JCECE

Engineering Entrance Exam

Solved Paper 2012

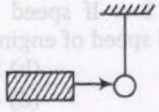
Physics

1. A monkey of 25 kg is holding a vertical rope. The rope does not break if a body of mass 30 kg is suspended from it, but the rope breaks if the mass of body suspended with the rope exceeds 30 kg. What will be the maximum acceleration with which the monkey can climb up along the rope? (Take $g = 10\ ms^{-2}$)

(a) $2.0\ ms^{-2}$ (b) $2.5\ ms^{-2}$

(c) $3.0\ ms^{2}$ (d) $4.0\ ms^{-2}$

2. A mass of 10 g moving horizontally with a velocity of $100\ cms^{-1}$ strikes a pendulum bob of same mass. The two masses after collision stick together. What will be the maximum height reached by the system now? (Take $g = 10\ ms^{-2}$)



(a) Zero (b) 1.25 cm

(c) 2.5 cm (d) 5 cm

3. The three physical quantities x, y and z have units $g\ cm^2s^{-5}$, $g\ s^{-1}$ and cms^{-2} respectively. The relation between x, y and z is

(a) $x = yz^2$ (b) $x = y^2z$

(c) $y^2 = xz$ (d) $z = x^2y$

4. If a vector **A** having a magnitude of 8 is added to a vector **B** which lies along x-axis, then the resultant of two vectors lies along y-axis and has magnitude twice that of **B**. The magnitude of **B** is

(a) $\frac{6}{\sqrt{5}}$ (b) $\frac{12}{\sqrt{5}}$

(c) $\frac{16}{\sqrt{5}}$ (d) $\frac{8}{\sqrt{5}}$

5. If the length of potentiometer wire is increased, then the length of the previously obtained balance point will

(a) increase (b) decrease

(c) remains unchanged (d) becomes two times

6. If a steel wire of length l and magnetic moment M is bent into a semicircular arc, then the new magnetic moment is

(a) $M \times l$ (b) $\frac{M}{l}$

(c) $\frac{2M}{\pi}$ (d) M

7. The amplitude and the periodic time of a SHM are 5 cm and 6 s respectively. At a distance of 2.5 cm away from the mean position, the phase will be

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{12}$

8. If all of a sudden the radius of the earth decreases, then which one of the following statements will be true?

(a) The angular momentum of the earth will become greater than that of the sun.

(b) The periodic time of the earth will increase.

(c) The energy and angular momentum will remain constant.

(d) The angular velocity of the earth will increase

9. If the external torque acting on a system is zero (*i. e.*, $\tau = 0$), then

(a) $J = 0$ (b) $F = 0$

(c) $\omega = 0$ (d) $\alpha = 0$

10. Two point charges $-q$ and $+q/2$ are situated at the origin and the point $(a, 0, 0)$ respectively. The point along the x-axis, where the electric field vanishes is

(a) $x = \sqrt{2}\,a$ (b) $x = \dfrac{a}{\sqrt{2}}$

(c) $x = \dfrac{\sqrt{2}\,a}{\sqrt{2}-1}$ (d) $x = \dfrac{\sqrt{2}a}{\sqrt{2}+1}$

11. When the kinetic energy of an electron is increased, the wavelength of the associated wave will

(a) decrease (b) increase

(c) remains unchanged (d) None of these

12. A current i ampere flows along the inner conductor of a coaxial cable and returns along the outer conductor of the cable, then the magnetic induction at any point outside the conductor at a distance r metre from the axis is

(a) ∞ (b) Zero

(c) $\dfrac{\mu_0}{4\pi}\dfrac{2i}{r}$ (d) $\dfrac{\mu_0}{4\pi}\dfrac{2\pi i}{r}$

13. Two parallel long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field midway between the wires is 10 μT. When the direction of i_2 is reversed, it becomes 40 μT. Then, ratio of i_1/i_2 is

(a) 3 : 4 (b) 5 : 3

(c) 7 : 11 (d) 11 : 7

14. The two coherent sources of equal intensity produce maximum intensity of 100 units at a point. If the intensity of one of the sources is reduced by 36% by reducing its width then the intensity of light at the same point will be

(a) 67 (b) 81 (c) 89 (d) 90

15. A convex mirror of focal length f forms an image which is $\dfrac{1}{n}$ times the object. The distance of the object from the mirror is

(a) $\left(\dfrac{n-1}{n}\right)f$ (b) $\left(\dfrac{n+1}{n}\right)f$

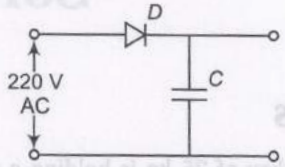
(c) $(n+1)f$ (d) $(n-1)f$

16. Consider a hydrogen like atom whose energy in nth excited state is given by $E_n = -\dfrac{13.6Z^2}{n^2}$, when this excited atom makes a transition from excited state to ground state, most energetic photons have energy $E_{max} = 52.224$ eV and least energetic photons have energy $E_{min} = 1.224$ eV. The atomic number of atom is

(a) 2 (b) 4

(c) 5 (d) None of these

17. A diode is connected to 220 V (rms) AC in series with a capacitor as shown in figure. The voltage across the capacitor is



(a) 220 V (b) 110 V

(c) 311.1 V (d) $\dfrac{220}{\sqrt{2}}$ V

18. A wave travelling along positive x-axis is given by $y = A\sin(\omega t - kx)$. If it is reflected from a rigid boundary such that 80% amplitude is reflected, then equation of reflected wave is

(a) $y = A\sin(\omega t + 0.8\,kx)$

(b) $y = -0.8A\sin(\omega t + kx)$

(c) $y = A\sin(\omega t + kx)$

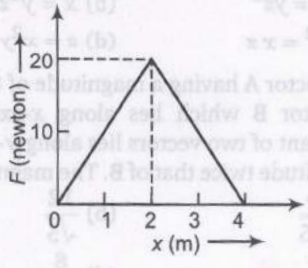
(d) $y = 0.8A\sin(\omega t + kx)$

19. An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km, it blows a whistle, whose echo is heard by the driver after 5 s. If speed of sound in air is 330 m/s, the speed of engine is

(a) 10 m/s (b) 20 m/s

(c) 30 m/s (d) 40 m/s

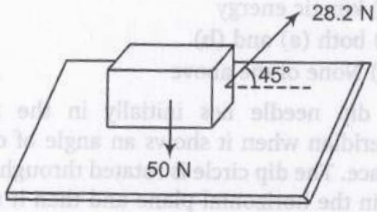
20. The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, its kinetic energy would be



(a) 10 J (b) 20 J

(c) 40 J (d) 50 J

21. A body of weight 50 N placed on a horizontal surface is just moved by a force of 28.2 N. The frictional force and normal reaction are



(a) 2N, 3N (b) 5N, 6N
(c) 10N, 15 N (d) 20 N, 30 N

22. On a railway curve, the outside rail is laid higher than the inside one so that resultant force exerted on the wheels of the rail car by the tops of the rails will
(a) equilibrate the centripetal force
(b) be vertical
(c) be decreased
(d) have a horizontal inward component

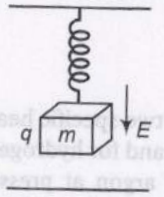
23. The mass of the moon is $\frac{1}{81}$ of the earth but the gravitational pull is $\frac{1}{6}$ of the earth. It is due to the fact that
(a) the radius of earth is $\frac{9}{\sqrt{6}}$ of the moon
(b) the radius of moon is $\frac{81}{6}$ of the earth
(c) moon is the satellite of the earth
(d) None of the above

24. Inertia is that property of a body by virtue of which the body is
(a) unable to change by itself the state of uniform motion
(b) unable to change by itself the state of rest and of uniform linear motion
(c) unable to change by itself the state of rest
(d) unable to change by itself the direction of motion

25. The radius of two metallic spheres A and B are r_1 and r_2 respectively ($r_1 > r_2$). They are connected by a thin wire and the system is given a certain charge. The charge will be greater
(a) equal on both
(b) zero on both
(c) on the surface of sphere A
(d) on the surface of sphere B

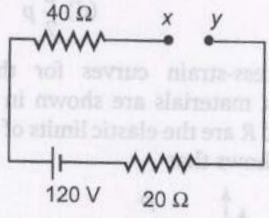
26. The temperature gradient in a rod of 0.5 m long is 80°C/m. If the temperature of hotter end of the rod is 30°C, then the temperature of the cooler end is
(a) 0°C (b) -10°C
(c) 10°C (d) 40°C

27. Time period of a block suspended from the upper plate of a parallel plate capacitor by a spring of stiffness K is T, when block is uncharged. If a charge q is given to the block then, the new time period of oscillation will be



(a) T (b) $> T$ (c) $< T$ (d) $\geq T$

28. In the circuit shown, potential difference between x and y will be



(a) Zero (b) 120 V (c) 60 V (d) 20 V

29. A wire of length 1 m is moving at a speed of $2\ ms^{-1}$ perpendicular to its length in a homogeneous magnetic field of 0.5T. If the ends of the wire are joined to a circuit of resistance 6Ω, then the rate at which work is being done to keep the wire moving at constant speed is
(a) 1 W (b) $\frac{1}{3}$ W (c) $\frac{1}{6}$ W (d) $\frac{1}{12}$ W

30. The transformation ratio in the step-up transformer is
(a) 1
(b) greater than one
(c) less than one
(d) the ratio greater or less than one depends on the other factors.

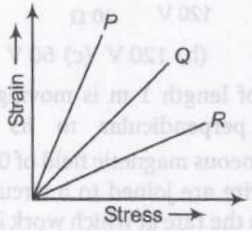
31. The focal length of the objective of a terrestrial telescope is 80 cm and it is adjusted for parallel rays, then its power is 20. If the focal length of

erecting lens is 20 cm, then full length of the telescope will be
(a) 164 cm (b) 124 cm
(c) 100 cm (d) 84 cm

32. Unpolarized light falls on two polarizing sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the final transmitted light is one-third the maximum intensity of the first transmitted beam
(a) 15°
(b) 35°
(c) 55°
(d) 75°

33. The ratio of two specific heats of gas C_p/C_V for argon is 1.6 and for hydrogen is 1.4. If adiabatic elasticity of argon at pressure p is E, then at what pressure the adiabatic elasticity of hydrogen will also be equal to E?
(a) p (b) $1.4p$
(c) $\frac{7}{8}p$ (d) $\frac{8}{7}p$

34. The stress-strain curves for three wires of different materials are shown in figure, where P, Q and R are the elastic limits of the wires. The figure shows that



(a) elasticity of wire P is maximum
(b) elasticity of wire Q is maximum
(c) elasticity of wire R is maximum
(d) None of the above is true

35. A body of density d_1 is counterpoised by Mg of weights of density d_2 in air of density d. Then, the true mass of the body is
(a) M (b) $\frac{M(1-d/d_2)}{(1-d/d_1)}$
(c) $M\left(1-\frac{d}{d_2}\right)$ (d) $M\left(1-\frac{d}{d_1}\right)$

36. According to kinetic theory of gases, total energy of a gas is equal to
(a) potential energy
(b) kinetic energy
(c) both (a) and (b)
(d) None of the above

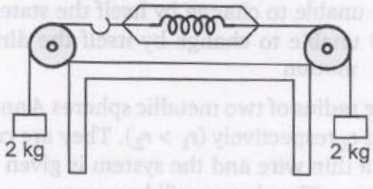
37. A dip needle lies initially in the magnetic meridian when it shows an angle of dip θ at a place. The dip circle is rotated through an angle x in the horizontal plane and then it shows an angle of dip θ′. Then, $\frac{\tan\theta'}{\tan\theta}$ will be
(a) $\cos x$ (b) $\frac{1}{\cos x}$
(c) $\frac{1}{\sin x}$ (d) $\frac{1}{\tan x}$

38. The rms current in an AC circuit is 2A. If the wattless current be $\sqrt{3}$ A, what is the power factor of the circuit ?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

39. An electric bulb rated as 500W – 100V is used in a circuit having 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb draws 500 W is
(a) 100Ω (b) 50Ω
(c) 20Ω (d) 10Ω

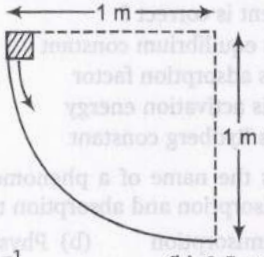
40. A string of length L and mass M hangs freely from a fixed point. Then, the velocity of transverse waves along the string at a distance x from the free end is
(a) gx (b) $\sqrt{gx}$ (c) gL (d) $\sqrt{gL}$

41. Two equal masses each of 2 kg are suspended from a spring balance as shown in figure. The reading of the spring balance will be



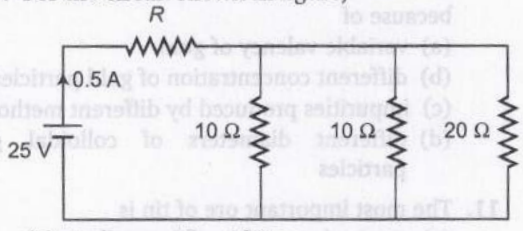
(a) Zero
(b) 2 kg
(c) 4 kg
(d) between zero and 2 kg

42. A body of mass 2 kg slides down a curved track which is quadrant of a circle of radius 1 m as shown in figure. All the surfaces are frictionless. If the body starts from rest, its speed at the bottom of the track is



(a) $2\ ms^{-1}$ (b) $0.5\ ms^{-1}$
(c) $19.6\ ms^{-1}$ (d) $4.43\ ms^{-1}$

43. For the circuit shown in figure,



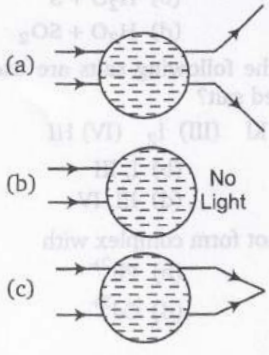
(a) resistance $R = 46\Omega$
(b) current through 20Ω resistance is 0.1A
(c) potential difference across the middle resistance is 2V
(d) All of the above are true

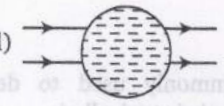
44. If 10000 V is applied across an X-ray tube, what will be the ratio of de-Broglie wavelength of the incident electrons to the shortest wavelength X-ray produced?

$\left(\frac{e}{m} \text{ for electron} = 1.8 \times 10^{11} C\ kg^{-1}\right)$

(a) 0.1 (b) 0.2 (c) 0.3 (d) 1.0

45. A water drop in air reflects the light rays as



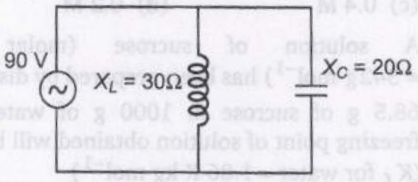


46. Two capillary tubes of same radius r but of lengths l_1 and l_2 are fitted in parallel to the bottom of a vessel. The pressure head is p. What should be the length of a single tube that can replace the two tubes so that the rate of flow is same as before ?

(a) $l_1 + l_2$ (b) $\frac{l_1 l_2}{l_1 + l_2}$ (c) $\frac{1}{l_1 + l_2}$ (d) $\frac{1}{l_1} + \frac{1}{l_2}$

47. An ideal thermometer should have
(a) small heat capacity
(b) large heat capacity
(c) medium heat capacity
(d) variable heat capacity

48. For the circuit shown in figure, the impedance of the circuit will be

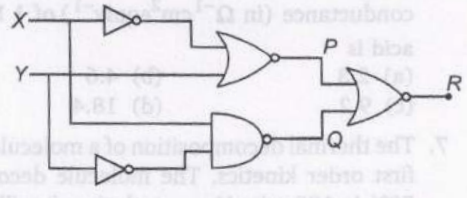


(a) 50Ω (b) 60Ω (c) 90Ω (d) 120Ω

49. An optical fibre communication system works on a wavelength of 1.3 μm. The number of subscribers it can feed if a channel requires 20 kHz are

(a) 1×10^5 (b) 2.3×10^{10}
(c) 1.15×10^{10} (d) None of these

50. A system of logic gates is shown in the figure. From the study of truth table it can be found that to produce a high output (1) at R, we must have



(a) $X = 0, Y = 1$ (b) $X = 1, Y = 1$
(c) $X = 1, Y = 0$ (d) $X = 0, Y = 0$

Chemistry

1. The reagent commonly used to determine hardness of water titrimetrically is
   (a) oxalic acid
   (b) disodium salt of EDTA
   (c) sodium citrate
   (d) sodium thiosulphate

2. In a solid lattice, the cation has left a lattice site and is located at an interstitial position. The lattice defect is
   (a) interstitial defect
   (b) vacancy defect
   (c) Frenkel defect
   (d) Schottky defect

3. The molarity of a solution obtained by mixing 800 mL of 0.5 M HCl with 200 mL of 1 M HCl will be
   (a) 0.8 M (b) 0.6 M
   (c) 0.4 M (d) 0.2 M

4. A solution of sucrose (molar mass = 342 g mol^{-1}) has been prepared by dissolving 68.5 g of sucrose in 1000 g of water. The freezing point of solution obtained will be (K_f for water = 1.86 K kg mol^{-1})
   (a) − 0.570°C (b) − 0.372°C
   (c) − 0.520°C (d) + 0.372°C

5. The standard electrode potential of three metals X, Y and Z are − 1.2 V, + 0.5 V and − 3.0 V respectively. The reducing power of these metals will be
   (a) $X > Y > Z$ (b) $Y > Z > X$
   (c) $Y > X > Z$ (d) $Z > X > Y$

6. The resistance of 1 N solution of acetic acid is 250 Ω, when measured in a cell having a cell constant of 1.15 cm^{-1}. The equivalent conductance (in $\Omega^{-1}cm^2equiv^{-1}$) of 1 N acetic acid is
   (a) 2.3 (b) 4.6
   (c) 9.2 (d) 18.4

7. The thermal decomposition of a molecule shows first order kinetics. The molecule decomposes 50% in 120 min. How much time it will take to decompose 90%?
   (a) 300 min (b) 360 min
   (c) 398.8 min (d) 400 min

8. In the respect of the equation, $k = Ae^{-Ea/RT}$ in chemical kinetics, which one of the following statement is correct ?
   (a) k is equilibrium constant
   (b) A is adsorption factor
   (c) E_a is activation energy
   (d) R is Rydberg constant

9. What is the name of a phenomenon in which both adsorption and absorption takes places ?
   (a) Chemisorption (b) Physisorption
   (c) Desorption (d) Sorption

10. Colloidal solutions of gold, prepared by different methods are of different colours because of
    (a) variable valency of gold
    (b) different concentration of gold particles
    (c) impurities produced by different methods
    (d) different diameters of colloidal gold particles

11. The most important ore of tin is
    (a) cassiterite (b) cryolite
    (c) cerussite (d) None of these

12. Which process of purifications is represented by the following scheme?

$$\underset{\text{impure}}{Ti} + 2I_2 \xrightarrow{250^\circ C} TiI_4 \xrightarrow{1400^\circ C} \underset{\text{Pure}}{Ti} + 2I_2$$

    (a) Cupellation
    (b) Zone refining
    (c) van-Arkel process
    (d) Poling

13. If the supply of oxygen is limited, H_2S reacts with O_2 to form
    (a) $H_2O + SO_3$ (b) $H_2O + S$
    (c) $H_2SO_4 + S$ (d) $H_2O + SO_2$

14. Which two of the following salts are used for preparing iodised salt?
    (I) KIO_3 (II) KI (III) I_2 (IV) HI
    (a) I, II (b) I, III
    (c) II, IV (d) III, IV

15. Ammonia will not form complex with
    (a) Ag^{2+} (b) Pb^{2+}
    (c) Cu^{2+} (d) Cd^{2+}

16. The magnetic moment of a transition metal ion is $\sqrt{15}$ BM. Therefore, the number of unpaired electrons present in it is
(a) 4 (b) 1
(c) 2 (d) 3

17. Ce^{4+} is stable. This is because
(a) half-filled *d*-orbitals
(b) all paired electrons in *d*-orbitals
(c) empty *d*-orbitals
(d) fully filled *d*-orbitals

18. $[Co(NH_3)_4Cl_2]NO_2$ and $[Co(NH_3)_4ClNO_2]Cl$ exhibit which type is isomerism?
(a) Geometrical (b) Optical
(c) Linkage (d) Ionisation

19. The π-bonded organometallic compound which has ethene as one of its component is
(a) Zeise's salt
(b) ferrocene
(c) dibenzene chromium
(d) tetraethyltin

20. Alcoholic KOH is used for
(a) dehydration
(b) dehydrogenation
(c) dehydrohalogenation
(d) dehalogenation

21. Freon used as refrigerant is
(a) $CF_2{=}CF_2$ (b) CH_2F_2
(c) CCl_2F_2 (d) CF_4

22. Which of the following reacts fastly with Na?
(a) 1° alcohol
(b) 2° alcoholo
(c) 3° alcohol
(d) The reactivity of all is equal

23. Consider the following reaction

$$\text{Phenol} \xrightarrow{\text{Zn dust}} X \xrightarrow[\text{anhy } AlCl_3]{CH_3Cl} Y \xrightarrow{\text{alk } KMnO_4} Z$$

The product *Z* is
(a) toluene (b) benzaldehyde
(c) benzoic acid (d) benzene

24. The formula of chloral is
(a) $CHCl_3$
(b) CH_2ClCHO
(c) CCl_3CHO
(d) $CHCl_2CHO$

25. One of the following named reaction is an example of disproportionation reaction. Identify it
(a) Birch reduction
(b) Aldol condensation
(c) Reimer-Tiemann reaction
(d) Cannizaro reaction

26. Calcium formate on dry heating yields
(a) acetone (b) formaldehyde
(c) acetic acid (d) acetaldehyde

27. Which amongst the following is the most stable carbocation ?

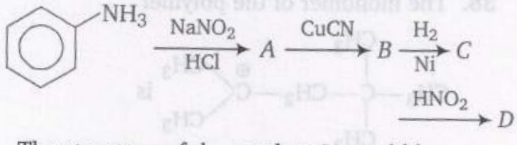
(a) $CH_3-\overset{+}{C}(CH_3)-H$ (b) $CH_3-\overset{+}{C}(CH_3)-CH_3$

(c) $\overset{+}{C}H_3$ (d) $CH_3\overset{+}{C}H_2$

28. At 25°C, the dissociation constant of a base, BOH, is 1.0×10^{-12}. The concentration of hydroxyl ions in 0.01 M aqueous solution of the base would be
(a) 2.0×10^{-6} mol L^{-1} (b) 1.0×10^{-5} mol L^{-1}
(c) 1.0×10^{-6} mol L^{-1} (d) 1.0×10^{-7} mol L^{-1}

29. Which one of the following pairs represents stereoisomerism ?
(a) Chain isomerism and rotational isomerism
(b) Structural isomerism and geometric isomerism
(c) Linkage isomerism and geometric isomerism
(d) Optical isomerism and geometric isomerism

30. Aniline in a set of reactions yielded a product *D*.



The structure of the product *D* would be
(a) $C_6H_5CH_2NH_2$ (b) $C_6H_5NHCH_2CH_3$
(c) C_6H_5NHOH (d) $C_6H_5CH_2OH$

31. The correct order in which the O—O bond length increases in the following is
(a) $H_2O_2 < O_2 < O_3$ (b) $O_3 < H_2O_2 < O_2$
(c) $O_2 < O_3 < H_2O_2$ (d) $O_2 < H_2O_2 < O_3$

32. The mass of carbon anode consumed (giving only carbondioxide) in the production of 270 kg of aluminium metal from bauxite by the Hall process is (Atomic mass Al = 27)

(a) 180 kg (b) 270 kg
(c) 540 kg (d) 90 kg

33. The number of moles of $KMnO_4$ reduced by one mole of KI in alkaline medium is

(a) one fifth (b) five
(c) one (d) two

34. Which of the following molecules has trigonal planar geometry ?

(a) IF_3 (b) PCl_3
(c) NH_3 (d) BF_3

35. Which one of the following forms micelles in aqueous solution above certain concentration?

(a) Urea
(b) Dodecyl trimethyl ammonium chloride
(c) Pyridinium chloride
(d) Glucose

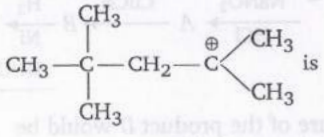
36. A nuclide of an alkaline earth metal undergoes radioactive decay by emission of three α-particles in succession. The group of the periodic table to which the resulting daughter element would belong is

(a) group 14 (b) group 16
(c) group 4 (d) group 6

37. Which of the following pairs of a chemical reaction is certain to result in a spontaneous reaction?

(a) Exothermic and decreasing disorder
(b) Endothermic and increasing disorder
(c) Exothermic and increasing disorder
(d) Endothermic and decreasing disorder

38. The monomer of the polymer



is

(a) $H_2C=C(CH_3)(CH_3)$
(b) $(CH_3)_2C=C(CH_3)_2$
(c) $CH_3CH=CH\cdot CH_3$
(d) $CH_3CH=CH_2$

39. The correct sequence of increasing covalent character is represented by

(a) $LiCl < NaCl < BeCl_2$
(b) $BeCl_2 < NaCl < LiCl$
(c) $NaCl < LiCl < BeCl_2$
(d) $BeCl_2 < LiCl < NaCl$

40. H_2S gas when passed through a solution of cations containing HCl precipitates the cations of second group of qualitative analysis but not those belonging to the fourth group. It is because

(a) presence of HCl decreases the sulphide ion concentration
(b) presence of HCl increases the sulphide ion concentration
(c) solubility product of group II sulphides is more than that of group IV sulphides
(d) sulphides of group IV cations are unstable in HCl

41. The energy of second Bohr orbit of the hydrogen atom is $-328\ kJ\ mol^{-1}$; hence the energy of fourth Bohr orbit would be

(a) $-41\ kJ\ mol^{-1}$
(b) $-1312\ kJ\ mol^{-1}$
(c) $-164\ kJ\ mol^{-1}$
(d) $-82\ kJ\ mol^{-1}$

42. Equilibrium constants K_1 and K_2 for the following equilibria

$$NO(g)+\frac{1}{2}O_2 \overset{K_1}{\rightleftharpoons} NO_2(g)$$

and $$2NO_2(g) \overset{K_2}{\rightleftharpoons} 2NO(g)+O_2(g)$$

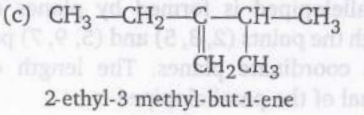
are related as

(a) $K_2=\frac{1}{K_1}$ (b) $K_2=K_1^2$
(c) $K_2=\frac{K_1}{2}$ (d) $K_2=\frac{1}{K_1^2}$

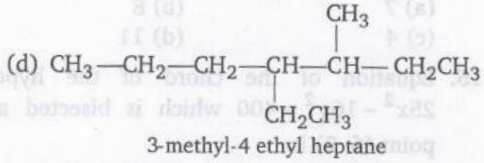
43. Names of some compounds are given. Which one is not correct in IUPAC system ?

(a) $CH_3-\underset{OH}{CH}-\underset{CH_3}{CH}-CH_3$
3-methyl-2-butanol

(b) $CH_3-CH_2\equiv C-CH(CH_3)_2$
4-methyl-2-pentyne

(c) 

2-ethyl-3 methyl-but-1-ene

(d) 

3-methyl-4 ethyl heptane

44. 10 g of hydrogen and 64 g of oxygen were filled in a steel vessel and exploded. Amount to water produced in this reaction will be
(a) 3 mol
(b) 4 mol
(c) 1 mol
(d) 2 mol

45. Dominance of strong repulsive forces among the molecules of the gas (Z = compressibility factor)
(a) depends on Z and indicated by $Z = 1$
(b) depends on Z and indicated by $Z > 1$
(c) depends on Z and indicated by $Z < 1$
(d) is independent of Z

Mathematics

1. If the lines represented by $x^2 - 2pxy - y^2$ are rotated about the origin through an angle θ, one in clockwise direction and other in anti-clockwise direction. Then, the equation of bisectors of the angles between the lines in the new position is
(a) $px^2 + 2xy + py^2 = 0$
(b) $px^2 - 2xy + py^2 = 0$
(c) $px^2 + 2xy - py^2 = 0$
(d) None of the above

2. If $z = i \log_e (2 - \sqrt{3})$, then find the value of $\cos z$.
(a) 2 (b) –2 (c) $2i$ (d) $-2i$

3. The sum of the roots of quadratic equation $ax^2 + bx + c = 0$ $(a, b, c \neq 0)$ is equal to the sum of squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in
(a) AP (b) GP
(c) HP (d) None of these

46. When glucose reacts with bromine water, the main product is
(a) acetic acid (b) saccharic acid
(c) glyceraldehyde (d) gluconic acid

47. Protein present in hair is
(a) albumin (b) globulin
(c) keratin (d) chromoprotein

48. The hydrolysis of 2-bromo-3-methyl butane by S_N1 mechanism gives mainly
(a) 3-methl-2-butanol
(b) 2-methyl-2-butanol
(c) 2,2-dimethyl-1-propanol
(d) 2 methyl-1-butanol

49. The pH value of an acid is 5 and its concentration is 1 M. What is the value of K_a for the acid ?
(a) 10^{-7} (b) 10^{-5} (c) 10^{-10} (d) 10^{-8}

50. If 1 mole of an ideal gas expands isothermally at 37°C from 15 L to 25 L, the maximum work obtained is
(a) 12.87 J (b) 6.43 J
(c) 8.57 J (d) 2.92 J

4. Find the value of
${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + 4 \cdot {}^4P_4 + \ldots + n\,{}^nP_n$.
(a) ${}^{n+1}P_{n+1}$ (b) ${}^{n+1}P_{n+1} - 1$
(c) ${}^{n+1}P_{n+1} - 2$ (d) ${}^{n+1}P_{n+1} + 1$

5. The eccentricity of an ellipse whose pair of a conjugate diameter are $y = x$ and $3y = -2x$ is
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{\sqrt{3}}$ (d) None of these

6. If the shortest distance between the lines
$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$
and
$$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$
is $\lambda\sqrt{30}$ units, then the value of λ is
(a) 1 (b) 2
(c) 3 (d) 4

7. Let u and v be two odd functions, then the function uov is
   (a) an even function
   (b) an odd function
   (c) neither even nor odd
   (d) a periodic function

8. The period of the function $f(x) = 2\sin x + 3\cos 2x$ is
   (a) π (b) 2π
   (c) $\frac{\pi}{2}$ (d) None of these

9. If $y = \frac{1}{t^2 - t - 6}$ and $t = \frac{1}{x-2}$, then the values of x which make the function y discontinuous, are
   (a) $2, \frac{2}{3}, \frac{7}{3}$
   (b) $2, \frac{3}{2}, \frac{7}{3}$
   (c) $2, \frac{3}{2}, \frac{3}{7}$
   (d) None of the above

10. The subtangent at any point of the curve $x^m y^n = a^{m+n}$ varies as
    (a) (abscissa)^2 (b) (abscissa)^3
    (c) abscissa (d) ordinate

11. $f(x) = 1 + [\cos x]\, x$, in $0 < x \le \frac{\pi}{2}$
    (a) has a minimum value 0
    (b) has a maximum value 2
    (c) is continuous in $\left[0, \frac{\pi}{2}\right]$
    (d) is not differentiable at $x = \frac{\pi}{2}$

12. If the force represented by $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is acting through the point $5\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, then its moment about the point (1, 2, 2)is
    (a) $14\mathbf{i} - 8\mathbf{j} + 12\mathbf{k}$
    (b) $-14\mathbf{i} + 8\mathbf{j} - 12\mathbf{k}$
    (c) $7\mathbf{i} + 9\mathbf{j} + 2\mathbf{k}$
    (d) $7\mathbf{i} - 9\mathbf{j} + 2\mathbf{k}$

13. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$, pass through a line, then find the value of $a^2 + b^2 + c^2 + 2abc$.
    (a) 0 (b) 1
    (c) -1 (d) $\frac{1}{2}$

14. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of the diagonal of the parallelopiped is
    (a) 7 (b) 8
    (c) 4 (d) 11

15. Equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at the point (6, 2) is
    (a) $16x - 75y = 418$ (b) $75x - 16y = 418$
    (c) $25x - 4y = 400$ (d) None of these

16. $\int \frac{1}{x^6 + x^4}\, dx$ is equal to
    (a) $-\frac{1}{3x^3} + \frac{1}{x} + \text{cosec}^{-1} x + C$
    (b) $-\frac{1}{3x^3} + \frac{1}{x} + \cot^{-1} x + C$
    (c) $-\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1} x + C$
    (d) None of the above

17. Find the sum of the series $\frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + \ldots$
    (a) $\log \frac{e}{2}$ (b) $\log \frac{e}{4}$ (c) $\log \frac{2}{3}$ (d) $\log \frac{2}{4}$

18. The equation $\tan^4 x - 2\sec^2 x + a^2 = 0$ will have atleast one solution, if
    (a) $|a| \le 4$ (b) $|a| \le 2$
    (c) $|a| \le \sqrt{3}$ (d) $|a| \le \sqrt{2}$

19. If $\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$, then find the value of $\cot\theta$.
    (a) $\sqrt{2}$ (b) $\sqrt{2} - 1$
    (c) $\sqrt{2} + 1$ (d) 0

20. If $2 - \cos^2\theta = 3\sin\theta\cos\theta$, $\sin\theta \ne \cos\theta$, then find the value of $\cot\theta$
    (a) $\frac{1}{2}$ (b) 0
    (c) -1 (d) 2

21. Two posts are x metres apart and the height of one is double that of the other. If from the mid-point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary, then the height (in metres) of the shorter post is
    (a) $x\sqrt{2}$ (b) $\frac{x}{\sqrt{2}}$ (c) $\frac{x}{2\sqrt{2}}$ (d) $\frac{x}{4}$

22. The coefficient of x^n in the expansion of $(1-x)^{-2}$ is

(a) $\frac{2^n}{2!}$ (b) $n+1$ (c) $n+2$ (d) $2n$

23. If $f(x)$ is an even function, then $\int_0^x f(t)\,dt$ is

(a) odd function
(b) even function
(c) neither even nor odd
(d) None of the above

24. Solve $(xy-1)\frac{dy}{dx}+y^2=0$.

(a) $xy+\log x=C$ (b) $xy+\log y=C$
(c) $xy-\log y=C$ (d) $xy-\log x=C$

25. Find the order and degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^{3/5}-5\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}-8\frac{dy}{dx}+5=0.$$

(a) 4, 3 (b) 3, 4 (c) 4, 5 (d) 5, 4

26. If the circle $x^2+y^2+2gx+2fy+c=0$ touches by the line $y=x$ at the point P such that $OP=6\sqrt{2}$, where O is the origin, then the value of c is equal to

(a) 74 (b) 62 (c) 64 (d) 72

27. The locus of the middle points of chords of the parabola $y^2=8x$ drawn through the vertex is a parabola whose

(a) focus is (2, 0) (b) latusrectum = 8
(c) focus is (0, 2) (d) latusrectum = 4

28. If $\sqrt{x}+\sqrt{y}=10$, find $\frac{dx}{dy}$ at $y=4$.

(a) 4 (b) −3 (c) −4 (d) 3

29. Find the value of $e^{iA}.e^{iB}.e^{iC}.e^{iD}$, where A, B, C and D are the angles of a quadrilateral.

(a) i (b) $-i$ (c) 1 (d) −1

30. The equation $z\bar{z}+z+\bar{z}+10=0$, represents

(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

31. The lengths of three unequal edges of a rectangular solid block are in GP. The volume and total surface area of the block are 216 cm^3 and 252 cm^2, respectively. Find the shortest edge of the block.

(a) 12 cm (b) 6 cm (c) 18 cm (d) 3 cm

32. If $\log(p+r)+\log(p+r-2q)=2\log(p-r)$, then p, q and r are in

(a) AP (b) GP
(c) HP (d) None of these

33. Find the sum of the real roots of the equation $x^2+5|x|+6=0$.

(a) 5 (b) 10
(c) −5 (d) None of these

34. If $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$ and $f(x)=\frac{1+x}{1-x}$, find the value of $f(A)$.

(a) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ (b) $\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}$
(c) $\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ (d) None of these

35. A skew-symmetric matrix M satisfies the relation $M+I=0$, where I is the unit matrix. Then, MM' is equal to

(a) I (b) $2I$
(c) $-I$ (d) None of these

36. Let X and Y be two random variables. The relationship $E(XY)=E(X)\cdot E(Y)$ holds

(a) always
(b) if $E(X+Y)=E(X)+E(Y)$ is true
(c) if X and Y are independent
(d) if X can be obtained from Y by a linear transformation

37. Find the number of solutions of the equation $\sin 2x+\cos 4x=2$.

(a) 0 (b) 1 (c) 2 (d) infinite

38. In any ΔABC, find the least value of $\frac{\sin^2 A+\sin A+1}{\sin A}$.

(a) 3 (b) $\sqrt{3}$ (c) 1 (d) 2

39. Find the critical points of the function $f(x)=(x-2)^{2/3}(2x+1)$.

(a) −1 and 2 (b) 1
(c) 1 and $-\frac{1}{2}$ (d) 1 and 2

40. If $\mathbf{a}=2\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $\mathbf{b}=-\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=3\mathbf{i}+\mathbf{j}$, then $\mathbf{a}+t\mathbf{b}$ is perpendicular to $\mathbf{c}$; if t is equal to

(a) 2 (b) 4
(c) 6 (d) 8

41. Find the angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$.

(a) 45° (b) 30° (c) 60° (d) 90°

42. The equation of the plane passing through the line of intersection of the planes $2x - y = 0$ and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$ is

(a) $2x + 17y + 9z = 0$ (b) $28x - 17y + 9z = 0$
(c) $2x + 17y - 9z = 0$ (d) None of these

43. The function defined by the equation $xy - \log y = 1$ satisfies $x(yy'' + y'^2) - y'' + kyy' = 0$. Find the value of k.

(a) -3 (b) 3
(c) 1 (d) -1

44. If $f(x) = [1 - (x-3)^4]^{1/7}$, find $f^{-1}(x)$.

(a) $3 + (1-x)^{7/4}$ (b) $3 + (1-x^4)^7$
(c) $3 + (1-x^7)^{1/4}$ (d) $3 - (1-x^4)^{1/7}$

45. Find the value of $\lim_{n\to\infty} \frac{a^n + b^n}{a^n + d^n}$, $a > b, d$.

(a) -1 (b) 1 (c) 0 (d) $\frac{1}{2}$

46. The function $y = x^2 + ax + b$ has a minimum at $x = 3$ and minimum value is 5. Find $a + b$.

(a) -6 (b) 14
(c) 20 (d) 8

47. The coefficient of x^n in the expansion of $(1 - 9x + 20x^2)^{-1}$ is

(a) $5^n - 4^n$ (b) $5^{n+1} - 4^{n+1}$
(c) $5^{n-1} - 4^{n-1}$ (d) None of these

48. The minimum value of $f(x) = |x-1| + |x-2| + |x-3|$ is equal to

(a) 1 (b) 2 (c) 3 (d) 0

49. $\frac{d^{20}y}{dx^{20}}(2\cos x \cos 3x)$ is equal to

(a) $2^{20}(\cos 2x - 2^{20}\cos 4x)$
(b) $2^{20}(\cos 2x + 2^{20}\cos 4x)$
(c) $2^{20}(\sin 2x + 2^{20}\sin 4x)$
(d) $2^{20}(\sin 2x - 2^{20}\sin 4x)$

50. The logically equivalent proposition of $p \Leftrightarrow q$ is

(a) $(p \vee q) \vee (p \wedge q)$
(b) $(p \Rightarrow q) \wedge (q \Rightarrow p)$
(c) $(p \wedge q) \vee (q \Rightarrow p)$
(d) $(p \wedge q) \Rightarrow (q \vee p)$

Answers

Physics

1. (a)	2. (c)	3. (a)	4. (d)	5. (a)	6. (c)	7. (c)	8. (d)	9. (d)	10. (c)
11. (a)	12. (b)	13. (b)	14. (b)	15. (d)	16. (a)	17. (d)	18. (b)	19. (c)	20. (a)
21. (d)	22. (d)	23. (a)	24. (b)	25. (c)	26. (b)	27. (a)	28. (b)	29. (c)	30. (b)
31. (a)	32. (c)	33. (d)	34. (d)	35. (b)	36. (b)	37. (b)	38. (a)	39. (c)	40. (b)
41. (b)	42. (d)	43. (d)	44. (a)	45. (c)	46. (b)	47. (a)	48. (b)	49. (c)	50. (c)

Chemistry

1. (b)	2. (c)	3. (b)	4. (b)	5. (d)	6. (b)	7. (c)	8. (c)	9. (d)	10. (d)
11. (a)	12. (c)	13. (b)	14. (a)	15. (b)	16. (d)	17. (c)	18. (d)	19. (a)	20. (c)
21. (c)	22. (a)	23. (c)	24. (c)	25. (d)	26. (b)	27. (b)	28. (d)	29. (d)	30. (d)
31. (c)	32. (d)	33. (d)	34. (d)	35. (b)	36. (a)	37. (c)	38. (a)	39. (c)	40. (a)
41. (d)	42. (d)	43. (d)	44. (b)	45. (b)	46. (d)	47. (c)	48. (b)	49. (c)	50. (a)

Mathematics

1. (c)	2. (a)	3. (c)	4. (b)	5. (c)	6. (c)	7. (b)	8. (b)	9. (b)	10. (c)
11. (c)	12. (d)	13. (b)	14. (a)	15. (b)	16. (c)	17. (a)	18. (c)	19. (b)	20. (d)
21. (c)	22. (c)	23. (a)	24. (c)	25. (a)	26. (d)	27. (d)	28. (c)	29. (c)	30. (a)
31. (d)	32. (c)	33. (d)	34. (b)	35. (a)	36. (c)	37. (a)	38. (a)	39. (b)	40. (d)
41. (d)	42. (b)	43. (b)	44. (c)	45. (b)	46. (d)	47. (b)	48. (b)	49. (b)	50. (b)

Hints *and* Solutions

Physics

1. The maximum weight which can be suspended with the rope without breaking it = 30 kg wt.

$= 30 \times 10 = 300$ N

$\therefore 300 \text{ N} = mg + ma$

or $ma = 300 - mg = 300 - 25 \times 10 = 50$ N

or $a = \dfrac{50}{m} = \dfrac{50}{25} = 2.0 \text{ ms}^{-2}$

2. Kinetic energy of mass of 10 g is converted into potential energy of the system

$\therefore \dfrac{1}{2} m_1 v_1^2 = (m_1 + m_2) gh$

or $h = \dfrac{m_1 v_1^2}{2(m_1 + m_2)g}$

$= \dfrac{10 \times (100)^2}{2(10 + 10) \times 1000} = \dfrac{10 \times 10000}{2 \times 20 \times 1000}$

$= \dfrac{10}{4} = 2.5$ cm

3. Given, $x = \text{g cm}^2 \text{ s}^{-5} = [\text{ML}^2\text{T}^{-5}]$

$y = \text{g s}^{-1} = [\text{ML}^0\text{T}^{-1}]$

and $z = \text{cms}^{-2} = [\text{M}^0\text{LT}^{-2}]$

Now, $z^2 = [\text{M}^0\text{L}^2\text{T}^{-4}]$

and $yz^2 = [\text{ML}^0\text{T}^{-1}][\text{M}^0\text{L}^2\text{T}^{-4}]$

$= [\text{ML}^2\text{T}^{-5}] = x$

i.e., $x = yz^2$

4. According to condition

$\mathbf{A} + \mathbf{Bi} = \mathbf{Rj}$ (where, $\mathbf{R}$ is the resultant vector)

Also, $\mathbf{R} = 2\mathbf{B}$

$\therefore \mathbf{A} + \mathbf{Bi} = 2\mathbf{Bj}$

or $\mathbf{A} = 2\mathbf{Bj} - \mathbf{Bi}$

$\therefore A^2 = 4B^2 + B^2 = 5B^2$

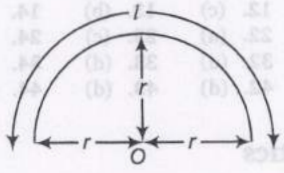
Here, $A = 8$

$\therefore 64 = 5B^2$

$\Rightarrow B = \sqrt{\dfrac{64}{5}} = \dfrac{8}{\sqrt{5}}$

5. When the length of potentiometer wire is increased, the potential gradient decreases and the length of previous balance point is increased.

6. When wire is bent in the form of semicircular arc then, $l = \pi r$



$\therefore$ The radius of semicircular arc, $r = l/\pi$

Distance between two end points of semicircular wire $= 2r = \dfrac{2l}{\pi}$

$\therefore$ Magnetic moment of semicircular wire

$= m \times 2r$

$= m \times \dfrac{2l}{\pi} = \dfrac{2}{\pi} ml$

But ml is the magnetic moment of straight wire

i.e., $ml = M$

$\therefore$ New magnetic moment $= \dfrac{2}{\pi} M$

7. The equation of motion is given as

$y = 5 \sin \dfrac{2\pi t}{6}$

Here, $y = 2.5$ cm

$\therefore 2.5 = 5 \sin \dfrac{2\pi t}{6}$

$\Rightarrow \dfrac{\pi}{6} = \dfrac{2\pi t}{6} \Rightarrow t = \dfrac{1}{2}$ s

$\therefore$ The phase $= \dfrac{2\pi t}{6} = \dfrac{2\pi}{6} \times \dfrac{1}{2} = \dfrac{\pi}{6}$

8. If the radius of earth decreases all of sudden, then its moment of inertia (I) decreases.

$\therefore$ By the relation, $L = I\omega$, we get

$\omega \propto \dfrac{1}{I}$ ($\because L$ is constant)

So, when moment of inertia decreases, angular velocity of earth will increases.

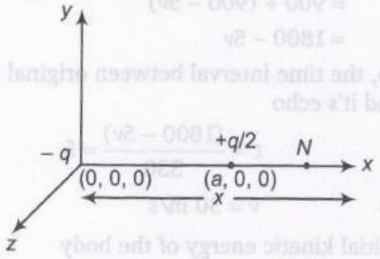
9. By the relation

$$\tau = I\alpha$$

if $\tau = 0$, then $\alpha = 0$

because moment of inertia of any body cannot be zero.

10. The given situation can be shown as



Suppose the field vanishes at a distance x, we have

$$\frac{kq}{x^2} = \frac{kq/2}{(x-a)^2}$$

$$\Rightarrow \quad 2(x-a)^2 = x^2$$

or $\sqrt{2}(x-a) = x$

$$\Rightarrow \quad \sqrt{2}x - \sqrt{2}a = x$$

$$\Rightarrow \quad \sqrt{2}x - x = \sqrt{2}a$$

$$\Rightarrow \quad (\sqrt{2}-1)x = \sqrt{2}a$$

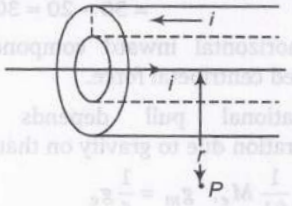
$$\Rightarrow \quad x = \left(\frac{\sqrt{2}a}{\sqrt{2}-1}\right)$$

11. We know that, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$\therefore \lambda \propto \frac{1}{\sqrt{E}}$ (as h and m will be constants)

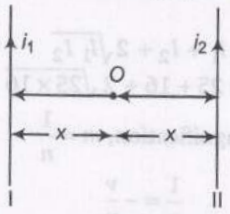
So, when the kinetic energy of electron increases, the wavelength associated with it decreases.

12. The described condition can be shown as



The magnetic field at P due to inner and outer conductors are equal and opposite. Hence, the net magnetic field at P will be zero.

13. When the current in the wires is in same direction. Magnetic field at mid point O due to I and II wires are respectively



$$B_I = \frac{\mu_o}{4\pi}\frac{2i_1}{x} \otimes$$

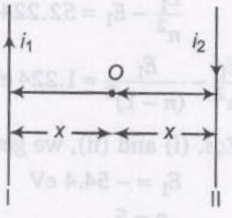
and $B_{II} = \frac{\mu_0}{4\pi} \cdot \frac{2i_2}{x} \odot$.

So, the net magnetic field at O

$$B_{net} = \frac{\mu_0}{4\pi} \times \frac{2}{x} \times (i_1 - i_2)$$

$$\Rightarrow \quad 10 \times 10^{-6} = \frac{\mu_0}{4\pi} \cdot \frac{2}{x}(i_1 - i_2) \qquad \ldots(i)$$

when the direction of i_2 is reversed



$$B_I = \frac{\mu_0}{4\pi} \cdot \frac{2i_1}{x} \otimes$$

and $B_{II} = \frac{\mu_0}{4\pi} \cdot \frac{2i_2}{x} \otimes$

So, net magnetic field at O

$$B_{net} = \frac{\mu_0}{4\pi} \times \frac{2}{x} \times (i_1 + i_2)$$

$$\Rightarrow \quad 40 \times 10^{-6} = \frac{\mu_0}{4\pi} \cdot \frac{2}{x}(i_1 + i_2) \qquad \ldots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{i_1 + i_2}{i_1 - i_2} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{5}{3}$$

14. Intensity of each source, $I = \frac{100}{4} = 25$ unit

If the intensity of one source is reduced by 36%, then $I_1 = 25$ unit and

$$I_2 = 25 - \frac{25 \times 36}{100} = 16 \text{ unit}$$

Hence, resultant intensity at the same point will be

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= 25 + 16 + 2\sqrt{25 \times 16} = 81 \text{ unit}$$

15. Given magnification, $m = \frac{1}{n}$

So, $\frac{1}{n} = -\frac{v}{u}$

or $v = -\frac{u}{n}$

By using mirror formula, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$= \frac{1}{-u/n} + \frac{1}{u}$$

$\Rightarrow$ $u = -(n-1)f$

16. Maximum energy is liberated for transition $E_n \to 1$ and minimum energy for $E_n \to E_{n-1}$

Hence, $\frac{E_1}{n^2} - E_1 = 52.224 \text{ eV}$ ...(i)

and $\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224 \text{ eV}$ ...(ii)

Solving Eqs. (i) and (ii), we get

$E_1 = -54.4 \text{ eV}$

and $n = 5$

But $E_1 = -\frac{13.6Z^2}{1^2}$

$\therefore$ $-54.4 = \frac{13.6}{1^2} Z^2$

$\Rightarrow$ $Z = 2$

17. The diode D will conduct for positive half cycle of AC supply because this is forward biased. For negative half cycle of AC supply, this is reverse biased and does not conduct. So output would be half wave rectified and for half wave rectified output will

$$V_{rms} = \frac{V_0}{2} = \frac{220\sqrt{2}}{2} = \frac{220}{\sqrt{2}}$$

18. On getting reflected from a rigid boundary the wave suffers.

Hence, if $y_{incident} = A \sin(\omega t - kx)$

Then, $y_{reflected} = (0.8A) \sin\{\omega t - k(-x) + \pi\}$

$= -0.8A \sin(\omega t + kx)$

an additional phase change of π.

19. If the speed of engine is v, the distance travelled by engine in 5 s will be $5v$ and hence the distance travelled by sound in reaching the hill and coming back to the moving driver

$= 900 + (900 - 5v)$

$= 1800 - 5v$

So, the time interval between original sound and it's echo

$$t = \frac{(1800 - 5v)}{330} = 5$$

$\Rightarrow$ $v = 30 \text{ m/s}$

20. Initial kinetic energy of the body

$$= \frac{1}{2} mv^2 = \frac{1}{2} \times 25 \times (2)^2 = 50 \text{ J}$$

Work done against resistive force = Area between F-x graph

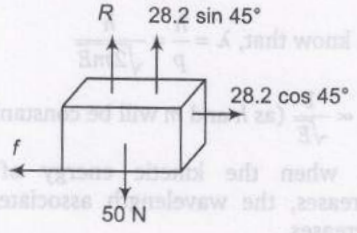
$$= \frac{1}{2} \times 4 \times 20 = 40 \text{ J}$$

Final KE = Initial KE – work done against resistive force

$= 50 - 40 = 10 \text{ J}$

21. Frictional force, $f = 28.2 \cos 45°$

$$= 28.2 \frac{1}{\sqrt{2}} = 20 \text{ N}$$



Normal reaction, $R = 50 - 28.2 \sin 45°$

$= 50 - 20 = 30 \text{ N}$

22. The horizontal inward component provides required centripetal force.

23. Gravitational pull depends upon the acceleration due to gravity on that planet.

$M_m = \frac{1}{81} M_e$, $g_m = \frac{1}{6} g_e$

By the relation, $g = \frac{GM}{R^2}$

$$\frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{\frac{1}{2}} = \left(81 \times \frac{1}{6}\right)^{\frac{1}{2}}$$

$$\therefore \quad R_e = \frac{9}{\sqrt{6}} R_m$$

24. Inertia is the inability of a body due to which body cannot change its state of rest or of motion by itself.

25. When the spheres are connected by a wire, then potential on both becomes equal.

$$\therefore \quad \frac{Q_1}{r_1} = \frac{Q_2}{r_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{r_1}{r_2}$$

when $r_1 > r_2$, then $Q_1 > Q_2$

26. Temperature gradient $= \frac{\theta_1 - \theta_2}{l}$

Here, $\theta_1 = 30°C$

$l = 0.5$ m

$$\therefore \quad 80 = \frac{30 - \theta_2}{0.5}$$

$$\theta_2 = 30 - 80 \times 0.5$$

$$\Rightarrow \quad \theta_2 = 30 - 40 = -10°C$$

27. The forces that act on the block are qE and mg. Since, qE and mg are constant forces, the only variable elastic force changes by kx (where, x is the elongation in the spring).

Unbalanced (restoring) force, $F = -kx$

$$\Rightarrow \quad -m\omega^2 x = -kx$$

$$\Rightarrow \quad \omega = \sqrt{\frac{k}{m}} = T$$

28. In open circuit of a cell $V = E$

$\therefore$ The potential difference between x and y is 120 V

29. Rate of work $= \frac{W}{t} = P$

But $P = Fv$,

Also, $F = Bil = B\left(\frac{Bvl}{R}\right)l$

$$\left[\because \text{ induced current, } i = \frac{Bvl}{R}\right]$$

$$\therefore \quad P = B\left(\frac{Bvl}{R}\right)lv = \frac{B^2v^2l^2}{R}$$

$$= \frac{(0.5)^2 \times (2)^2 \times (1)^2}{6} = \frac{1}{6} \text{ W}$$

30. Transformation ratio, $K = \frac{N_s}{N_P} = \frac{V_s}{V_P}$

For step-up transformer, $N_s > N_P$ *i.e.*, $V_s > V_P$

Hence, $K > 1$.

31. Magnification for parallel rays,

$$m = \frac{f_o}{f_e}$$

$$\Rightarrow \quad 20 = \frac{80}{f_e}$$

or $f_e = 4$cm

If the focal length of erecting lens is 20 cm, then the length of the telescope

$L_\infty = f_o + 4f + f_e$

[where, f is the focal length of erecting lens]

$= 80 + 4 \times 20 + 4$

$= 80 + 80 + 4 = 164$ cm

32. Intensity of unpolarized light, $I' = \frac{I}{2}\cos^2\theta$

$$\therefore \quad \frac{I}{2}\cos^2\theta = \frac{I}{6}$$

$$\Rightarrow \quad \cos^2\theta = \frac{1}{3}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \theta = 55°$$

33. Adiabatic elasticity $E = \gamma p$,

For argon, $E_{Ar} = 1.6\, p$ ...(i)

For hydrogen, $E_H = 1.4p'$ ...(ii)

As elasticity of argon and hydrogen are equal

$$\therefore \quad 1.6\, p = 1.4\, p'$$

or $$p' = \frac{1.6}{1.4} = \frac{8}{7} p$$

34. As stress is shown on x-axis and strain on y-axis.

So, we can say that $Y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$

So, elasticity of wire P is minimum and of wire R is maximum.

35. Let M_0 = mass of body in vacuum

Apparent weight of body in air = Apparent weight of standard weights in air

$\Rightarrow$ Actual weight – upthrust due to displaced air = Actual weight – upthrust due to displaced air

$$\Rightarrow M_0 g - \left(\frac{M_0}{d_1}\right)dg = Mg - \left(\frac{M}{d_2}\right)dg$$

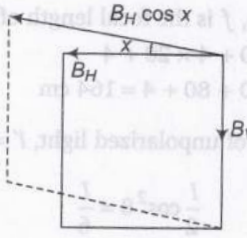
$$\Rightarrow \quad M_0 = \frac{M\left(1-\frac{d}{d_2}\right)}{\left(1-\frac{d}{d_1}\right)}$$

36. According to kinetic theory of gases, there is no attraction force between the molecules of gases,

So, potential energy = 0

37. In first case,

$$\tan\theta = \frac{B_V}{B_H} \quad \ldots(i)$$



In second case,

$$\tan\theta' = \frac{B_V}{B_H \cos x} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{\tan\theta'}{\tan\theta} = \frac{1}{\cos x}$$

38. We know that, $i_{wL} = i_{rms} \sin\phi$

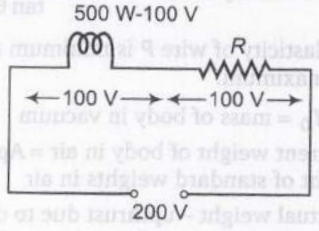
$$\Rightarrow \quad \sqrt{3} = 2\sin\phi$$

or $$\sin\phi = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \phi = 60°$$

So, power factor = $\cos\phi = \cos 60° = \frac{1}{2}$

39. We know that, $P = Vi$



$\therefore$ Current through the circuit,

$$i = \frac{500}{100} = 5\text{ A}$$

Potential difference across, $R = 100$ V

$\therefore$ Relation, $V = IR$

$$100 = 5 \times R$$

$$\Rightarrow \quad R = \frac{100}{5} = 20\Omega$$

40. Total length of string = L

Mass of the string = M

The velocity of transverse wave $v = \sqrt{\frac{T}{m}}$

where, T = weight of part of string hanging below the point under consideration

$$= \left(\frac{M}{L}\right)xg$$

$$\Rightarrow \quad v = \sqrt{\frac{\left(\frac{M}{L}\right)xg}{\left(\frac{M}{L}\right)}} = \sqrt{xg}$$

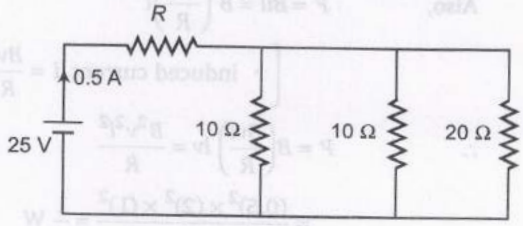
41. According to the shown condition, one 2 kg-wt on the right will act as the support for the spring balance. Hence, its reading will be 2 kg.

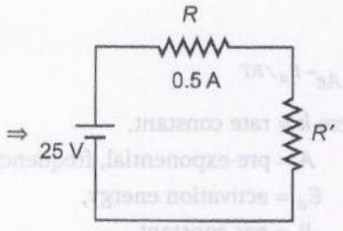
42. By conservation of energy, $mgh = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1}$$

$$= \sqrt{19.6} = 4.43\text{ ms}^{-1}$$

43. $\frac{1}{R'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{20}$





$$\Rightarrow \quad R' = \frac{20}{5} = 4\Omega$$

Now, using Ohm's law, $i = \frac{25}{R + R'}$

$$\Rightarrow \quad 0.5 = \frac{25}{R + 4}$$

$$\Rightarrow \quad R + 4 = \frac{25}{0.5}$$

$$\Rightarrow \quad R = 50 - 4 = 46\Omega$$

Current through 20 Ω resistor

$$= \frac{0.5 \times 5}{20 + 5} = \frac{2.5}{25} = 0.1\text{A}$$

Potential difference across middle resistor

= Potential difference across 20 Ω

$= 20 \times 0.1 = 2\text{ V}$

44. For incident electron, $\frac{1}{2}mv^2 = eV$

or $\quad p^2 = 2meV$

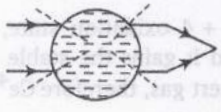
∴ de-Broglie wavelength, $\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

Shortest X-rays wavelength, $\lambda_2 = \frac{hc}{eV}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{c}\sqrt{\left(\frac{V}{2}\right)\left(\frac{e}{m}\right)}$$

$$= \frac{\sqrt{\frac{10^4}{2} \times 1.8 \times 10^{11}}}{3 \times 10^8} = 0.1$$

45. A water drop in air behaves as converging lens.



46. For parallel combination

$$\frac{1}{R_{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{\pi r^4}{8\eta l} = \frac{\pi r^4}{8\eta l_1} + \frac{\pi r^4}{8\eta l_2} \quad \Rightarrow \frac{1}{l} = \frac{1}{l_1} + \frac{1}{l_2}$$

$$\therefore \quad l = \frac{l_1 l_2}{l_1 + l_2}$$

47. The thermometer has to attain the temperature of the body. So it should draw as little heat from the body as possible, so that the existing temperature of the body is not disturbed. Therefore, the heat capacity of a thermometer should be small.

48. Current through inductance, $i_L = \frac{90}{30} = 3\text{ A}$

Current through capacitor, $i_C = \frac{90}{20} = 4.5\text{ A}$

Net current through circuit

$$i = i_C - i_L = 4.5 - 3 = 1.5\text{ A}$$

∴ Impedance of the circuit,

$$Z = \frac{V}{i} = \frac{90}{1.5} = 60\Omega$$

49. Optical source frequency,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{1.3 \times 10^{-6}}$$

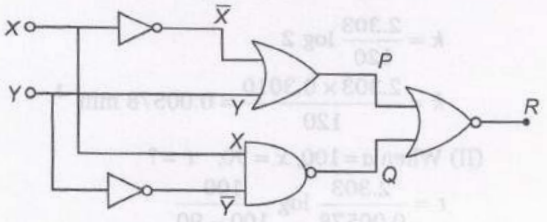
$$= 2.3 \times 10^{14}\text{ Hz}$$

∴ Number of channels or subscribers

$$= \frac{2.3 \times 10^{14}}{20 \times 10^3} = 1.15 \times 10^{10}$$

50. The truth table can be written as

X	Y	$\overline{X}$	$\overline{Y}$	$P = \overline{X} + Y$	$Q = \overline{X \cdot Y}$	$R = \overline{P + Q}$
0	1	1	0	1	1	0
1	1	0	0	1	1	0
1	0	0	1	0	0	1
0	0	1	1	1	1	0



So, $X = 1$, $Y = 0$ gives output $R = 1$.

Chemistry

1. The hardness of water is estimated by simple titration of ethylene diamine tetraacetate (EDTA) solution. EDTA forms stable complexes with metal ions present in hard water.

2. If an ion is missing from its lattice site and it occupies the interstitial site, this type of defect is called Frenkel defect.

3. $\underset{\text{Initial}}{M_1V_1} + \underset{\text{Final}}{M_2V_2} = M_3V_3$

$$V_3 = V_1 + V_2 = 800 + 200 = 1000 \text{ mL}$$
$$= 0.5 \times 800 + 1 \times 200 = M_3 \times 1000$$
$$M_3 = \frac{600}{1000} = 0.6 \text{ M}$$

4. $$\Delta T_f = \frac{1000 \times K_f \times w_2}{M_2 \times w_1}$$
$$= \frac{1000 \times 1.86 \times 68.5}{342 \times 1000} = 0.372°\text{C}$$
$$\Delta T_f = T_f^\circ - \Delta T_f = 0 - 0.372 = -0.372°\text{C}$$

5. Higher the reduction potential lesser the reducing power.

Hence, $Z > X > Y$

6. Equivalent conductivity $(\Lambda_{eq}) = \frac{\kappa \times 1000}{c}$

$$\kappa = \frac{\text{cell constant}}{\text{resistance}} = \frac{1.15}{250} \text{ S cm}^{-1}$$
$$\Lambda_{eq} = \frac{1.15 \times 1000}{250 \times 1} = 4.6\ \Omega^{-1}\text{cm}^2 \text{ equiv}^{-1}$$

7. $$t = \frac{2.303}{k} \log \frac{a}{a-x}$$

(I) When $a = 100$, $x = 50$ and $t = 120$ min

$$k = \frac{2.303}{120} \log \frac{100}{100 - 50}$$
$$k = \frac{2.303}{120} \log 2$$
$$k = \frac{2.303 \times 0.3010}{120} = 0.00578 \text{ min}^{-1}$$

(II) When $a = 100$, $x = 90$, $t = ?$

$$t = \frac{2.303}{0.00578} \log \frac{100}{100 - 90}$$
$$t = \frac{2.303}{0.00578} \times \log 10 = 398.4 \approx 398.8 \text{ min}$$

8. $k = Ae^{-E_a/RT}$

where k = rate constant,
A = pre-exponential, frequency factor,
E_a = activation energy,
R = gas constant,
T = temperature

9. When adsorption and absorption both takes place simultaneously, the phenomenon is called sorption.

10. Colour of the colloidal sol depends upon the particle size of dispersed phase.

11. The chief ore of tin is cassiterite or tin stone (SnO_2).

12. This process of purification is represented by van-Arkel process.

13. In limited supply of O_2, H_2S burns with blue flame and S and H_2O are main products.

$$2H_2S + O_2 \longrightarrow 2H_2O + 2S$$

14. To prepare iodised salt, KIO_3 and KI are used.

15. Generally *d*-block elements form complex and lead is a *p*-block element.

16. Magnetic moment $= \sqrt{n(n+2)}$

where, n = number of unpaired electrons

Given $\mu = \sqrt{15}$

$$\sqrt{15} = \sqrt{n(n+2)}$$
$$15 = n(n+2)$$
$$n^2 + 2n - 15 = 0$$
$$(n+5)(n-37) = 0$$
$$n = 3 \qquad (\because n \neq -5)$$

17. $\text{Ce}(58) = [\text{Xe}]\ 4f^1 5d^1 6s^2$ predicted

$[\text{Xe}]\ 4f^2 5d^0 6s^2$ observed

$\text{Ce}^{4+} = [\text{Xe}]\ 4f^0 5d^0 6s^0$

Since, in + 4 oxidation state, all orbitals are empty and it gains the stable configuration of nearest inert gas, therefore Ce^{4+} is most stable.

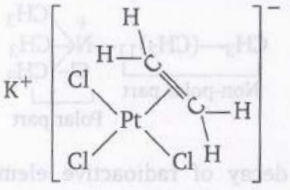
18. Both give different ions when subjected to ionisation.

$$[Co(NH_3)_4Cl_2]NO_2 \longrightarrow [Co(NH_3)_4Cl_2]^+ + NO_2^-$$

$[Co(NH_3)_4ClNO_2]Cl \rightarrow [Co(NH_3)_4ClNO_2]^+ + Cl^-$

They exhibit ionisation isomerism.

19. Zeise's salt $k[PtCl_3(C_2H_4)]$ is a π-bonded organometallic compound. Its structure is



20. $CH_3—CH_2Br + KOH\,(alc.) \xrightarrow{\text{dehydrohalogenation}} CH_2=CH_2 + KBr + H_2O.$

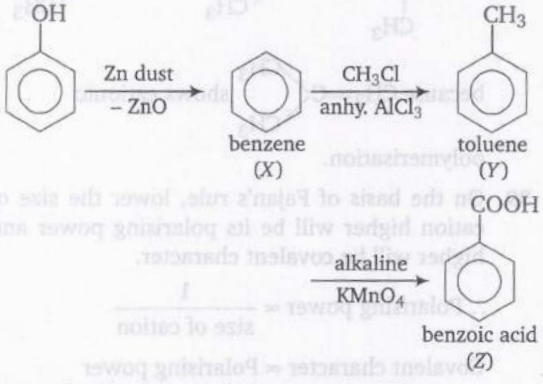
In alcoholic KOH, alkoxide ion ($\overline{R}O$) is present which is a strong base favour elimination reaction.

21. Freon used as refrigerant is CCl_2F_2.

$$CCl_4 + 2HF \xrightarrow{Sb_2F_5} \underset{\text{freon}}{CF_2Cl_2} + 2HCl$$

22. 1° alcohols are more acidic than 2° and 3° alcohols and hence they react faster with Na.

23.



24. The formula of chloral is CCl_3CHO.

25. $HCHO + HCHO \xrightarrow{OH^-} CH_3OH + HCOO^-$ (oxidation; reduction)

26. Calcium formate $(HCOO)_2Ca \xrightarrow{\text{dry distillation}} 2HCHO$ (formaldehyde) $+ CaCO_3$

27. The most stable carbocation is t-alkyl carbocation because the order of stability of alkyl carbocation is t-alkyl > s-alkyl > p-alkyl > CH_3^+ carbocation. This stability is due to the hyperconjugation.

28. Base BOH is dissociated as follows

$$BOH \rightleftharpoons B^+ + OH^-$$

So, the dissociation constant of base BOH

$$K_b = \frac{[B^+][OH^-]}{[BOH]} \quad \ldots(i)$$

At equilibrium,

$$[B^+] = [OH^-] \quad \therefore K_b = \frac{[OH^-]^2}{[BOH]}$$

Given that $k_b = 1.0 \times 10^{-12}$

and $[BOH] = 0.01$ M

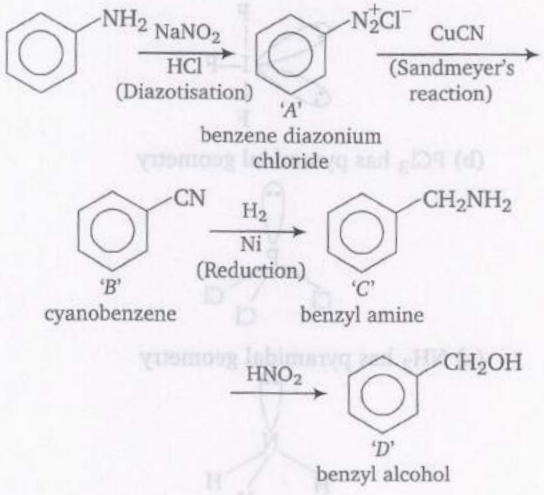
Thus, $1.0 \times 10^{-12} = \frac{[OH^-]^2}{0.01}$

$$[OH^-]^2 = 1 \times 10^{-14}$$

$$[OH^-] = 1.0 \times 10^{-7} \text{ mol } L^{-1}$$

29. Pair of optical isomerism and geometrical isomerism are able to exhibit the phenomenon of stereoisomerism because both type of isomers differ only in their orientation in space.

30.



31. Bond length of O—O in $O_2 = 1.21$ Å

Bond length of O—O in $O_3 = 1.278$ Å

Bond length of O—O in $H_2O_2 = 1.49$ Å

Therefore, correct order of O—O bonds length is (bond order decrease in the same order)

$$O_2 < O_3 < H_2O_2$$

Bond lensth of 0-0 in H_2O_2 = 1.49 Å

32. In hall and Heroult process

$$2Al_2O_3 \longrightarrow 4Al + 3O_2$$
$$4C + 3O_2 \longrightarrow 2CO_2 + 2CO\uparrow$$

Only for removal of CO_2, following equation is possible

$$2Al_2O_3 + \underset{3\times12=96}{3C} \longrightarrow \underset{4\times27=108}{4Al} + 3CO_2$$

$\because$ For 108 kg of Al, 36 g of C is required in above reaction

$\therefore$ For 270 kg of Al require amount of C

$$= \frac{36}{108} \times 270 = 90 \text{ kg}$$

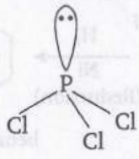
33. In alkaline solution, $KMnO_4$ is reduced to MnO_2 (colourless).

$$2KMnO_4 + 2H_2O \longrightarrow MnO_2 + 2KOH + 3[O]$$
$$KI + 3[O] \longrightarrow KIO_3$$
$$2KMnO_4 + 2H_2O + KI \longrightarrow 2MnO_2 + 2KOH + KIO_3$$

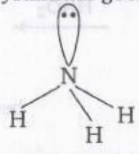
Hence, two moles of $KMnO_4$ are reduced by one mole of KI.

34. (a) IF_3 has bent-T geometry

(b) PCl_3 has pyramidal geometry



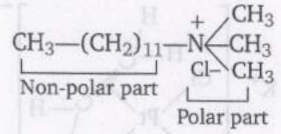
(c) NH_3 has pyramidal geometry



(d) BF_3 has trigonal planar geometry

($\mu = 0$)

35. Surfactants detergents form micelles in aqueous solution above to their CMC (critical micelle concentration). Dodecyl trimethyl ammonium chloride is an example of surfactant (cationic surfactant), so it shows a following phenomena.



36. After decay of radioactive element, the last element (stable) Pb is obtained which belongs to group 14 of periodic table.

37. For a spontaneous reaction, $\Delta G = -ve$.

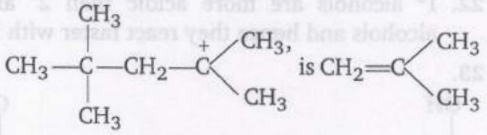
From Gibbs -Helmholtz equation

$$\Delta G = \Delta H - T\Delta S$$

$\Delta G = -$ ve, when $\Delta H = -$ve and $\Delta S = +$ve

For $\Delta H = -$ve, reaction should be exothermic and $\Delta S = +$ve for increase in disorder.

38. The monomer of polymer



because $CH_2=C(CH_3)(CH_3)$ shows cationic polymerisation.

39. On the basis of Fajan's rule, lower the size of cation higher will be its polarising power and higher will be covalent character.

$$\therefore \text{Polarising power} \propto \frac{1}{\text{size of cation}}$$

Covalent character $\propto$ Polarising power

So the correct order is NaCl < LiCl < $BeCl_2$

(The order of size of cation $Na^+ > Li^+ > Be^{2+}$)

40. In qualitative analysis of cations of second group, H_2S is passed in presence of HCl. Due to common ion effect, ionisation of H_2S decreases and less sulphide ions are obtained. These sulphide ions are sufficient for the precipitation of second group cations in form of their sulphides due to lower value of their solubility product (K_{sp}). Here, fourth group cations are not precipitated because they require for

exceeding their ionic product to their solubility products and higher sulphide ions concentration due to their higher K_{sp} which is not obtained here due to common ion effect.

41. The energy of second Bohr orbit of hydrogen atom (E_2) is -328 kJ mol^{-1} therefore

$$-328 = \frac{E}{2^2} \Rightarrow E = -328 \times 4 = -1312 \text{ kJ mol}^{-1}$$

$$\therefore \quad E_n = \frac{-1312}{n^2} \text{ kJ mol}^{-1}$$

If $n = 4$,

$$\therefore \quad E_4 = -\frac{1312}{4^2} \text{ kJ mol}^{-1} = -82 \text{ kJ mol}^{-1}$$

42. (i) $NO(g) + \frac{1}{2}O_2(g) \overset{K_1}{\rightleftharpoons} NO_2(g)$

So, $$K_1 = \frac{[NO_2]}{[NO][O_2]^{1/2}} \quad \ldots(i)$$

(ii) $2NO_2(g) \overset{K_2}{\rightleftharpoons} 2NO + O_2(g)$

So, $$K_2 = \frac{[NO]^2[O_2]}{[NO_2]^2} \quad \ldots(ii)$$

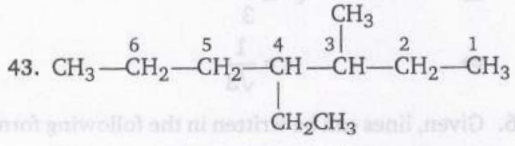
From Eq. (i)

$$K_1^2 = \frac{[NO_2]^2}{[NO]^2[O_2]}$$

or $$\frac{1}{K_1^2} = \frac{[NO]^2[O_2]}{[NO_2]^2} \quad \ldots(iii)$$

From Eqs. (ii) and (iii)

$$K_2 = \frac{1}{K_1^2}$$

43. 

Correct IUPAC name is 4-ethyl-3-methyl heptane because substituents are written in alphabatical order.

44. Moles of $H_2 = \frac{10}{2} = 5$ mol

Moles of $O_2 = \frac{64}{32} = 2$ mol

$$\underset{(5 \text{ mol})}{2H_2} + \underset{(2 \text{ mol})}{O_2} \longrightarrow \underset{(4 \text{ mol})}{2H_2O}$$

Here O_2 is limiting reagent. So, amount of product, H_2O obtained depends on the amount of O_2. According to equation,

1 mole O_2 on reaction with H_2 gives = 2 mole of H_2O

2 mole O_2 on reaction with H_2 will give

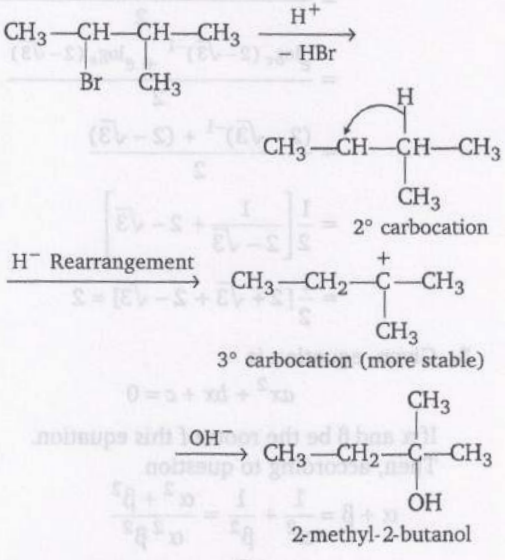
$$= \frac{2 \times 2}{1} = 4 \text{ mole of } H_2O$$

Note 1 mole of H_2 is in excess.

45. $Z > 1$ shows that the gas is less compressible than expected. This is due to dominance of strong repulsive forces among the molecules.

46. $\underset{\text{glucose}}{C_6H_{12}O_6} + [O] \xrightarrow[H_2O]{Br_2} \underset{\text{gluconic acid}}{CH_2OH(CHOH)_4COOH}$

47. Protein present in hair is keratin.

48. 

49. By using $pH = -\log \sqrt{K_a} \cdot C$

$$5 = -\log \sqrt{K_a \times 1} = -\frac{1}{2} \log K_a$$

$$\log K_a = -10$$

$$k_a = 10^{-10}$$

Hence, dissociation constant (K_a) $= 10^{-10}$.

50. $W_{rev.} = -2.303\, nRT \log \frac{V_2}{V_1}$

$$= -2.303 \times 1 \times 0.0821 \times (273 + 37) \log \frac{25}{15}$$

$$= -2.303 \times 0.0821 \times 310 \log \frac{5}{3}$$

$$= -13 \text{ J} \approx -12.87 \text{ J}$$

Mathematics

1. The bisectors of the angles between the lines in the new position are same as the bisectors of the angles between their old position. Therefore, required equation is

$$\frac{x^2 - y^2}{1 - (-1)} = \frac{xy}{-p}$$

$$\Rightarrow \quad -px^2 + py^2 = 2xy$$

$$\Rightarrow \quad px^2 + 2xy - py^2 = 0$$

2. Given, $z = i \log_e (2 - \sqrt{3})$

$$\because \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$= \frac{e^{i^2 \log_e (2-\sqrt{3})} + e^{-i^2 \log_e (2-\sqrt{3})}}{2}$$

$$= \frac{e^{\log_e (2-\sqrt{3})^{-1}} + e^{\log_e (2-\sqrt{3})}}{2}$$

$$= \frac{(2-\sqrt{3})^{-1} + (2-\sqrt{3})}{2}$$

$$= \frac{1}{2}\left[\frac{1}{2-\sqrt{3}} + 2 - \sqrt{3}\right]$$

$$= \frac{1}{2}[2 + \sqrt{3} + 2 - \sqrt{3}] = 2$$

3. Given, equation is

$$ax^2 + bx + c = 0$$

If α and β be the roots of this equation. Then, according to question

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2 \beta^2}$$

$$\Rightarrow \quad \frac{-b}{a} = \frac{\frac{b^2}{a^2} - 2\frac{c}{a}}{\frac{c^2}{a^2}}$$

$$\Rightarrow \quad \frac{-b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow \quad \frac{2a}{c} = \frac{b^2}{c^2} + \frac{b}{a} = \frac{ab^2 + bc^2}{ac^2}$$

$$\Rightarrow \quad 2a^2 c = ab^2 + bc^2$$

$$\Rightarrow \quad \frac{2a}{b} = \frac{b}{c} + \frac{c}{a}$$

$\Rightarrow \frac{c}{a}, \frac{a}{b}$ and $\frac{b}{c}$ are in AP.

$\Rightarrow \frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in HP.

4. rth term of the given series

$$r \cdot {}^r p_r = r \cdot r! = \{(r+1) - 1\} r!$$

$$= (r+1)! - r!$$

On putting r = 1, 2, 3,... n and adding, we get

$${}^1 p_1 + 2 \cdot {}^2 p_2 + 3 \cdot {}^3 p_3 + \ldots + n \cdot {}^n p_n$$

$$= (2! - 1!) + (3! - 2!) + (4! - 3!) + \ldots + \{(n+1)! - n!\}$$

$$= (n+1)! - 1!$$

$$= {}^{n+1} P_{n+1} - 1$$

5. Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Since, $y = x$ and $3y = -2x$ is a pair of conjugate diameters. Therefore,

$$m_1 m_2 = -\frac{b^2}{a^2}$$

$$\Rightarrow \quad (1)\left(-\frac{2}{3}\right) = -\frac{b^2}{a^2}$$

$$\Rightarrow \quad 2a^2 = 3b^2$$

$$\Rightarrow \quad 2a^2 = 3a^2 (1 - e^2)$$

$$\Rightarrow \quad 2 = 3(1 - e^2)$$

$$\Rightarrow \quad e^2 = \frac{1}{3}$$

$$\Rightarrow \quad e = \frac{1}{\sqrt{3}}$$

6. Given, lines can be written in the following form

$\mathbf{r} = (3\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}) + \mu_1 (3\mathbf{i} - \mathbf{j} + \mathbf{k})$

and $\mathbf{r} = (-3\mathbf{i} - 7\mathbf{j} + 6\mathbf{k}) + \mu_2 (-3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$

[If two given lines are

$\mathbf{r} = \mathbf{a}_1 + \mu_1 \mathbf{b}_1$

and $\mathbf{r} = \mathbf{a}_2 + \mu_2 \mathbf{b}_2$

then shortest distance is given by

$$\left| \frac{(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)}{|\mathbf{b}_1 \times \mathbf{b}_2|} \right|$$
]

$\therefore$ Shortest distance between two lines

$$= \left| \frac{\{(-3\mathbf{i} - 7\mathbf{j} + 6\mathbf{k}) - (3\mathbf{i} + 8\mathbf{j} + 3\mathbf{k})\} \cdot \{(3\mathbf{i} - \mathbf{j} + \mathbf{k}) \times (-3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})\}}{|(3\mathbf{i} - \mathbf{j} + \mathbf{k}) \times (-3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})|} \right|$$

$$= \left| \frac{(-6\mathbf{i} - 15\mathbf{j} + 3\mathbf{k}) \cdot (-6\mathbf{i} - 15\mathbf{j} + 3\mathbf{k})}{|-6\mathbf{i} - 15\mathbf{j} + 3\mathbf{k}|} \right|$$

$$\left[\because (3\mathbf{i} - \mathbf{j} + \mathbf{k}) \times (-3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} \right.$$

$$\left. = \mathbf{i}(-4-2) - \mathbf{j}(12+3) + \mathbf{k}(6-3) \right]$$

$$= -6\mathbf{i} - 15\mathbf{j} + 3\mathbf{k}$$

$$= \frac{(36 + 225 + 9)}{\sqrt{36 + 225 + 9}}$$

$$= \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$$

Hence, $\lambda = 3$

7. We have, $uov(-x)$

$= u[v(-x)]$

$= u[-v(x)]$ [$\because v$ is odd]

$= -u[v(x)]$ [$\because u$ is odd]

$= -uov(x)$

Thus, uov is an odd function.

8. $2\sin x$ is a periodic function of period 2π and $3\cos 2x$ is a periodic function of period π.

$\therefore$ $f(x) = 2\sin x + 3\cos 2x$

has period LCM of $\{\pi, 2\pi\} = 2\pi$

9. Clearly, the function $t = \frac{1}{x-2}$ is discontinuous at $x = 2$ and the function $y = \frac{1}{t^2 - t - 6}$ is discontinuous at the points, where

$$t^2 - t - 6 = 0$$

$\Rightarrow$ $(t+2)(t-3) = 0$

$\Rightarrow$ $t = -2, 3$

But $t = -2$

$\Rightarrow$ $\frac{1}{x-2} = -2$

$\Rightarrow$ $x - 2 = -\frac{1}{2}$

$\Rightarrow$ $x = \frac{3}{2}$

and $t = 3$

$\Rightarrow$ $\frac{1}{x-2} = 3$

$\Rightarrow$ $x - 2 = \frac{1}{3}$

$\Rightarrow$ $x = \frac{7}{3}$

Therefore, the values of x which make the function y discontinuous are $x = 2, \frac{3}{2}$ and $\frac{7}{3}$.

10. We have, $x^m y^n = a^{m+n}$

Taking logarithm on both sides, we get

$\Rightarrow m \log x + n \log y = (m+n) \log a$

Differentiating both sides w.r.t. x, we get

$\therefore$ $\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0$

$\Rightarrow$ $\frac{dx}{dy} = -\frac{nx}{my}$

$\therefore$ Subtangent $= \left| y\frac{dx}{dy} \right| = \frac{nx}{m} \propto x$

11. Since, $f(x) = 1$ in $0 < x < \frac{\pi}{2}$ (as $[\cos x] = 0$)

$\therefore f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$.

12. We have,

$\mathbf{F} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

Let $P(5, 4, -3)$ and $O(1, 2, 2)$ be given points.

Then, $\mathbf{OP} = 4\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$

Now, moment of $\mathbf{F}$ about O

$$= \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 2 & -5 \\ 1 & 1 & 1 \end{vmatrix}$$

$= (2+5)\mathbf{i} - (4+5)\mathbf{j} + (4-2)\mathbf{k}$

$= 7\mathbf{i} - 9\mathbf{j} + 2\mathbf{k}$

13. Given, planes are

$x - cy - bz = 0$ ...(i)

$cx - y + az = 0$ ...(ii)

$bx + ay - z = 0$ ...(iii)

Equation of plane passing through the line of intersection of planes (i) and (ii) may be taken as

$(x - cy - bz) + \lambda(cx - y + az) = 0$

$(1 + c\lambda)x + y(-c - \lambda) + z(-b + a\lambda) = 0$ ...(iv)

Now, planes (iii) and (iv) are same.

$$\therefore \quad \frac{1+c\lambda}{b} = \frac{-(c+\lambda)}{a} = \frac{-b+a\lambda}{-1}$$

By eliminating λ, we get

$$a^2 + b^2 + c^2 + 2abc = 1$$

14. The length of the edges are given by $a = 5-2 = 3$, $b = 9-3 = 6$ and $c = 7-5 = 2$, so length of the diagonal

$$= \sqrt{a^2+b^2+c^2}$$

$$= \sqrt{9+36+4} = 7 \text{ units}$$

15. Equation of hyperbola is

$$25x^2 - 16y^2 = 400$$

$$\Rightarrow \quad \frac{x^2}{16} - \frac{y^2}{25} = 1$$

Chord is bisected at (6, 2).

$$\therefore \quad \frac{6x}{16} - \frac{2y}{25} = \frac{6^2}{16} - \frac{2^2}{25}$$

$$\Rightarrow \quad 75x - 16y = 418$$

16. Let $I = \int \frac{1}{x^6 + x^4}\,dx = \int \frac{1}{x^4(x^2+1)}\,dx$

On putting $x = \tan\theta$ and $dx = \sec^2\theta\, d\theta$, we get

$$I = \int \frac{1}{\tan^4\theta(1+\tan^2\theta)} \sec^2\theta\, d\theta$$

$$= \int \cot^4\theta\, d\theta$$

$$= \int \cot^2\theta\,(\operatorname{cosec}^2\theta - 1)\, d\theta$$

$$= \int \cot^2\theta \operatorname{cosec}^2\theta\, d\theta - \int \cot^2\theta\, d\theta$$

$$= -\int \cot^2\theta\, d(\cot\theta) - \int (\operatorname{cosec}^2\theta - 1)\, d\theta$$

$$= -\frac{\cot^3\theta}{3} + \cot\theta + \theta + C$$

$$= -\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}x + C$$

17. $\frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + \ldots$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \ldots$$

$$= 1 - 1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots$$

$$= 1 - \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \ldots\right)$$

$$= 1 - \log 2 = \log e - \log 2$$

$$= \log\frac{e}{2}$$

18. $\tan^4 x - 2\sec^2 x + a^2 = 0$

$$\Rightarrow \quad \tan^4 x - 2(1+\tan^2 x) + a^2 = 0$$

$$\Rightarrow \quad \tan^4 x - 2\tan^2 x + 1 = 3 - a^2$$

$$\Rightarrow \quad (\tan^2 x - 1)^2 = 3 - a^2$$

$$\Rightarrow \quad 3 - a^2 \geq 0$$

$$\Rightarrow \quad a^2 \leq 3$$

$$\Rightarrow \quad |a| \leq \sqrt{3}$$

19. $\sin\theta + \cos\theta = \sqrt{2}\cos(90^\circ - \theta)$

$$\sin\theta + \cos\theta = \sqrt{2}\sin\theta$$

$$\Rightarrow \quad \cos\theta = (\sqrt{2}-1)\sin\theta$$

$$\Rightarrow \quad \cot\theta = \sqrt{2} - 1$$

20. $2 - \cos^2\theta = 3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$, we get

$$2\sec^2\theta - 1 = 3\tan\theta$$

$$2(1+\tan^2\theta) - 1 = 3\tan\theta$$

$$\Rightarrow \quad 2 + 2\tan^2\theta - 1 = 3\tan\theta$$

$$\Rightarrow \quad 2\tan^2\theta - 3\tan\theta + 1 = 0$$

$$\Rightarrow \quad 2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$$

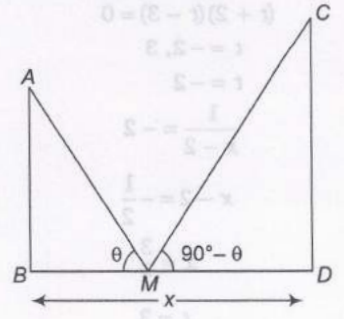
$$\Rightarrow \quad 2\tan\theta(\tan\theta - 1) - (\tan\theta - 1) = 0$$

$$\Rightarrow \quad (2\tan\theta - 1)(\tan\theta - 1) = 0$$

$$\Rightarrow \quad \tan\theta = \frac{1}{2} \text{ and } 1$$

$$\Rightarrow \quad \cot\theta = 2 \text{ and } 1$$

21. Let the height of the shorter and the longer pole be h and $2h$, respectively.



In ΔABM,

$$\tan\theta = \frac{AB}{BM} = \frac{h}{x/2} = \frac{2h}{x} \quad ...(i)$$

In ΔCDM,

$$\tan(90^\circ - \theta) = \frac{CD}{MD}$$

$$= \frac{2h}{x/2} = \frac{4h}{x}$$

$$\Rightarrow \quad \cot\theta = \frac{4h}{x} \quad ...(ii)$$

On multiplying both the equations, we get

$$1 = \frac{2h}{x} \cdot \frac{4h}{x}$$

$$\Rightarrow \quad \frac{x^2}{8} = h^2$$

$$\Rightarrow \quad h = \frac{x}{2\sqrt{2}}$$

22. We have, $(1-x)^{-2} = 1 + 2x + 3x^2 + \ldots$

$$+ (n+1)x^n + (n+2)x^{n+1} + \ldots$$

$\therefore$ Coefficient of x^{n+1} is $n+2$.

23. Let $\phi(x) = \int_0^x f(t)\,dt$,

$$\phi(-x) = \int_0^{-x} f(t)\,dt = -\int_0^x f(-z)\,dz$$

(putting $t = -z \Rightarrow dt = -dz$)

$$= -\int_0^x f(z)\,dz \qquad (\because f \text{ is an even function})$$

$$= -\int_0^x f(t)\,dt = \phi(x)$$

$\therefore \phi(x)$ is an odd function.

24. Given, equation can be written as

$$xy\,dy + y^2 dx - dy = 0$$

On multiplying by $\frac{1}{y}$, we get

$$x\,dy + y\,dx - \frac{1}{y}dy = 0$$

$$\Rightarrow \quad d(xy) - \frac{1}{y}dy = 0$$

On integrating, we get

$$\Rightarrow \quad \int d(xy) - \int \frac{dy}{y} = 0$$

$$\Rightarrow \quad xy - \log y = C$$

25. $\left(\frac{d^4y}{dx^4}\right)^{3/5} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 5 = 0$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^3 = \left(5\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 5\right)^5$$

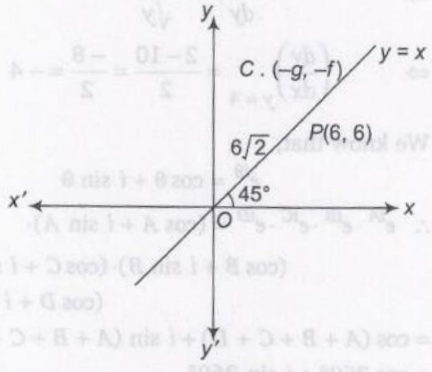
Which is the differential equation of order 4 and degree 3.

(because order is equal to highest order derivative and degree is equal to highest power of highest order derivative).

26. The equation of the line $y = x$ in parametric form is

$$\frac{x}{\cos\pi/4} = \frac{y}{\sin\pi/4}$$

$\because \quad OP = 6\sqrt{2}$



$\therefore$ Coordinates of P are given by

$$\frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2}$$

$$\Rightarrow \quad x = y = 6$$

$\therefore$ Coordinates of P are (6, 6).

The equation of circle touching $y = x$ at $P(6, 6)$ is

$$(x-6)^2 + (y-6)^2 + \lambda(x-y) = 0$$

$$\Rightarrow x^2 + y^2 + x(\lambda - 12) - y(\lambda + 12) + 72 = 0$$

Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$

$$\lambda - 12 = 2g, -(\lambda + 12) = 2f \text{ and } c = 72$$

Hence, required value of c is 72.

27. If the middle point of a chord is (α, β), then

$$\alpha = \frac{2t^2 + 0}{2}, \beta = \frac{4t + 0}{2}$$

On eliminating t, we get

$$\alpha = \left(\frac{\beta}{2}\right)^2$$

$\therefore$ Locus is $x = \frac{y^2}{4}$

$\Rightarrow$ $y^2 = 4x$

This is the equation of parabola with latusrectum 4.

28. $\sqrt{x} + \sqrt{y} = 10$

$$\frac{1}{2\sqrt{x}}\frac{dx}{dy} + \frac{1}{2\sqrt{y}} = 0$$

$$\Rightarrow \quad \frac{dx}{dy} = -\frac{\sqrt{x}}{\sqrt{y}} = -\frac{10-\sqrt{y}}{\sqrt{y}}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{\sqrt{y}-10}{\sqrt{y}}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{y=4} = \frac{2-10}{2} = \frac{-8}{2} = -4$$

29. We know that,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\therefore \quad e^{iA}\cdot e^{iB}\cdot e^{iC}\cdot e^{iD} = (\cos A + i\sin A)\cdot (\cos B + i\sin B)\cdot (\cos C + i\sin C)\cdot (\cos D + i\sin D)$$

$$= \cos(A+B+C+D) + i\sin(A+B+C+D)$$

$$= \cos 360^\circ + i\sin 360^\circ$$

$$= 1$$

30. Let $z = x + iy$

Then, $\bar{z} = x - iy$

Given, equation is $z\bar{z} + z + \bar{z} + 10 = 0$

$\Rightarrow (x+iy)(x-iy) + (x+iy) + (x-iy) + 10 = 0$

$\Rightarrow x^2 + y^2 + 2x + 10 = 0$

It represents the equation of a circle.

31. Let the edges of the block are $\frac{a}{r}$, a and ar.

Then, $\frac{a}{r}\cdot a\cdot ar = 216$

$\Rightarrow$ $a^3 = 216$

$\Rightarrow$ $a = 6$

and $2\left[\frac{a}{r}\cdot a + a\cdot ar + ar\cdot\frac{a}{r}\right] = 252$

$$\Rightarrow \quad 2a^2\left[\frac{1}{r} + r + 1\right] = 252$$

$$\Rightarrow \quad \frac{1}{r} + r + 1 = \frac{252}{2\times 6\times 6} = \frac{7}{2}$$

$$r + \frac{1}{r} = \frac{5}{2}$$

$$\Rightarrow \quad r = 2$$

Thus, shortest edge $= \frac{a}{r} = \frac{6}{2} = 3$ cm

32. $\log(p+r) + \log(p+r-2q) = 2\log(p-r)$

$\Rightarrow \log[(p+r)(p+r-2q)] = \log(p-r)^2$

$\Rightarrow p^2 + pr - 2pq + pr + r^2 - 2qr = p^2 + r^2 - 2pr$

$\Rightarrow 4pr - 2pq - 2qr = 0$

$\Rightarrow 2pr = pq + qr$

$\Rightarrow 2pr = q(p+r)$

$\Rightarrow q = \frac{2pr}{p+r}$

$\Rightarrow p$, q and r are in HP.

33. Since, all the three terms of the equation are positive. So, it does not have any real root.

Hence, required sum does not exist.

34. We have, $f(x) = \frac{1+x}{1-x}$

$$\therefore \quad f(A) = \frac{I+A}{I-A}$$

$$= (I+A)(I-A)^{-1}$$

$$= \begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}0 & -2\\ -2 & 0\end{bmatrix}^{-1}$$

$$\Rightarrow \quad f(A) = \begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}0 & -1/2\\ -1/2 & 0\end{bmatrix}$$

$$= \begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}$$

35. Since, M is skew-symmetric matrix.

$\therefore \quad M' = -M$

Also, $M^2 + I = 0$

$\Rightarrow$ $M^2 = -I + 0 = -I$

$\Rightarrow$ $M\cdot M = -I$

$\Rightarrow M\,M\,M' = -IM' = I(-M') = IM = M$

$\Rightarrow M^{-1}MMM' = M^{-1}M$

$\Rightarrow IMM' = I$

$\Rightarrow MM' = I$

36. We know that the relationship

$$E(XY) = E(X) \cdot E(Y)$$

holds, when X and Y are independent variables.

37. Given, $\sin 2x + \cos 4x = 2$

$\Rightarrow \sin 2x = 1, \cos 4x = 1$

$\Rightarrow 1 - 2\sin^2 2x = 1$

$\Rightarrow 1 - 2 = 1$

which is not possible.

Thus, there is no solution of the given equation.

38. $\dfrac{\sin^2 A + \sin A + 1}{\sin A}$

$$= \sin A + \frac{1}{\sin A} + 1$$

$$= \left(\sqrt{\sin A} - \frac{1}{\sqrt{\sin A}}\right)^2 + 3$$

$$\geq 3$$

39. We have,

$$f(x) = (x-2)^{2/3}(2x+1)$$

$$\Rightarrow f'(x) = \frac{2}{3}(x-2)^{-1/3}(2x+1) + (x-2)^{2/3} \cdot 2$$

$$= \frac{2(2x+1)}{3(x-2)^{1/3}} + 2(x-2)^{2/3}$$

$$= \frac{2(2x+1) + 6(x-2)}{3(x-2)^{1/3}}$$

$$= \frac{4x + 2 + 6x - 12}{3(x-2)^{1/3}}$$

$$= \frac{10(x-1)}{3(x-2)^{1/3}}$$

For critical points,

$$f'(x) = 0$$

$\Rightarrow x = 1$

and $f'(x)$ is not defined at $x = 2$.

40. $\mathbf{a} + t\mathbf{b} \perp \mathbf{c}$

$\Rightarrow (\mathbf{a} + t\mathbf{b}) \cdot \mathbf{c} = 0$

$\Rightarrow \mathbf{a} \cdot \mathbf{c} + t\mathbf{b} \cdot \mathbf{c} = 0$

$$\Rightarrow t = -\frac{\mathbf{a} \cdot \mathbf{c}}{\mathbf{b} \cdot \mathbf{c}}$$

$$\Rightarrow t = -\frac{(2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j})}{(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j})}$$

$$\Rightarrow t = -\frac{6 + 2 + 0}{-3 + 2 + 0} = 8$$

41. The angle between the given lines is given by

$$\cos\theta = \frac{(2)(1) + (5)(2) + (4)(-3)}{\sqrt{4 + 25 + 16} \cdot \sqrt{1 + 4 + 9}}$$

$$= 0$$

$\Rightarrow \theta = 90°$

42. The equation of plane passing through the intersection line of the planes $2x - y = 0$ and $3z - y = 0$ is

$$(2x - y) + \lambda(3z - y) = 0$$

$\Rightarrow 2x - (1 + \lambda)y + 3\lambda z = 0$ ...(i)

According to the question, plane (i) will be perpendicular to

$$4x + 5y - 3z = 8 \quad \text{...(ii)}$$

$\therefore 2 \cdot 4 - (1 + \lambda)5 + 3\lambda(-3) = 0$

$\Rightarrow 8 - 5 - 5\lambda - 9\lambda = 0$

$\Rightarrow 14\lambda = 3 \Rightarrow \lambda = \dfrac{3}{14}$

Thus, required plane is

$$(2x - y) + \frac{3}{14}(3z - y) = 0$$

$\Rightarrow 28x - 17y + 9z = 0$

43. Given, equation is

$$xy - \log y = 1$$

On differentiating it w.r.t. x, we get

$$xy' + y \cdot 1 - \frac{1}{y} \cdot y' = 0$$

$\Rightarrow xyy' + y^2 - y' = 0$

Again, differentiating w.r.t. x, we get

$$(xy - 1)y'' + y'(xy' + y \cdot 1) + 2yy' = 0$$

$\Rightarrow x(yy'' + y'^2) - y'' + 3yy' = 0$

Comparing it with the given equation, we get

$$k = 3$$

44. Let $y = f(x) = [1 - (x-3)^4]^{1/7}$

$\Rightarrow y^7 = 1 - (x-3)^4$

$\Rightarrow (x-3)^4 = 1 - y^7$

$$\Rightarrow \quad x = 3 + (1 - y^7)^{1/4}$$

$$\Rightarrow \quad f^{-1}(y) = 3 + (1 - y^7)^{1/4}$$

$$\Rightarrow \quad f^{-1}(x) = 3 + (1 - x^7)^{1/4}$$

45. $\lim\limits_{n \to \infty} \dfrac{a^n + b^n}{a^n + d^n}$

$$= \lim_{n \to \infty} \frac{1 + \left(\frac{b}{a}\right)^n}{1 + \left(\frac{d}{a}\right)^n}$$

$$= \frac{1 + 0}{1 + 0}$$

$$\left[\because \left(\frac{b}{a}\right)^n \to 0 \text{ and } \left(\frac{d}{a}\right)^n \to 0 \text{ as } n \to \infty\right]$$

$$= 1$$

46. We have, $y = x^2 + ax + b$

$$\Rightarrow \quad \frac{dy}{dx} = 2x + a$$

Given, $\dfrac{dy}{dx} = 0$ at $x = 3$

$$\therefore \quad 2x + a = 0$$

$$\Rightarrow \quad a = -2x = -6$$

Also, at $x = 3$ and $y = 5$

$$\therefore \quad 9 - 6(3) + b = 5$$

$$b = 5 - 9 + 18 = 14$$

Hence, $a + b = -6 + 14 = 8$

47. We have,

$$(1 - 9x + 20x^2)^{-1} = [(1 - 5x)(1 - 4x)]^{-1}$$

$$= \frac{1}{(1 - 5x)(1 - 4x)} = \frac{5}{1 - 5x} - \frac{4}{1 - 4x}$$

$$= 5(1 - 5x)^{-1} - 4(1 - 4x)^{-1}$$

$$= 5[1 + 5x + (5x)^2 + \ldots + (5x)^n + \ldots]$$

$$- 4[1 + 4x + (4x)^2 + \ldots + (4x)^n + \ldots]$$

$\Rightarrow$ The coefficient of x^n is $5^{n+1} - 4^{n+1}$

48. Given function can be rewritten as

$$f(x) = \begin{cases} 6 - 3x, & x < 1 \\ 4 - x, & 1 \le x < 2 \\ x, & 2 \le x < 3 \\ 3x - 6, & x \ge 3 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -3, & x < 1 \\ -1, & 1 < x < 2 \\ 1, & 2 < x < 3 \\ 3, & x > 3 \end{cases}$$

Thus, $f(x)$ is increasing for $x < 2$ and increasing for $x > 2$.

Hence, $f(x)$ is minimum for $x = 2$

and $\quad [f(x)]_{\min} = f(2) = 2$

49. $y = 2 \cos x \cos 3x = \cos 4x + \cos 2x$

$$\frac{dy}{dx} = -4 \sin 4x + (-2) \sin 2x$$

$$\frac{d^2y}{dx^2} = (-4)^2 \cos 4x + (-2)^2 \cos 2x$$

Similarly,

$$\frac{d^{20}y}{dx^{20}} = (-4)^{20} \cos 4x + (-2)^{20} \cos 2x$$

$$= 4^{20} \cos 4x + 2^{20} \cos 2x$$

$$= 2^{20}[2^{20} \cos 4x + \cos 2x]$$

50. $(p \Rightarrow q) \wedge (q \Rightarrow p)$ means $p \Leftrightarrow q$