kR)^2 + 4kR^2}}{2k}$$

### 15. Symmetrical Circuits:

Some circuits can be modified to have simpler solution by using symmetry if they are solved by traditional method of KVL and KCL then it would take much time.

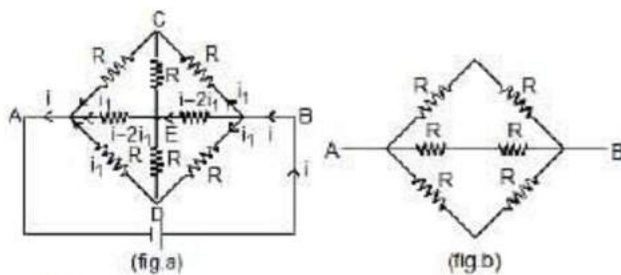
**Ex.26 Find the equivalent Resistance between A and B**



**Sol. I Method: mirror symmetry**

The branches AC and AD are symmetrical  
Therefore, current through them will be same.

The circuit is also similar from left side and right side like mirror images with a mirror placed along CD therefore current distribution while entering through B and an exiting from A will be same. Using all these facts the currents are as shown in the figure. It is clear that current in resistor between C and E is 0 and also in ED is 0. Its equivalent is shown in figure (b)

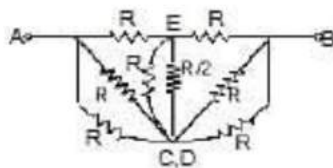


$$R_{eq} = \frac{2R}{3}$$

## II Method: folding symmetry

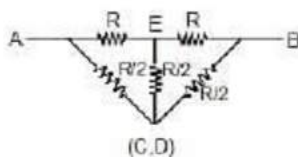
**Therefore,** The potential difference in R between (B, C) and between (B, D) is same  
 $V_C = V_D$

Hence the point C and D are same hence circuit can be simplified as



This is called folding.

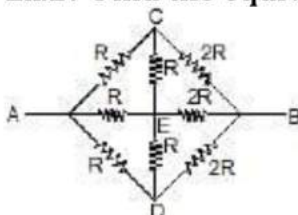
Now, it is Balanced Wheatstone bridge



$$R_{eq} = \frac{2R \times R}{2R + R} = \frac{2R}{3}$$

■ In II Method it is not necessary to know the currents in CA and DA

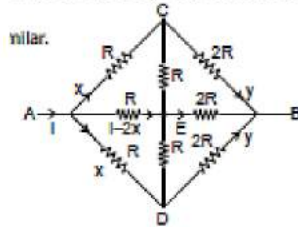
**Ex.27 Find the equivalent Resistance between A and B**



**Sol.** In this case the circuit has symmetry in the two branches AC and AD at the input  
 Therefore, current in them are same but from input and from exit the circuit is not similar

(Because, on left R and on right 2R)

Therefore, on both sides the distribution of current will not be similar.

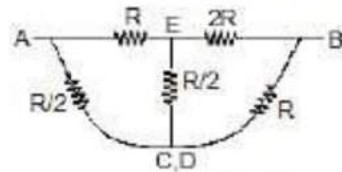


Here  $V_c = V_d$

hence C and D are same point

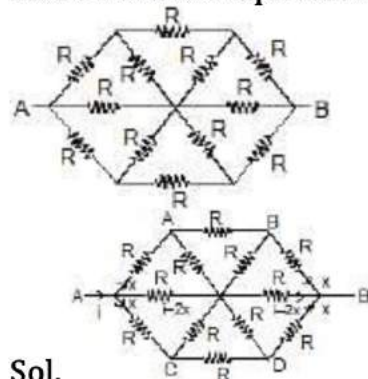
So, the circuit can be simplified as

Now it is balanced wheat stone bridge.



$$R_{eq} = \frac{3R \times \frac{3R}{2}}{3R + \frac{3R}{2}} = \frac{\frac{9}{2}R}{\frac{9}{2}} = R$$

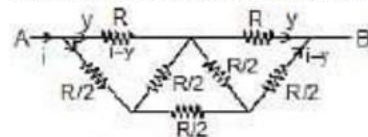
Ex.28 Find the equivalent Resistance between A and B



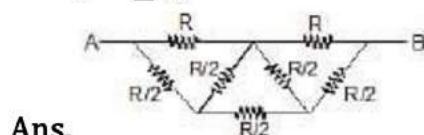
Sol.

Here  $V_A = V_C$  and  $V_B = V_D$

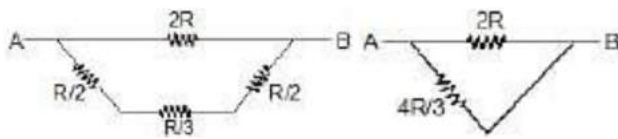
Here the circuit can be simplified as



$$R_{eq} = \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}} = \frac{4R}{5}$$

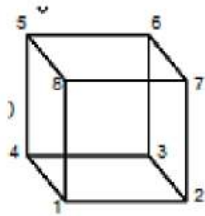


Ans.

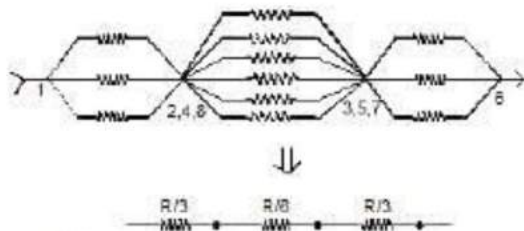


**Ex.29** Twelve equal resistors each  $R$  W are connected to form the edges of a cube. Find the equivalent resistances of the network.

(a) When current enters at 1 & leaves at 6 (body diagonal)



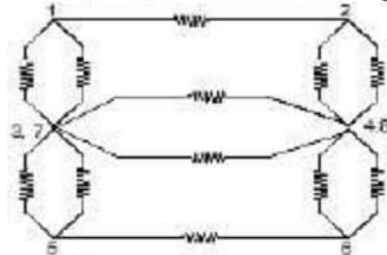
**Sol.** Here 2, 4, 8 are equipotential points (if we move from  $1 \rightarrow 2, 4, 8$  it comes along the edge &  $6 \rightarrow 2, 4, 8$  it comes along face diagonal). Similarly 3, 5, 7 are equipotential points.



$$R_{eq} = \frac{5R}{6}$$

(b) When current enters at 1 and leaves at 2

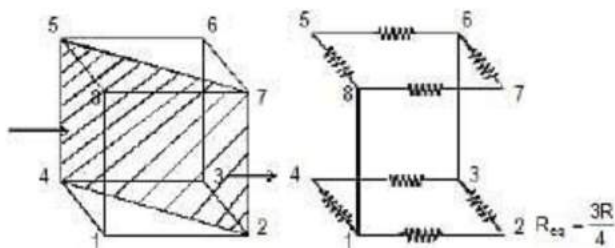
**Sol.** Here 3, 7 are equipotential surface (if we move from  $1 \rightarrow 3, 7$  we have along face and  $2 \rightarrow 3, 7$  we move along edge) similarly 4, 8 are equipotential surface.



$$R_{eq} = \frac{7R}{12}$$

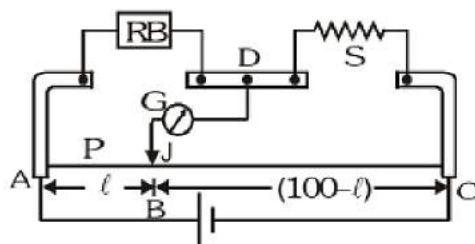
(c) When current enters at 1 and leaves at 3

**Sol.** If we cut the cube along the plane passing through 2, 4, 5, 7 then by mirror symmetry, the final configuration will be



## Meter Bridge

It is based on principle of whetstone bridge. It is used to find out unknown resistance of wire. AC is 1 m long uniform wire R.B. is known resistance and S is unknown resistance. A cell is connected across 1 m long wire and Galvanometer is connected between Jockey and midpoint D. To find out unknown resistance we touch jockey from A to C and find balance condition. Let balance is at B point on wire.



$$AB = l \text{ cm } P = r l$$

$$BC = (100 - l) \text{ cm } Q = r (100 - l) \text{ where } r = \text{resistance per unit length on wire.}$$

At balance condition:

$$\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{r l}{r(100 - l)} = \frac{R}{S} \Rightarrow S = \frac{(100 - l)}{l} R$$

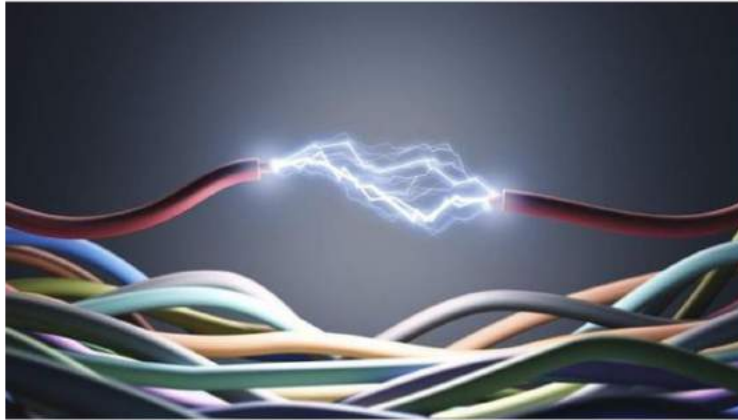
## Resistivity of Various Materials

### Resistivity of Materials

We know there are three types of materials; conductors, semiconductors and insulators. Conductors are the materials that can pass electricity through them. In this document, before we learn about electrical resistivity, let us know what is meant by electrical conductivity and its units.

Electrical Conductivity is an intrinsic property of a material which is defined as the measure of the amount of electrical current a material can carry. Electrical conductivity is also known as specific conductance, and the SI unit is Siemens per

meter (S/m). It is also defined as the ratio of the current density to the electric field strength. It is represented by the Greek letter  $\sigma$ .



**Electrical conductivity is a measure of how readily a material transmits an electrical current.**

### **What is Electrical Resistivity?**

Electrical resistivity is the reciprocal of electrical conductivity. It is the measure of the ability of a material to oppose the flow of current.

- Metals are good conductors of electricity. Hence, they have low resistivity.
- The insulators like rubber, glass, graphite, plastics, etc. have very high resistivity when compared to the metallic conductors.
- The third type is the semiconductor which comes in between the conductors and insulators. Their resistivity decreases with the increase in temperature and is also affected by the presence of impurities in them.

### **Resistivity of Different Materials**

The table below lists the electrical resistivity of several conductors, semiconductors, and insulators.

	Material	Resistivity ( $\Omega \cdot m$ )
<b>Conductors</b>	Silver	$1.60 \times 10^{-8}$
	Copper	$1.62 \times 10^{-8}$
	Aluminium	$2.63 \times 10^{-8}$
	Tungsten	$5.20 \times 10^{-8}$
	Nickel	$6.84 \times 10^{-8}$
	Iron	$10.0 \times 10^{-8}$
	Chromium	$12.9 \times 10^{-8}$
	Mercury	$94.0 \times 10^{-8}$
	Manganese	$1.84 \times 10^{-6}$
<b>Alloys</b>	Constantan (alloy of Cu and Ni)	$49 \times 10^{-6}$
	Manganin (alloy of Cu, Mn and Ni)	$44 \times 10^{-6}$
	Nichrome (alloy of Ni, Cr, Mn and Fe)	$100 \times 10^{-6}$
<b>Insulators</b>	Glass	$10^{10} - 10^{14}$
	Hard rubber	$10^{13} - 10^{16}$
	Ebonite	$10^{15} - 10^{17}$
	Diamond	$10^{12} - 10^{13}$
	Paper (dry)	$10^{12}$

### Resistivity Formula

Materials having electric field and current density will have the following resistivity formula:

$$\rho = \frac{E}{J}$$

Where

- $\rho$  is the resistivity of the material in  $\Omega \cdot m$
- $E$  is the magnitude of the electric field in  $V \cdot m^{-1}$
- $J$  is the magnitude of current density in  $A \cdot m^{-2}$

Conductors with a uniform cross-section and uniform flow of electric current will have the following resistivity formula:

$$\rho = R \frac{A}{l}$$

Where

- $\rho$  is the resistivity of the material in  $\Omega \cdot m$
- $R$  is the electrical resistance of uniform cross-sectional material in  $\Omega$
- $l$  is the length of a piece of material in  $m$

- A is the cross-sectional area of the material in  $m^2$

### Resistivity Unit

Following is the unit of resistivity:

CGS unit	$\Omega.cm$
SI unit	$\Omega.m$

Dimension of resistivity  $M^1L^3T^{-3}A^{-2}$

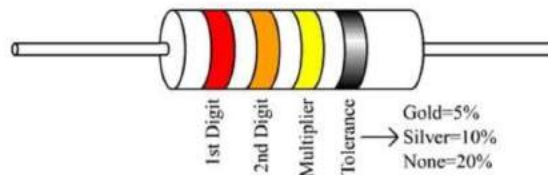
### Factors determining Resistivity of Materials

Following are the factors that determine the resistivity of materials:

- Nature of material
- Temperature

### Resistor Colour-Coding

Resistors are used in electrical circuits to control or reduce the flow of current in the circuit. Their resistance is indicated by using electronic color codes. Different color bands or rings are marked on these resistors for different values of resistance.



Color	Digit	Multiplier	Tolerance (%)
Black	0	$10^0$ (1)	
Brown	1	$10^1$	1
Red	2	$10^2$	2
Orange	3	$10^3$	
Yellow	4	$10^4$	
Green	5	$10^5$	0.5
Blue	6	$10^6$	0.25
Violet	7	$10^7$	0.1
Grey	8	$10^8$	
White	9	$10^9$	
Gold		$10^{-1}$	5
Silver		$10^{-2}$	10
(none)			20

## Resistor Colour Code and Resistor Tolerance explained

Highlights of the resistor color-coding are as follows:

- It usually contains four bands.
- The first band is indicative of the first significant figure of the resistance.
- The second band is the second significant figure. (At times there is a third band to have more precision and hence, they are 5 band resistors.)
- The third band is the decimal multiplier.
- The fourth band is indicative of the tolerance (in percentage) that the resistor can withstand the indicated values.
- In the absence of the fourth band, a default tolerance of 20% is taken.

Let us take an example to understand **resistor color coding**.

