

Chapter 2 Polynomial

Question-1

Find all the common zeroes of the polynomials: $x^3 + 5x^2 - 9x - 45$ and $x^3 + 8x^2 + 15x$.

Solution:

Let $P(x) = x^3 + 5x^2 - 9x - 45$ and $Q(x) = x^3 + 8x^2 + 15x$.

Let $P(x) = x^3 + 5x^2 - 9x - 45 = x^2(x + 5) - 9(x + 5)$

$$= (x + 5)(x^2 - 9)$$

$$= (x + 5)(x + 3)(x - 3)$$

$Q(x) = x^3 + 8x^2 + 15x = x(x^2 + 8x + 15)$

$$= x(x + 5)(x + 3)$$

\therefore The common zeroes of the polynomials are $(x + 5)$ and $(x + 3)$.

Question-2

Determine whether the given value of x is a zero of the polynomial $3x^2 - 2x - 1$ or not; if $x = 1$

Solution:

Let $p(x) = 3x^2 - 2x - 1$

$$\Rightarrow p(1) = 3(1)^2 - 2(1) - 1 = 3 - 2 - 1 = 0.$$

Therefore the given value of x is a zero of the given polynomial.

Question-3

Determine whether the given value of x is a zero of the given polynomial or not: $2x^2 - 6x + 3$; $x = \frac{1}{2}$

Solution:

Let $p(x) = 2x^2 - 6x + 3$

$$\Rightarrow p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 3 = \frac{1}{2} - 3 + 3 = \frac{1}{2}.$$

Therefore the given value of x is not a zero of the given polynomial.

Question-4

Determine whether the given value of x is a zero of the given polynomial or not: $(2x + 3)(3x - 2)$; $x = \frac{2}{3}$

Solution:

$$\text{Let } p(x) = (2x + 3)(3x - 2)$$

$$\Rightarrow p\left(\frac{2}{3}\right) = p\left(2 \times \frac{2}{3} + 3\right)\left(3 \times \frac{2}{3} - 2\right) = \left(\frac{4}{3} + 3\right)(2 - 2) = \left(\frac{4}{3} + 3\right) 0 = 0.$$

Therefore the given value of x is a zero of the given polynomial

Question-5

Determine whether the given value of x is a zero of the given polynomial or not: $x^2 + x + 1$; $x = -1$

Solution:

$$\text{Let } p(x) = x^2 + x + 1$$

$$\Rightarrow (-1) = (-1)^2 + (-1) + 1 = 1 - 1 + 1 = 1.$$

Therefore the given value of x is not a zero of the given polynomial

Question-6

Determine whether the given values of x are zeroes of the given polynomial or not: $x^2 + 6x + 5$; $x = -1$, $x = -5$

Solution:

$$\text{Let } p(x) = x^2 + 6x + 5;$$

$$\text{Put } x = -1$$

$$\Rightarrow p(-1) = (-1)^2 + 6(-1) + 5 = 1 - 6 + 5 = 0$$

$$\text{Put } x = -5$$

$$p(-5) = (-5)^2 + 6(-5) + 5 = 25 - 30 + 5 = 0$$

Therefore the given value of x is a zeroes of the given polynomial.

Question-7

In the following, determine whether the given values of x are zeroes of the given polynomial or not: $6x^2 - x - 2$; $x = -\frac{1}{2}$, $x = \frac{2}{3}$

Solution:

$$\text{Let } p(x) = 6x^2 - x - 2$$

$$\text{Put } x = -\frac{1}{2}$$

$$p\left(-\frac{1}{2}\right) = 6\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 2 = \frac{3}{2} + \frac{1}{2} - 2 = \frac{3+1-4}{2} = 0$$

Put $x = \frac{2}{3}$

$$p\left(\frac{2}{3}\right) = 6x^2 - x - 2 = 6\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right) - 2 = \frac{4}{3} - \frac{2}{3} - 2 = \frac{4-2-6}{3} = \frac{-4}{3}$$

Therefore the given value of $x = -\frac{1}{2}$ is a zero and $x = \frac{2}{3}$ is not the zero of the given polynomial.

Question-8

In the following, determine whether the given values of x are zeroes of the given polynomial or not:

$$x^2 + \sqrt{2}x - 4; x = \sqrt{2}, x = -2\sqrt{2}$$

Solution:

$$\text{Let } p(x) = x^2 + \sqrt{2}x - 4$$

$$\text{Put } x = \sqrt{2}$$

$$\Rightarrow p(\sqrt{2}) = x^2 + \sqrt{2}x - 4 = (\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 2 + 2 - 4 = 0$$

$$\text{Put } x = -2\sqrt{2}$$

$$p(-2\sqrt{2}) = x^2 + \sqrt{2}x - 4 = (-2\sqrt{2})^2 + \sqrt{2}(-2\sqrt{2}) - 4 = 8 - 4 - 4 = 0$$

Therefore the given value of x is a zero of the given polynomial.

Question-9

Determine whether the given values of x are zeroes of the given polynomial or not:

$$9x^2 - 3x - 2; x = -\frac{1}{3}, x = \frac{2}{3}$$

Solution:

$$\text{Let } p(x) = 9x^2 - 3x - 2$$

$$\text{Put } x = -\frac{1}{3}$$

$$\Rightarrow p\left(-\frac{1}{3}\right) = 9\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) - 2 = 1 + 1 - 2 = 0$$

$$\text{Put } x = \frac{2}{3}$$

$$\Rightarrow p\left(\frac{2}{3}\right) = 9x^2 - 3x - 2 = 9\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right) - 2 = 4 - 2 - 2 = 0$$

Therefore the given value of x is a zero of the given polynomial.

Question-10

Determine whether the given values of x are zeroes of the given polynomial or not: $(x+4)(x-5); x = -4, x=5$

Solution:

$$\text{Let } p(x) = (x+4)(x-5)$$

$$\text{Put } x = -4$$

$$\Rightarrow p(-4) = (x+4)(x-5) = (-4+4)(-4-5) = 0(-9) = 0$$

$$\text{Put } x = 5$$

$$\Rightarrow p(5) = (x+4)(x-5) = (5+4)(5-5) = 9(0) = 0$$

Therefore the given value of x is a zero of the given polynomial.

Question-11

Determine whether the given values of x are zeroes of the given polynomial or not: $(3x+8)(2x+5)$; $x = 2\frac{2}{3}$, $x = 2\frac{1}{2}$

Solution:

$$\text{Let } p(x) = (3x+8)(2x+5)$$

$$\text{Put } x = 2\frac{2}{3} = \frac{8}{3}$$

$$\Rightarrow p\left(\frac{8}{3}\right) = \left(3 \times \frac{8}{3} + 8\right)\left(2 \times \frac{8}{3} + 5\right) = (8+8)\left(\frac{16}{3} + 5\right) = 16 \times \frac{31}{3} = \frac{496}{3}$$

$$\text{Put } x = 2\frac{1}{2} = \frac{5}{2}$$

$$\Rightarrow p\left(\frac{5}{2}\right) = \left(3 \times \frac{5}{2} + 8\right)\left(2 \times \frac{5}{2} + 5\right) = \left(\frac{15}{2} + 8\right)(5+5) = \frac{31}{2} \times 10 = 155$$

Therefore the given value of x is not the zero of the given polynomial.

Question-12

Find the sum and product of the zeroes of the polynomial $x^2 - 6x + 5$;

Solution:

$$x^2 - 6x + 5$$

$$\text{Sum of the zeroes of the polynomial} = \frac{-b}{a} = \frac{-(-6)}{1} = 6$$

$$\text{Product of the zeroes of the polynomial} = \frac{c}{a} = \frac{5}{1} = 5$$

Question-13

Find the sum and product of the zeroes of the polynomial $px^2 + qx + pq$.

Solution:

$$px^2 + qx + pq$$

$$\text{Sum of the zeroes of the polynomial} = \frac{-b}{a} = \frac{-q}{p}$$

$$\text{Product of the zeroes of the polynomial} = \frac{c}{a} = \frac{pq}{p} = q.$$

Question-14

Find the sum and product of the zeroes of the polynomial $x^2 - 25$.

Solution:

$$x^2 - 25$$

\therefore Sum of the zeroes of the polynomial $= \frac{-b}{a} = \frac{-0}{1} = 0$ \therefore Product of the zeroes of the polynomial $= \frac{c}{a} = \frac{-25}{1} = -25$.

Question-15

Find the sum and product of the zeroes of the polynomial $4x^2 - 7x$.

Solution:

$$4x^2 - 7x$$

\therefore Sum of the zeroes of the polynomial $= \frac{-b}{a} = \frac{-(-7)}{4} = \frac{7}{4}$ \therefore Product of the zeroes of the polynomial $= \frac{c}{a} = \frac{0}{4} = 0$

Question-16

Form the polynomial whose zeroes are 5, 6.

Solution:

The roots are 5 and 6. \therefore Sum of the zeroes $= 5 + 6 = 11$

\therefore Product of the zeroes $= 5 \times 6 = 30$

The required polynomial is $x^2 - (\text{sum of the roots})x + \text{Product of the zeroes}$
 $\Rightarrow x^2 - 11x + 30$ is the required polynomial.

Question-17

Form the polynomial whose zeroes are 2, -2.

Solution:

The zeroes are 2 and -2.

\therefore Sum of the zeroes $= 2 + (-2) = 0$

\therefore Product of the zeroes $= 2 \times (-2) = -4$

The required polynomial is $x^2 - (\text{sum of the zeroes})x + \text{Product of the zeroes}$
 $\Rightarrow x^2 - (0)x + (-4)$

$$\Rightarrow x^2 - 4$$

Question-18

Form the polynomial whose zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$.

Solution:

The zeroes are $3 + \sqrt{3}$ and $3 - \sqrt{3}$. \therefore Sum of the zeroes = $(3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$

$$\begin{aligned}\therefore \text{Product of the zeroes} &= (3 + \sqrt{3})(3 - \sqrt{3}) = 9 - (\sqrt{3})^2 \\ &= 9 - 3 = 6\end{aligned}$$

\ The required polynomial is

$$x^2 - (\text{sum of the zeroes})x + \text{Product of the zeroes} \Rightarrow x^2 - (6)x + (6)$$

$$\therefore x^2 - 6x + 6$$

Question-19

Form the polynomial whose zeroes are $\frac{4+\sqrt{2}}{2}$, $\frac{4-\sqrt{2}}{2}$.

Solution:

$$\begin{aligned}\text{The zeroes are } \frac{4+\sqrt{2}}{2} \text{ and } \frac{4-\sqrt{2}}{2}. \therefore \text{Sum of the zeroes} &= \frac{4+\sqrt{2}}{2} + \frac{4-\sqrt{2}}{2} \\ &= \frac{4+\sqrt{2}+4-\sqrt{2}}{2} \\ &= \frac{8}{2} = 4\end{aligned}$$

$$\begin{aligned}\therefore \text{Product of the zeroes} &= \frac{4+\sqrt{2}}{2} \times \frac{4-\sqrt{2}}{2} \\ &= \frac{(4+\sqrt{2})(4-\sqrt{2})}{2 \times 2} \\ &= \frac{16 - (\sqrt{2})^2}{4} \\ &= \frac{16-2}{4} \\ &= \frac{14}{4} \\ &= \frac{7}{2}\end{aligned}$$

\therefore The required polynomial is $x^2 - (\text{sum of the zeroes})x + \text{Product of the zeroes}$

$$\Rightarrow x^2 - (4)x + \left(\frac{7}{2}\right) \Rightarrow 2x^2 - 8x + 7$$

Question-20

If α and β are the roots of the polynomial $ax^2 + bx + c$, then find the value of $\alpha^2 + \beta^2$

Solution:

The polynomial is $ax^2 + bx + c$ whose zeroes are α and β .

$$\text{The sum of the zeroes} = \alpha + \beta = \frac{-b}{a}$$

$$\text{The product of the zeroes} = \alpha \beta = \frac{c}{a}$$

$$\begin{aligned}
 \therefore \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a} \\
 &= \frac{b^2}{a^2} - \frac{2c}{a} \\
 &= \frac{b^2 - 2ac}{a^2}
 \end{aligned}$$

Question-21

Form the polynomial whose zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$.

Solution:

The zeroes are $3 + \sqrt{3}$, $3 - \sqrt{3}$. \therefore Sum of the zeroes $= (3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$ \therefore

$$\begin{aligned}
 \text{Product of the zeroes} &= (3 + \sqrt{3})(3 - \sqrt{3}) \\
 &= 9 - (\sqrt{3})^2 \\
 &= 9 - 3 \\
 &= 6
 \end{aligned}$$

\therefore The required polynomial is

$$x^2 - (\text{sum of the zeroes})x + \text{Product of the zeroes} \Rightarrow x^2 - (6)x + (6) \Rightarrow x^2 - 6x + 6$$

Question-22

Form the polynomial whose zeroes are $2 + \frac{1}{\sqrt{2}}$, $2 - \frac{1}{\sqrt{2}}$.

Solution:

The zeroes are $2 + \frac{1}{\sqrt{2}}$ and $2 - \frac{1}{\sqrt{2}}$

$$\begin{aligned}
 \text{Sum of the zeroes} &= \left(2 + \frac{1}{\sqrt{2}}\right) + \left(2 - \frac{1}{\sqrt{2}}\right) \\
 &= 2 + \frac{1}{\sqrt{2}} + 2 - \frac{1}{\sqrt{2}} \\
 &= 2 + 2 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Product of the zeroes} &= \left(2 + \frac{1}{\sqrt{2}}\right) \times \left(2 - \frac{1}{\sqrt{2}}\right) \\
 &= 2\left(2 - \frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(2 - \frac{1}{\sqrt{2}}\right) \\
 &= 2(2) - 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}(2) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)
 \end{aligned}$$

$$= 4 \cdot \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= 4 \cdot \frac{1}{2}$$

$$= \frac{8-1}{2}$$

$$= \frac{7}{2}$$

The required polynomial $x^2 - (\text{sum of the zeroes}) x + \text{Product of the zeroes}$

$$\Rightarrow x^2 - 4x + \frac{7}{2}$$

$$\therefore 2x^2 - 8x + 7$$