ALGEBRA & PROBABILITY

TIME : 3hrs   **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

Very Short Answer Type Questions.   **10X2 =20**

Note : Attempt all questions. Each question carries 2 marks.

1.   Prove that the roots of $(x-a)(x-b) = h^2$.

2.   If α, β and 1 are the roots of $x^3 - 2x^2 - 5x + 6 = 0$ then find α and β.

3.   If $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 4 & 0 \\ 4 & -2 & -1 \end{bmatrix}$ then show that $(A+B)' = A' + B'$

4.   If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and det $A = 45$, find x.

5.   If $^nP_7 = 42\,^nP_5$ and $^nC_r = 210$, find n.

6.   Find the number of ways of selecting 3 girls and 3 boys out of 7 girls and 6 boys.

7.   Find the range of the infinite series $\dfrac{1}{1!} + \dfrac{1+2}{2!} + \dfrac{1+2+3}{3!} + ...$

8.   A problem in calculus is given to two students A and B whose chances of solving it are 1/3 and ¼. Find the probability of the problem being solved, if both of them try independently.

9.   Find the range of x for which the binomial expansion $(7+3x)^{-5}$ is valid.

10.   If the mean and variance of a binomial variable X are 2.4 and 1.44 respectively then find p and q.

SECTION B

Short Answer Type Questions.   **5x4 =20**

Note: Answer any FIVE questions. Each question carries 4 marks.

11.   If the expression $\dfrac{x-p}{x^2-3x+2}$ takes all real values for $x \in R$, then find the bounds for p.

12.   If $\theta - \phi = \dfrac{\pi}{2}$, then show that $\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} \begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{pmatrix} = 0$

13.   Find the number of numbers that are greater than 4000 which can be formed using 0,2,4,6,8.

14.   Prove that $3 \le r \le n$, $n\text{-}3_{c_r} + 3.\, n\text{-}3_{c_{r-1}} + 3.\, n\text{-}3_{c_{r-2}} + n\text{-}3_{c_{r-3}} = n_{c_r}$

15.   Resolve $\dfrac{x^2-3}{(x+2)\left(x^2+1\right)}$ into partial fractions.

16.   If $y = x + \dfrac{x^3}{2} + \dfrac{x^3}{3} + \dfrac{x^4}{4} + ...,$ then show that $x = y - \dfrac{y^2}{2!} + \dfrac{y^3}{3!} - \dfrac{y^4}{4!} + ...$

17.   If E_1, E_2 are any two events of a random experiment and P is a probability function, then

       $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

SECTION C

Long Answer Type Questions.   **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18.   Solve the equation $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, the product of two of the roots being 6.

19.   Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{bmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{bmatrix} = \left(a^3 + b^3 + c^3 - 3abc\right)^2$

20. Solve the following simultaneous linear equations by using 'Cramer's rule.

$3x + 4y + 5z = 18$

$2x - y + 8z = 13$

$5x - 2y + 7z = 20$

21. Find the sum of the series $\dfrac{3.5}{5.10} + \dfrac{3.5.7}{5.10.15} + \dfrac{3.5.7.9}{5.10.15.20} + \ldots\ldots$

22. If the 2nd, 3rd and 4th terms in the expansion of $(a + x)^n$ are respectively, 240, 720, 1080 find a, x, n.

23. A, B, C are three horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A, B and C to win the race.

24. The range of random variable X is {0, 1, 2} given that $P(X = 0) = 3c^3$
$P(X = 1) = 4c - 10c^2$; $P(X = 2) = 5c - 1$
i) Find the value of c ii) $P(X < 1)$, $P(1 < X \leq 2)$ and $P(0 < X \leq 3)$

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