Chapter 10

Stability of Slopes

CHAPTER HIGHLIGHTS

- *Introduction*
- *Types of slopes*
- *Type of slope failure*
- *Different definitions of factor of safety (F_s)*
- *Stability of an infinite slope of cohesionless soils*
- *Stability analysis of an infinite slope of cohesive soils*
- *Finite slopes*
- *Swedish circle method or method of slices*
- *Location of most critical circle*
- *Effective stress analysis*
- *Bishop's method*
- *Friction circle method*
- *Taylor's method*

Introduction

Earth slope is an unsupported inclined surface of soil mass required in the construction of highways, railways, earth dams, etc. The stability of earth slope is, therefore, important as the failure of a slope may lead to loss of life and property. The present chapter outlines the concept of stability of both finite and infinite slopes based on factor of safeties given by various methods. Gravitational forces and forces due to seepage of water in the soil mass are the reasons for failure of earth slopes.

Types of Slopes

1. **Infinite slope:** An infinite slope is one which represents the boundary surface of a semi-infinite soil mass inclined to the horizontal soil have uniform soil properties below the free surface.
2. **Finite slope:** A slope of a finite extent bounded by a top surface is said to be finite.

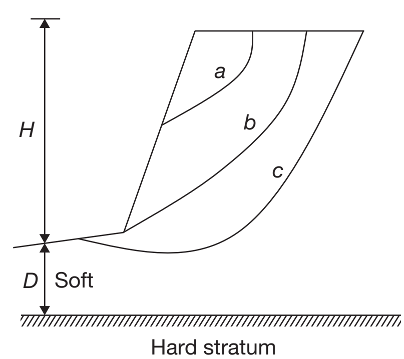
Type of Slope Failure

1. **Rotational failure:** Occurs by rotation along a slip surface by downward and outward movement of the soil mass.

- It occurs in finite slopes.
- Slip surface is circular in case of homogeneous conditions and non-circular in case of non-homogeneous conditions.
- Based on depth factor, rotational slips are further divided into three types.

$$D_f = \frac{H + D}{D}$$

$$D_f = \frac{\text{Depth of hard stratum below top}}{\text{Height of slope}}$$



Rotational failure

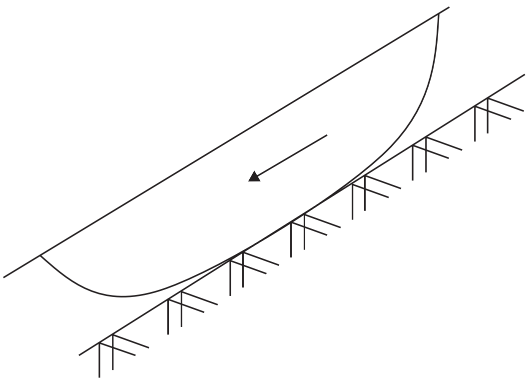
If $D_f < 1 \Rightarrow$ Face failure
$D_f = 1 \Rightarrow$ Toe failure
$D_f > 1 \Rightarrow$ Base failure

(a) **Toe failure:** Failure occurs along the surface that passes through the toe.

(b) **Face failure:** Failure occurs along a surface that intersects the slope above the toe.

(c) **Base failure:** Failure surface passes below the toe.

2. **Translational failure:**
   - Translational failure occurs in an infinite slope along a long failure surface parallel to the slope.
     In case of layered material, translational failure occurs along the slope of layered material.



Translational failure

- The shape of translational failure depends on the hard stratum at shallow depth below the slope surface.

Different Definitions of Factor of Safety (F_s)

1. **Factor of safety with respect to shear strength:** It is defined as the ratio of shear strength to shear stress along the surface of failure:

$$F_s = \frac{S}{\tau_m}$$

Where

F_s = Factor of safety with respect to shear strength

S = Shear stress

τ_m = Mobilized shear strength (equal to applied shear stress)

$$F_s = \frac{C + \sigma \tan\phi}{C_m + \sigma \tan\phi_m}$$

C_m = Mobilized cohesion

ϕ_m = Mobilized angle of internal friction

$$F_s = F_c + F_\phi$$

Factor of safety with respect to shear strength is equal to the factor of safety with respect to cohesion and that with respect to angle of internal friction.

2. **Factor of safety with respect to cohesion (F_c):** It is defined as the ratio of available cohesion intercept (C) and the mobilized cohesion intercept (C_m).

$$F_c = \frac{C}{C_m}$$

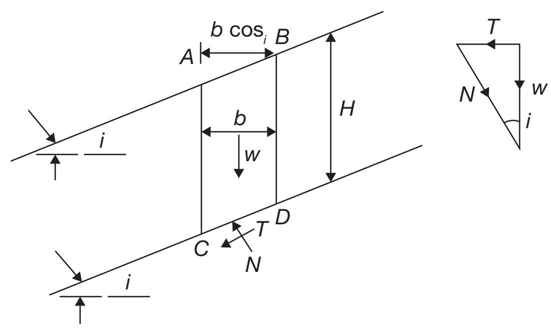
3. **Factor of safety with respect to friction (F_ϕ):** It is defined as the ratio of the available frictional strength to the mobilized frictional strength.

$$F_\phi = \frac{\tan\phi}{\tan\phi_m}$$

Stability of an Infinite Slope of Cohesionless Soils

The stability criteria of an infinite slope of cohesion less soils will depend on whether the soil is dry, submerged or steady seepage.

1. **Dry soil:** Consider a section of infinite slope making an slope angle of i.



The normal and shear stresses are given by:

$$\sigma = \frac{N}{b} = \frac{w\cos i}{b} = \frac{\gamma H b \cos^2 i}{b} = \gamma H \cos^2 i$$

$$C = \frac{T}{b} = \frac{w \sin i}{b} = \frac{\gamma H b \cos i \sin i}{b} = \gamma H \cos i \sin i$$

In the above equation, w is called 'weight of prism per unit length'.

$w = \gamma \times$ volume of prism per unit length

$w = \gamma \times Hb\cos i$

Factor of safety against shear failure is given by:

$$F_s = \frac{S}{\tau} = \frac{\text{Shear strength}}{\text{Shear stress}}$$

$S = \sigma \tan\phi = \gamma H \cos^2 i \tan\phi$

$C = \gamma H \cos i \sin i$

$$F_s = \frac{\gamma H \cos^2 i \tan\phi}{\gamma H \cos i \sin i}$$

$$F_s = \frac{\tan\phi}{\tan i}$$

For dry cohesion less soil:

For $i < \phi \rightarrow$ slope is stable

$i = \phi \rightarrow$ slope is just stable

$i > \phi \rightarrow$ slope is not stable

NOTE

Factor of safety of an infinite slope of cohesionless soil is independent of the height 'H' of slope.

2. **Submerged slope:**
   - In case of slope submerged under water, the normal effective stress and the shear stress are calculated using the submerged unit weight and not the bulk unit weight as used for dry soil.

     $\sigma = \gamma' H \cos^2 i$

     $C = \gamma' H \cos i \sin i$

     Where, γ' is the submerged unit weight.

     Factor of safety,

$$F_s = \frac{S}{\tau} = \frac{(\gamma' H \cos^2 i)\tan\phi}{\gamma' H \cos i \sin i}$$

For submerged soil,

$$F_s = \frac{\tan\phi}{\tan i}$$

NOTE

Factor of safety of an infinite slope for cohesionless soil is same for dry and submerged conditions.

3. **Steady seepage along the slope:** Factor of safety in case of seepage parallel to the slope is given by

$$F_s = \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi}{\tan i}$$

NOTE

Factor of safety in case of seepage parallel to the slope is reduced to one-half of the condition when there is no seepage.

Stability Analysis of an Infinite Slope of Cohesive Soils

1. **Dry soil:**

   Factor of safety:

$$F_s = \frac{S}{\tau} = \frac{C + (\gamma' H \cos^2 i)\tan\phi}{\gamma' H \cos i \sin i}$$

2. **Submerged slope:**
   - Normal and tangential components of weight are taken for submerged unit weight and not for bulk unit weights.

$$F_s = \frac{C + \gamma' H \cos^2 i \tan\phi}{\gamma' H \cos i \sin i}$$

3. **Steady seepage along the slope:**

$$F_s = \frac{C + \gamma' H \cos^2 i \tan\phi}{\gamma_{sat} H \cos i \sin i}$$

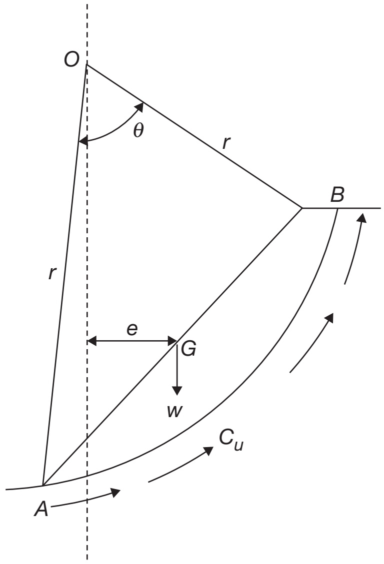
Finite Slopes

The following methods are used to analyze finite slopes.

1. $\phi_u = 0$ analysis
2. Swedish circle method or method of slices
3. Bishop's method
4. Friction circle method
5. Stability number method

$\phi_u = 0$ Analysis

- The stability of the slope can be checked in terms of total stress for fully-saturated clay under undrained condition.
- Suitable for slopes immediately after construction.
- In $\phi_u = 0$, analysis failure surface is assumed as on arc of circle.
- The following figure shows a failure surface with centre 'O' and radius 'r'.



$$\text{Factor of safety} = \frac{\text{Restraining moment}}{\text{Sliding moment}} = \frac{C_u L_a r}{w \times e}$$

Where

L_a = Length of arc $AB = r \cdot \theta^\circ = r \cdot \theta \times \frac{\pi}{180}$ radians

r = Radius of slip circle

W = Weight of soil within slip surface

C_u = Undrained cohesion

= shearing strength of soil since $\phi_u = 0$

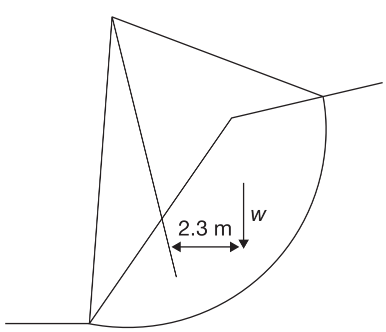
e = Lever arm of w about 'O'

SOLVED EXAMPLE

Example 1

The critical slip circle for a slope is shown below along the soil properties

$$\phi = 0$$
$$C_u = 30 \text{ kpa}$$
$$\gamma = 20 \text{ kN/m}^3$$



The length of the arc of the slip circle is 15.6 m and the area of soil within the slip circle is 82 m^2. The radius of the slip circle is 10.3 m. The factor of safety against the slip circle failure is nearly equal to

(A) 1.05 (B) 1.22
(C) 0.78 (D) 1.28

Solution

$$F_s = \frac{C_u L_a r}{w \cdot x}$$

Where $C_u = 30$ kPa, $\gamma = 20$ kN/m^3

$\bar{x} = 2.3$ m,

$\gamma = 10.3$ m

w = Weight of soil with in slip surface

= Area × 1 × γ

= 82 × 1 × 20

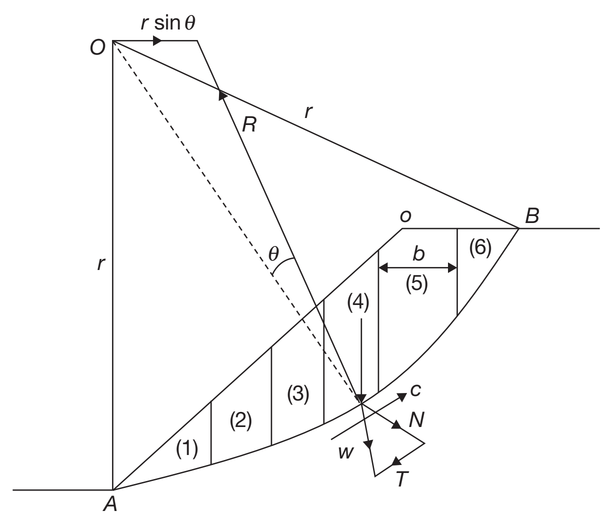
$w = 1640$ kN

$$F = \frac{30 \times 15.6 \times 10.3}{1640 \times 2.30}$$

$F = 1.28$.

Swedish Circle Method or Method of Slices

In this method, the failure wedge is assumed as an arc of circle and is divided into vertical slices by drawing vertical lines of equal width (b).



Swedish circle method

Each slice is in equilibrium under the following forces:

1. Weight (w) acting vertically through its centre of gravity.
2. Cohesive force (C) acting along the curved surface in direction opposite to the direction of probable movement of wedge.
3. Reaction 'R' at the base inclined at an angle ϕ to the normal.

In the Swedish circle method, forces acting between the slices are neglected. But the error due to this assumption is a positive side of factor of safety, i.e., F_s is generally less than that obtained from more accurate methods, such as Bishop's method.

The factor of safety is determined by considering the moments about the centre of rotation '*O*'.

Factor of safety for the slice:

$$F_s = \frac{\text{Resisting moment } (M_r)}{\text{Overturning moment } (M_0)}$$

Resisting moment $(M_R) = (\bar{c}\,\Delta \bar{L}) \times r + R(r \sin\phi)$

$= (C\,\Delta L) \times r + N \cdot r \tan\phi$

Actuating or overturning moment (M_0)

$= T \times r$ (normal component passes through '*O*' and do not cause any moment)

Factor of safety of the entire wedge

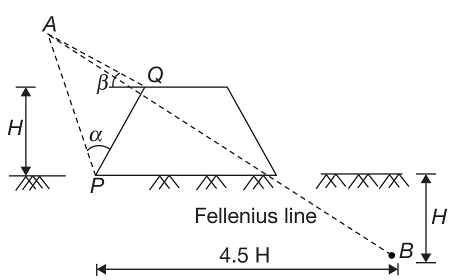
$$F_s = \frac{CL + \sum N \tan\phi}{\sum T} \text{ without seepage}$$

$$F_s = \frac{C'L + \sum (N - U)\tan\phi}{\sum T} \text{ with seepage}$$

- The circle which gives the minimum factor of safety is the most critical circle.
- This method is equally applicable to homogeneous soils, stratified deposits, partially submerged cases and non-uniform slopes and seepage effects can also be considered.

Location of Most Critical Circle

- The Fellenius method is used for location of the centre of most critical circle.
- The centre of most critical circle lines on Fellenius line *AB*.
- Fellenius line '*AB*' can be obtained by locating the point '*B*' at a distance '*H*' and at a distance 4.5 *H* from point *P* at the toe of slope and point '*A*' is located by drawing two lines '*PA*' and '*QA*' making an angle α with slope '*PQ*', and making an angle β with the horizontal at '*Q*'.



Fellenius line

- For purely cohesive soil, the point '*A*' is the centre of the most critical circle. For other soils, it lies on the line or on the extension of line *AB*.

Effective Stress Analysis

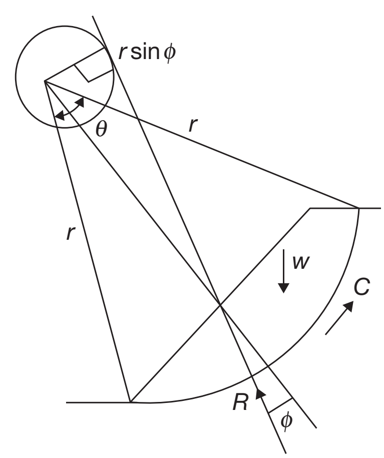
- Total stress analysis (Example: $\phi_u = 0$ analysis and Swedish method) is applied for the analysis of stability of a slope soon after construction under undrained conditions.
- Effective stress analysis is appropriate if seepage or drainage takes place.
- In case of steady seepage through an earth dam or embankment, the stability of downstream slope is considered to be most critical.
- In case of rapid or sudden draw down, the upstream slope of an embankment or on earth dam represents the critical condition.
- In case of immediately after construction, the stability of upstream and downstream slopes are to be considered.

Bishop's Method

The Bishop's method considers the forces acting on the sides of slices. It is an accurate method compared to the Swedish method in which the forces acting on the sides of slices is neglected.

Friction Circle Method

- This method is useful for stability analysis of slopes made of homogeneous soils.
- In this method also, the failure surface is assumed as an arc of circle.
- Friction circle is a smaller circle of radius $r.\sin\phi$. Also called ϕ circle.
- The resultant of normal reaction and frictional force is tangential to the friction circle.



Taylor's Method

Stability number (S_n) is given by:

$$S_n = \frac{C_m}{\gamma H} = \frac{C}{F_c \gamma\ H}; F_c = \frac{C}{C_m}$$

The reciprocal of stability number is known as stability factor. It is a dimensionless quantity.

- The values of S_n given by Taylor are based on the friction circle method, and for values of ϕ_m and i.
- For purely cohesive soil ($\phi_m = 0°$), stability number depends on i and D_f.
- For slope angle (i) greater than 53°, the toe failure occurs and for $i \leq 53°$, and small values of ϕ_m, the failure surface extends below the toe, i.e., base failure occurs.
- Stability number is used to determine the factor of safety given by:

$$F_c = \frac{C}{C_m} = \frac{C}{S_n \gamma H}$$

Maximum value of $S_n = 0.261$, for $i = 90°$, $\phi_m = 0°$, and for $i = 90°$ and $D_f = 1$.

NOTES

1. For purely frictional soils, the cohesion intercept (C) is zero. As the stability number reduces to zero, the stability charts cannot be used for such soils.
2. For submerged slope:

$$S_n = \frac{C}{F_c \gamma' H}$$

3. If the slope is saturated by capillary water:

$$S_n = \frac{C}{F_c \gamma_{sat} H}$$

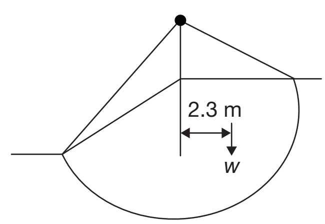
4. In case of sudden drawdown, weighed friction angle (ϕ_w) and γ_{sat} should be used,

$$\phi_w = \left(\frac{\gamma'}{\gamma_{sat}}\right)$$

S_n is read against the ϕ_w value.

Exercises

1. With respect to a c-ϕ soil in an infinite slope, identify if the following two statements are true or false.
   I. The stable slope angle can be greater than ϕ.
   II. The factor of safety of the slope does not depend on the height of soil in the slope.
   (A) Both statements are false.
   (B) I is true but II is false.
   (C) I is false but II is true.
   (D) Both statements are true.
2. A granular soil possesses saturated density of 20 kN/m^3. Its effective angle of internal friction is 35°. If the desired factor of safety is 1.5, the safe angle of slope for this soil, when seepage occurs at and parallel to the slope surface, will be
   (A) 25° (B) 23°
   (C) 20° (D) 13°
3. A 40° slope is excavated to a depth of 10 m in a deep layer of saturated clay of unit weight 20 kN/m^3; the relevant shear strength parameters are $c_u = 72$ kN/m^2 and $\phi_u = 0$. The rock ledge is at a great depth. The Taylor's stability coefficient for $\phi_u = 0$ and 40° slope angle is 0.18. The factor of safety of the load is:
   (A) 2.0 (B) 2.1
   (C) 2.2 (D) 2.3
4. A deep cut of 7 m has to be made in a clay with unit weight 16 kN/m^3 and a cohesion of 25 kN/m^3. What will be the factor of safety if one has to have a slope angle of 30°? Stability number is given to 0.178 (from Taylor's chart) for a depth factor of 3.
   (A) 0.80 (B) 1.1
   (C) 1.25 (D) 1.0
5. The critical slip circle for a slope is shown in the given figure along with the soil properties.



$\phi = 0$
$C_u = 30$ kPa
$\gamma = 20$ kN/m^3
The length of the arc of the slip circle is 15.6 m and the area of soil within the slip circle is 82 m^2. The radius of the slip circles is 10.3 m. The factor of safety against the slip circle failure is nearly equal to
   (A) 1.05 (B) 1.22
   (C) 0.78 (D) 1.28

6. An infinite slope is to be constructed in a soil. The effective stress strength parameters of the soil are $c' = 0$ and $\phi' = 30°$, the saturated unit weight of the slope is 20 kN/m^3 and the unit weight of water is 10 kN/m^3. Assuming that seepage is occurring parallel to the slope, the maximum slope angle for a factor of safety of 1.5 would be

(A) 10.89° (B) 11.30°
(C) 12.48° (D) 14.73°

Direction for questions 7 and 8:

A canal having side slopes 1 : 1 is proposed to be constructed in a cohesive soil to a depth of 10 m below the ground surface. The soil properties are: $\phi_u = 15°$, $C_u = 12$ kPa, $e = 1.0$, $G_s = 2.65$.

7. If Taylor's stability number S_n is 0.08 and if the canal flows full, the factor of safety with respect to cohesion against failure of the canal bank slope will be

(A) 3.7 (B) 1.85
(C) 1.0 (D) None of these

8. If an infinite slope of clay at a depth 5 m has cohesion of 1t/m^2 and unit weight of 2t/m^3, then the stability number will be

(A) 0.1 (B) 0.2
(C) 0.3 (D) 0.4

9. Consider the following statements associated with stability of slope:

I. Stability number is inversely proportional to cohesion and directly proportional to height.
II. Swedish method of analysis is based on circular failure surfaces.
III. The Culmann method assumes that rupture will occur in a plane.

Which of these statements are correct?

(A) II and III (B) I and III
(C) I and II (D) I, II and III

10. Taylor's stability number S_n is given by which one of the following expressions? (c is cohesion, F_c is factor of safety, γ is density of soil and H is height of the slope).

(A) $\frac{c}{F_c\gamma}$ (B) $\frac{c}{\gamma H}$
(C) $\frac{c}{F_c\gamma H}$ (D) $\frac{c}{F_c(\gamma + H)}$

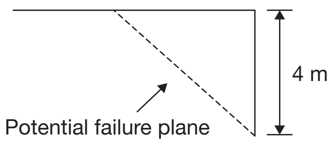
11. If there is a sudden drawdown of water in the canal and if Taylor's stability number for the reduced value of ϕ_w is 0.126, the factor of safety with respect to cohesion against the failure of bank slopes will be

(A) 1.85 (B) 1.18
(C) 0.84 (D) 0.53

12. An infinite soil slope with an inclination of 35° is subjected to seepage parallel to its surface. The soil has $c' = 100$ kN/m^2 and $\phi' = 30°$. Using the concept of mobilized cohesion and friction, at a factor of safety of 1.5 with respect to shear strength, the mobilized friction angle is

(A) 20.02° (B) 21.05°
(C) 23.33° (D) 30.00°

13. Using $\phi_u = 0$ analysis and assuming planar failure as shown, the minimum factor of safety against shear failure of a vertical cut of height 4 m in a pure clay having $C_u = 120$ kN/m^2 and $\phi_{sat} = 20$ kN/m^3 is



(A) 1 (B) 6
(C) 10 (D) 20

14. For two infinite slopes (one in dry condition and other in submerged condition) in a sand deposit having the angle of shearing resistance 30°, factor of safety was determined as 1.5 (for both slopes). The slope angles would have been

(A) 21.05° for dry slope and 21.05° for submerged slope.
(B) 19.47° for dry slope and 18.40° for submerged slope.
(C) 18.4° for slope and 21.05° for submerged slope.
(D) 22.6° for dry slope and 19.47° for submerged slope.

15. List I below gives the possible types of failure for a finite soil slope and List II gives the reasons for these different types of failure. Match the items in List I with the items in List II.

	List I		List II
P.	Base failure	1.	Soils above and below the toe have same strength
Q.	Face failure	2.	Soil above the toe is comparatively weaker
R.	Toe failure	3.	Soil above the toe is comparatively stronger

Codes:

	P	Q	R		P	Q	R
(A)	1	2	3	(B)	2	3	1
(C)	2	1	3	(D)	3	2	1

16. For toe failure, the depth factor D_F is

(A) < 1 (B) > 1
(C) = 1 (D) None of these

17. The maximum value of stability number is

(A) 0.261 (B) 0.281
(C) 0.241 (D) 0.291

18. A canal having side slopes 1 : 1 is proposed to be constructed in a cohesive soil to a depth of 10 m below ground surface. The soil properties are $\phi_U = 20°$, $C_u = 25$ kPa, $e = 1$, $G_S = 2.65$.

If taylor's stability number S_n is 0.08 and canal is full, the factor of safety with respect to cohesion against failure of canal bank slopes is

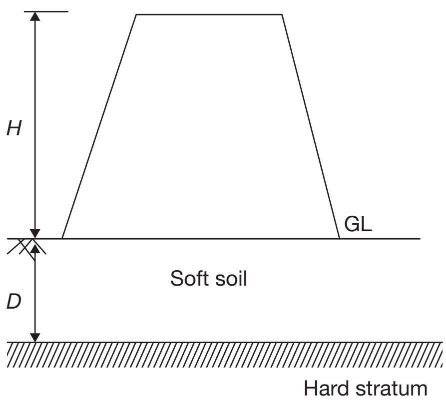
(A) 3.85 (B) 3.65
(C) 7.85 (D) 1.7

19. The cohesion and density of a soil are $4t/m^2$ and $8\ t/m^2$ respectively. For a factor of safety of 2 and stability number 0.1 the safe height of slope is

(A) 5 m (B) 50 m
(C) 25 m (D) 2.5 m

20. The cohesion and density of a soil are $1.8t/m^2$ and $2\ t/m^2$. If Stability number is taken as maximum, for a factor of safety against 2.5 what will be the safe height of the slope in metres?

21.



For the given slope, the failure expected to be ______.

(A) toe failure
(B) base failure
(C) face failure
(D) Cannot be determined/Data inadequate

22. Identify the false statements from the list given.

I. Skempton's theory is suitable for clays only.
II. The discharge between any two adjacent flow lines is constant.
III. If water table rises σ' and u increases but σ decreases.
IV. The westergaard analysis is suitable for stratified soils.

(A) III and IV
(B) I and III
(C) III only
(D) IV only

23. A trench is to be cut with vertical sides. It is a pure cohesive soil having bulk density $1.8\ t/m^3$ and cohesion $C = 2.4\ t/m^2$, if $i = 90°$, $S_n = 0.261$, the depth upto which the trench can be excavated without any lateral support is

(A) 4.6 m B) 3.4 m
(C) 4.9 m (D) 5.1 m

24. Determine the factor of safety with respect to cohesion for a submerged embankment of 25 m high and having a slope of 40°.

($C = 40\ kN/m^2$, $\phi_m = 10°$, $\gamma_{sat} = 18\ kN/m^3$)

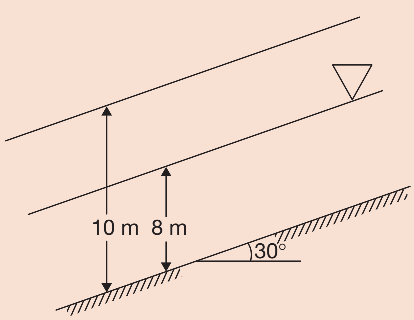
stability numbers

i \ ϕ_m	0°	5°	10°
30°	0.156	0.110	0.075
45°	0.170	0.136	0.108
60°	0.191	0.162	0.138

(A) 2 (B) 2.6
(C) 3.2 (D) 4.1

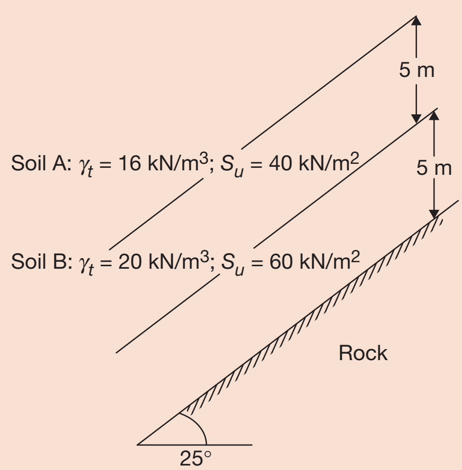
Previous Years Questions

1. The factor of safety of an infinite soil slope shown in the figure having the properties $\phi = 35°$, $\gamma_{dry} = 16\ kN/m^3$ and $\gamma_{sat} = 20\ kN/m^3$ is **[GATE, 2007]**



(A) 0.70 (B) 0.80
(C) 1.00 (D) 1.20

2. The soil profile above the rock surface for a 25° infinite slope is shown in the figure, where s_u is the undrained shear strength and γ_t is total unit weight. The slip will occur at a depth of **[GATE, 2013]**



(A) 8.83 m
(B) 9.79 m
(C) 7.83 m
(D) 6.53 m

3. An infinitely long slope is made up of a c-ϕ soil having the properties: cohesion $C = 20$ kPa, and dry unit weight $\gamma_d = 16$ kN/m^3. The angle of inclination and critical height of the slope are 40° and 5 m, respectively. To maintain the limiting equilibrium, the angle of internal friction of the soil (in degrees) is ________. **[GATE, 2014]**

4. A long slope is formed in a soil with shear strength parameters: $c' = 0$ and $\phi' = 34°$. A firm stratum lies below the slope and it is assumed that the water table may occasionally rise to the surface, with seepage taking place parallel to the slope. Use $\gamma_{sat} = 18$ kN/m^3 and $\gamma_w = 10$ kN/m^3. The maximum slope angle (in degrees) to ensure a factor of safety of 1.5, assuming a potential failure surface parallel to the slope, would be **[GATE, 2014]**

(A) 45.3 (B) 44.7
(C) 12.3 (D) 11.3

5. In friction circle method of slope stability analysis, if r defines the radius of the slip circle, the radius of friction circle is **[GATE, 2015]**

(A) $r \sin \phi$ (B) r
(C) $r \cos \phi$ (D) $r \tan \phi$

Answer Keys

Exercises

1. B	**2.** D	**3.** A	**4.** C	**5.** D	**6.** A	**7.** B	**8.** A	**9.** A	**10.** C
11. D	**12.** B	**13.** B	**14.** A	**15.** D	**16.** C	**17.** A	**18.** A	**19.** D	
20. 1.37 to 1.40		**21.** B	**22.** C	**23.** D	**24.** A				

Previous Years' Questions

1. A	**2.** A	**3.** 22.44°	**4.** D	**5.** A