## CBSE Test Paper 02
## Chapter 7 Integrals

1. $\int_{0}^{\pi} \sqrt{1+\cos x}$ dx is equal to

   a. 2
   b. $\sqrt{2}$
   c. $2\sqrt{2}$
   d. 1

2. $\int \frac{x^2-1}{x^4+3x^2+1} dx$ is equal to

   a. $\tan\left(x+\frac{1}{x}\right)+C$
   b. $\tan^{-1}\left(x+\frac{1}{x}\right)+C$
   c. $\tan^{-1}\left(3x^2+2x\right)+C$
   d. $\tan^{-1}\left(x^2+1\right)+C$

3. $\int \frac{\cos 4x+1}{\cot x+\tan x} dx$ is equal to

   a. $-\frac{1}{6}\cos^3 2x+C$
   b. $\frac{1}{6}\cos^3 2x+C$
   c. $-\frac{1}{6}\sin^3 2x+C$
   d. $-\frac{1}{2}\sin^2 6x+C$

4. $\int \frac{1}{\sqrt{1-x}} dx$ is equal to

   1. $-2\sqrt{1-x}$ + C
   2. $3\sqrt{x-2}+C$
   3. $2\sqrt{1-x}+C$
   4. $\sqrt{1-x}$ + C

5. $\int (1-\cos x)\cos ec^2\ x$ dx is equal to

   a. $\frac{1}{2}\tan x+C$
   b. $\cot \frac{x}{2}+C$

c. $\tan \frac{x}{2} + C$
d. $3\cot \frac{2x}{3} + C$

6. The value of integral $\int_0^1 \frac{xdx}{\sqrt{1+x^2}}$ is _______.
7. The indefinite integral of $2x^3 + 4$ is _______.
8. The definite integral of $\int_1^3 \left(x^2 + 3x + 2\right) dx$ is _______.
9. Evaluate $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$.
10. Evaluate $\int_0^{\pi/2} tan^2 x dx$.
11. Write the value of $\int \frac{\sec^2 x}{cosec^2 x} dx$.
12. Evaluate $\int \frac{x^3 + x}{x^4 - 9} dx$.
13. Integrate the function $\tan^2(2x - 3)$.
14. Evaluate $\int \frac{x}{\sqrt{x}+1} dx$. **(2)**
15. Integrate the function $\sqrt{1 + 3x - x^2}$.
16. Integrate the function $\frac{(x-3)e^x}{(x-1)^3}$.
17. Evaluate $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$.
18. Evaluate $\int_1^3 \left(3x^2 + 1\right) dx$ by the method of limit of sum.

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### Solution

1. c. $2\sqrt{2}$

   **Explanation:** $= \sqrt{2}\int_0^{\pi} \cos\frac{x}{2}dx$

   $= \sqrt{2}\left[\frac{\sin\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi}$

   $= 2\sqrt{2}(sin\frac{\pi}{2} - sin0) = 2\sqrt{2}$

2. b. $\tan^{-1}\left(x + \frac{1}{x}\right) + C$

   **Explanation:** Divide num. and deno. by $x^2$

   Substitute $x + \frac{1}{x} = t$ then $(1 - \frac{1}{x^2})dx = dt$

   $\Rightarrow \int \frac{dt}{t^2+1}$

   $\Rightarrow \tan^{-1}(x + \frac{1}{x}) + C$

3. a. $-\frac{1}{6}\cos^3 2x + C$

   **Explanation:** $\int \frac{\cos 4x+1}{\tan x+\cot x}dx$

   $= \int \frac{2\ \cos^2 2x}{2\ cosec\ 2x}dx$

   $= \int \frac{2\ \cos^2 2x}{2\ cosec\ 2x}dx$

   substitute cos2x = t , then -2 sin2x dx = dt

   $\frac{-1}{2}\int\frac{t^2}{2}dt \Rightarrow = \frac{-1}{2}\left(\frac{\cos^3 2x}{3}\right) + C$

4. a. $-2\sqrt{1-x}$ + C

   **Explanation:** $\int(1-x)^{-1/2}dx = \frac{(1-x)^{1/2}}{(1/2)(-1)}$ +C$= -2\sqrt{1-x}$+ C

5. c. $\tan\frac{x}{2} + C$

   **Explanation:** $\int(cosec^2 x - cot\ x\ cosec\ x)dx$$= (-cot\ x + cosec\ x) + C$

   $= \frac{1-\cos x}{\sin x} + C = \tan\frac{x}{2} + C$

6. $\sqrt{2} - 1$
7. $\frac{1}{2}x^4 + 4x + c$
8. $\frac{74}{3}$
9. $\int\frac{x^3-x^2+x-1}{x-1}dx$

$= \int \frac{x^2(x-1)+(x-1)}{x-1} dx$

$= \int \frac{(x-1)(x^2+1)}{(x-1)} dx$

$= \int (x^2+1) dx$

$= \frac{x^3}{3} + x + c$

10. $I = \int_0^{\frac{\pi}{4}} tan^2 x dx = \int_0^{\frac{\pi}{4}} (sec^2 x - 1) dx$

$= [tan x - x]_0^{\pi/4}$

$= (tan \frac{\pi}{4} - \frac{\pi}{4}) - (0-0) = 1 - \frac{\pi}{4}$

11. Let I = $\int \frac{\sec^2 x}{cosec^2 x} dx = \int \frac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{1}{\sin^2 x}\right)} dx$

$= \int \frac{\sin^2 x}{\cos^2 x} dx$

$= \int \tan^2 x dx = \int \left(\sec^2 x - 1\right) dx \ \left[\because \tan^2 x = \sec^2 x - 1\right]$

$= \int \sec^2 x dx - \int 1 dx = \tan x - x + c$

12. we have

$I = \int \frac{x^3+x}{x^4-9} dx = \int \frac{x^3}{x^4-9} dx + \frac{x dx}{x^4-9} = I_1 + I_2$

Now $I_1 = \int \frac{x^3}{x^4-9}$

Put $t = x^4 - 9$ so that $4x^3 dx = dt$. Therefore

$I_1 = \frac{1}{4} \int \frac{dt}{t} = \frac{1}{4} \log|t| + C_1 = \frac{1}{4} \log\left|x^4 - 9\right| + C_1$

Again, $I_2 = \int \frac{x dx}{x^4-9}$

Put $x^2 = u$ so that $2x\,dx = du$. Then

$I_2 = \frac{1}{2} \int \frac{du}{u^2-(3)^2} = \frac{1}{2\times 6} \log\left|\frac{u-3}{u+3}\right| + C_2$

$= \frac{1}{12} \log\left|\frac{x^2-3}{x^2+3}\right| + C_2$

Thus I = $I_1$ + $I_2$

$= \frac{1}{4} \log\left|x^4 - 9\right| + \frac{1}{12} \log\left|\frac{x^2-3}{x^2+3}\right| + C . [\because C = C_1 + C_2]$

13. $\int \tan^2 (2x-3) dx$

$= \int \left\{\sec^2 (2x-3) - 1\right\} dx$

$= \int \sec^2 (2x-3) dx - \int 1 dx$

Using $\int \sec^2 (ax+b) dx = \frac{\tan(ax+b)}{a} + c$

$= \frac{\tan(2x-3)}{2} - x + c$

14. Let $I = \int \frac{x}{\sqrt{x}+1} dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\Rightarrow dx = 2\sqrt{x}\,dt$$
$$\therefore I = 2\int\left(\frac{x\sqrt{x}}{t+1}\right)dt = 2\int\frac{t^2.t}{t+1}dt = 2\int\frac{t^3}{t+1}dt = 2\int\frac{t^3+1-1}{t+1}dt$$
$$= 2\int\frac{t^3+1}{t+1}dt - 2\int\frac{1}{t+1}dt = 2\int\frac{(t+1)\left(t^2-t+1\right)}{t+1}dt - 2\int\frac{1}{t+1}dt$$
$$= 2\int\left(t^2-t+1\right)dt - 2\int\frac{1}{t+1}dt$$
$$= 2\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|(t+1)|\right] + C$$
$$= 2\left[\frac{x\sqrt{x}}{2} - \frac{x}{2} + \sqrt{x} - \log|\sqrt{x}+1|\right] + C$$

15. $\int\sqrt{1+3x-x^2}dx$

$$= \int\sqrt{-x^2+3x+1}dx$$
$$= \int\sqrt{-\left(x^2-3x-1\right)}dx$$
$$= \int\sqrt{-\left(x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2-1\right)}dx$$
$$= \int\sqrt{-\left[\left(x-\frac{3}{2}\right)^2-\left(\frac{\sqrt{13}}{2}\right)^2\right]}dx$$
$$= \int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}dx$$
$$= \frac{x-\frac{3}{2}}{2}\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2} + \frac{\left(\frac{\sqrt{13}}{2}\right)^2}{2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + c$$
$$\left[\because \int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{2}\right]$$
$$= \left(\frac{2x-3}{4}\right)\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + c$$

16. Let $I = \frac{(x-3)e^x}{(x-1)^3}dx$

$$= \int\frac{(x-1)-2}{(x-1)^3}e^x dx$$
$$= \int e^x\left[\frac{(x-1)}{(x-1)^3} - \frac{2}{(x-1)^3}\right]dx$$
$$\Rightarrow I = \int e^x\left[\frac{1}{(x-1)^2} + \frac{-2}{(x-1)^3}\right]dx$$
$$\left[\int e^x\left\{f(x)+f'(x)\right\}dx\right]$$

It is in the form of $\int e^x\left\{f(x)+f'(x)\right\}dx$ since here $f(x) = \frac{1}{(x-1)^2}$ and

$$f'(x) = \frac{d}{dx}\left\{(x-1)^{-2}\right\}$$
$$= -2(x-1)^{-3}$$

$= \frac{-2}{(x-1)^3}$.

$\Rightarrow I = \frac{e^x}{(x-1)^2} + c$

$[\because \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c]$

17. Let $I = \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

Put $x = a\tan^2\theta$

$\Rightarrow dx = 2a \tan\theta \sec^2\theta d\theta$

$\therefore I = \int \sin^{-1} \sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}} \left(2a \tan\theta . \sec^2\theta\right) d\theta$

$= 2a \int \sin^{-1} \left(\frac{\tan\theta}{\sec\theta}\right) \tan\theta . \sec^2\theta d\theta$

$= 2a \int \sin^{-1} (\sin\theta) \tan\theta . \sec^2\theta d\theta$

$= 2a \int \theta . \tan\theta \sec^2 \theta d\theta$

$= 2a \left[\theta . \int \tan\theta . \sec^2\theta d\theta - \int \left(\frac{d}{d\theta}\theta . \int \tan\theta . \sec^2\theta d\theta\right) d\theta\right]$

Let $\tan\theta = t$

$\sec^2\theta d\theta = dt$

$\int \tan\theta \sec^2\theta d\theta = \int t dt = \frac{t^2}{2} = \frac{\tan^2\theta}{2}$

I$= 2a \left[\theta . \frac{\tan^2\theta}{2} - \int \frac{\tan^2\theta}{2} d\theta\right]$

$= a\theta\tan^2\theta - a \int \left(\sec^2\theta - 1\right) d\theta$

$= a\theta\tan^2\theta - a \tan\theta + a\theta + C$

$= a \left[\frac{x}{a} \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1} \sqrt{\frac{x}{a}}\right] + C$

18. According to given question , $I = \int_1^3 \left(3x^2 + 1\right) dx$

On comparing the given integral with $\int_a^b f(x)dx$, we get

Here,$a = 1, b = 3$and f(x) = $3x^2 + 1$

We know that ,

$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+(n-1)h)\ ]$

where, $h = \frac{b-a}{n} \Rightarrow nh = b - a.$

f(a) = f(1) = $3(1)^2 + 1 = 4$

f(a+h) = f(1 + h) = $3(1 + h)^2 + 1 = 4 + 6h(1) + 3h^2(1)^2$

f(a+2h) = f(1 + 2h) = $3(1 + 2h)^2 + 1 = 4 + 6h(2) + 3h^2(2)^2$

so on

f[a + (n - 1)h] = f[1 + (n - 1)h] = $3[1 + (n - 1)h]^2 + 1 = 4 + 6h(n - 1) + 3h^2(n - 1)^2$

Now, $\int_1^3 \left(3x^2+1\right)dx = \lim\limits_{h \to 0}$ h [ f (1) + f (1 + h) + f (1 + 2h)+........+ f(1 + (n - 1)h]

$= \lim\limits_{h \to 0}$[4 + 4 + 6h(1) + $3h^2(1)^2$ + 4+ 6h(2) + $3h^2(2)^2$ +......+4 + 6h(n - 1) + $3h^2(n-1)^2$]

On rearranging terms , we get

$= \lim\limits_{h \to 0}$[4 + 4 + 4 +......+ 4 + 6h{1 + 2 + 3.......+(n - 1)} + $3h^2$ {$1^2 + 2^2 + 3^2$.......+$(n-1)^2$}]

=$\lim\limits_{h \to 0} h\left[4n + 6h \cdot \frac{n(n-1)}{2} + 3h^2 \frac{n(n-1)(2n-1)}{6}\right]$

$[\because \sum n = \frac{n(n+1)}{2}, \sum n^2 = \frac{n(n+1)(2n+1)}{6}]$

$= \lim\limits_{h \to 0}\left[4nh + \frac{6nh(nh-h)}{2} + \frac{3hn(nh-h)(2nh-h)}{6}\right]$

$= 4(2) + \frac{6(2)(2-0)}{2} + \frac{3\times2(2-0)(2\times2-0)}{6}$

= 8 + 12 + 8

= 28 sq units.