Chapter 8: Comparing Quantities

Introduction of Comparing Quantities

What is Comparison?

It is sometimes necessary and also nice to know if one number is the same as, or smaller than,

or bigger than, another number. We will use some signs to compare numbers.

When two values are equal, we use the equal to (=) symbol. For example, 7 + 2 = 9 (7 plus 2 equals to 9) When one value is greater than another, we use the equal to (>) symbol.

For example, 25 > 2 (25 is greater than 2)

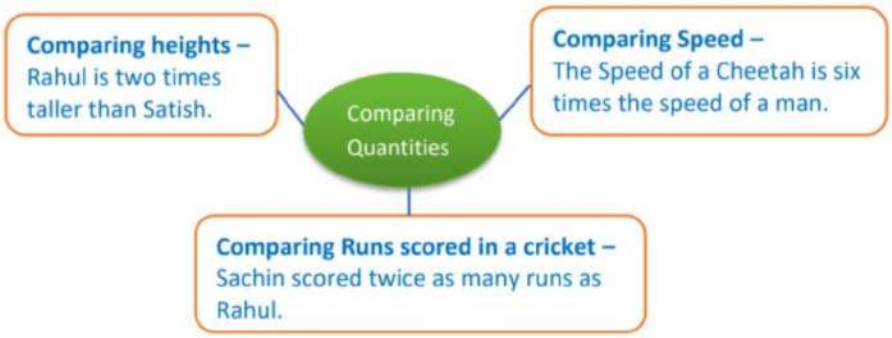
When one value is smaller than another, we will use a less than (<) symbol.

For example, 14 < 35 (14 is less than 35)

When we compare the quantities using concepts of ratio, proportion, percentage, etc. that is called comparing quantities

A figure by which one person or thing is compared to another, or the two are considered with regard to some property or quantity, which is common to both of them.

When we compare the two quantities, the units must be the same.



What is Ratio?

Relationship between two quantities of the same units expressed as “a to b”

or

“a : b”, sometimes expressed as the quotient of the two quantities that indicates how many times the first number contains the second.

Ratio is represented in the form of a fraction of two quantities. The units of quantities must be the same.

A unit may be Objects, Persons, Students, Quality, Quantity, etc.

For example,

Mohan has 15 pens and 29 pencils, then the ratio of pens to pencils would be 15:29 and the ratio of pencils to pens would be 29:15.

If 2A = 3B and 4B = 5C, then A : C is what?

$$\Rightarrow \frac{A}{B} = \frac{3}{2} \quad \text{and} \quad \frac{B}{C} = \frac{5}{4}$$

$$\Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right)$$

$$\Rightarrow \frac{A}{C} = \left(\frac{3}{2} \text{ X } \frac{5}{4}\right)$$

$$\Rightarrow \frac{A}{C} = \frac{15}{8}$$

$\Rightarrow$ A : C = 15 : 8

What is Percentage?
A fraction can be expressed as a percentage by multiplying it by 100. A change is a quantity from a to b is a change of 100(b - a)/a percent. The term percent means per hundred or for every hundred.

We denote x percent by x%.

$$x\% = \frac{x}{100}$$

Finding the percent of a given number, we proceed as follows:

Step 1: Obtain the number, let's say a.

Step 2: Obtain the required percent, let's say P%.

Step 3: Multiply a by P and divide by 100 to obtain the required P% of a i.e.,

$$P\% \text{ of } a = \frac{p}{100} \times a$$

For example,

a) Find 30% of ₹ 150.

$$30\% \text{ of } ₹ 150 = \frac{30}{100} \times 150 = ₹ 45$$

Expressing Percent as a fraction

The term percent means per hundred or for every hundred.

Express each of the following as a fraction:

a) 36%

$$= \frac{36}{100}$$

$$\frac{18}{50}$$

$$\frac{9}{25}$$

b) 120%

$$= \frac{120}{100}$$

$$= \frac{60}{50}$$

$$= \frac{6}{5}$$

$$= 1\frac{1}{5}$$

Expressing Percent as a fraction

Any fraction can be expressed into a percentage by multiplying it with 100

Express each of the following fractions as a percentage –

a) $\frac{7}{25}$ b) $\frac{5}{8}$

a) $\frac{7}{25}$

$= (\frac{7}{25} \text{ X } 100)\%$

$= 28\%$

b) $\frac{5}{8}$

$= (\frac{5}{8} \text{ X} 100)\ \%$

$= \frac{500}{8}\ \%$

$= \frac{125}{2}\ \%$

$= 62\frac{1}{2}\%$

Increase or Decrease Percent

We calculate the increase or decrease percent as follows:

$$\text{Increase \%} = \frac{Increase}{Original\ Value} \text{ X } 100\%$$

$$= \frac{Decrease}{Original\ Value} \text{ X } 100\%$$

For example,

The price of a watch which was ₹ 3000 last year increases by 20% this year. What is the price of the watch now?

New price of the watch = Old price + Increase Old price = ₹ 3000

$$\text{Increase \%} = \frac{Increase\ in\ price}{Original\ price} \times 100\%$$

$$20\ \% = \frac{Increase\ in\ price}{3000} \times 100\%$$

$$\text{Increase in price} = \frac{3000X20}{100} = 600$$

Increased price = ₹ 600

Increased price of the scooter = 3000 + 600 = ₹ 3,600

Profit and Loss

What is Cost Price?

The amount paid to purchase an article or the price at which an article is made is known as its cost price. The cost price is abbreviated as C.P.

The overhead expenses like taxes, labour charge, etc. are included in the cost price. If overhead expenses are not included in the cost price, then

Effective Cost Price = Payment made while purchasing the goods + Overhead expenses.

What is the Selling Price?

The price at which an article is sold is known as its selling price.

The selling price is abbreviated as S.P.

What is a Profit?

If the selling price (S.P.) of an article is greater than the cost price (C.P.),

then the difference between the selling price and cost price is called

profit.

Profit = S.P. – C.P.

Profit Percentage

The profit percent is the profit that would be obtained for a C.P. of ₹ 100 i.e.,

Profit percent = $\left(\frac{Profit}{C.P.} \text{ X } 100\right)$ %

Formulas for calculating S.P. and C.P., when profit % is given.

S.P. = $\left(\frac{100 + Profit\%}{100}\right)$ X C.P

C.P = $\left(\frac{100XS.P}{100 + Profit\%}\right)$

What is Loss?

If the selling price (S.P.) of an article is less than the cost price (C.P.) then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.

Loss = C.P. – S.P.

Loss Percentage

The loss percent is the loss that would be made for a C.P. of ₹ 100.

That is, loss percent = $\left(\frac{loss}{C.P} \text{ X } 100\right)\%$

Formulas for calculating S.P. and C.P. when loss % is given:

S.P= $\left(\frac{100 - loss\%}{100}\right)$ X C.P

C.P. = $\left(\frac{100XS.P}{100 - loss\%}\right)$

To find SP when CP and profit% or loss% are given:

$SP = CP\left(1 + \frac{\text{Profit \%}}{100}\right)$ $SP = CP\left(1 - \frac{\text{Loss \%}}{100}\right)$

To find CP when SP and profit% or loss% are given:

$CP = \frac{100}{(100+\text{profit \%})} \times SP$ $CP = \frac{100}{(100-\text{Loss \%})} \times SP$

For example,

Ranu bought a shirt for ₹ 200 and sold it for ₹ 180. Find loss and loss percent.

We have, C.P. of watch = ₹ 200

S.P. of watch = ₹ 180

Since S.P. < C.P. so, there is loss given by

Loss = C.P. – S.P. = 200 – 180 = ₹ 20

Now, Loss% = $\left(\frac{loss}{C.P} \text{ X } 100\right)\%$

= $\left(\frac{20}{200} \text{ X } 100\right)\% = 10\%$

Hence, Loss = ₹ 20 and Loss% = 10%

Ranvi bought a phone for ₹ 7000 and spent ₹ 400 on its repairs. Then, he sold it for ₹ 7622 Find his profit or loss percent.

CP of phone = ₹7000.

Total Cost Price = (7000+400) = ₹7400

SP of phone = ₹7622

As SP > CP, there is Profit given by

Profit = SP - CP

Profit = 7622 – 7400 = ₹222

Profit% = $\left(\frac{(Profit}{C.P} \text{ X } 100\right)\%$

Profit% = $\left(\frac{222}{7400} \text{ X } 100\right) \% = 3\%$

Discount

What is Marked Price or List Price?

The price written on the article or tagged on the article is called Marked Price (MP) or List Price (LP).

What is Selling Price?

The difference between the Marked Price and Discount is the Selling Price of the Article.

What is Discount?

The deduction made on the Marked Price is called the Discount. Discount is generally given as a certain percent of the marked price. It is always calculated on the marked price or list price.

Discount = M.P. – S.P.

Rate of Discount = Discount % = $(\frac{Discount}{M.P.} \times 100)\%$

S.P. = M.P. - Discount

$\Rightarrow$ S.P. = M.P - $\frac{Discount\% \times M.P}{100}$

$\Rightarrow$ S.P. = M.P $\times(1 - \frac{Discount\%}{100})$

$\Rightarrow$ S.P. = M.P $\times(\frac{100 - Discount\%}{100})$

$\Rightarrow$ M.P = $(\frac{100 \times S.P}{100 - Discount\%})$

For example,

The marked Price of a fan is ₹ 1,100 and the shopkeeper allows a discount of 6% on it. Find the selling price of the fan.

Marked Price (MP) = ₹ 1,100

Discount = 6% of MP = 6% of ₹ 1,100

Marked Price (MP) = ₹1100

Discount = 6% of MP = 6% of ₹1100

Discount = $\frac{6}{100} \times 1100$

= ₹66

Selling Price = Marked Price - Discount

= 1100- 66

= ₹1034

Taxes

What is Tax?

Tax is the money collected by the government from the citizens of the country in order to provide them the services.

There are various types of taxes:

a) Income Tax
b) Sales Tax
c) Value-added Tax etc.

a) Income Tax
An income tax is a tax that governments impose on income generated by businesses and individuals within their jurisdiction.

b) Sales Tax

It is collected by the government on the sale of different commodities.

c) Value Added Tax

It is charged by the shopkeeper from the customer and given to the government. Therefore, this is always on the selling price of an article and is added to the value of the bill. These

days, the prices include

Value Added Tax (VAT).

GST

The Goods and Service Tax Act was passed in the Parliament on 29th March 2017. The Act came into effect on 1st July 2017; Goods & Services Tax Law in India is a comprehensive, multi-stage, destination-based tax that is levied on every value addition.

i) The Rate of this tax is depending upon the item sold

ii) When C.P. is given exclusive of tax.

Tax to be paid = $\frac{Tax\%}{100}$ X 100

So, bill amount = C.P. + Sales Tax.

iii)When C.P. is given inclusive of tax or VAT.

Original Price = $\frac{100}{100+Tax\%}$ X C.P

Ram bought furniture for ₹ 5200 including VAT at 4%. Find the original price of the furniture.

Let the original price of furniture be ₹x

VAT = 4% of x = $\frac{4}{100} \times x = \frac{x}{25}$

Price including VAT = $x + \frac{x}{25} = \frac{26x}{25}$

$\frac{26x}{25} = 5200$

$x = \frac{25}{26} \times 5200$

x = ₹5000

The cost of a pair of socks at a shop was ₹ 400. The sales tax charged was 5%. Find the bill amount.

On ₹ 100, the tax paid was ₹ 5.

On ₹ 400, the tax paid would be = $\frac{5}{100}$ X 400 = ₹ 20.

Bill amount = Cost of item + Sales Tax = ₹ 400 + ₹ 20= ₹ 420

Simple Interest

Simple Interest

Principal or Sum

The money borrowed from an agency or an individual for a certain period of time is called the principal or sum.
Amount
The principal together with the interest is called the amount.

Amount (A) = Principal (P) + Interest (I)

Rate of interest

The interest on 100 per annum is called the rate of interest per annum.

Simple Interest

If the principal remains the same for the entire loan period, then the interest paid is called simple interest.

P = Principal
T = Time Period
R = Rate of Interest

Simple Interest = $\frac{P \times T \times R}{100}$

Amount (A) =Principal (P) + Simple Interest (SI)

For example,

Find the simple interest on ₹2500 at 15% per annum for 2 years.

Principal (P) = ₹2500

Rate (R) = 15%

Time period (T) = 2 years

Simple Interest (SI)= $\frac{P \times T \times R}{100} = \frac{2500 \times 2 \times 15}{100} = 750$

Simple Interest = 750

A certain sum has been borrowed at 16% per annum under simple interest. If the sum amounts to ₹12000 in 1 year 3 months, then find the sum borrowed.

Amount (A) = ₹12000

Rate of Interest (R) = 16%

Time Period(T) = 1 year 3 months = $1\frac{3}{12}$ years $= 1\frac{1}{4}$ years = $\frac{5}{4}$ years

Let the sum borrowed be P

SI = $\frac{P \times T \times R}{100} = \frac{P \times 5 \times 16}{100 \times 4} = \frac{P}{5}$

A = SI + P

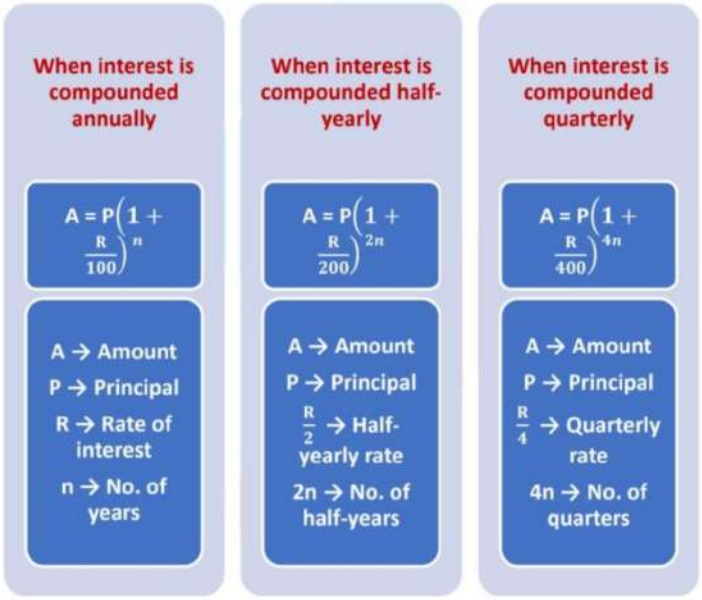
12000 = $\frac{P}{5} + P$

$$\frac{6P}{5} = 12000$$

$$P = \frac{5 \times 12000}{6} = ₹10000$$

Compound Interest

Compound Interest
Compound Interest is the interest calculated on the previous year's amount (A = P + I)



Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1 + \frac{R}{100}\right)^{n}$$

$$C.I. = A - P = P\left\{\left(1 + \frac{R}{100}\right)^{n} - 1\right\} \text{ respectively.}$$

Let P be the principal and the rate of interest be R% per annum. If the interest is compounded k-times in a year, then the amount A and the compound interest C.I. at the end of n years are

given by

$$A = P\left(1 + \frac{R}{100}\right)^{nk}$$

$$C.I. = A - P = P\left\{\left(1 + \frac{R}{100}\right)^{nk} - 1\right\} \text{ respectively.}$$

Let P be the principal and the rate of interest be R1%, for the first year, R2%, for the second year, R3%, for the third year and so on and in last Rn%, for the nth year. Then, the amount A and

the compound interest

C.I. at the end of n years are given by

$$A = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right) \ldots\ldots\ldots \left(1 + \frac{R_n}{100}\right) \text{ and, } C.I. = A - P$$

respectively.

Find the compound interest on ₹21000 for 3 years at 10% per annum, when compounded annually.

P = ₹ 21000

R = 10%

n = 3 years

$$A = P(1 + \frac{R}{100})^n$$

$$A = 21000(1 + \frac{10}{100})^3$$

$$A = 21000(\frac{110}{100})^3$$

$$A = 21000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$$

= ₹27951

CI = A-P

= ₹27951 - 21000

=₹6951

Find the compound interest on ₹25000 for $1\frac{1}{2}$ years at 16% per annum, when compounded half-yearly.

P = ₹25000

R = 16% per annum

n= $1\frac{1}{2}$ years = $\frac{3}{2}$ years

2n = $\frac{3}{2} \times 2 = 3$

A = $P(1+\frac{R}{200})^{2n}$

A = $25000(1+\frac{16}{200})^{3}$

A = $25000(\frac{216}{200})^{3}$

A = $25000 \times \frac{27}{25} \times \frac{27}{25} \times \frac{27}{25}$

= ₹157464

CI =A-P

= ₹157464-25000

= ₹132464