* LINEAR ALGEBRA *

- The expression $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called determinant of 2^{nd} order and stands for $a_1 b_2 - b_1 a_2$ where a_1, a_2, b_1, b_2 are called as elements which are arranged in two horizontal lines (rows) & two vertical lines (columns).

- MATRIX is an arrangement of number (objects) in rows and columns. It may be of any order (rectangular or square) whereas determinant is a value associated with square matrix.

NOTE: ① To find determinant of higher order make adjacent element zero and expand along non-zero elements.

② Operation is always less than order

Cofactor: $(-1)^{i+j} \underbrace{\text{Minor}}_{\text{Determinant}}$

$$A = \begin{bmatrix} \overset{+}{a_1} & \overset{-}{b_1} & \overset{+}{c_1} \\ \overset{-}{a_2} & \overset{+}{b_2} & \overset{-}{c_2} \\ \overset{+}{a_3} & \overset{-}{b_3} & \overset{+}{c_3} \end{bmatrix}$$

$$A_1 = (-1)^{1+1} \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = b_2 c_3 - b_3 c_2$$

$$\boxed{\Delta = a_1 A_1 + b_1 B_1 + c_1 C_1}$$

laplace equation

NOTE:

$$A_1 = (-1)^{1+1} \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

$$= b_2 c_3 - b_3 c_2$$

$$\Delta = a_i A_j + b_i B_j + c_i C_j$$

If $i = j = \Delta$ If $i \neq j = 0$

Q:- Find determinant of matrix

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix} \text{ is ?}$$

$$0\begin{vmatrix} 0 & 3 & 0 \\ 3 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 3 & 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 1 \end{vmatrix}$$

$$0 - 1\left(1(-1) - 3(4-3)\right) + 2(6) - 3\left(3 + 3(-9)\right)$$

$$-1(-1-3) + 12 - 3(-24)$$

$$4 + 12 + 72 = 88$$

* Properties of Determinant

① A determinant remains unaltered by changing its row into column and column into rows

$$A = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \qquad A^T = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$|A| = |A^T|$$

② If two column or rows are interchanged, the determinant retains its ~~re~~ numerical value but changes its sign.

$$\Delta = \begin{vmatrix} a_1 & b_2 \\ a_2 & b_1 \end{vmatrix}, \quad \Delta' = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = -\Delta$$

③ If two column or row are identical then Determinant $= 0$

④ If each element of row or column be mutiplied by the same factor, the whole determinant is multiplied by the same factor.

$$\begin{vmatrix} a_1 & pb_1 & c_1 \\ a_2 & pb_2 & c_2 \\ a_3 & pb_3 & c_3 \end{vmatrix} = p \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(5) If each element of row or column consists of small 'm' terms, the determinant can be expressed as sum of 'm' determinants

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 + d_1 - e_1 \\ a_2 & b_2 & c_2 + d_2 - e_2 \\ a_3 & b_3 & c_3 + d_3 - e_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} - \begin{vmatrix} a_1 & b_1 & e_1 \\ a_2 & b_2 & e_2 \\ a_3 & b_3 & e_3 \end{vmatrix}$$

(6) If two each element of row or column be added equimultiples of a corresponding element of 1 or more parallel rows or columns, the determinant remains unaltered.

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta' = \begin{vmatrix} a_1 + pb_1 - kc_1 & b_1 & c_1 \\ a_2 + pb_2 - kc_2 & b_2 & c_2 \\ a_3 + pb_3 - kc_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} pb_1 & b_1 & c_1 \\ pb_2 & b_2 & c_2 \\ pb_3 & b_3 & c_3 \end{vmatrix} + (-) \begin{vmatrix} kc_1 & b_1 & c_1 \\ kc_2 & b_2 & c_2 \\ kc_3 & b_3 & c_3 \end{vmatrix}$$

$$= \Delta + 0 + 0 = \Delta$$

(7) If the element of determinant Δ are function of x and two rows or column becomes identical when $x=a$, then $(x-a)$ is a factor of Δ

$$\Delta = \begin{vmatrix} a^3 & a^2 & a & 1 \\ b^3 & b^2 & b & 1 \\ c^3 & c^2 & c & 1 \\ d^3 & d^2 & d & 1 \end{vmatrix}$$

$a=b$ $(a-b)$

$a=c$ $(a-c)$

$a=d$ $(a-d)$ are factors

$b=c$ $(b-c)$

$b=d$ $(b-d)$

$c=d$ $(c-d)$

$$\Delta = (a-b)(a-c)(a-d)(b-c)(b-d)(c-d)$$