CHAPTER-13
LIMITS AND DERIVATIVES

LIMITS

1. Find $\lim_{x\to 2} f(x)$ where $f(x)=3$. (K)

2. Find $\lim_{x\to 1}(x^2+x)$. (U)

3. Evaluate: $\lim_{x\to 1}\left[x^3-x^2+1\right]$. (U)

4. Evaluate: $\lim_{x\to 3}\left[x(x+1)\right]$. (U)

5. Evaluate: $\lim_{x\to 3}\left[x+3\right]$. (U)

6. Evaluate: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$. (U)

7. Evaluate: $\lim_{r\to 1}\pi r^2$. (U)

8. Evaluate: $\lim_{x\to 4}\frac{4x+3}{x-2}$. (U)

9. Evaluate: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$. (U)

10. Evaluate: $\lim_{x\to 0}\frac{ax+b}{cx+1}$. (U)

11. Evaluate: $\lim_{x\to 0}\frac{\cos x}{(\pi-x)}$. (U)

12. Evaluate: $\lim_{x\to 0} x\sec x$. (U)

TWO MARK QUESTIONS

1. Discuss the limit of the function $f(x)=x+10$ at $x=5$. (K)

2. Discuss the limit of the function $f(x)=x^3$ at $x=1$. (K)

3. Find $\lim_{x\to 2} f(x)$ where $f(x)=3x$. (K)

4. Evaluate: $\lim_{x\to 1}\left[\frac{x^2+1}{x+100}\right]$. (U)

5. Evaluate: $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$. (S)

6. Evaluate: $\lim_{x\to 2}\frac{\sqrt{1+x}-1}{x}$. (S)

7. Evaluate: $\lim_{x\to 0}\frac{1-\cos x}{x}$. (S)

8. Evaluate: $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$. (U)

9. Evaluate: $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$. (S)

10. Evaluate: $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$, $a+b+c\neq 0$. (K)

11. Evaluate: $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$. (S)

12. Evaluate: $\lim_{x\to 0}\frac{\sin ax}{bx}$. (U)

13. Evaluate: $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$. *(S)*

THREE MARK QUESTIONS

1. Find $\lim_{x\to 0} f(x)$, where $f(x)=\begin{cases} x-2, & x<0 \\ 0, & x=0 \\ x+2, & x>0 \end{cases}$. (K)

2. Find $\lim_{x\to -1}\left[1+x+x^2+.....+x^{10}\right]$. (U)

3. Evaluate: $\lim_{x\to 2}\left[\frac{x^3-4x^2+4x}{x^2-4}\right]$. (S)

4. Evaluate: $\lim_{x\to 2}\left[\frac{x^2-4}{x^3-4x^2+4x}\right]$. (S)

5. Evaluate: $\lim_{x\to 2}\left[\frac{x^3-2x^2}{x^2-5x+6}\right]$. (S)

6. Evaluate: $\lim_{x\to 1}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]$. (S)

7. For any positive integer n, prove that, $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$. (K)

8. Evaluate: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$. (S)

9. Evaluate: $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}$. (S)

10. Evaluate: $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$. (S)

11. Evaluate: $\lim_{x\to \pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$. (U)

12. Evaluate: $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$. (S)

13. Evaluate: $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$. (A)

14. Evaluate: $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}, a,b \text{ and } a+b\neq 0$. (S)

15. Evaluate: $\lim_{x\to 0}(\operatorname{cosec} x-\cot x)$. (K)

16. Evaluate: $\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}$. (U)

17. Find $\lim_{x\to 5} f(x)$, where $f(x)=|x|-5$. (K)

FOUR MARK QUESTIONS

1. Find $\lim_{x\to 0} f(x)$ where $f(x)=\begin{cases}2x+3, & if\ \ x\leq 0\\ 3(x+1), & if\ \ x>0\end{cases}$. (K)

2. Find $\lim_{x\to 1} f(x)$, where $f(x)=\begin{cases}2x+3, & if\ \ x\leq 1\\ 3(x+1), & if\ \ x>1\end{cases}$. (K)

3. Find $\lim_{x\to 1} f(x)$, where $f(x)=\begin{cases}x^2-1, & if\ \ x\leq 1\\ -x^2-1, & if\ \ x>1\end{cases}$. (K)

4. Find $\lim_{x\to 0} f(x)$, where $f(x)=\begin{cases}\frac{|x|}{x}, & if\ \ x\neq 0\\ 0, & if\ \ x=0\end{cases}$. (K)

5. Find $\lim_{x\to 0} f(x)$, where $f(x)=\begin{cases}\frac{x}{|x|}, & if\ \ x\neq 0\\ 0, & if\ \ x=0\end{cases}$. (K)

6. Suppose $f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$ and $\lim_{x\to 1} f(x)=f(1)$ what are possible values of 'a' and 'b'? (S)

7. If $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$, for what value(s) of 'a' does $\lim_{x\to a} f(x)$ exists? (S)

8. If the function $f(x)$ satisfies $\lim_{x\to 1}\frac{f(x)-2}{x^2-1}=\pi$, evaluate $\lim_{x\to 1} f(x)$. (S)

FIVE MARK QUESTIONS

1. Prove geometrically that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$, θ is in radian and hence deduce that $\lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1$. (K)

DERIVATIVES

ONE MARK QUESTIONS

1. Find the derivative at $x=2$ of the function $f(x)=3x$. (K)

2. Find the derivative of the constant function $f(x) = a$ for a fixed real number 'a'.(U)

3. Find the derivative of $f(x) = 2x - \frac{3}{4}$. (K)

4. Find the derivative of the function: $f(x) = \sec x$. (K)

5. Find the derivative of the function: $f(x) = \operatorname{cosec} x$. (K)

6. Find the derivative of $f(x) = x + \frac{1}{x}$. (U)

7. Find the derivative of $f(x) = \sin x + \cos x$. (U)

8. Find the derivative of $y = -x$. (U)

9. Find the derivative of $f(x) = (-x)^{-1}$. (K)

10. Find the derivative of $y = (x + a)$. (K)

TWO MARK QUESTIONS

1. Find the derivative of $sin\, x \ at\ x = 0$. (U)

2. Find the derivative of $f(x) = 3 \ at\ x = 0$. (U)

3. Find the derivative of $f(x) = 3 \ at\ x = 3$. (U)

4. Find the derivative of $f(x) = 10x$. (K)

5. Find the derivative of $f(x) = x^2$. (K)

6. If $f(x) = 10x$, find $f'(x)$. (K)

7. Compute the derivative of $6x^{100} - x^{55} + x$. (S)

8. Find the derivative of $x^2 - 2 \ at\ x = 10$. (K)

9. Find the derivative of $99x \ at\ x = 100$. (K)

10. Find the derivative of $x \ at\ x = 1$. (K)

11. Find the derivative of the function: $f(x) = x^3 - 27$. (U)

12. Find the derivative of the function: $f(x) = (x-1)(x-2)$. (U)

13. Find the derivative of the function: $f(x) = \sin x \cos x$. (U)

14. Find the derivative of the function: $f(x) = 5\sec x + 4\cos x$, (U)

15. Find the derivative of the function: $f(x) = 3\cot x + 5\operatorname{cosec} x$. (U)

16. Find the derivative of the function: $f(x) = 5\sin x - 6\cos x + 7$. (U)

17. Find the derivative of the function: $f(x) = 2\tan x - 7\sec x$. (U)

18. Find the derivative of $f(x) = x\sin x$. (U)

19. Find the derivative of $y = 4\sqrt{x} - 2$. (U)

THREE MARK QUESTIONS

1. Find the derivative of $f(x) = \frac{1}{x}$. (U)

2.Prove that derivative of the function $f(x)=x$ is the constant function 1. (K)

3. Prove that derivative of $f(x)=x^n$ is $n\,x^{n-1}$ for any positive integer n. (K)

4. Find the derivative of $f(x)=1+x+x^2+x^3+.....+x^{50}$ at $x=1$. (S)

5. Find the derivative of $f(x)=\dfrac{x+1}{x}$. (U)

6. Compute the derivative of $\sin x$. (U)

7. Compute the derivative of $\cos x$. (U)

8. Compute the derivative of $\tan x$. (U)

9. Compute the derivative of $\cot x$. (U)

10. Compute the derivative of $\sec x$. (U)

11. Compute the derivative of $\operatorname{cosec} x$. (U)

12. Compute the derivative of $\sin(x+1)$. (S)

13. Compute the derivative of $\cos\left(x-\dfrac{\pi}{8}\right)$. (S)

14. Compute the derivative of $f(x)=\sin^2 x$. (U)

15. For some constant 'a' and 'b', find the derivative of $f(x)=(x-a)(x-b)$. (U)

16. For some constant 'a' and 'b', find the derivative of $f(x)=(ax^2+b)^2$. (U)

17. For some constant 'a' and 'b', find the derivative of $f(x)=\dfrac{x-a}{x-b}$. (U)

18. Find the derivative of $\dfrac{x^n-a^n}{x-a}$ for some constant 'a'. (U)

19. Find the derivative of $\dfrac{2}{x+1}-\dfrac{x^2}{3x-1}$. (S)

20. Find the derivative of $(5x^3+3x-1)(x-1)$. (U)

21. Find the derivative of $x^{-3}(5+3x)$. (U)

22. Find the derivative of $x^5\left(3-6x^{-9}\right)$. (U)

23. Find the derivative of $x^{-4}\left(3-4x^{-5}\right)$. (U)

24. Find the derivative of the function: $f(x)=\dfrac{1}{x^2}$. (U)

25. Find the derivative of the function: $f(x)=\dfrac{x+1}{x-1}$. (U)

26. Find the derivative of $f(x)=\dfrac{2x+3}{x-2}$. (U)

27. Compute derivative of $f(x)=\sin 2x$. (S)

28. Compute derivative of $f(x)=\cot x$. (U)

29. Find the derivative of $y=(px+q)\left(\dfrac{r}{x}+s\right)$. (U)

30. Find the derivative of $y=(ax+b)\left(cx+d\right)^2$. (U)

31. Find the derivative of $y=\dfrac{a}{x^4}-\dfrac{b}{x^2}+\cos x$. (U)

32. Find the derivative of $y = \sin(x+a)$. (S)

33. Find the derivative of $y = \operatorname{cosec} x \cot x$. (U)

34. Find the derivative of $y = x^4(5\sin x - 3\cos x)$. (U)

35. Find the derivative of $y = (x^2+1)\cos x$. (U)

36. Find the derivative of $y = (ax^2 + \sin x)(p + q\cos x)$. (U)

37. Find the derivative of $y = (x+\cos x)(x+\tan x)$. (U)

38. Find the derivative of $y = (x+\sec x)(x-\tan x)$. (U)

FOUR MARK QUESTIONS

1. Find the derivative of the function $f(x) = 2x^2 + 3x - 5$ *at* $x = -1$. Also prove that $f'(0) + 3f'(-1) = 0$. (A)

2. For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots\ldots\ldots \frac{x^2}{2} + x + 1.$ prove that $f'(1) = 100 f'(0)$.(A)

3. Find the derivative of $f(x) = \frac{x^5 - \cos x}{\sin x}$. (U)

4. Find the derivative of $f(x) = \frac{x + \cos x}{\tan x}$. (U)

5. Find the derivative of $f(x) = \frac{x + \cos x}{\tan x}$. (U)

5. Find the derivative of $y = \frac{ax+b}{cx+d}$. (U)

6. Find the derivative of $y = \frac{1+\frac{1}{x}}{1-\frac{1}{x}}$. (U)

7. Find the derivative of $y = \frac{1}{ax^2+bx+c}$. (U)

8. Find the derivative of $y = \frac{ax+b}{px^2+qx+r}$. (U)

9. Find the derivative of $y = \frac{px^2+qx+r}{ax+b}$. (U)

16. Find the derivative of $y = \frac{\cos x}{1+\sin x}$. (U)

17. Find the derivative of $y = \frac{\sin x + \cos x}{\sin x - \cos x}$. (U)

18. Find the derivative of $y = \frac{\sec x - 1}{\sec x + 1}$. (U)

20. Find the derivative of $y = \frac{a + b\sin x}{c + d\cos x}$. (U)

21. Find the derivative of $y = \frac{\sin(x+a)}{\cos x}$. (S)

26. Find the derivative of $y = \dfrac{4x + 5\sin x}{3x + 7\cos x}$. (U)

27. Find the derivative of $y = \dfrac{x^2 \cos\left(\dfrac{\pi}{4}\right)}{\sin x}$. (U)

28. Find the derivative of $y = \dfrac{x}{1 + \tan x}$. (U)

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