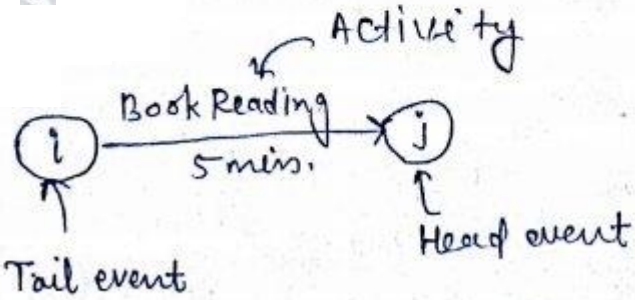
PERT-CPM

PERT & CPM

Project :- It is the group or combination of interelated activities that must be excuted in a certain fixed order before the entire task is completed. Activities are interelated related in a logical sequence in the sence that some activities can only be started when all the activities earlier to get completed

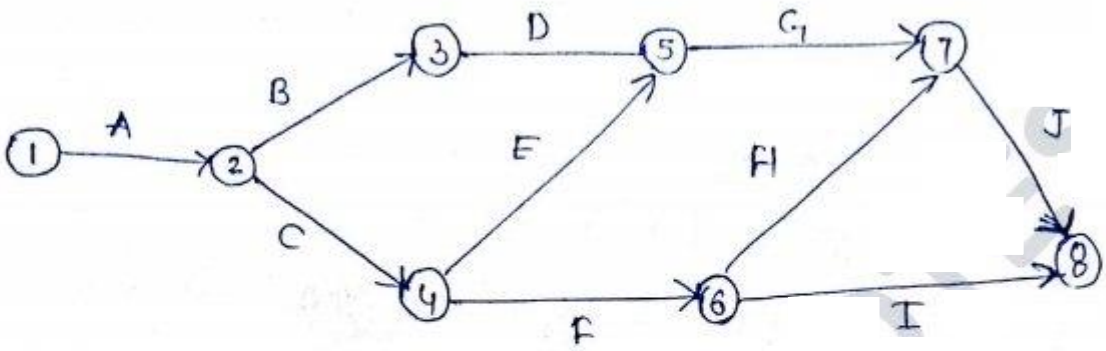
Event :- Event denote ~~of~~ point of time or Complisment occuring at a moment and is used to denote starting and end point of an activity event neither consume any time nor resources for its ~~Completion~~ completion.

Activity : It is a recognizable part of a project which consume time and resources for its Complition and it may involve ~~phyc~~ physical or mental work. When all the activities are executed then only a project gets Completed



PERT & CPM (4)

Network diagram! -



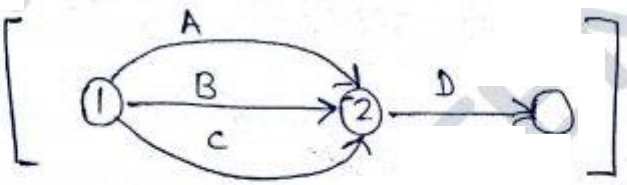
It is a graphical representation of the logical sequence in which different activities are inter related to each other while completing a project.

Rules for Network Construction!-

① An activity can only be started when all the activities earlier to ~~get~~ it are completed

② No two or more activity may have same head and tail event

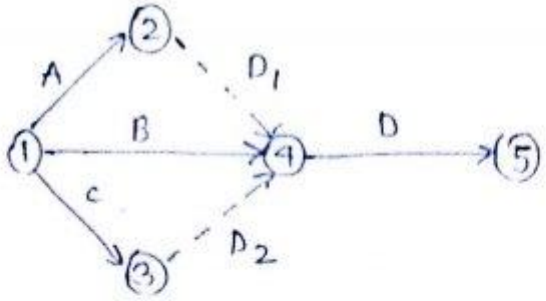
A → ① - ②
B → ① - ②
C → ① - ②



X (wrong)

In these condition to represent the same logic we need to use dummy activity

Dummy Activity: An activity which is used to show logic, dependency or relationship of one activity over the other but does not consume any time or resources for its completion this termed as dummy Activity.



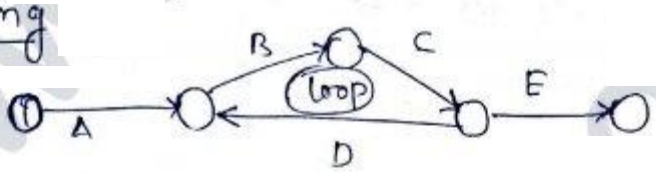
A - (1) - (2)
B - (1) - (4)
C - (1) - (3)

(3) Dummy activity should only be use when it is very necessary but there is no restriction on the number of dummy activity used.

(4) The left hand direction of arrow is indicative one a time flow from left to right on the network diagram.

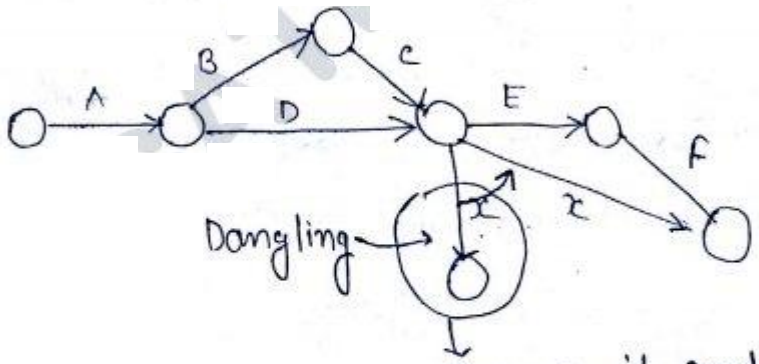
(5) There should be no looping and Dangling on the network diagram

Looping



there is no solution for such problems.

Dangling



Remove it and Connect to end event point

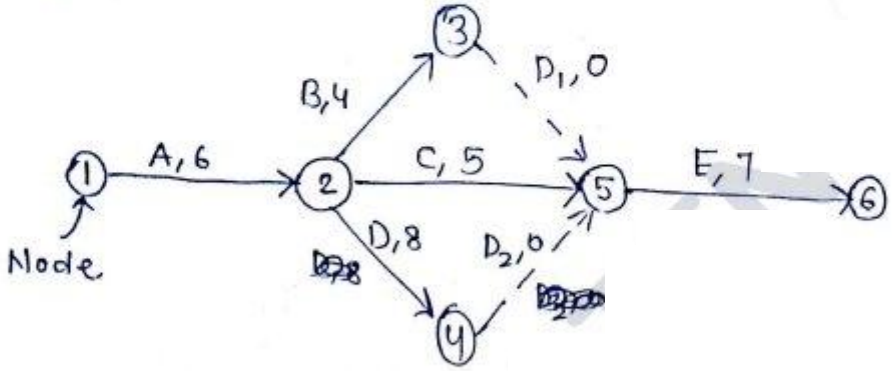
when activity others then the final activity does not have any succeseving activity then situation called dangling such activity should be connected to the last event on the network diagram.

Types of Network diagram:-

There are two types of network diagram

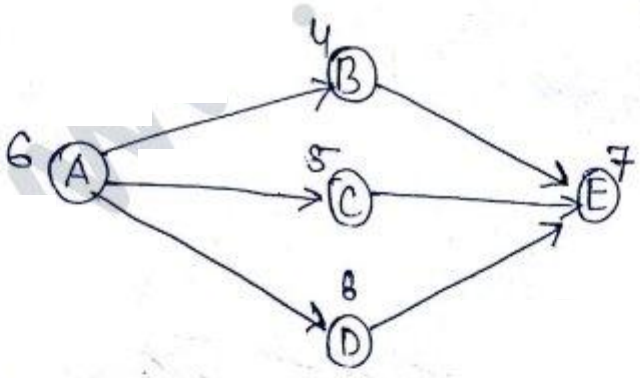
1) Event on Node (EON) / Activity on Arrow (AOA)

May be ⬡ or □ mainly use in PERT CPM



2) Activity on node (AON)

use in line Balancing

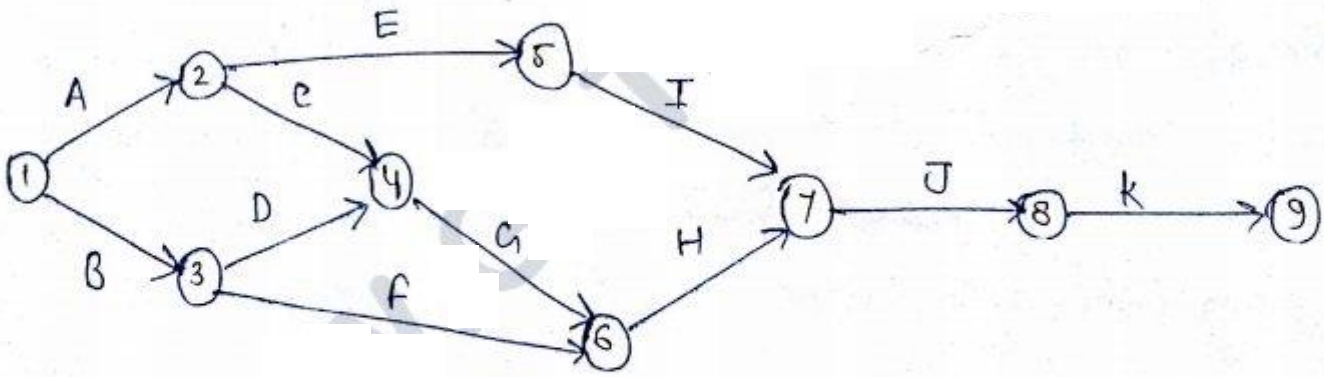


Activity on node diagram does not require dummy activity and it is consider to be simple and easy irrecpective of these advantages event on node diagram is preffered in PERT & CPM

Problem (20) Draw the network diagram for the following set of activities

Activity	Precedence
A	–
B	–
C	A
D	B
E	A
F	B
G	C, D
H	G, F
I	E
J	H, I
K	J

Soln



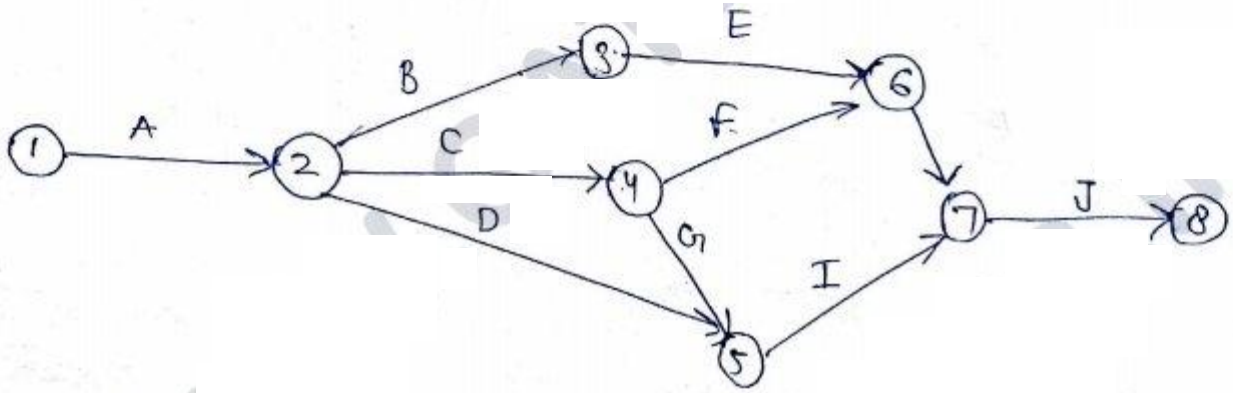
2, 3 & 4, 5 can interchange

Fulkerson's Rule:- Numbering of event

Problem (22) Draw the network diagram for the following set of activities.

Activity	A	B	C	D	E	F	G	H	I	J
Precedence	–	A	A	A	B	C	C	E,F	D,G	H,I

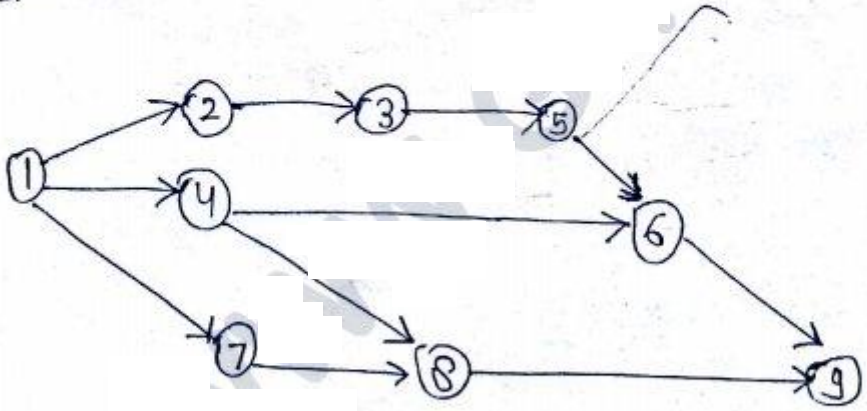
Soln



Problem (23) Draw the network diagram for the following set of activities IFS2015

Activity 1-2, 1-4, 1-7, 2-3, 3-5, 4-6, 4-8, 5-6, 6-9, 7-8, 8-9

Soln

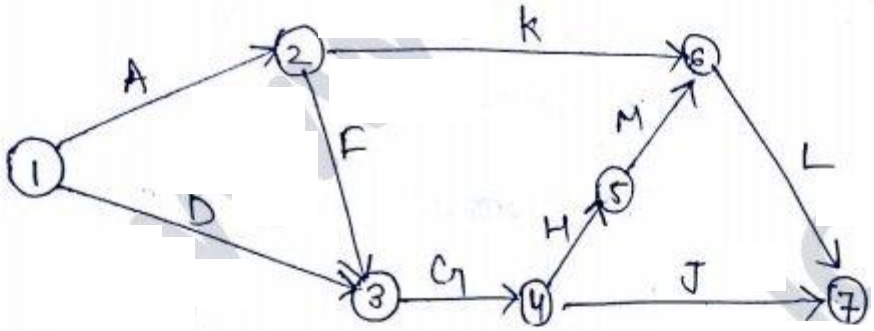


Problem (24) Draw the network diagram for the following conditions

(i) A & D start at same time
(ii) F follows A
(iii) K follows A but precedes L — Before starting of L, K should be completed
(iv) G follows D but precedes J
(v) G follows F but precedes H
(vi) M follows H but precedes L (M must be completed before starting of L)
(vii) J and L terminate at the same time

Soln

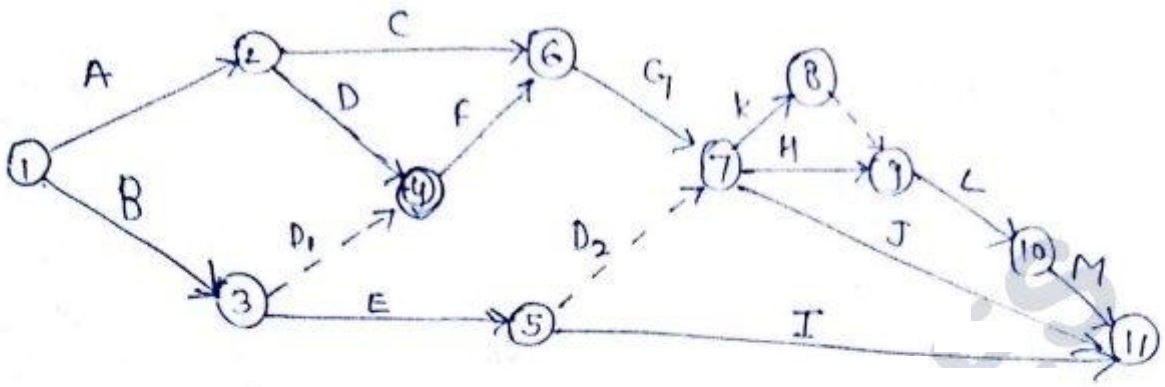
A	D	F	G	H	J	K	M	L
-	-	A	D, F	G	G	A	H	K, M



Problem (25) Draw the network diagram for the following set of activities

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M
Precedence	-	-	A	A	B	B, D	C, F	G, E	E	G, E	G, E	H, K	L

Soln Dummy Activities are required



Difference between PERT & CPM.

Pert	CPM
Program (Project) Evaluation & Review technique	Critical path method
① Event oriented	① it is activity oriented
② it is associated with probabilistic activities	② Associated with deterministic activities
③ it is based upon three time estimate to complete an activity	③ Based upon single time to complete an activity
④ it is used where time require to complete varies activities is not certain	④ Used for repeatative job where one has prior experience of handling similar project
⑤ it usally doesn't consider cost analysis	⑤ it gives importance to cost analysis and crashing is done to minimize the cost of CPM project
⑥ it is used mainly for research and development projects	⑥ used mainly for construction projects

PERT:- it is used for Uncertain project and it is based upon three time estimate and these are

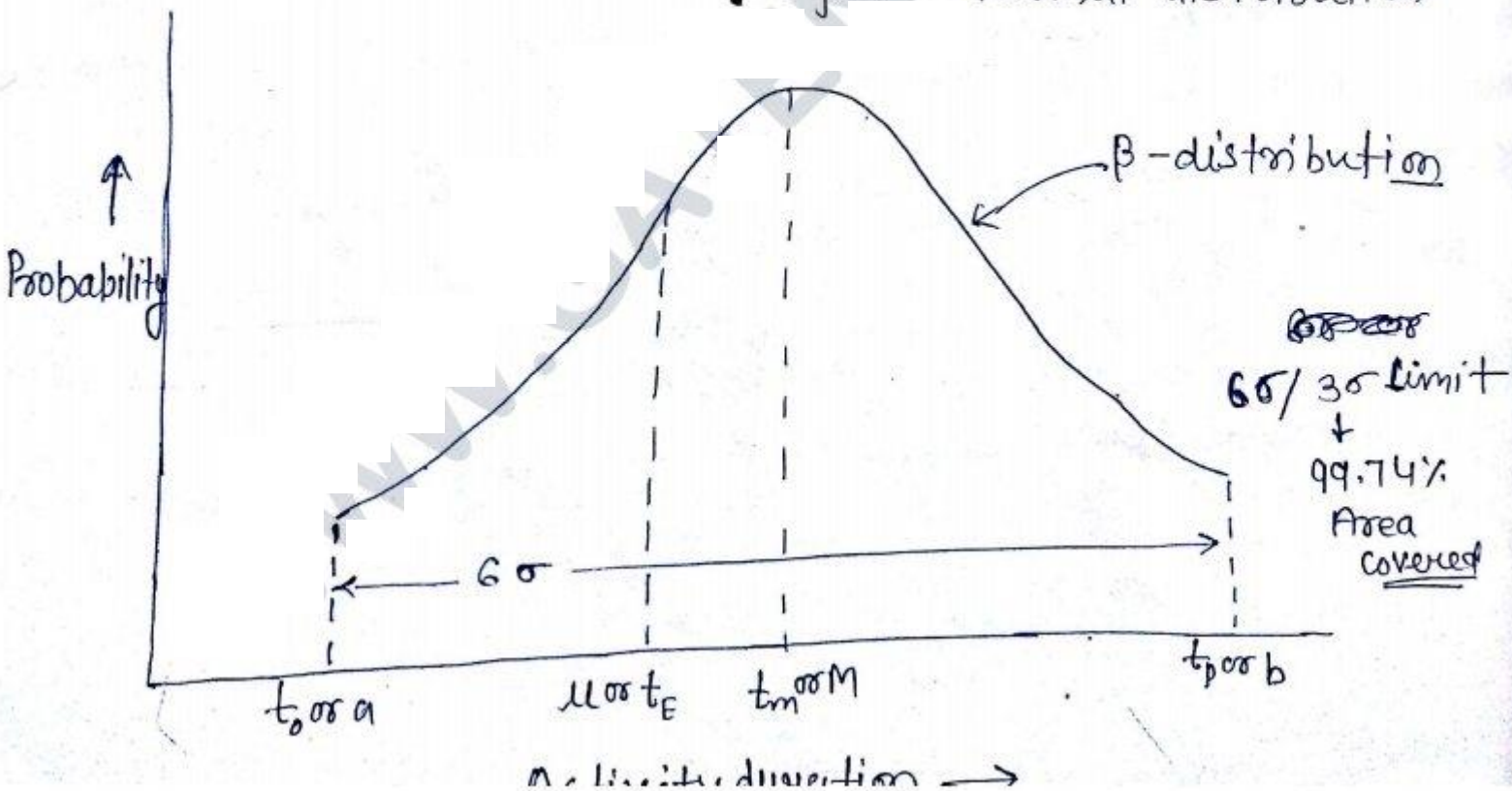
(i) optimistic time (t_o or a) — optimistic आशावादी
(ii) Pessimistic time (t_p or b) — pessimistic निराशावादी
(iii) Most likely time (t_m or m) — Most likely – सबसे अधिक संभावना

1) Optimistic time (t_o or a): it is the minimum time require to complete an activity when everything goes according to the plan.

2) Pessimistic time (t_p or b):- it is the maximum time required to complete an activity when everything goes against the plan.

3) Most likely time (t_m or m) it is the time required to complete an activity when executed under normal working condition.

{ Activity - β distribution
Project - Normal distribution



The fundmental assumption in PERT is that three time estimate form the end point of the distribution curve and activity is assumed to follow β-distribution. it is also assume that the probability of completing activity in time a and b is equal and the probability of completing in time m (t_m) is four times of a or b.

The average or expected time to complete an activity is given by t_E or μ

$$\mu \text{ or } t_E = \left(\frac{a+4m+b}{6}\right) = \left(\frac{t_o+4t_m+t_p}{6}\right)$$

$$\sigma = \left(\frac{b-a}{6}\right) = \left(\frac{t_p-t_o}{6}\right) \Rightarrow (b-a) = 6\sigma \Rightarrow \sigma = \frac{b-a}{6}$$

$$\text{Variance 'V'} = \sigma^2 = \left(\frac{t_p-t_o}{6}\right)^2$$

Note:- Variance Give the measure of uncertainty of activity completion Higher the value of variance larger the uncertainty will be

Critical Path (T_E) it is the maximum time Consuming path from the first event to last event in a network. The time take along the Critical path is termed as expected project completion time.

The activities along the path are critical activities and are represented by double line (⇒) arrow.

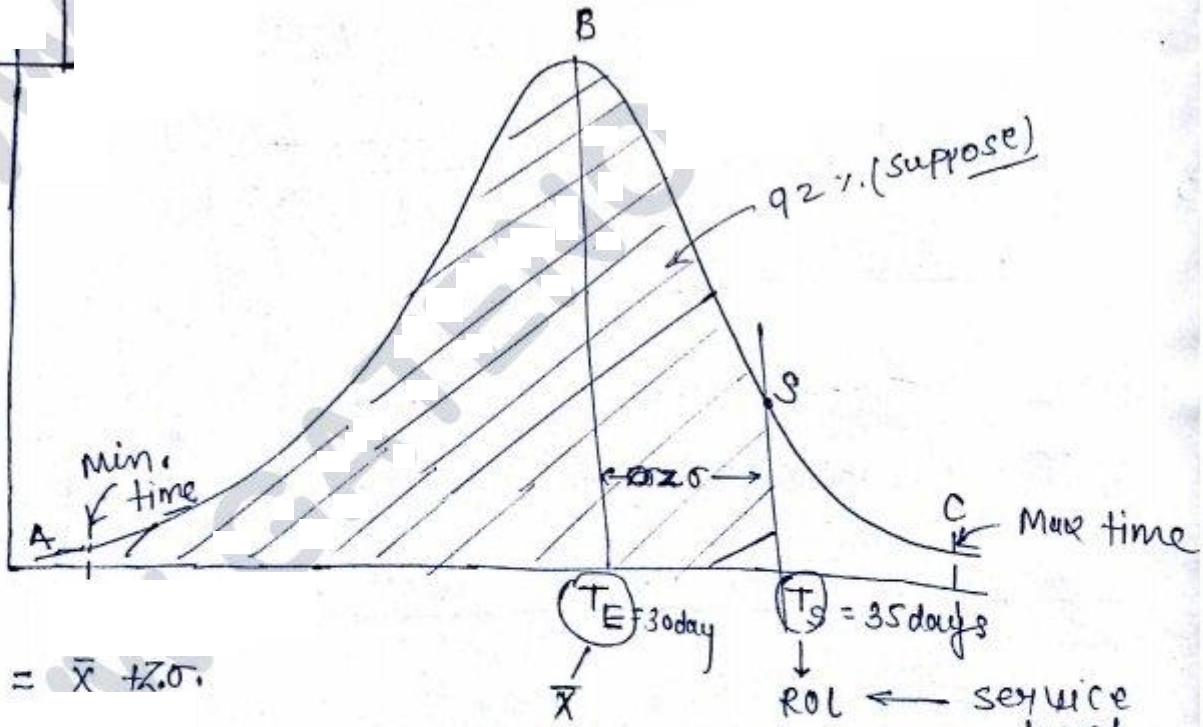
Probability of completing within scheduled time –

if T_E is the expected project completion time, σ standard deviation along the critical path then the probability of completing project within schedulod time (T_s) is given by following

$$Z = \frac{T_S - T_E}{\sigma}$$

Z	P(Z)
1	84.13%
0	50%
-1	15.86%
2	97%

where Z - standard normal variant.



$ROL = \bar{X} + Z.\sigma$

$T_S = T_E + Z.\sigma$

$$Z = \frac{T_S - T_E}{\sigma}$$

T_S

$$P(T_S) = \frac{\text{Area ABS}}{\text{Area ABC}}$$

Find Z and after that probability

e.g. (σ_1), σ_2, (σ_3) σ_4, σ_5, (σ_6), σ_7, (σ_8), (σ_9)

$$\sigma = \sqrt{\text{sum of variance along critical path}}$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_3^2 + \sigma_6^2 + \sigma_8^2 + \sigma_9^2}$$

T_E

↘ Max. time $= T_E + 3\sigma$

↘ Min. time $= T_E - 3\sigma$

Problem (25) For the following set of activities Draw the network diagram and determine

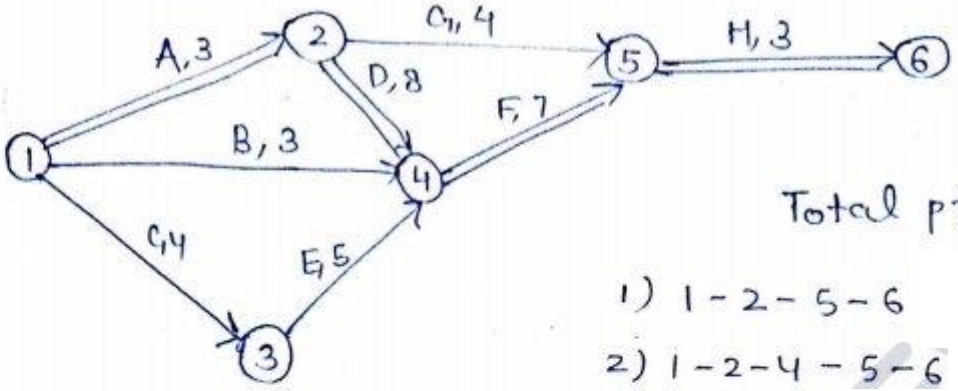
i) critical path and the expected time for project

ii) determine the probability of completing project in 19 day

(iii) if a company make and agreement to complete a project in 22 day and for failing the will pay Rs ~~50,000~~ per day as fine. find the probability that a fine may be paid but not exceeding Rs 1,00,000

Activity	Preceding	Time a	Time m	Time b	$t_e = \frac{a+4m+b}{6}$	$\sigma = \left(\frac{b-a}{6}\right)$
A	–	1	3	5	(3)	$0.66 = 2/3$
B	–	2	3	4	3	$0.33 = 1/3$
C	–	3	4	5	4	$1/3$
D	A	2	9	10	(8)	$4/3$
E	C	4	5	6	5	$1/3$
F	B, D, E	5	6	13	(7)	$4/3$
G	A	2	4	6	4	$2/3$
H	G, F	0	3	6	(3)	1

Sol^n



Total ptath

1) 1-2-5-6 = 10 day
2) 1-2-4-5-6 = 21 days
3) 1-4-5-6 = 13 days
4) 1-3-4-5-6 = 19 days

critical path ⇒ 1-2-4-5-6

i) Critical path A-D-F-H

& Project duration T_E = 21 days

(ii) $\sigma_A = 4/6$, $\sigma_D = 8/6$, $\sigma_F = 8/6$, $\sigma_H = 6/6$

$$\sigma = \sqrt{\sigma_A^2 + \sigma_D^2 + \sigma_F^2 + \sigma_H^2}$$

$$\sigma = \sqrt{(4/6)^2 + (8/6)^2 + (8/6)^2 + (6/6)^2}$$

$$\sigma = \sqrt{5} \Rightarrow \sigma = 2.236 \text{ days}$$

T_S = 19 days

T_E = 21 days

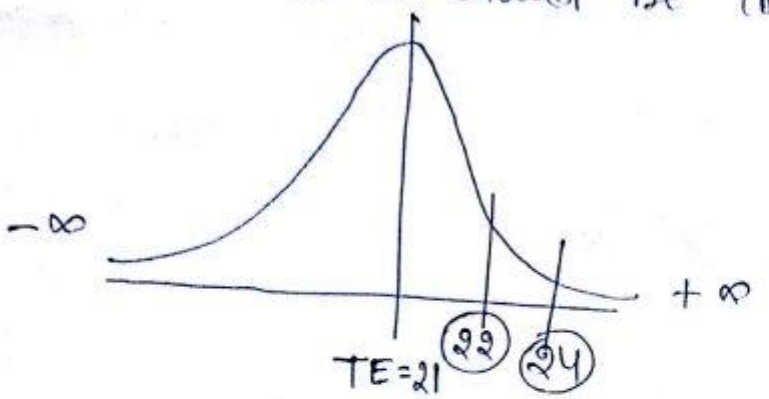
$$Z = \frac{19-21}{2.236}$$

$$Z = -0.8944$$

↓ Prob.

Probability 18.55%

(iii) Fine should not exceed 1 lakh & fine Rs 50,000/day
So it should be completed before 24 days



$$z = \frac{T_S - T_E}{\sigma} \quad \text{Fixed}$$

$$z_1 = \frac{24-21}{2.236} = 1.341$$

↓ Prob.
91.01 %.

$$z_2 = \frac{22-21}{2.236} = 0.4472$$

↓ Prob
67.26%.

So the probability = 91.01 − 67.26
= 23.75 %.

* Probability of project work completed in expected time (T_E) is always 50 %.

$$z = \frac{T_S - T_E}{\sigma} \qquad z = \frac{T_E - T_E}{\sigma}$$

$$z = 0$$

↓

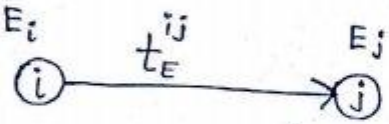
Prob.= 50 %.

Critical Path:- The procedure for finding critical path is similar in both PERT & CPM and it consist of two phase

1) forward path computation
2) Backward path computations

1) Forward Path Computation :-

In this we compute the time by which an event is expected is to be completed as the earliest.



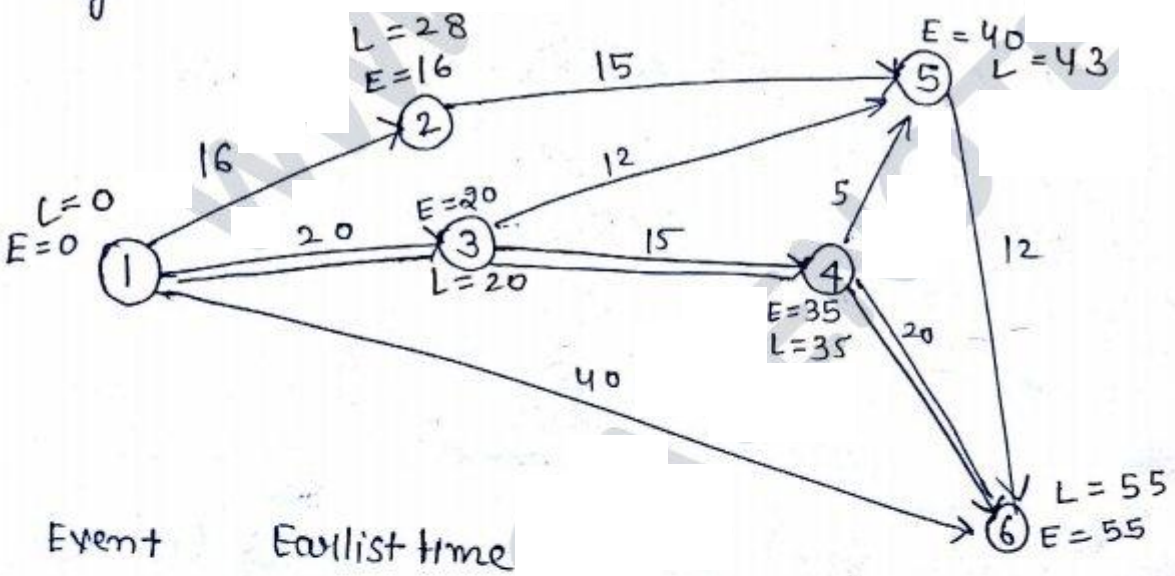
E_j = Maximum of all $[E_i + t_E^{ij}]$

E_i = Earliest expected time for event i

E_j = Earliest expected time for event j

t_E^{ij} = Expected time for activity ij

eg.

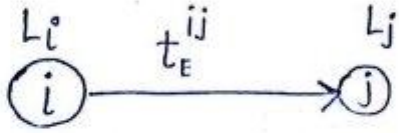


Event	Earliest time
1	0
2	16
3	20
4	35
5	Max.[31, 32, 40] = 40
6	Max.[52, 55, 40] = 55

[maximum of all incoming entries on an event]

2) Backward Path Computation

In this we compute the time by which an event must be completed at the latest.

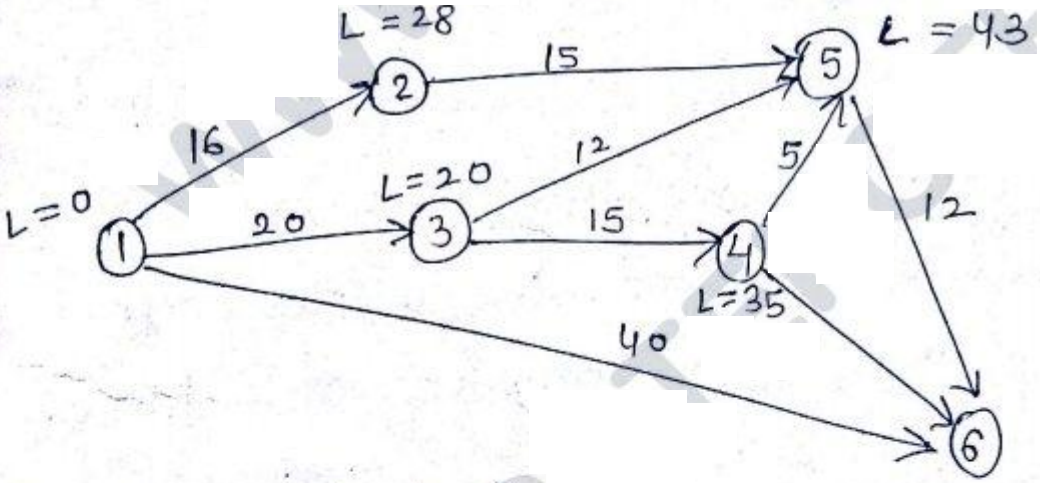


$$L_i = \text{Minimum of All } \left[L_j - t_E^{ij} \right]$$

L_i - latest allowable time for event i

L_j - latest allowble time for event j

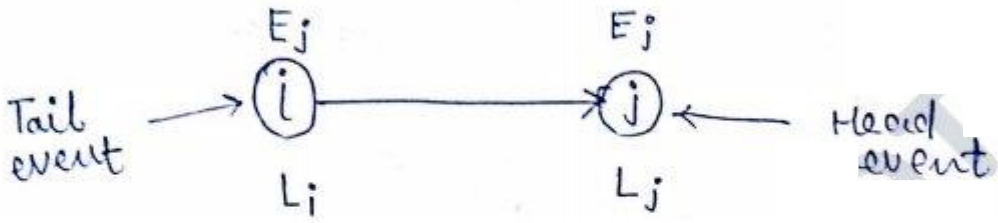
t_E^{ij} - expected time for activity ij



Event	latest time
6	55
5	43
4	Min. [38, 35] = 35
3	Min. [31, 20] = 20
2	28
1	Min. [12, 0, 15] = 0

[Minimum of outgoing entries on an event

⇒ for any activity to be critical the following three conditions must be satisfied



1> Head event slack = 0, $L_j - E_j = 0$

2> Tail event slack = 0 $L_i - E_i = 0$

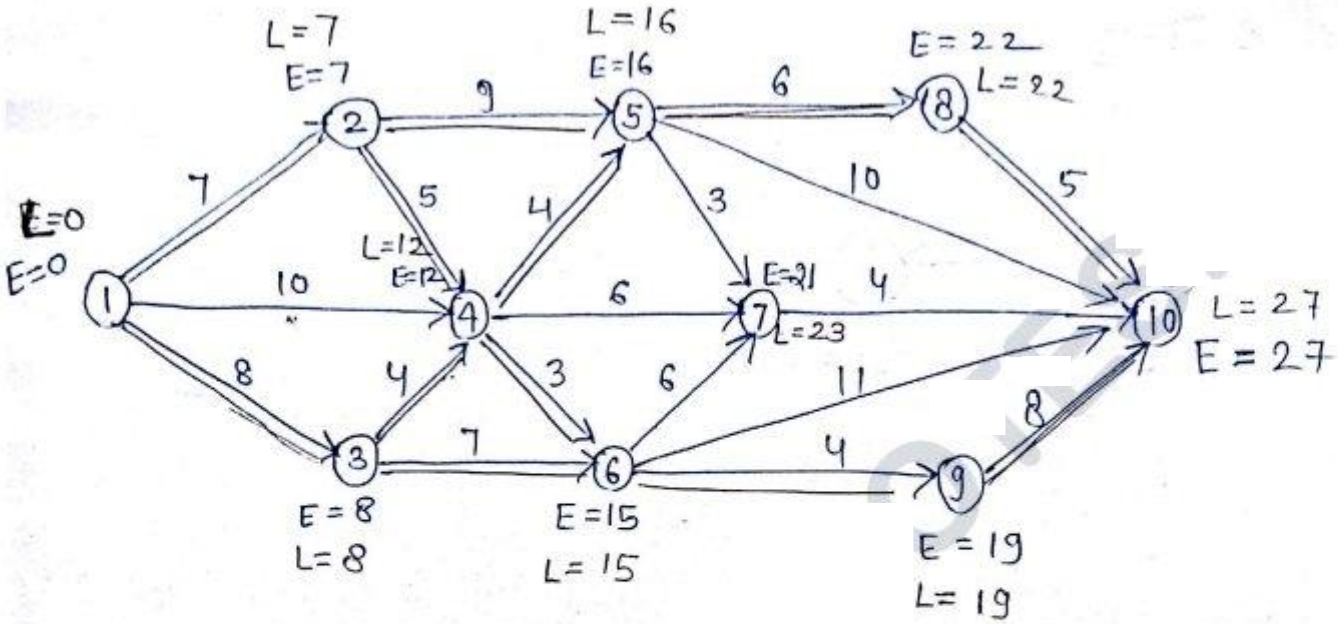
3> $L_j - L_i = E_j - E_i = t_E^{ij}$

So the critical path for previous example ①-③-④-⑥

Critical Path- critical path is termed as critical because any activity on this path is delayed by certain amount of time the whole project is delayed by the same amount of time.

Problem (26) For the network diagram shown below find the critical and expected project completion time.

Sol^n E = 27



1) 1 - 2 - 5 - 8 - 10

2) 1 - 2 - 4 - 5 - 8 - 10

3) 1 - 2 - 4 - 6 - 9 - 10

4) 1 - 3 - 6 - 9 - 10

5) 1 - 3 - 4 - 6 - 9 - 10

6) 1 - 3 - 4 - 5 - 8 - 10

Always Find Critical Activity and double them

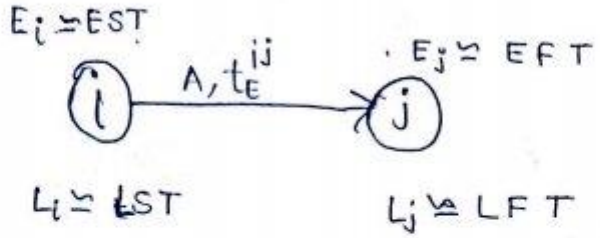
Note:- In case of PERT if there are more than one critical path then in order to determine probability we select the path having maximum standard deviation. (Maximum σ)

$$Z = \frac{T_S - T_E}{\sigma}$$ const.

e.g. $\sigma_1 = 4.8$ $\sigma_2 = 3.9$ $\sigma_3 = 5.5$

σ_1 → 76%. σ_2 → 85%. σ_3 → 62%.

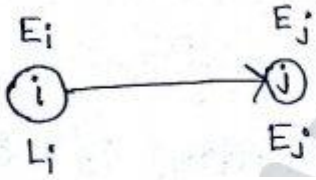
Slack & Float :-



$$E_i + t_E^{ij} = E_j \Rightarrow \boxed{EST + t_E^{ij} = EFT}$$

The term like earliest expected time, latest allowable time and slack corrosponing to event in pert while the terms like Earliest start time (EST), Earliest finish time (EFT), Latest start time (LST), Latest finish time (LFT), Flot corrosponding to activities in CPM.

1) Slack or Event float :-



$$\boxed{S_i = L_i - E_i}$$

$$\boxed{S_j = L_j - E_j}$$

Slack denote the amount of time by which a particular event can be delayed without delaying the project schedule

2) Float :-

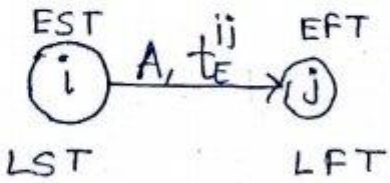
Three types of Float Values

(a) Total float.

(b) Free float.

(c) Independend float.

a) Total Float



for Any Activity

$$TF = L_j - (E_i + t_E^{ij})$$

$$TF = LFT - EFT = LST - EST$$

Total Float denotes the amount of time by which an activity can be delayed without delaying the project complition date. it is the extra time available for an activity without delaying the project schedule. If total flot value -

Positive - Resources are surplus and can be allocated for other activities

Negative - If resources are not sufficient and activity may not Complete on time

Zero - Resources are just sufficient to Completed activity on time.

b) Free float : (FF) :-

it is that part of total float which can be use without affecting the float of succeeding activity, it is extra time by which an activity can be delayed so that the succeeding activity can be started at their earlies start time (EST).

$$FF = TF - \text{Head event slack} \qquad FF = E_j - (E_i + t_E^{ij})$$

c) Independend float (IF) :-

It is the amount of time which can be used without affecting either Head or the tail event

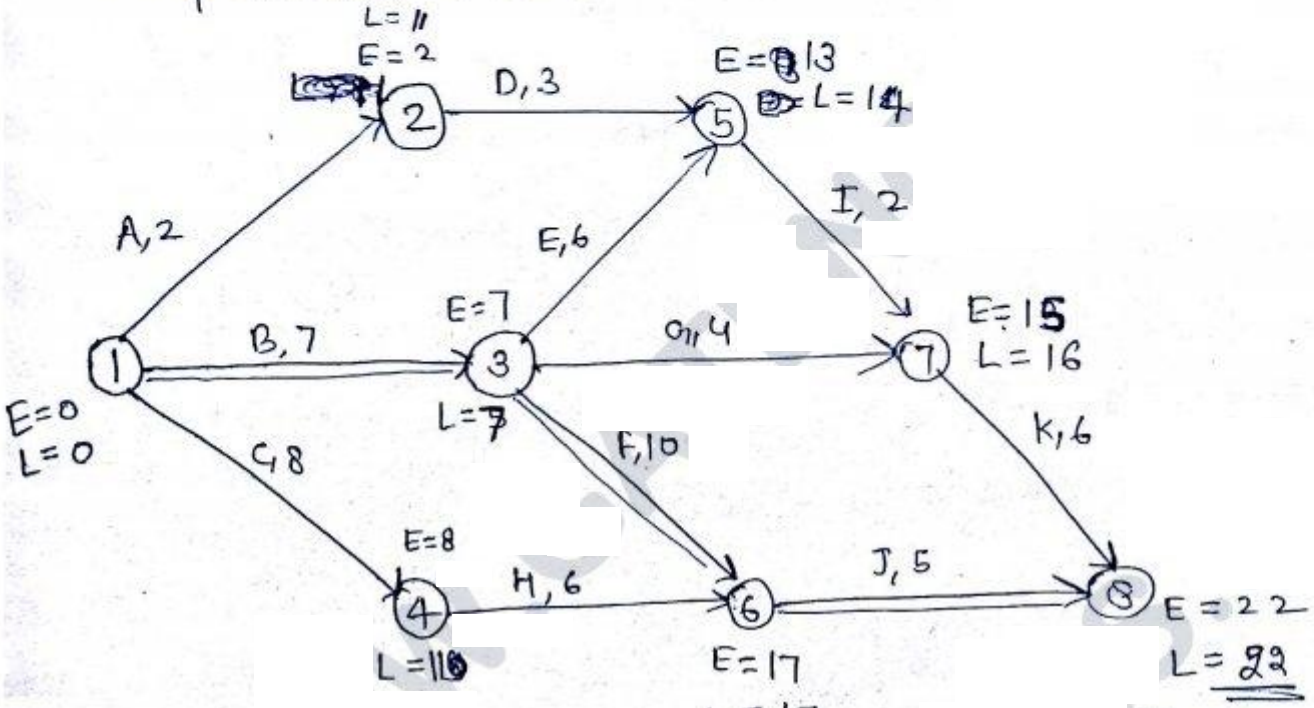
$$IF = FF - \text{Tail event slack} \qquad IF = E_j - (L_i + t_E^{ij})$$

$$If = TF - \text{Head event slack} - \text{Tail event Slack}$$

$$TF \geq FF \geq If \quad \text{for}$$

* for critical path activity $\{TF = FF = IF = 0\}$ Always

Problem (27) for the network diagram shown below find the critical path and Expected project completion time (T_E). Draw a table showing the detail for each activity along with Total, free, independent float.



Activity	t_E^{ij}	Earliest EST	Earliest EFT	latest LST	latest LFT	Float TF	Float FF	Float IF
A, 1-2	2	0	2	9	11	9	0	0
B, 1-3	7	0	7	0	7	0	0	0
C, 1-4	8	0	8	3	11	3	0	0
D, 2-5	3	2	5	11	14	9	8	-1* (0)
E, 3-5	6	7	13	8	14	1	0	0
F, 3-6	10	7	17	7	17	0	0	0
G, 3-7	4	7	11	12	16	5	4	4
H, 4-6	6	8	14	11	17	3	3	0
I, 5-7	2	13	15	14	16	1	0	-1* (0)
J, 6-8	5	17	22	17	22	0	0	0
K, 7-8	6	15	21	16	22	1	1	0

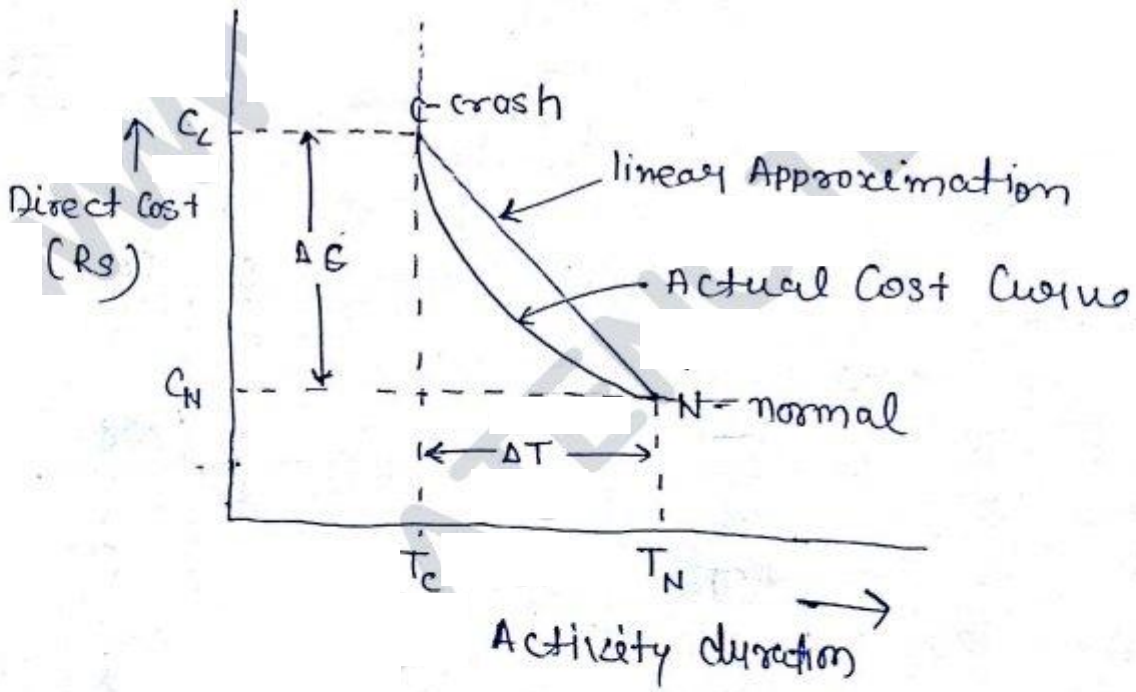
start

D → IF = 8 - 9 = -1 (for objective) in conventional ⇒ -1*(0) (-1) doesn't make any sense

CRASHING / Time Cost model :—

It is an extension of critical path method that considers a compromise between the time and cost required to complete a project. The total cost of any project consist of direct and indirect cost involve in its execution

1) direct Cost:- it is the cost directly involve in the completion of an activity and it include direct material, direct labour, cost of machine, equipment etc.



Crash time is the minimum activity duration to which an activity can be compressed by increasing the resources and hence increasing direct cost. The slope of line give the amount of decrease in the direct cost per unit time for crashing an activity.

$$\text{Cost time Slope} = \frac{\Delta C}{\Delta T} = \frac{C_C - C_N}{T_N - T_C}$$

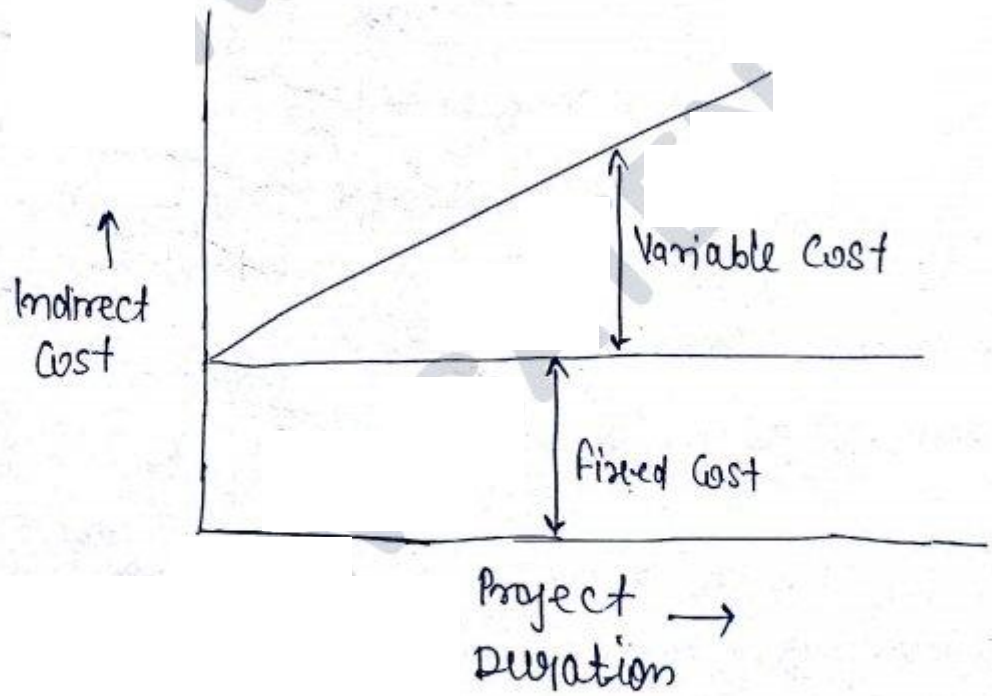
for e.g.

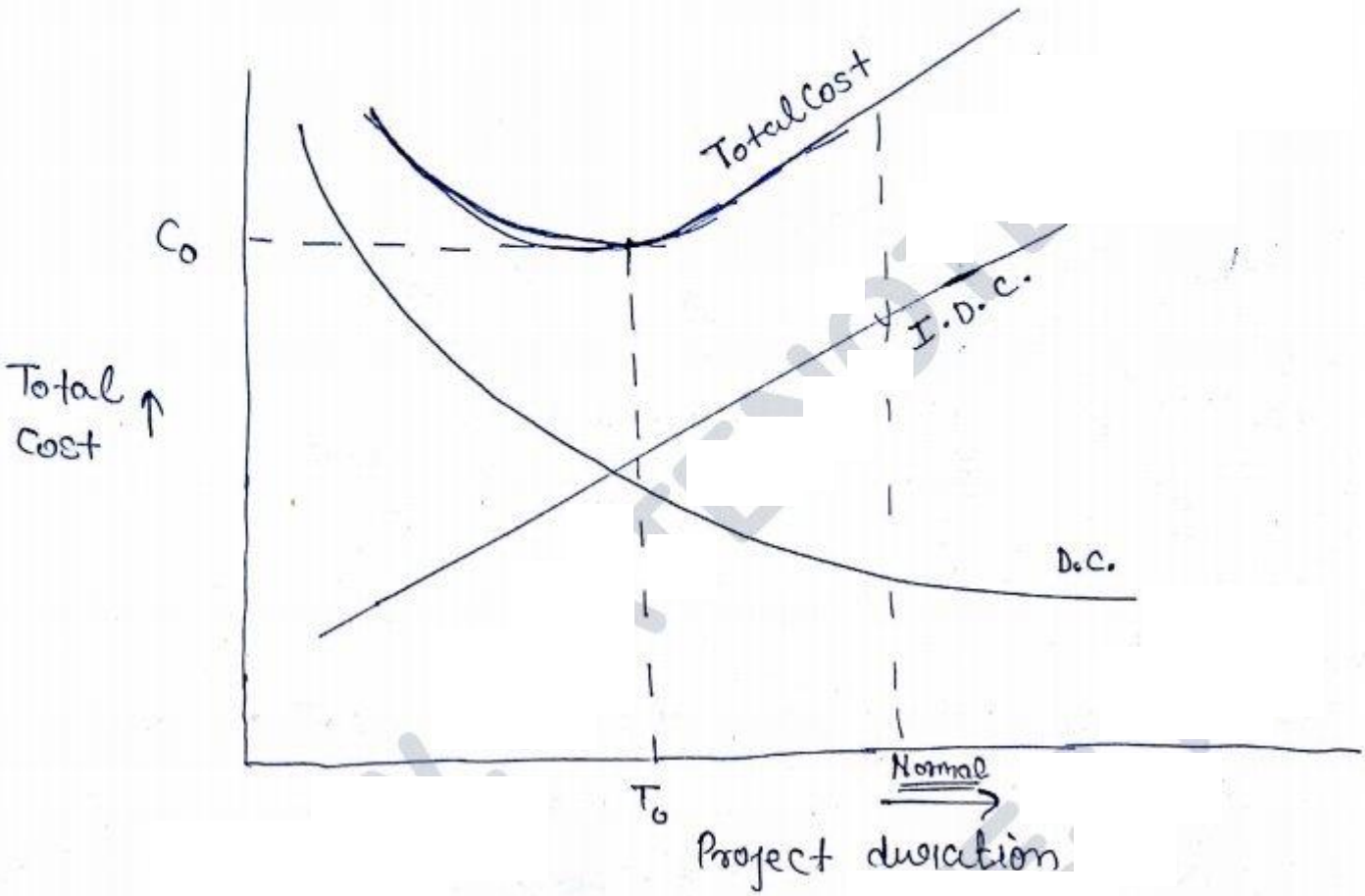
Normal		Crash	
10day	Rs 8000	6 days	Rs 18000

$$\text{Cost time Slope} = \frac{\Delta C}{\Delta T} = \frac{\text{Rs } 10000}{4 \text{ days}} = 2500 \text{ Rs/day}$$

2) Indirect Cost:-

This is the cost not directly invalve in the execution of an activity but it's complusory for safe and timely complition of project





The objective of crashing a network is to determine optimum project duration corresponding to minimum cost of project and the steps involved are -

i) In the critical path select the critical activity having minimum cost slope

ii) Reduce the duration of this activity by one time unit

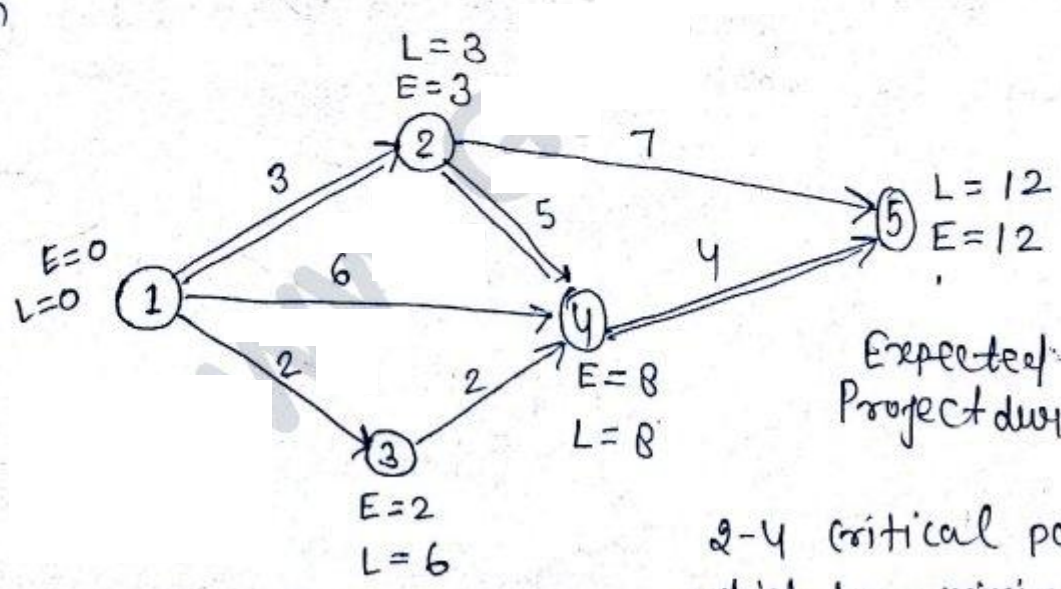
iii) Revise the network diagram by adjusting the time and cost of crashed activity. Again find critical path, project duration and total cost of project

iv) If the optimum project duration is obtain stop. otherwise repeat from step(i).

Problem (28) Draw the network diagram and crash the network to optimum project duration corresponding to minimum cost of project. It is given that indirect cost is Rs 900 per day

i.j Activity	Normal Time(day)	Normal Cost(Rs)	Crash Time(day)	Crash Cost(Rs)	$\frac{\Delta C}{\Delta T}$
1-2	3	500	2	1000	500
1-3	2	750	1	1500	750
1-4	6	1400	4	2600	600
2-4	5	1000	3	1800	400
2-5	7	1150	6	1450	300
3-4	2	800	2	800	–
4-5	4	1000	2	2400	700

Sol^{n}



Expected Project duration T_E = 12 days

2-4 critical path Activity which have minimum cost slop.

T_E = 12 days

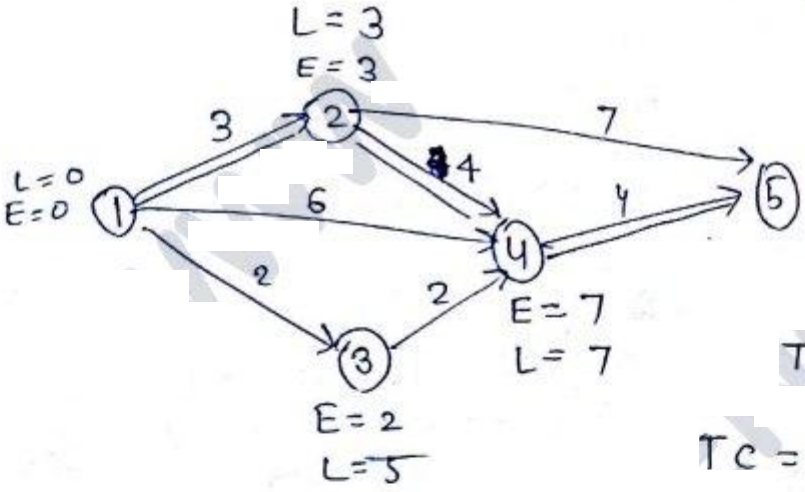
TC = D.C. + I.D.C.

D.C. = Rs 6600

IDC = 12 × 900 = Rs 10800

TC = Rs 17400

Now crashing the minimum cost activity along the critical path. it is activity 2–4. So crasing it by 2 days (1+1) the revised network diagram and cost of project is as given below



[2–4 = 4 day]

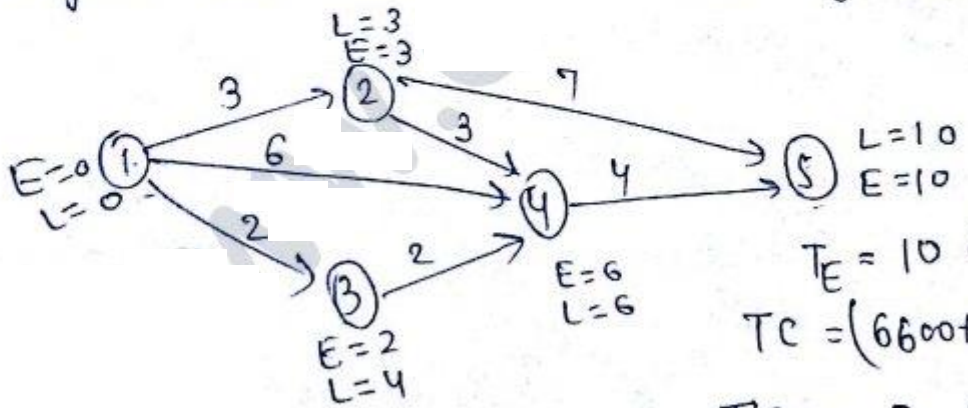
This step not require just crash by 2 days

T_E = ~~11~~ 11 days

DC = 6600 + 400 × 1

TC = Rs 16500 + 400 = 16900 Rs

Again we have to crash it by 1 day.



T_E = 10 days

TC = (6600 + 400 × 2) + (10 × 900)

TC = Rs 16400

Now the network has three critical path and by crashing any one activity Project duration does not change so we need to crash at-least two activities simultaneously which give three option Out of these we select the option for which the sumation of Cost Slop is minimems

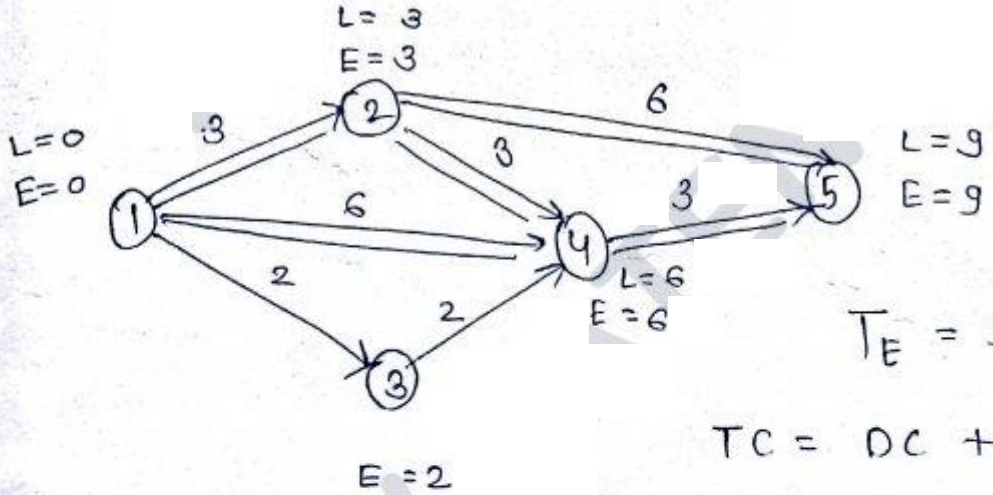
critical paths

→ 1-2-5
→ 1-2-4-5
→ 1-4-5

O Crash only common activities

1) 1-2 & 1-4 → 500+600 = 1100
2) 2-5 & 4-5 → 300+700 = (1000)
3) 1-2 & 4-5 → 500+700 = 1200

Crash the Activity 2-5 & 4-5 each by one(1) day.



T_E = 9 days

TC = DC + IDC

DC = Rs 6600 + 400×2 + 1000
(1st crash, 2nd crash)

DC = Rs 8400

IDC = 9 × 900 = Rs 8100

TC = Rs 16500 {Cost increase}

T_o = 10 day
C_o = 16400 Rs

1st crash

DC ↑ Rs 8000

IDC ↓ Rs 18000

TC ↓ 10000 Rs

2nd crash

DC ↑ Rs 1000

IDC ↓ Rs 900

TC ↑ Rs 100