

Circumference of a Circle and Area

15.01 Introduction

In previous chapters, we have learnt about circle related definitions and properties. In this chapter we shall study about the circumference and area of the some circular figures.

A circle is a locus of a point which moves in a plane in such a manner that its distance from a fixed point in the same plane remains constant.

15.02 Circumference of the Circle

To find the circumference (approximately) mark a point C at the rim of a circular disc and put it in a plane such that the point C touches a point A in the plane. Now rotate the circular disc carefully.

That the point C may again touch the plane at the point B (See the fig. 15.01)

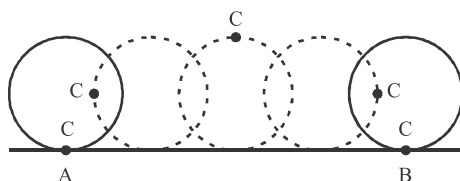


Fig. 15.01

Now measure the line segment AB. The length of the line segment AB is equal to the circumference of circular disc. So we can say that the distance covered by a wheel in one complete rotation is called its circumference. It is also known as the perimeter.

Circumference of a circle = $2 \pi r$ or $\pi \times d$

where r stands for radius and d stands for diameter of the circle

So, $d = 2r$

or
$$\frac{\text{Circumference}}{\text{diameter}} = \pi$$

Note : Words "the circle and the circumference" of the circle are completely different. A circle is a figure in the plane while circumference is a length measured. The ratio of the circumference of a circle with its diameter is a constant which is denoted by the Greek letter π (Pie)

The value of π is calculated up to 5,00,000 (half million) decimal places by computer. We will take the value of $\pi = \frac{22}{7}$. A more accurate value of π is $\frac{62832}{200000}$ approximately 3.1416 which was given by the great Indian astronomer Aryabhatta (499 AD). π is a irrational number. The value of $\pi = 3.1416$ is correct to four decimal places.

The circumferences of the circle $C(0, r)$ is $2 \pi r$ see (fig. 15.02).

$$\frac{\text{Circumference}}{\text{diameter}} = \pi$$

or Circumference = $\pi \times \text{diameter}$

If C represents the circumference and D represents the diameter, then

$$\begin{aligned} C &= \pi \times D = \pi \times 2r \quad \text{since diameter} = 2 \times \text{radius} \\ &= 2\pi r \end{aligned}$$

15.03. Area of a Circle

If a circle is drawn on a graph paper, we can obtain the area of the circle by counting the square surrounded by it. In this way, we get

$$\frac{\text{Area of the circle}}{(\text{radius})^2} = \pi$$

or $\frac{A}{r^2} = \pi$ where A is the area and r represents the radius of the circle.

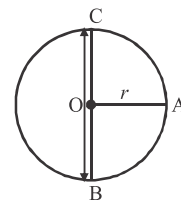


Fig. 15.02

or $A = \pi r^2$

15.04. Area between Two Concentric Circles

Circles having the same centre but with different radii are called concentric circles (fig. 15.03). If r_1 and r_2 are the radii of two concentric circles where ($r_1 > r_2$), then the area between two circles.

$$\begin{aligned} &= \pi r_1^2 - \pi r_2^2 \\ &= \pi (r_1^2 - r_2^2) \end{aligned}$$

Following results can be obtained from the above given facts :

- (i) The distance covered by a wheel in one complete rotations is equal to circumference of the wheel.
- (ii) Number of rotations made by a wheel in one minute

$$= \frac{\text{distance covered in per minute}}{\text{circumference}}$$

- (iii) Area of the circle is πr^2

Example 1. Find the radii of the circles given below, when

- (i) The circumference of circle is 132 cm.
- (ii) The circumference of circle is 176 cm.

Solution : (i) We have, circumference of circle = 132 cm.

or $2\pi r = 132$ (where r is the radius of circle)

or $2 \times \frac{22}{7} \times r = 132$

or $r = \frac{132 \times 7}{2 \times 22} = \frac{42}{2}$

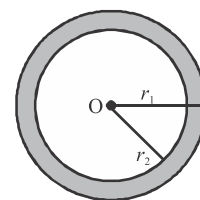


Fig. 15.03

or radius = 21 cm.

(ii) We have, circumference of the circle = 176 cm

or $2\pi r = 176$ (where, r = radius of the circle)

or $2 \times \frac{22}{7} \times r = 176$

$$r = \frac{176 \times 7}{2 \times 22} = \frac{56}{2} = 28$$

hence radius of the circle = 28 cm.

Example 2. Find the area of a circle whose radius is 7 cm.

Solution : Given, radius of the circle = 7 cm

\therefore Area of the circle = πr^2

$$= \frac{22}{7} \times 7 \times 7 = 154$$

Hence, area of the circle = 154 cm²

Example 3. A wheel of a bicycle makes 5000 rotations to complete a distance of 11 km. Find the diameter of the wheel.

Solution : We have, number of rotations = 5000

Distance covered = 11 km

Distance covered in one rotation by wheel

$$= \frac{\text{Total distance covered}}{\text{Number of rotations}}$$

$$= \frac{11}{5000} \text{ km}$$

$$= \frac{11}{5000} \times 1000 \times 100 \text{ cm}$$

$$= 220 \text{ cm}$$

Let radius of the wheel be r

Circumference = 220 cm

$$2\pi r = 220 \text{ cm}$$

$$2 \times \frac{22}{7} \times r = 220 \text{ cm}$$

$$r = \frac{220 \times 7}{2 \times 22} \text{ cm} = 35 \text{ cm}$$

diameter = $2r = 2 \times 35 = 70 \text{ cm}$

Exercise 15.1

1. Radius of a circle is 3.5 cm. Find its circumference and area.
2. The circumference of a circle is 44 m. Find the area of the circle.
3. The radius of a semi-circle shaped plot is 21 metre. Find the radius of the wheel.
4. A wheel covers a distance of 88 metres in 100 rotation. Find the radius of the wheel.
5. The area of a circular plates is 154 cm^2 . Find its circumference.
6. The circumference of a circle is equal to the perimeter of a square. If area of the square is 484 m^2 , then find area of the circle.
7. The cost of constructing a boundry wall of a circular field is ₹ 5280 at the rate of ₹ 24 per metre. Find the cost of ploughing this field at rate of ₹ 0.50 per metre².
8. The radius of a circular grass land is 35 m. There is a foot-path of width 7 m. around in it. Find the area of the foot path.
9. The area between two concentric circles will be
 (a) πR^2 (b) $\pi(R+r)(R-r)$ (c) $\pi(R^2-r^2)$ (d) None of these.
10. The radii of two concentric circles are 4 cm and 3 cm. The area between these two circles will be :
 (a) 22 cm^2 (b) 12 cm^2 (c) 32 cm^2 (d) 18 cm^2

15.05. Area of Sector and Segment of a Circle

The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a sector of the circle.

In the fig. 15.04., take a sector AOB in the circle (O, r) . Let $\angle AOB = \theta$ and $\theta < 180^\circ$. When angle at the centre is increased, then the length of the arc AB is also increased in the same ratio. When an arc subtends an angle of θ at the centre then the length of arc = length of the arc of semi circle = πr .

\therefore The length of the arc that subtends an angle of 180° at the centre is $= \pi r$.

\therefore The length of the arc that subtends an angle of $\theta = \frac{\pi r \theta}{180^\circ} = 2\pi r \times \frac{\theta}{360^\circ}$

$$\text{or} \quad L = 2\pi r \times \frac{\theta}{360^\circ} \quad \dots (i)$$

Similarly, when angle at the centre is 180° , then the area of its corresponding sector is $= \frac{\pi r^2}{2}$

When the angle at the centre is θ , then area of the corresponding sector

$$A = \frac{\pi r^2 \theta}{2 \times 180} = \frac{\pi r^2 \theta}{360}$$

$$\text{or} \quad A = \pi r^2 \times \frac{\theta}{360} \quad \dots (ii)$$

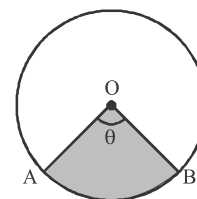


Fig. 15.04

From (i) and (ii) we get

$$A = \frac{1}{2} L \times r$$

Note : Here angle θ is taken in degree.

Some Important Results

- (i) The minute hand of a watch moves round 6° during one minute.
- (ii) The hour hand of a watch moves round $\left(\frac{1}{2}\right)^\circ$ during one minute.

Illustrative examples

Example 1. The length of an arc of a circle is 4 cm and its radius is 6 cm. Find the area of this sector of the circle.

Solution : We have, length of the arc of the circle = 4 cm.

and radius = 6 cm.

We know that the length of the sector $= \frac{\pi r \theta}{180^\circ}$

or
$$4 = \frac{\pi \times 6 \times \theta}{180}$$

or
$$\theta = \frac{4 \times 180^\circ}{\pi \times 6}$$

or
$$\theta = \frac{4 \times 30^\circ}{\pi} = \frac{120^\circ}{\pi}$$

$$\therefore \text{Area of the sector} = \frac{\pi r^2 \theta}{360^\circ} = \frac{\pi \times (6)^2 \times \left(\frac{120^\circ}{\pi}\right)}{360^\circ}$$

or
$$\frac{6 \times 6 \times 120^\circ}{360^\circ} = \frac{6 \times 6}{3} = \frac{36}{3} = 12$$

Hence, area of the given sector = 12 cm²

This problem can also be solved with the help of formula $A = \frac{1}{2} L \times r$

Example 2. The angle subtended at the centre by an arc of a circle is 50° . If the length of the arc is 5π cm, find the area of the minor sector made by this arc of the circle. (Take $\pi = 3.14$)

Solution : Given, the length of the arc $l = 5\pi$ cm

and angle of the sector $\theta = 50^\circ$

Length of the arc
$$L = \frac{\pi \theta}{180^\circ}$$

$$r = \frac{5\pi \times 180^\circ}{50\pi} \text{ cm} = 18 \text{ cm}$$

Area of the sector
$$= \frac{1}{2} l \times r$$

$$\begin{aligned}
 &= \frac{1}{2} \times 5\pi \times 18 \text{ cm}^2 = 45\pi \text{ cm}^2 \\
 &= 45 \times 3.14 \text{ cm}^2 = 140.3 \text{ cm}^2
 \end{aligned}$$

Example 3. Radius of a circle is 7 cm and the angle of the sector is 90° . Find the length of the arc and the area of the minor sector of the circle. (Take $\pi = \frac{22}{7}$).

Solution : Given, radius of the circle = 7 cm
and angle of sector = 90°

$$\text{Length of the arc of the sector } L = \frac{\pi r \theta}{180^\circ} = \frac{22}{7} \times \frac{7 \times 90^\circ}{180^\circ}$$

$$\text{or } l = 11 \text{ cm}$$

$$\begin{aligned}
 \text{Area of the sector} &= \frac{1}{2} \times L \times r \\
 &= \frac{1}{2} \times 11 \times 7 = \frac{77}{2} = 38.5 \text{ cm}^2
 \end{aligned}$$

Example 4. Given figure is an equilateral triangle, whose length of one side is 20 cm. From each of the vertex of the triangle three arcs are drawn with a radius 10 cm. Find out the area of the shaded portion. (Take $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Solution : Given, length of the side of equilateral triangle = 20 cm.

$$\begin{aligned}
 \therefore \text{ area of the triangle} &= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (20)^2 \text{ cm}^2 \\
 &= \frac{\sqrt{3}}{4} \times 20 \times 20 \text{ cm}^2 \\
 &= 1.73 \times 100 \text{ cm}^2 = 173 \text{ cm}^2
 \end{aligned}$$

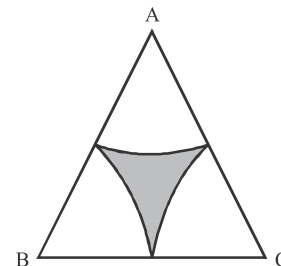


Fig.15.05

We know that, measured degree of each of the angle of an equilateral triangle = 60°
There for, area of three sectors will be equal.

$$\begin{aligned}
 \therefore \text{ Area of three sectors} &= 3 \times \frac{\pi r^2 \theta}{360^\circ} \\
 &= \frac{3 \times 3.14 \times 10^2 \times 60^\circ}{360^\circ} \text{ cm}^2 \\
 &= 157 \text{ cm}^2
 \end{aligned}$$

$$\text{Hence, area of the shaded portion} = (173 - 157) \text{ cm}^2 = 16 \text{ cm}^2$$

Example 5. The length of an hour hand of a clock is 6 cm. Find the area of the sector swept by this hour hand with in 90° minutes.

Solution : Given, length of hour hand = 6 cm

∴ The radius of the sector = 6 cm

The angle subtended by this hand in 12 hours = 360°

The angle subtended by this hand in 1 hour = $\frac{360}{12} = 30^\circ$

The angle subtended by this hand in 1 minutes = $\frac{30^\circ}{60} = \left(\frac{1}{2}\right)$

∴ The angle subtended by this hand in 90 minutes = $\frac{1}{2} \times 90 = 45^\circ$

The area swept by this hand = Area of the sector

$$\begin{aligned} &= \frac{\pi r^2 \theta}{360^\circ} \\ &= \frac{\frac{22}{7} \times 6^2 \times 45^\circ}{360^\circ} \text{ cm}^2 = \frac{22 \times 6 \times 6 \times 45^\circ}{7 \times 360^\circ} \text{ cm}^2 \\ &= \frac{22 \times 36}{7 \times 8} \text{ cm}^2 = \frac{792}{56} \text{ cm}^2 = 14.14 \text{ cm}^2 \end{aligned}$$

Hence, the area swept by the hour hand of the clock is 90 minutes = 14.14 cm^2

15.06. Area of the Segment of Circle

Every chord of a circle divides the circle into two parts and each of the part is called the segment. The greater part is called major segment and smaller part is known as minor segment of the circle.

The centre of the circle is O and radius is r (see fig 15.06). Let the chord PQ divides the circle into two segments. We are to find the area of the minor segment (PQR) of the circle.

Let $\angle POQ = \theta^\circ$

Then $\angle POM = \angle QOM = \frac{\theta}{2}$

Area of sector $OPRQ$ = area of sector PRQ + area of ΔPOQ

∴ area of segment PRQ = area of sector $OPRQ$ – area of ΔPOQ

$$\begin{aligned} &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} PQ \times OM \\ &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} \times 2 PM \times OM = \frac{\pi r^2 \theta}{360^\circ} - r \sin \frac{\theta}{2} \times r \cos \frac{\theta}{2} \end{aligned}$$

$$\Rightarrow \text{Area segment } PRQ = \frac{\pi r^2 \theta}{360^\circ} - \frac{r^2}{2} \sin \theta \quad \left[\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right]$$

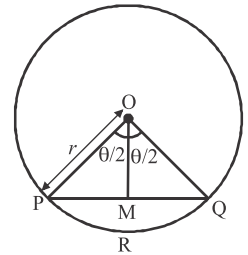


Fig. 15.06

Example 6. A chord of the circle with radius 5 cm subtends a right angled at the centre. Find the area of the minor segment by this chord.

Solution : Given, radius of the circle = 5 cm and the angle subtended at the centre by given chord = 90°

$$\begin{aligned}\text{Area of sector } OPRQ &= \frac{\pi r^2 \theta}{360^\circ} \\ &= \frac{\frac{22}{7} \times 5^2 \times 90^\circ}{360^\circ} \\ &= \frac{22 \times 25 \times 90^\circ}{7 \times 360^\circ} = \frac{550}{28} = 19.64\end{aligned}$$

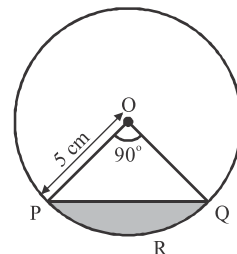


Fig. 15.07

$$\begin{aligned}\text{Area of } \Delta POQ &= \frac{1}{2} \times OP \times OQ \\ &= \frac{1}{2} \times 5 \times 5 = \frac{25}{2} = 12.50\end{aligned}$$

\therefore Area of minor segment of the circle = Area of sector OPRQ - area of ΔPOQ
 $= 19.64 - 12.50 = 7.14$

Hence, area of minor segment of the circle = 7.14 cm^2

This result also can be obtained with formula area of minor segment of circle

$$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta.$$

Example 7. A chord of a circle radius 14 cm subtends an angle of 30° at the centre. Find the areas of both, minor segments and major segment of the circle. (Take $\pi = \frac{22}{7}$).

Solution : Given, radius of the circle $r = 14 \text{ cm}$.

Angle subtended at the centre by chord $\theta = 30^\circ$

We know that area of minor segment of the circle

$$\begin{aligned}&= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta \\ &= \frac{\frac{22}{7} \times 14 \times 14 \times 30^\circ}{360^\circ} - \frac{1}{2} \times 14 \times 14 \sin 30^\circ \\ &= \frac{22 \times 2 \times 14}{12} - \frac{1}{2} \times 196 \times \frac{1}{2} \\ &= \frac{616}{12} - 49 = 51.33 - 49 \\ &= 2.33\end{aligned}$$

Area of the major segment of the circle

= area of the circle - area of the minor segment of the circle

$$= \pi r^2 - 2.33 = \frac{22}{7} \times (14)^2 - 2.33$$

$$= \frac{22 \times 14 \times 14}{7} - 2.33$$

$$= 22 \times 2 \times 14 - 2.33$$

$$= 616 - 2.33$$

$$= 613.67 \text{ cm}^2$$

Example 8. A chord in a circle radius 12 cm subtends an angle of 120° at the centre. Find the area of corresponding segment of the circle. (Take $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Solution : Here. $r = 12$ cm and $\theta = 120^\circ$

Area of the corresponding segment = area of the minor segment

= area of sector OAB – area of ΔOAB .

$$\text{Area sector } OAB = \frac{\theta}{360} \times \pi r^2 = \frac{120}{360} \times 3.14 \times 12 \times 12 \text{ cm}^2$$

$$= 3.14 \times 48 \text{ cm}^2$$

To find the area of ΔOAB , draw $OM \perp AB$

$AM = BM$ and $\angle AOM = \angle BOM = 60^\circ$

$$\text{Not } \frac{OM}{OA} = \cos 60^\circ \quad \therefore \quad OM = OA \cos 60^\circ = 12 \times \frac{1}{2} = 6$$

$$\frac{AM}{OA} = \sin 60^\circ$$

$$\therefore \quad AM = OA \sin 60^\circ = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$$

$$AB = 2 \times 6\sqrt{3} \text{ cm} = 12\sqrt{3} \text{ cm}$$

$$\text{Area of } \Delta OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3}$$

... (iii)

So from equation (i), (ii) and (iii), we get

$$\text{Area of corresponding segment} = (3.14 \times 48 - 36\sqrt{3}) \text{ cm}^2$$

$$= (3.14 \times 48 - 36 \times 1.73) \text{ cm}^2$$

$$= 12(12.56 - 3 \times 1.73) \text{ cm}^2$$

$$= 12(12.56 - 5.19) \text{ cm}^2 = 88.44 \text{ cm}^2$$

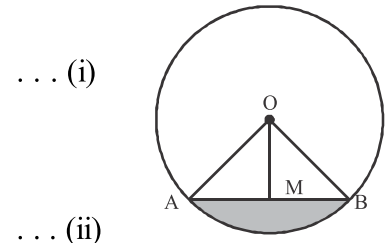


Fig. 15.08

Exercise 15.2

1. Radius of a circle is 7 cm and the angle subtended at the centre by an arc is 60° . Find the length of the arc.
2. Radius of a circle is 10.5 cm and angle of the sector is 45° . Find the area of the minor sector of the circle. $\left(\pi = \frac{22}{7}\right)$
3. The length of an arc is 12 cm and radius of a circle is 7 cm respectively. Find the area of the minor sector of circle.
4. An arc of a circle with radius 21 cm subtends an angle of 60° at the centre. Find :
 - (i) Length of the arc
 - (ii) Area of the sector formed by this arc
 - (iii) Area of the segment formed by corresponding chord.
5. The length of the minute hand of a clock is 10.5 cm. Find the area of the sector swept by the minute hand in 10 minutes $\left(\pi = \frac{22}{7}\right)$
6. Radius of a circle is 3.5 cm and the angle subtended by a chord at the centre is 90° . Find the area of the minor segment of the circle formed by this chord. $\left(\pi = \frac{22}{7}\right)$
7. The circumference of a circle is 22 cm. Find the area of its one quadrant.
8. Minute hand of a clock is 5 cm long. Find the area of the sector formed by this hand in 7 minutes.
9. Given figure 15.09 $ABCD$ is a rectangle, in which side $AB = 10$ cm and $BC = 7$ cm. From the each vertex of the rectangle four circles with radii 3.5 cm each are drawn. Find the area of shaded portion. $\left(\pi = \frac{22}{7}\right)$

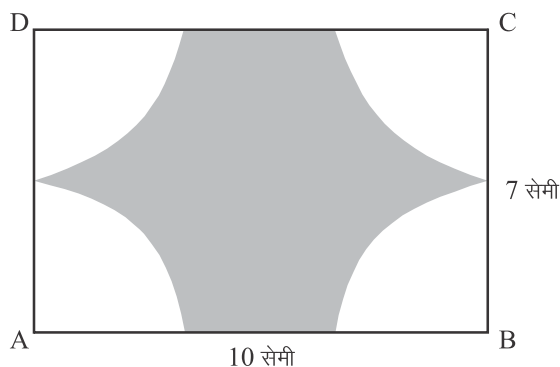


Fig.15.09

15.07. Area of Combination of Plane Figures

Combination means forming a new shape by joining two or more plane figures. Now, we will calculate the area of different type of figures related to the circles. Now we shall try to calculate the areas of some other figures. We come across these types of figures in our daily life and also in the form of various interesting designs. Flower beds, drain covers, window designs on table covers are some of such examples. To find the areas of these figures several examples are given below.

Example 9. There are two circular portions on two sides of a square lawn of side 58 m. The centre of each circular portion is the point of intersection of the diagonals of the square lawn. Find the area of the lawn in whole.

Solution : We have, side of the square = 58 m

$$\begin{aligned}\therefore \text{ length of its diagonal} &= \sqrt{58^2 + 58^2} \\ &= 58\sqrt{2} \text{ m}\end{aligned}$$

So the radius of the circle whose centre is the intersecting point of its diagonals

$$= \frac{58\sqrt{2}}{2} = 29\sqrt{2}$$

The area of one circular end = area of the sector of the circle with radius $29\sqrt{2}$
and centre angle is 90°

$$\begin{aligned}&= \left[\frac{\pi r^2 \theta}{360^\circ} - r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right] = \left[\frac{\pi \theta}{360^\circ} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right] r^2 \\&= \left[\frac{22}{7} \times \frac{90^\circ}{360^\circ} - \sin 45^\circ \cos 45^\circ \right] (29\sqrt{2})^2 \text{ m}^2 \\&= \left[\frac{11}{14} - \frac{1}{2} \right] \times 29 \times 29 \times 2 \text{ m}^2 \\&= 29 \times 29 \times 2 \times \frac{4}{14} \text{ m}^2 \\&= \frac{3364}{7} \text{ m}^2\end{aligned}$$

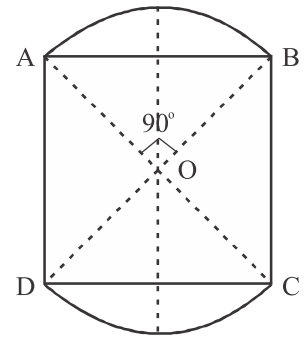


Fig. 15.10

Area of the lawn with two circular portion.

= area of the square + 2 (area of the one end)

$$\begin{aligned}&= \left[58 \times 58 + 2 \times \frac{3364}{7} \right] \text{ m}^2 \\&= 3364 \left[1 + \frac{2}{7} \right] \text{ m}^2 \\&= 3364 \times \frac{9}{7} \text{ m}^2 \\&= 4325.14 \text{ m}^2\end{aligned}$$

Example 10. Diameter of a circular grass land is 42 m. It has 3.5 metre wide path around it on the outside. Find the cost of gravelling the path at the rate of a ₹ 4 per metre².

Solution : Given, diameter of the grass land = 42 m

∴ Radius of the grass land = 21 m

Radius of the grass land with path = $(21 + 3.5) = 24.5$ m

$$\begin{aligned}
 \text{Area of the path} &= \left[\pi (24.5)^2 - \pi (21)^2 \right] \text{ m}^2 \\
 &= \pi \left[(24.5)^2 - (21)^2 \right] \text{ m}^2 \\
 &= \pi \left[(24.5 + 21)(24.5 - 21) \right] \text{ m}^2 \\
 &= \pi [45.5 \times 3.5] \text{ m}^2 \\
 &= \frac{22}{7} \times 45.5 \times 3.5 \text{ m}^2 = 500.5 \text{ m}^2
 \end{aligned}$$

Cost of the gravelling the path = ₹ $500.5 \times 4 = ₹ 2002$

Example 11. In the fig. 15.11 is a square $ABCD$ with side 14 cm. Find the area of shaded portion.

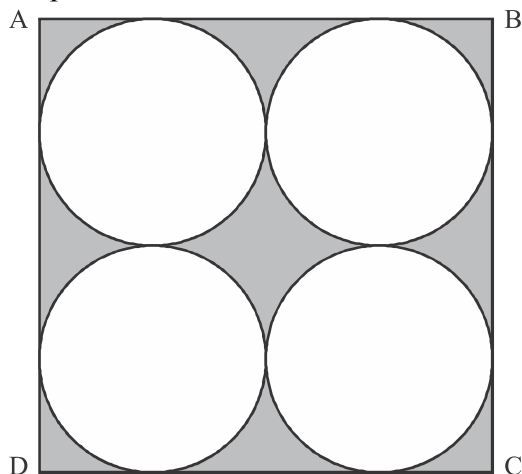


Fig. 15.11

Solution : We have side of the square $ABCD = 14$ cm

$$\therefore \text{Area} = (\text{side})^2 = 14 \times 14 = 196 \text{ cm}^2$$

$$\text{Diameter of every circle} = \frac{14}{2} \text{ cm} = 7 \text{ cm}$$

$$\therefore \text{Radius of every circle} = \frac{7}{2} \text{ cm}$$

So, area of one circle = πr^2

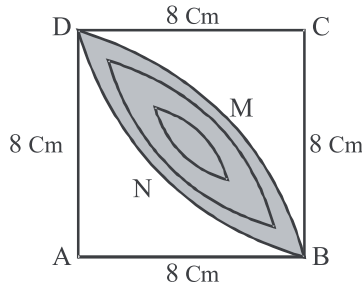
$$\begin{aligned}
 &= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2 \\
 &= \frac{154}{4} = \frac{77}{2} \text{ cm}^2
 \end{aligned}$$

$$\text{Area of all four circles} = 4\pi r^2$$

$$= 4 \times \frac{77}{2} \text{ cm}^2 = 154 \text{ cm}^2$$

$$\text{Area of shaded portion} = (196 - 154) \text{ cm}^2 = 42 \text{ cm}^2$$

Example 12. Find the area of shaded portion in following figure which is the region between the two quadrants of the circle with radii 8 cm each.



Solution : Given, radii of quadrants $ABMD$ and $BNDC = 8$ cm each.

$$\text{The sum of their areas} = 2 \times \frac{1}{4} \pi r^2 = \frac{1}{2} \pi r^2$$

$$= \left[\frac{1}{2} \times \frac{22}{7} \times 64 \right]$$

$$= \frac{704}{7} \text{ cm}^2$$

$$\text{Area of the square } ABCD = (8 \times 8) \text{ cm}^2 = 64 \text{ cm}^2$$

Area of the shaded region

= sum of the area of two quadrants - area of the square $ABCD$

$$= \left[\frac{704}{7} - 64 \right] \text{ cm}^2 = \left[\frac{704 - 448}{7} \right] \text{ cm}^2 = \frac{256}{7} \text{ cm}^2$$

Exercise 15.3

- Find the circumference of the incircle of square with side 14 cm.
- The difference between the circumference and the radius of a circle is 74 cm, find the area of the circle.
- In the given fig. 15.14 O is the centre of a circle. $\angle AOB = 90^\circ$ and $OA = 3$ cm, then find the area of the shaded region.

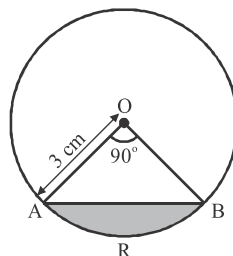


Fig. 15.14

4. If perimeter of a circle is equal to the perimeter of a square, then find the ratio of their areas.
5. The radius of a circular park 3.5 metre. There is a footpath of width 1.4 m around the park. Find the area of the footpath (see fig. 15.15).

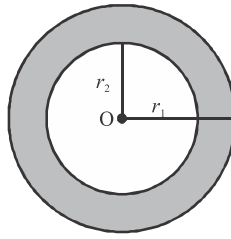


Fig. 15.15

6. Find the area of the square inscribed in a circle of radius 8 cm.
7. In the fig. 15.16, there is a quadrant $ABMC$ of a circle with radius 14 cm and with BC as diameter a semicircle is drawn. Find the area of shaded port.

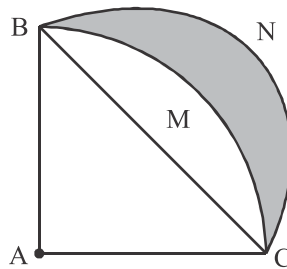


Fig. 15.16

8. In the given figure 15.17 AB is the diameter of a circle, $AC = 6$ cm and $BC = 8$ cm. Find the area of the shaded portion.

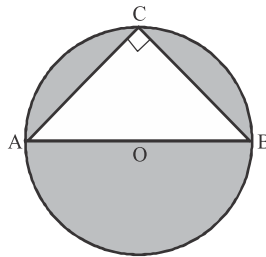


Fig. 15.17

9. Find the area of the shaded design of the given figure 15.18 where $ABCD$ is a square with side 10 cm and taking every side of the square as the diameter four semi-circles are drawn. ($\pi = 3.14$)

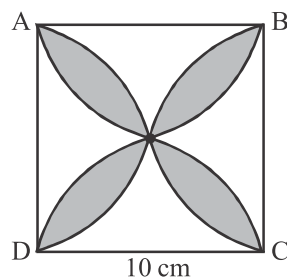


Fig. 15.18

10. In the given figure 15.19 radius of the semicircle is 7 cm. Find area of the circle formed inside the semicircle.

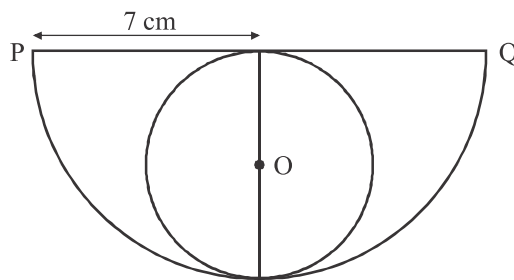


Fig. 15.19

11. The sum of the circumferences of two circles with radii R_1 and R_2 is equal to the perimeter of the circle with radius R . Choose the correct option.
- (A) $R_1 + R_2 = R$ (B) $R_1 + R_2 > R$
 (C) $R_1 + R_2 < R$ (D) Nothing can be said
12. The circumference of the incircle of a square with side 14 cm is :
- (A) 22 cm (B) 44 cm (C) 33 cm (D) 55 cm

Important Points

1. Circumference of the circle $= 2\pi r = \pi d$ (r = radius, d = diameter)
2. Area of the circle $= \pi r^2$
3. Area of the portion between two concentric circles $= \pi(r_1^2 - r_2^2)$ where $r_1 > r_2$
4. Area of a sector of the circle $A = \frac{\pi r^2 \theta}{360^\circ}$
5. Length of the arc of the sector of the circle $= L = \frac{2\pi r \theta}{360^\circ}$
6. Area of a sector of the circle $= \frac{1}{2} Lr$
7. Area of a segment of a circle $= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta$
8. Area of the major segment of a circle
 $= \text{area of the circle} - \text{area of the minor segment of the circle}.$

Answer Sheet

Exercise 15.1

1. 22 cm, 38.5 cm²
2. 154 sq.m.
3. 693 sq. metre, 108 metre
4. 14 cm,
5. 144 cm
6. 616 sq. metre
7. 1925
8. 1386 sq. metre
9. B
10. A

Exercise 15.2

1. 7.3 cm
2. 43.31 cm²
3. 42 cm²
4. (i) 22 cm (ii) 231 cm²
- (iii) 40.047cm²
5. 57.75 cm²
6. 3.5 cm²
7. 9.625cm²
8. 7.64 cm²
9. 31.5 cm²

Exercise 15.3

1. 44 cm
2. 616 cm²
3. 2.57 cm²
4. 14 : 11
5. 36.96m²
6. 128 cm²
7. 98 cm²
8. 54.57 cm²
9. 57 cm²
10. 38.5 cm²
11. A
12. B