# 7 INFORMATION PROCESSING

## Learning Objectives

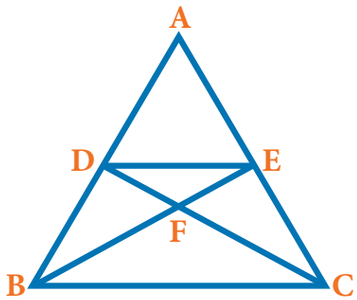
- To determine the number of possible orderings of an arbitrary number of objects following certain procedures.
- To learn a SET game for developing logical thinking .
- To investigate the role of map colouring in representing and modelling of mathematical ideas.
- To observe the Fibonacci pattern in physical and biological phenomena.
- To choose the best method in finding the HCF of numbers.
- To understand how information can be processed in, encryption and decryption.
- To consider alternatives in shopping before making a purchase, calculate the unit price for each items and make purchase in limited budget.
- To understand how to pack things efficiently in a given space and find the optimal solution.

### Recap

Before to learn, we recall the concepts like listing, counting, pattern of Fibonacci series and calculate the unit price of the products by answering the following questions

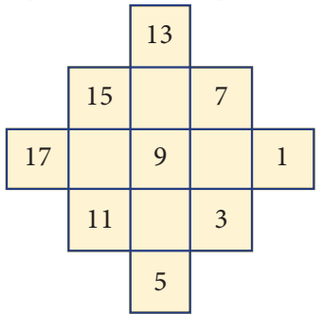
1. Find the number of all possible triangles that can be formed from the triangle given below.

3ZBIXY



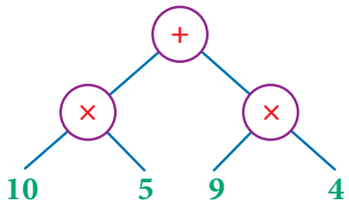

Ans:_____

2. Use the numbers given in the figure to form a 3 x 3 magic square.



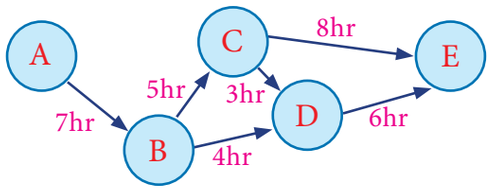

Ans:

3. Convert the tree diagram into a numeric expression.




Ans:_____

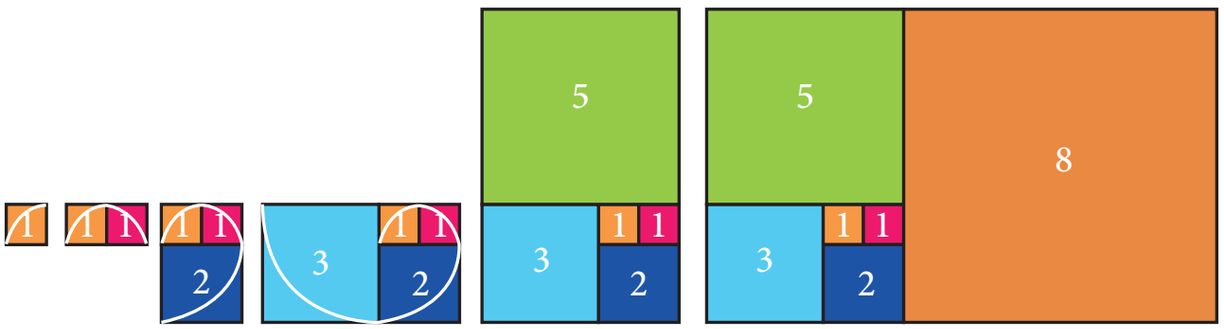
4. (i) Find the total time taken by the bus to reach from A to E via B , C and D.
(ii) Find which is the shortest route from A to E.




(i) .....hrs.

(ii) A → ... → ... → E

5. Connect the Fibonacci squares through diagonals by curve from corner to corner across each square to draw a Golden Spiral.



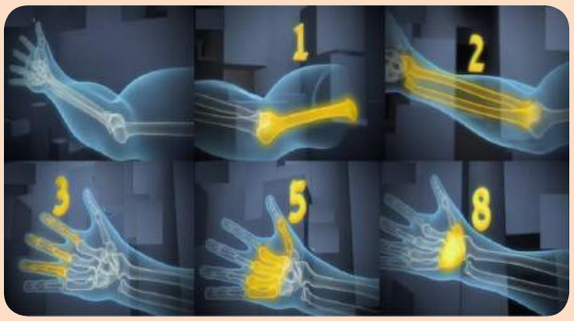

FIBONACCI SQUARES

6. When you plan to buy a shirt, one shop offers a discount of ₹200 on MRP ₹1000 and another shop offers 15% discount on the same MRP. Where would you buy?
7. Amazing park is offers a package deal of 5 entrance passes for ₹130. If one entrance pass normally costs ₹30, how much will you save by taking advantage of this special deal?

## 7.1 Introduction

Success in Mathematics depends on the development of the number sense, logical thinking and cognitive skills. In this class, we find practical solutions for day to day life situations like selection procedures from various forms, colour the given map with minimum number of colours. We are also going to see Fibonacci number pattern in physical and biological phenomena and also discuss some cipher for the higher order thinking which is to do well in the competitive exams. We will also discuss how will you be a wise consumer while shopping and packing. Meanwhile we will play a game to support your mind blooming activities. All the above discussions are very helpful to you, to explore other chapters and improve your interest in Mathematics.

**MATHEMATICS ALIVE – INFORMATION PROCESSING IN REAL LIFE**




Biological example for the Fibonacci numbers

Mason packing bricks for constructing a wall between two pillars

## 7.2 Principles of Counting

There are some basic counting techniques which will be useful in determining the number through different ways of arranging or selecting objects. The basic counting principles are given below.

### 7.2.1 Addition principle

If there are two selections such that they can be done **independently** in **m** ways and **n** ways respectively, then either of the two selections can be done in (**m** + **n**) ways.

Let us learn about this addition principle of counting as given below :

***Situation:***

In class VIII, there are 16 boys and 9 girls. The teacher wants to select either a boy or a girl as the class leader. Let us see, in how many ways can the teacher select the class leader.

Fig. 7.1

The teacher can select the class leader in any one of the following ways.

(i) In the first choice, the teacher can select **a boy among 16 boys in 16 ways** (who ever may be of the 16 boys).

(ii) In the second choice, the teacher can select **a girl among 9 girls in 9 ways** (who ever may be of the 9 girls).

Hence, the teacher can select the class leader who is **a boy** or **a girl** in **25 different ways (16 boys + 9 girls).**

Thus, we come to know, if a selection **A** can occur in ***m*** ways and another selection **B** can occur in ***n*** ways, and suppose that **both cannot occur together**, then **A** or **B** can occur in (***m*** + ***n***) ways. Let us see an example.

## Example 7.1

If you are going to a hotel to have food and the hotel offers different food items as shown in Fig 7.2. Find how many ways are possible to have either tiffin or meals?

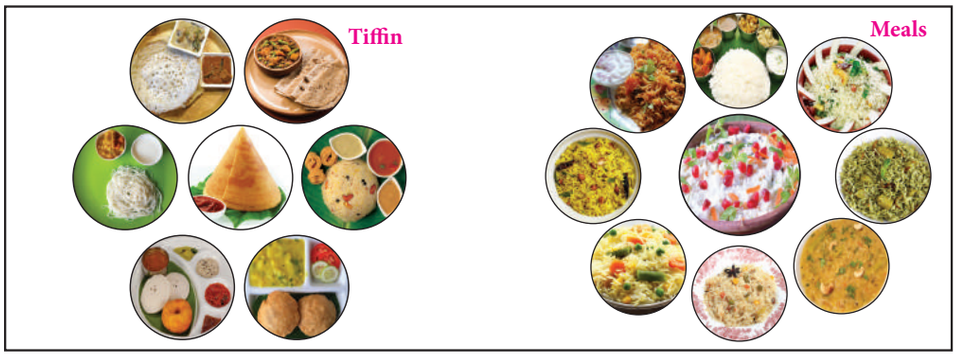



Fig. 7.2

***Solution:***

From the above Fig. 7.2, we come to know

(i) For tiffin, we can choose one among **7 items** in 7 ways

(ii) For meals, we can choose one among **9 items** in 9 ways.

Therefore, there are **16** (**7 tiffin items** + **9 meals items**) **different ways** by which we can choose any one food.

### 7.2.2 Multiplication principle

If a selection can be performed in ***m*** ways, following which another selection can be performed in ***n*** ways, and both the selections are **dependent** on each other then, the two selections together can be performed in exactly ($m \times n$) different ways.

Now, we shall learn about multiplication principle of counting from the following situation.

***Situation:***

There are 3 places in a city namely A,B and C. There are 3 routes **a1, a2 and a3** from A to B. There are 2 different routes, **b1 and b2** from B to C as shown in the Fig 7.3.

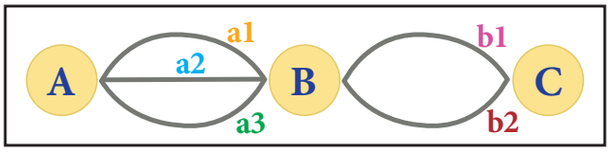



Fig. 7.3

Suppose a person wants to travel from A to C via B. Lets us see the number of ways he can go from place A to C via B.

(i) In the first way, he can go from **A to B in 3 routes** and

(ii) In the second way, he can go from **B to C in 2 different routes.**

Therefore, the total number of ways in which he can travel is **6** (**3 × 2**) routes as shown in Fig 7.4.

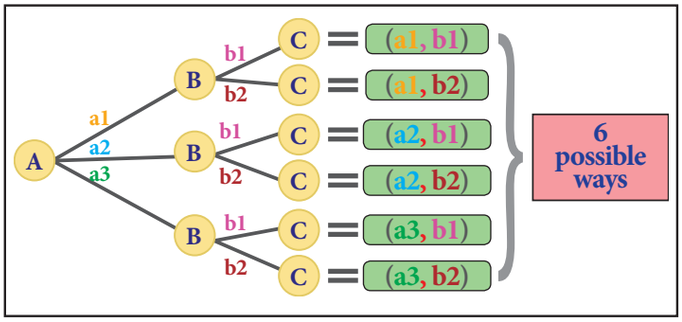



Fig. 7.4

Thus, we come to know, if a selection **A** can occur in ***m*** ways and another selection **B** can occur in ***n*** ways, and both the selections are **dependent** on each other then, the two selections can be performed in exactly ($m \times n$) different ways. Let as learn more about from the following examples.

## Example 7.2

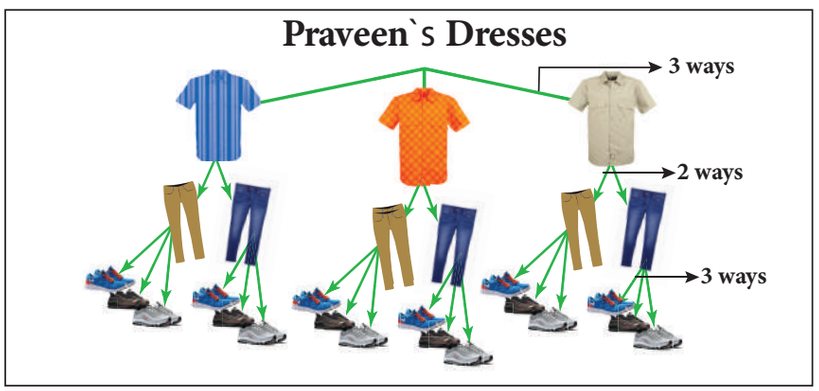
Praveen bought **3 shirts, 2 jeans and 3 pairs of shoes** for his birthday. In Fig. 7.5 shows the different ways of wearing the dress. In how many different ways can Praveen wear a dress on his birthday?

Fig. 7.5

***Solution:***

Here, Praveen has 3 shirts  , 2 jeans  and  3 pairs of shoes.

He can wear a dress either this way  or he can have the choices as shown in the Fig. 7.6




**3 shirts × 2 jeans × 3 pairs of shoes = 18 ways**

Fig. 7.6

Therefore, Praveen can wear his dress in **18** **(3×2×3)** **different possible ways** on his birthday.

## Example 7.3

In class VIII, a math club has four members **M,A,T** and **H.** Find the number of different ways, the club can elect

(i) a leader,

(ii) a leader and an assistant leader.

***Solution:***

**(i) To elect a leader**

In class VIII, a math club has four members namely **M, A, T** and **H.**

Therefore, there are **4** **(4 × 1)** **different ways** by which they can be elected a leader.

### (ii) To elect a leader and an assistant leader

In the Fig. 7.7, the **red** shaded boxes show that same member comes twice. As, one person cannot have two leadership, therefore, the red shaded boxes cannot be counted. So there are only **12** **((4 × 1 × 4) - 4) different ways** (shown in yellow boxes and green boxes) to choose a leader and an assistant leader for the math club.

| LEADER \ ASSISTANT LEADER | | M | A | T | H |
|---|---|---|---|---|---|
| | | M | A | T | H |
| | M | MM | MA | MT | MH |
| | A | AM | AA | AT | AH |
| | T | TM | TA | TT | TH |
| | H | HM | HA | HT | HH |

(1 (M or A or T or H) × 4 (Leader) × 4 (Assistant Leader)) – 4 (Leader and Assistant Leader) = 12 ways

Fig. 7.7

**Activity**

1. Determine the number of two digit numbers that can be formed using the digits 1, 3 and 5 with repetition of digits allowed.

The activity consists of two parts

(i) Choose a one's digit.

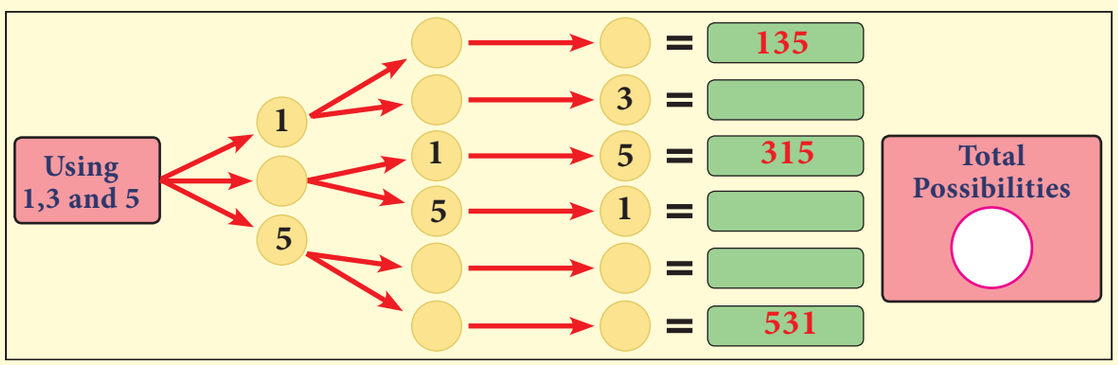
(ii) Choose a ten's digit.

Complete the table given beside

| Ten's Digit \ One's Digit | 1 | 3 | 5 |
|---|---|---|---|
| 1 | | | 15 |
| 3 | | 33 | |
| 5 | 51 | | |

2. Find the three digit numbers that can be formed using the digits 1, 3 and 5 without repetition of digits.

Complete the tree diagram given below to the numbers




**DO YOU KNOW?**

A **password** using 6 characters is created where the first 2 characters are any of the **alphabets**, the third character is any one special character **like @, #, $, %, &, _,+,~, * or -** and the last 3 characters are any of the numbers from **0 to 9**. For that, there are **26 × 26 × 10 × 10 × 10 × 10 = 67,60,000** number of different ways possible to create that password.

## Example 7.4

In how many ways, can the students answer 3 true or false type questions in a slip test?

***Solution:***

(i) Assuming that the question $Q_1$ is answered True, questions $Q_2$ and $Q_3$ can be answered as **TT, TF, FT** and **FF** in 4 ways.

(ii) Assuming that the question $Q_1$ is answered False, $Q_2$ and $Q_3$ can also be answered as **TT, TF, FT,** and **FF** in 4 ways.

Thus, as each question has only two options (True or False) in 2 ways, the number of ways of answering these 3 questions in a slip test is **8 (2 x 2 x 2) possible ways** as shown in Fig. 7.8.

**Number of ways of answering the questions**

| | Questions | | |
|---|---|---|---|
| | $Q_1$ | $Q_2$ | $Q_3$ |
| | T/F | T/F | T/F |
| Answer1 | T | T | T |
| Answer2 | T | T | F |
| Answer3 | T | F | T |
| Answer4 | T | F | F |
| Answer5 | F | T | T |
| Answer6 | F | T | F |
| Answer7 | F | F | T |
| Answer8 | F | F | F |

$[(1\ (Q_1) \times 2\ (T/F)) \times (1\ (Q_2) \times 2\ (T/F)) \times (1\ (Q_3) \times 2\ (T/F))] = 8$ ways

Fig. 7.8

## Example 7.5

Madhan wants to a buy a new car. The following choices are available for him.

- There are **2 types** of cars as shown in the Fig. 7.9
- There are **5 colours** available in each type as shown in Fig. 7.9.
- There are **3 models** available in each colour
  - (i) **GL** (standard model)
  - (ii) **SS** (sports model)
  - (iii) **SL** (luxury model)

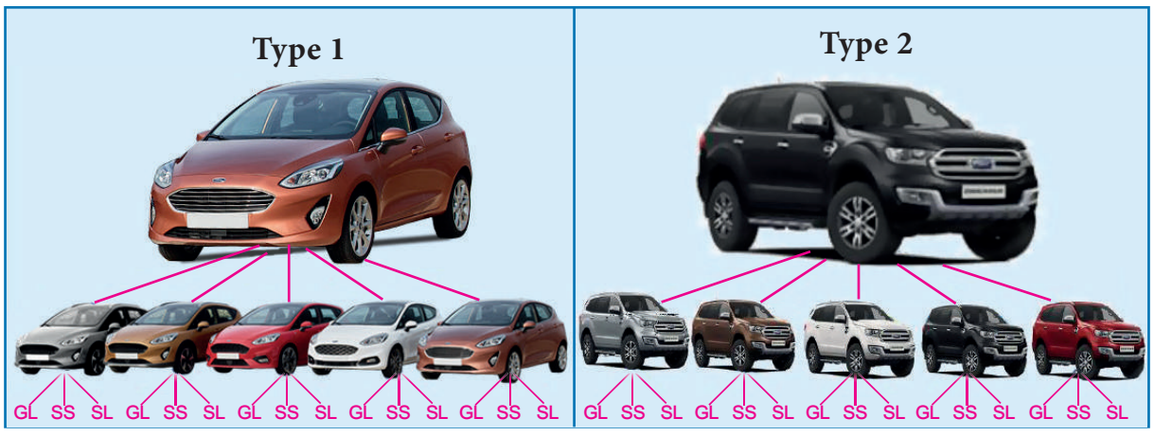



Fig. 7.9

(i) In how many different ways can Madhan buy any one of the new car?

(ii) If the **white colour** is not available in **Type 2**, then in how many ways can Madan buy a new car among the given option?

***Solution:***

(i) To buy any one of the new car from the given choices

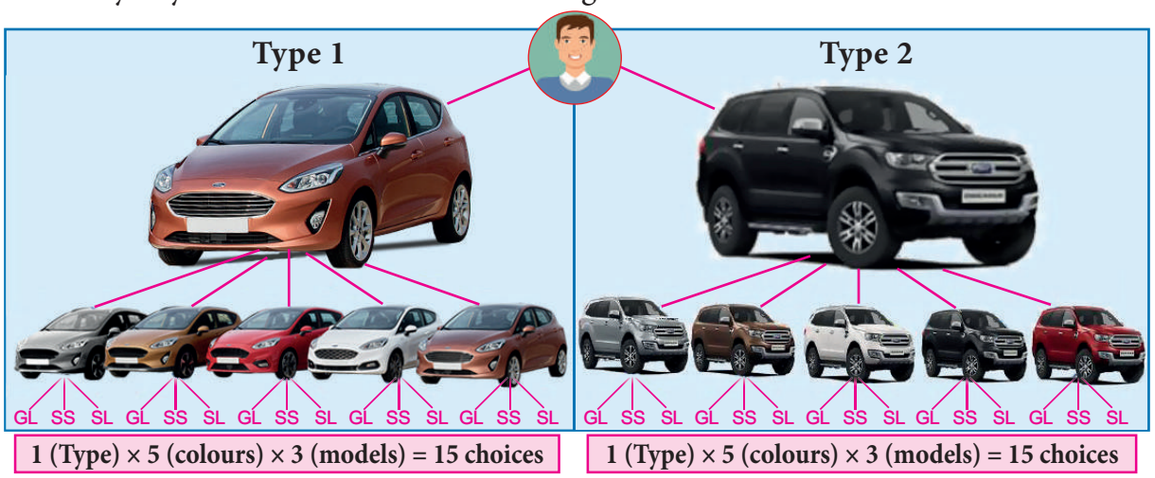



Fig. 7.10

Here, we have **2 types** of car with **5 different colours** and **3 models** in each colour. Therefore, there are **30** **[2 (1 × 5 × 3)]** **different ways** to buy a new car by Madhan.

(ii) If the **white colour** is not available in **Type 2,** then...

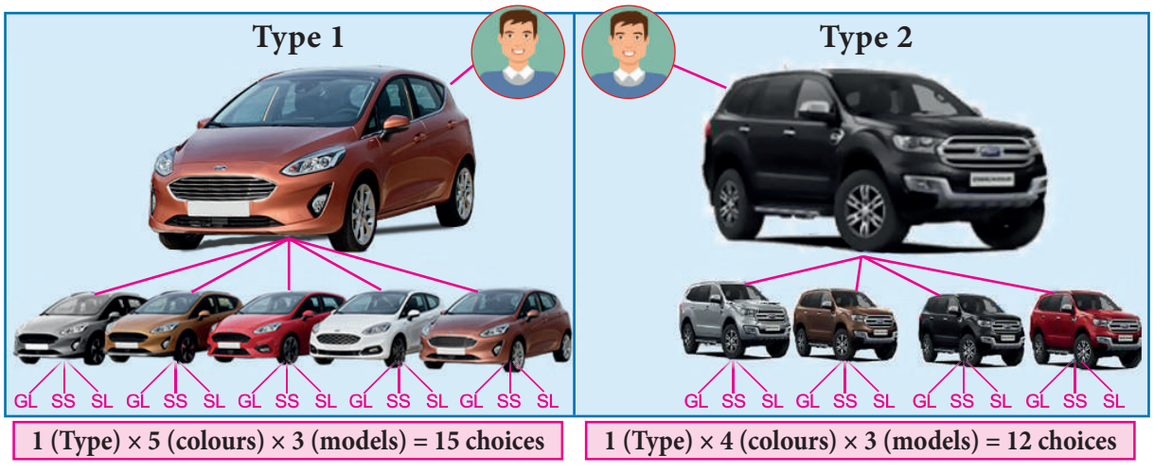



Fig. 7.11

(i) For **Type 1,** we have 5 colours and 3 models and hence there are **1 × 5 × 3 = 15 choices.**

(ii) For **Type 2,** we have only 4 colours and 3 models and hence there are **1 × 4 × 3 = 12 choices.**

Therefore, there are **27** **(15 + 12)** **different ways** to buy a new car by Madhan

The above example illustrates both the addition and multiplication principles.

## 7.3 SET - Game

Any game that uses features can be used to stimulate logical thinking and it provides an interesting and challenging context for exploring ideas in discrete mathematics.

Now, let us learn about a SET game

A **SET** game proves to be an excellent extension for activities that involve organizing the objects by attributes. The **SET** game builds the cognitive, logical, spatial reasoning as well as visual perception skills.

The **SET** game is a puzzle that uses cards which have four features on them. They are shapes, colours, shades and the number of shapes.

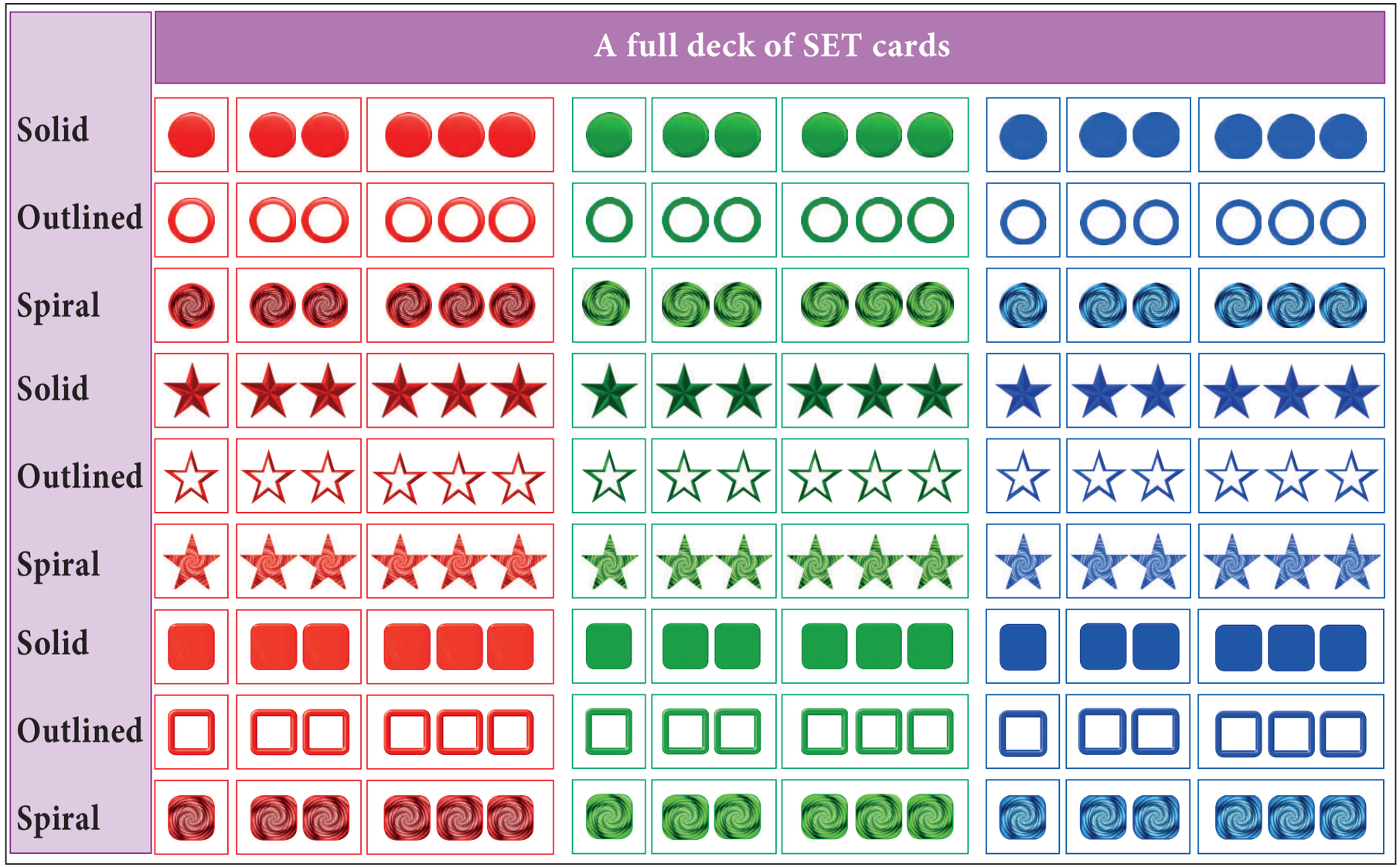



Fig. 7.12

In one full deck of **SET** cards, there are **3 different shapes**: circle, star, square and in **3 different colours**: red, green, blue. **3** Each of these 9 cards **(3 shapes × 3 colours)** have **3 different shades:** solid, outlined, spiral and also they can be paired **3 with different numbers:** one, two and three. So there are totally **81 cards (3 shapes × 3 colours × 3 shades × 3numbers)** in the deck as shown in Fig. 7.12.

A **SET** which consists of **three cards** should satisfy all the **four** following conditions:

(i) All the three cards have the **same shape** or have **three different shapes**.
(ii) All the three cards have the **same colour** or have **three different colours**.
(iii) All the three cards have the **same shade** or have **three different shades**.
(iv) All the three cards have the **same number** or have **three different numbers**.

***Situation:***

The teacher displays **12 cards** as shown in Fig. 7.13 and explains how to form a **SET** using these 2 cards  and  taken from them. Now, follow the step by step procedure to figure out the third card to complete this **SET** is as follows.

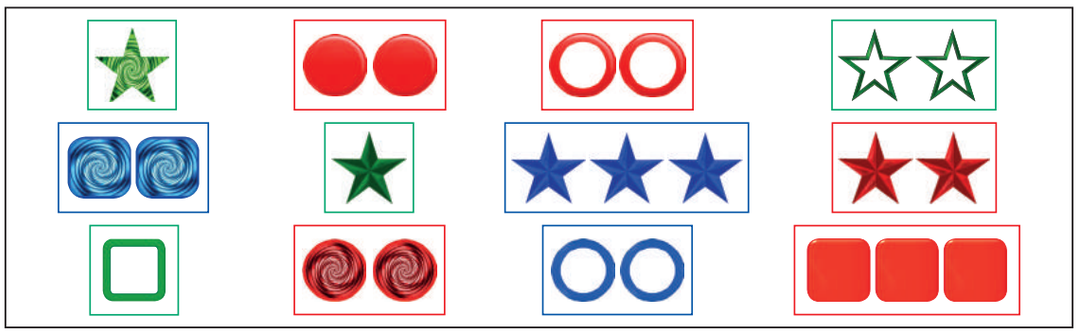


Fig. 7.13

Remember, a **SET** consists of 3 set of cards.

**STEP 1:** If you look at the **shape** then, **one is star**  and the other one is also **star**. 
These two cards have the **same shape**. So, the last card also should have the **same shape**. 

**STEP 2:** If you look at the **colour** then, **one is green**  and the **other is red** 
These two cards have **different colours**. So, the last card also should have a **different colour** that is **blue**. 

**STEP 3:** If you look at the **shade** then, **one is solid**  and the other is also **solid**  .
These two cards have the **same shade**. So, the last card also should have the **same shade**. 

**STEP 4:** If you look at the **number** then, **one star card is green solid**  and the other has **two star red solid cards**  . So, the last card also should have a **different number three blue cards**.  Therefore these three set of cards have different numbers and different colours with same shape and shade.

Finally, 

**Shape** : All same ✓ **Colour** : All different ✓
**Shade** : All same ✓ **Number** : All different ✓

**Now, this completes the rules for a SET.**

Now, the teacher asks the students to find two more **SET**s from Fig.7.13. Let us check the **SET** again.

1. 

**Shape** : All same ✓ **Colour** : All same ✓
**Shade** : All different ✓ **Number** : All same ✓

**Yes, This is a SET.**

2. 

**Shape** : All different ✓ **Colour** : All different ✓
**Shade** : All different ✓ **Number** : All different ✓

**Yes, This is a SET.**

**Think**

For the given set of **12 cards** from Fig.7.13 **7 SETs** are possible. Explore. (repetition of cards allowed).

Again , the teacher makes an arrangement of cards and asks them to check **whether, it forms a SET?**

3. 

**Shape** : All same or all different ✗ **Colour** : All same or all different ✗
**Shade** : All same or all different ✗ **Number** : All same or all different ✗

**No, this is not a SET.**

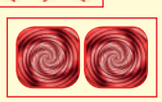
Since, the above **SET** does not complete the **SET** rule.

Thus,we come to know a **SET** consists of **3 set of cards** in which each individual feature is either **all same** on each card or **all different** on each card.

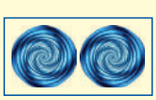
**Activity**

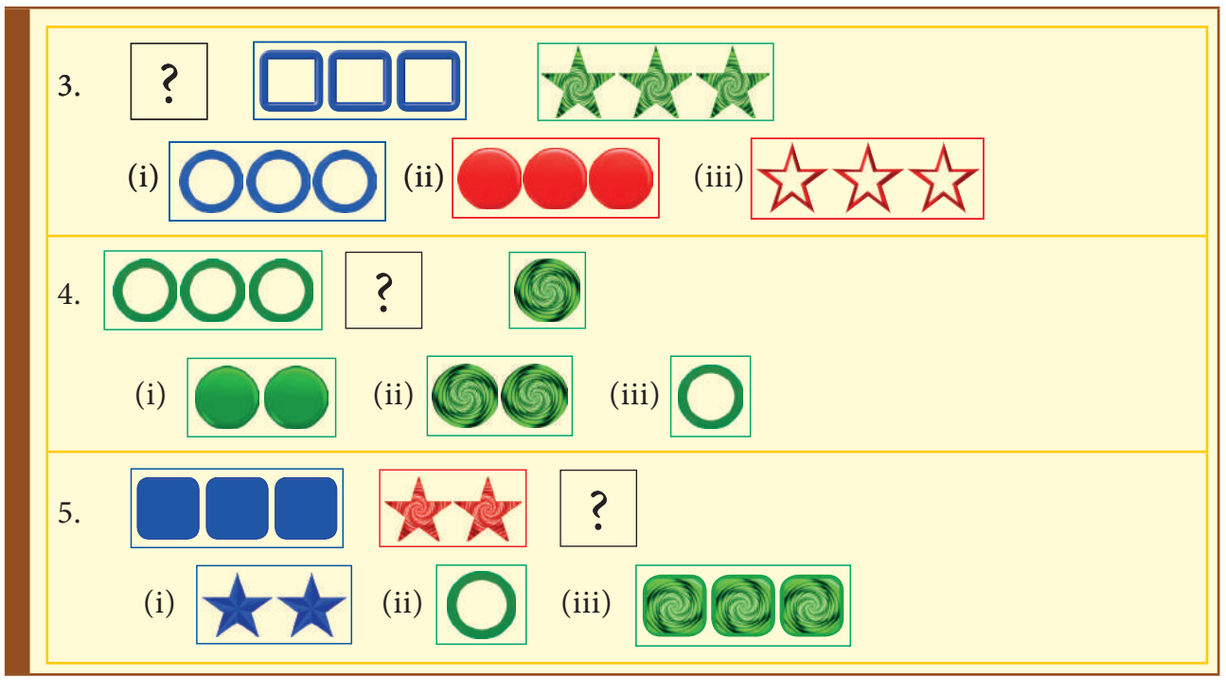
Choose the correct card to complete the perfect **SET**. One is done for you.

1. 

(i)  (ii)  (iii)  Ans: 

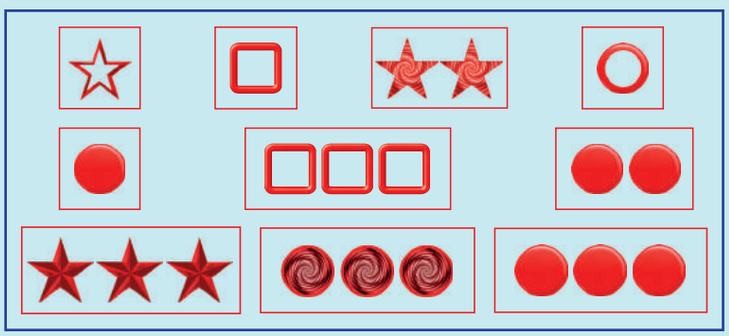
2. 

(i)  (ii)  (iii) 

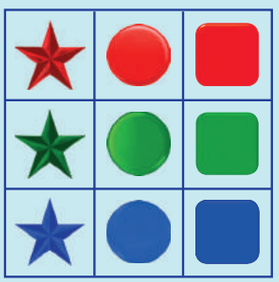



**Try these**

1. Find any five **SET**s among these set of cards (repetition of cards allowed).



2. This is an example for a magic square in **SET**s. Can you make another two?



## 7.4 Map Colouring

Map colouring is the act of assigning different colours to different features on a map. In Mathematics, the problem is to determine the minimum number of colours required to colour a map,so that no two adjacent regions have the same colour. Let us learn about the role of map colouring.

***Situation:***

The teacher divides the class into two groups and instructs one group to use as many colours possible and another group use minimum number of colours for the given patterns.

The only rule is that, shaded regions cannot share the same colour at edges, although they are allowed to meet at a corner.

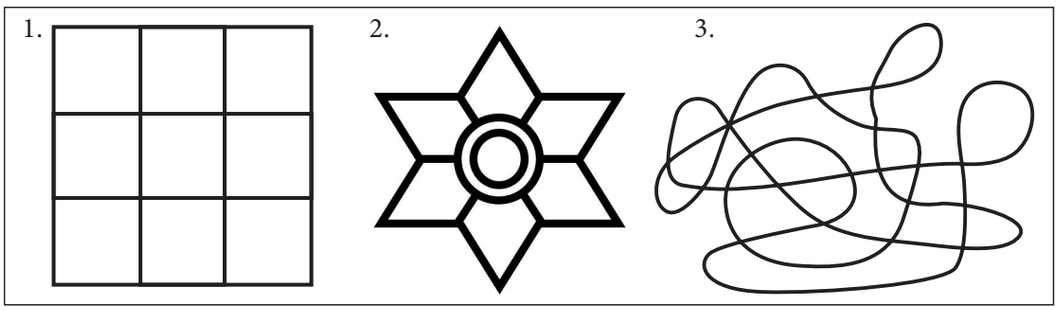



Fig. 7.14

Fig. 7.15 , shows how each group is coloured

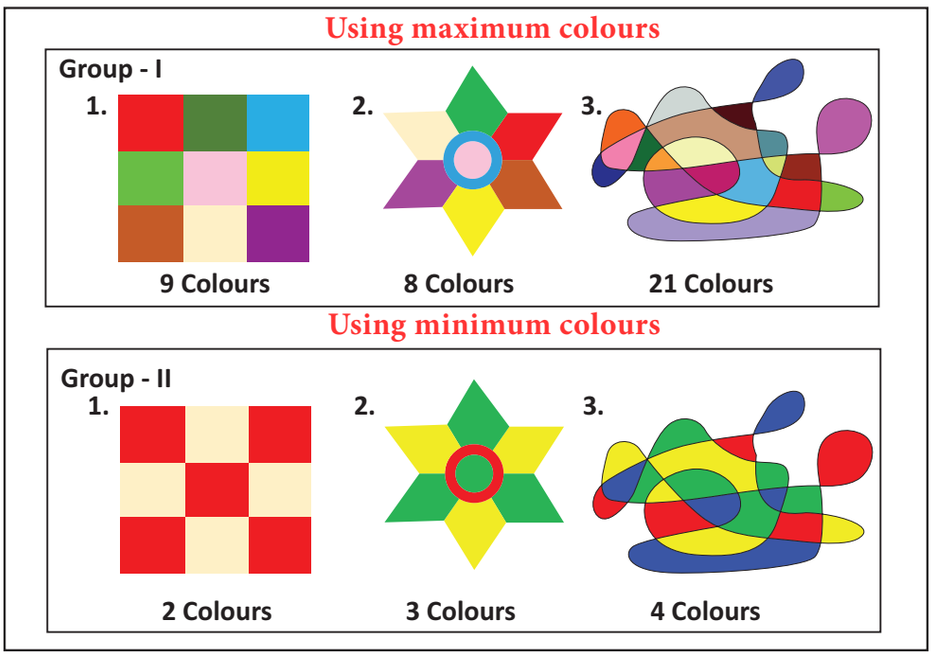



Fig. 7.15

From this investigation, we will get results. Which could be more interesting , if we want to colour a map.

Map colouring is about the colours that must be chosen for regions in a map, which make bordering regions with different colours. Let us learn more from the following examples.

**Example 7.6**

Colour a map of South India (Fig. 7.16) with the fewest number of colours.

***Solution:***

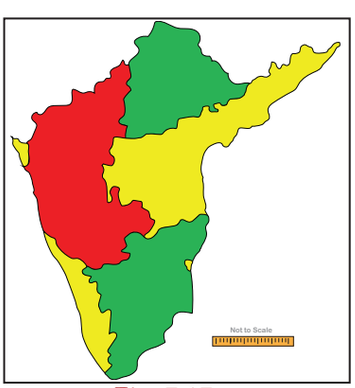



Fig. 7.17

This is one of the solutions with minimum number of colours . Try for more solutions.

Fig. 7.16

## Activity

Try to colour the INDIA-States map with the fewest number of colours.

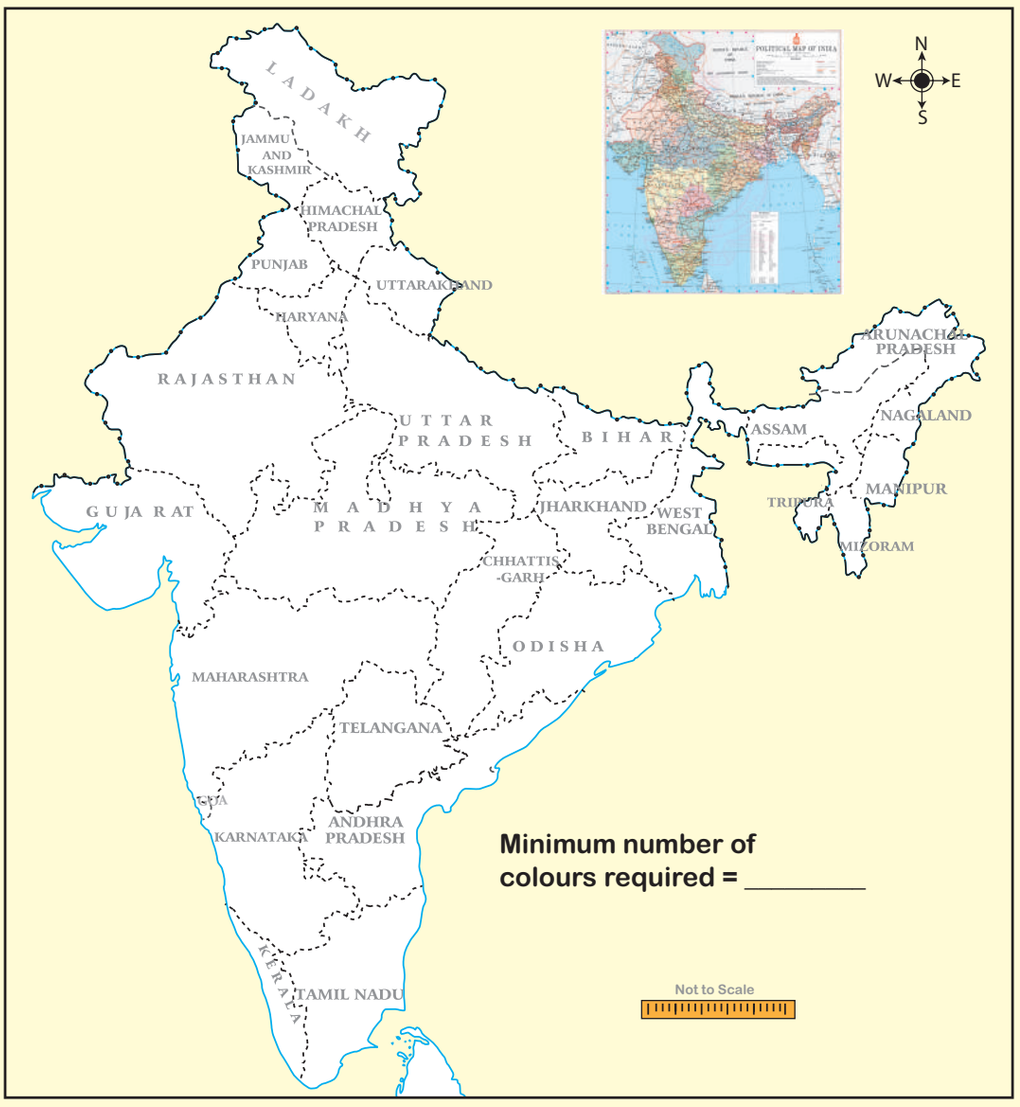



Fig. 7.18

## Try these

Draw your school building map showing the HM room, class rooms, staff room, science lab, PET room, computer lab, office room etc., and use graph colouring to detetermine the minimum number of colours that can be used to colour the map of your school.

## Exercise 7.1

1. You want to have an ice cream or a cake. There are three flavours (chocolate, strawberry and vanilla) in ice creams, and two flavours (orange and red velvet) in the cakes. In how many possible ways can you choose an ice cream or a cake?
2. Shanthi has 5 chudithar sets and 4 frocks. In how many possible ways, can she wear either a chudithar or a frock ?

3. In a Higher Secondary School, the following groups are available in XI standard

I. Science Group:

(i) Physics, Chemistry, Biology and Mathematics

(ii) Physics, Chemistry, Mathematics and Computer Science

(iii) Physics, Chemistry, Biology and Home Science

9XLYAW

II. Arts Group:

(i) Accountancy, Commerce, Economics and Business Maths

(ii) Accountancy, Commerce, Economics and Computer Science

(iii) History, Geography, Economics and Commerce

III. Vocational Group:

(i) Biology, Nursing Theory, Nursing Practical I and Nursing Practical II

(ii) Home Science, Textiles and Dress Designing Theory, Textiles and Dress Designing Practical I and Textiles and Dress Designing Practical II

In how many possible ways, can a student choose a group?

4. If you have 2 school bags and 3 water bottles then, in how many different ways can you choose each one of them, while going to school ?

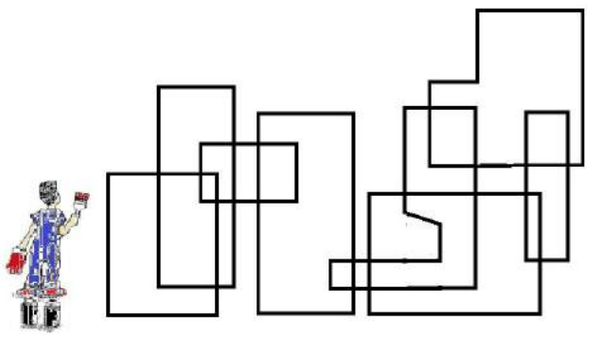
5. Roll numbers are created with a letter followed by 3 digits in it, from the letters A, B, C, D and E and any 3 digits from 0 to 9. In how many possible ways can the roll numbers be generated? (except A000, B000, C000, D000 and E000)


6. A safety locker in a jewel shop requires a 4 digit unique code. The code has the digits from 0 to 9. How many unique codes are possible ?

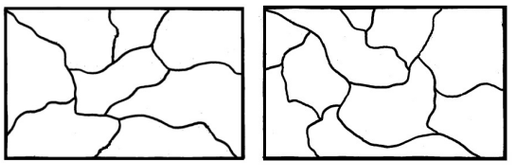
7. An examination paper has 3 sections, each with five questions and students are instructed to answer one question from each section. In how many different ways of can the questions be answered?

8. The given spinner is spun twice and the two numbers got are used to form a 2 digit number. How many different 2 digits numbers are possible?


9. Ramya wants to paint a pattern in her living room wall with minimum budget. Help her to colour the pattern with 2 colours but make sure that no two adjacent boxes are the same colour. The pattern is shown in the picture.



10. Colour the regions in the maps with few colours as possible but make sure that no two adjacent countries are of the same colour.



## Objective Type Questions

11. In a class there are 26 boys and 15 girls. The teacher wants to select a boy or a girl to represent a quiz competition. In how many ways can the teacher make this selection?
(A) 41 (B) 26 (C) 15 (D) 390

12. How many outcomes can you get when you toss three coins once?
(A) 6 (B) 8 (C) 3 (D) 2

13. In how many ways can you answer 3 multiple choice questions, with the choices A,B,C and D?
(A) 4 (B) 3 (C) 12 (D) 64

14. How many 2 digit numbers contain the number 7 ?
(A) 10 (B) 18 (C) 19 (D) 20

## 7.5 Fibonacci Numbers

We have learnt in earlier classes, on how all beautiful things in nature as well as man made things are connected with Mathematics. Now, we just refresh everyone's memory and show how Math can be beautiful when seen in physical and biological things everywhere around us.

Fibonacci (real name Leonardo Bonacci) was an Italian mathematician who developed the Fibonacci Sequence. Remember the pattern of the Fibonacci sequence we already studied in standard VI it looks like this: 1, 1, 2, 3, 5, 8, 13, 21, 34... and it goes on.

Let us tabulate the Fibonacci sequence and find a rule.

| Term (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F (n) | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 | 233 | 377 | 610 | ... |

We observe that the **3rd** term of the Fibonacci sequence is the sum of **2nd term** and the **1st term**.

That is, **F(3) = F(2) + F(1)** and so we can extend and write the rule is

| F(1) | F(2) | F(3) | F(4) | F(5) | F(6) | F(7) | ... |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 8 | 13 | ... |

+ =

**F(3) = F(2)+F(1)**

**F(n)= F(n–1) + F(n–2)**

where **F(n)** is the **nth** term

**F(n–1)** is the previous term to the **nth** term

**F(n–2)** is the term before the **(n–1)th** term

This is how the Fibonacci Sequence is obtained. Let us learn more from the following real life examples.

***Situation:***

Let us look at the family tree of a male drone bee and a female bee as shown Fig 7.19.

Here, female bees have 2 parents, male (drone) bees have just one parent, a female. (Male bee (drone) are produced by the queen's unfertilized eggs, so male(drone) bees only have a mother but no father!)

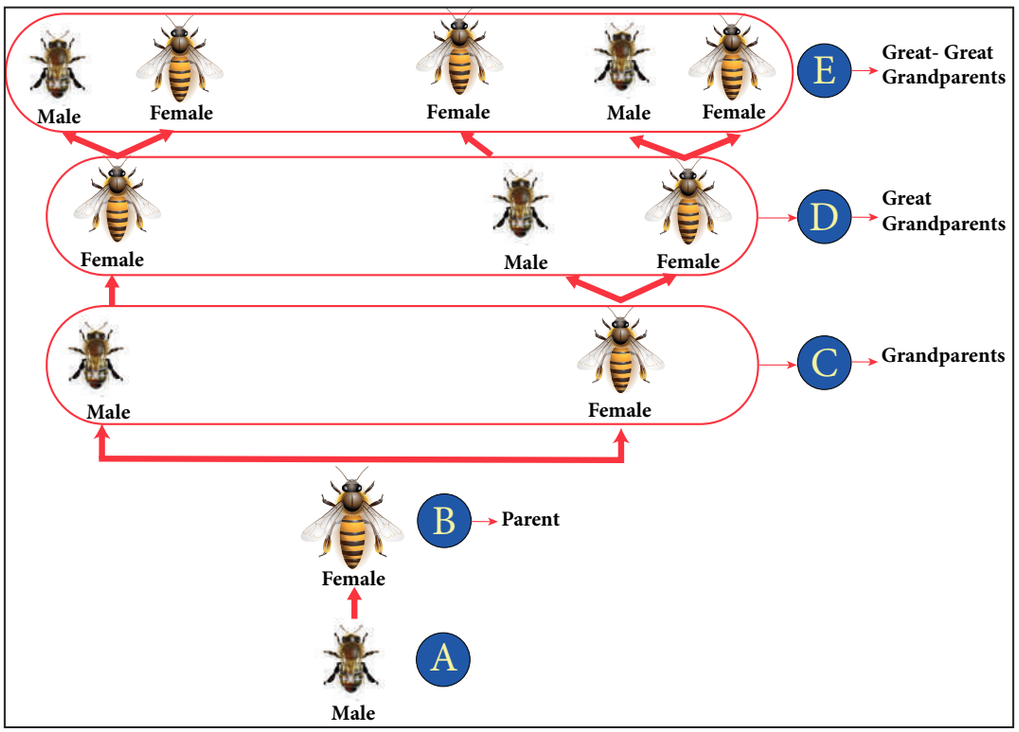



Fig. 7.19

From the picture the following points are noted:

1. The male Ⓐ has 1 parent, a female Ⓑ.
2. The male Ⓐ has **2** grandparents Ⓒ, since his mother had parents Ⓒ, a male and a female.
3. The male Ⓐ has **3** great-grandparents Ⓓ: since his grandmother has two parents but his grandfather has only one.

Now, answer, how many great-great-grandparents did the male Ⓐ have?

Let us try to find the relationship among the pattern of bees family by representing in the tabular form given below,

| Number of Ⓐ | Parents Ⓑ | Grandparents Ⓒ | Great Grandparents Ⓓ | Great- Great Grandparents Ⓔ | Great- Great-Great Grandparents Ⓕ |
|---|---|---|---|---|---|
| a Male bee (1) | 1 | 2 | 3 | 5 | 8 |
| a Female bee (1) | 2 | 3 | 5 | 8 | 13 |

We see the Fibonacci numbers 1, 1, 2, 3, 5, 8, 13… in the above table.

**Note**

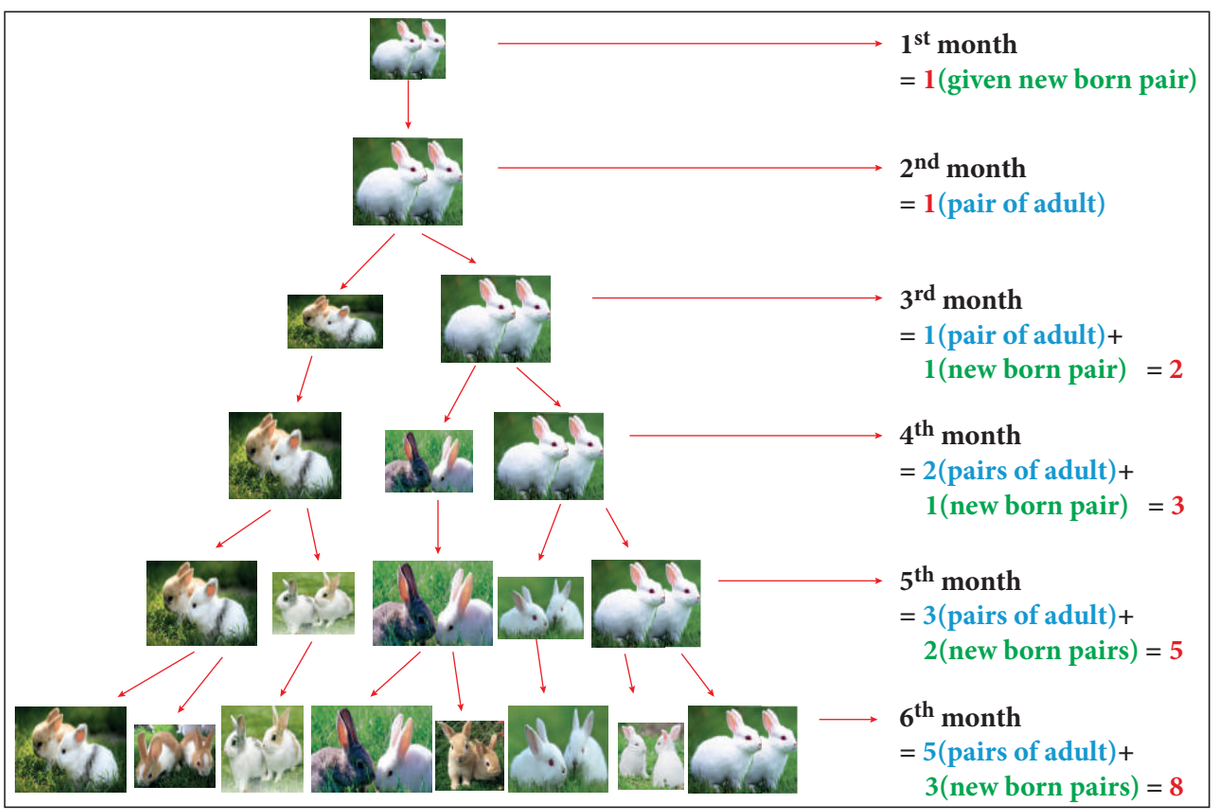
The difference between two consecutive numbers of the Fibonacci sequence increase very quickly. ((For example F(5) – F(4) = 5 – 3 = 2 ; F(10) – F(9) = 55 – 34 = 21 ; F(15) – F(14) = 610 – 377 = 233)(1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946 ...))

## Example 7.7

Given that one pair of new born rabbits they produce a new pair each month and from the second month, each new pair can breed themselves. Find how many pairs of rabbits are bred from one pair in a year, and find the relationship between the number of months and the number of pairs of rabbits by tabulation (a pair means (a male and a female)).

***Solution:***

The below picture clearly forms the sequence is **1,1,2,3,5,8...** Here, we find the pattern in which each number is in the Fibonacci sequence, obtained by adding together with previous two. Going on like this to find subsequent numbers at the **twelfth month,** we will get **144 pairs of rabbits.** In the other words, twelfth Fibonacci number is 144.




| Number of months | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of pairs of rabbits | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 |

## Activity

Using the given Table I, find the pattern, answer the following questions and colour the values in the given Table II. One is done for you.

Table I

| Term(n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F(n) | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 | 233 | 377 | 610 | |

1. Where are the even Fibonacci Numbers? Colour both the term n and where **F(n) is even** in **yellow.** Do you find any pattern? Every **Third** Fibonacci number is **a multiple of 2(even).**
2. Where there are Fibonacci numbers which are **multiple of 3 ?** Colour both the term **n and** where **F(n) is multiple of 3** in **red.**

   Write down the pattern you find

   Every ……Fibonacci number is …….
3. What about the **multiple of 5**? Colour both the term **n** and where **F(n) is multiple of 5** in **blue.** Write down the pattern you find.

   Every ……i.e. ……
4. What about the **multiple of 8?** Colour both the term **n** where **F(n) is multiple of 8** in **green.** Write down the pattern you find.

   Every ……i.e. ……

Table II

| Term(n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F(n) | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 | 233 | 377 | 610 | |

From the above activity, we conclude that
Every Fibonacci number is a factor of (a term number of) Fibonacci numbers in miltiples.

| | n | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | F(n) | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 | 233 | 377 | 610 | ... |
| Factors | 2=F(3) | ✓ | × | × | ✓ | × | × | ✓ | × | × | ✓ | × | × | ✓ | Every 3rd Fib number |
| | 3=F(4) | × | ✓ | × | × | × | ✓ | × | × | × | ✓ | × | × | × | Every 4th Fib number |
| | 5=F(5) | × | × | ✓ | × | × | × | × | ✓ | × | × | × | × | ✓ | Every 5th Fib number |
| | 8=F(6) | × | × | × | ✓ | × | × | × | × | × | ✓ | × | × | × | Every 6th Fib number |
| | F(k) | | | | | | | | | | | | | | Every Kth Fib number |

From the above table, we get a general rule as **Every $k^{th}$ Fibonacci number is a multiple of F(k).**

## 7.6 Highest Common Factor

We have learnt in class VI that iteration is a process wherein a set of instructions or structures are repeated in a sequence for a specified number of times or until a condition is met. Here, we are going to learn to find HCF by listing all factors and find the biggest, then to find HCF by repeated subtraction and see to how much faster the iteration goes (and how in fewer steps you get the HCF) and then how to improve further by repeated division and remainder and that both lead to the same solution but one is faster than the other.

We know that, HCF is used in simplifying or reducing fractions. To understand how this concept applies in real life, imagine the following situation.

### 7.6.1 Methods to find HCF (Highest Common Factor):

#### 1. Factorisation Method:

***Situation:***

Let us assume that you have 20 mangoes and 15 apples. You want to donate them together equally among the orphan children. How many orphan children can you help at the maximum?

Here, basically question demands finding HCF of two numbers. HCF is Highest Common Factor, also known as GCD (Greatest Common Divisor). HCF of two or more than two numbers is such that, it is the largest possible number which divides given numbers completely.

Here, let us find the HCF of 20 mangoes and 15 apples.

Factors of 20 = 1,2,4,(5),10,20

Factors of 15 = 1,3,(5),15

So, the HCF of 20 and 15 is 5. That is, you can a help maximum of 5 orphan children.

So that for **5 children** you can give **4 mangoes** (20 ÷ 5 = 4)  and **3 apples** (15 ÷ 5 = 3)  to each of them. In this way you can distribute equally the mangoes and the apples to each child.

#### 2. Prime Factorisation Method:

***Situation:***

Suppose there are 18 students in Class VII and 27 in Class VIII and each class is divided into teams to prepare for an upcoming sports tournament, with the winning teams from each class play each other in the final. What would be the biggest possible team size that you could divide both these classes such that each team has exactly the same number of students and that no one is left behind.

The problem here is to find the HCF of 18 and 27.

Prime Factors of 18 = 2 x **3** x **3**

Prime Factors of 27 = 3 x **3** x **3**

Common prime factors of 18 and 27 = 3 x 3 = 9

So, the HCF of 18 and 27 is **9.**

Now, let us learn some more methods of finding the HCF. The largest team member each group is **9**, Std VII has 2 teams and Std VIII has 3 teams.

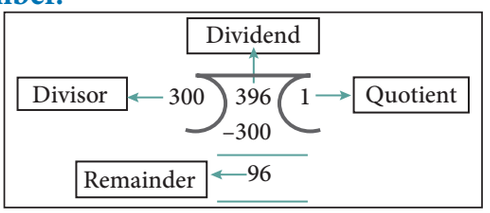
### 3. Repeated Division Method:

The above methods are easy to finding HCF, but for larger numbers these methods are tedious to find factors of the given numbers. In that case, alternatively we have some more methods to find HCF. Let us learn more about the other methods of finding the HCF.

For the above Situation, what if the Class VII had 396 students and Class VIII had 300 students? Then, what would be the biggest possible team size? Well, the above said two methods may not help us quickly. So, we can use continuous division method for finding the highest common factor.

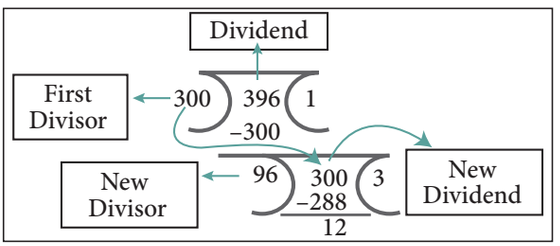
**STEP 1: Divide the larger number by the smaller number.**

Here, 360 is the larger number. So, we divide 360 (Dividend) by 300 (Divisor). We get the Remainder as 96.




**STEP 2: The remainder from Step 1 becomes the new divisor, and divisor of Step 1 becomes the new dividend.**

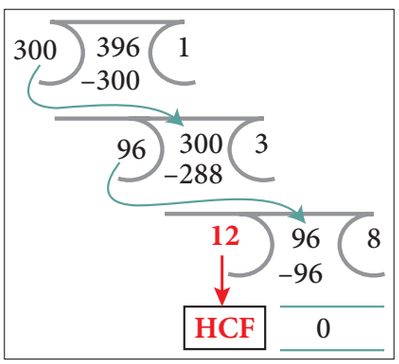
From the step 1, we got 96 as remainder. So, in the second step 96 becomes the new divisor and 300 becomes the new dividend.




**STEP 3: Repeat this division process till remainder becomes zero. The divisor of the last division (when remainder is zero) is the HCF.**

From step 2, we got 12 as the new remainder which will become the new divisor. In the third step 12 becomes the new divisor and 96 becomes the new dividend. **Now, the remainder is zero when 12 is the last divisor of the division. Therefore,12 is the required HCF.**

Hence, the **HCF of 396 and 300 is 12.** So each team would be **12 students.**



### 4. Repeated Subtraction Method:

To find the HCF for the given two numbers say **m** and **n** we do the subtraction continuously until m and n are equal. For example,

Find the HCF of 144 and 120

**STEP 1: Check whether m = n**

Here, take **m** = 144 and **n** = 120

Check whether **m = n** or **m > n** or **m < n** Here **m > n (144 > 120).**

**STEP 2: m > n perform m – n repeat the process till m = n or m < n perform n – m repeat the process till m = n**

If m is greater than **n,** then we perform **m – n** and assign the result (the difference) as **m.** Again we check whether **m and n are equal or not** and repeat the process. If **m** is less than **n,** then we perform **n – m** and we assign the result (the difference) as **n**. Again we check whether **m and n are equal or not** and the process is repeated.

Subtract **120(n)** from **144(m)** till **m = n.**

| | | | | | |
|---|---|---|---|---|---|
| First | 144 – 120 = 24 | Repeat | 120 – 24 = 96 | Repeat | 96 – 24 = 72 |
| Repeat | 72 – 24 = 48 | Repeat | 48 – 24 = 24 | Repeat | 24 – 24 = 0 |

**STEP 3: When m and n values are equal then that equal value will be the HCF (m, n).**

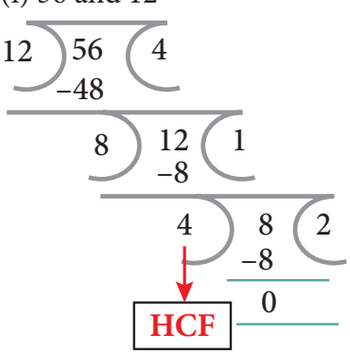
Now m = n, Hence, we conclude that the **HCF of 144 and 120 is 24.**

Comparing both the repeated division and repeated subtraction methods, in finding the HCF, we can conclude that the repeated subtraction, in one way is easier and gives the HCF faster that the repeated division and also that one would want to easily do subtraction rather than division. Isn't it ?

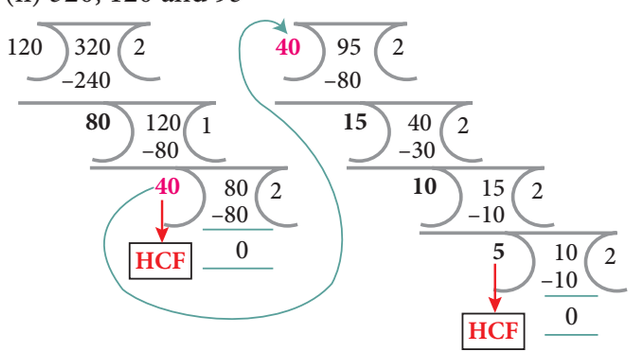
## Exercise 7.2

1. Using repeated division method, find the HCF of the following:
   (i) 455 and 26 (ii) 392 and 256 (iii) 6765 and 610 (iv) 184, 230 and 276
2. Using repeated subtraction method, find the HCF of the following:
   (i) 42 and 70 (ii) 36 and 80 (iii) 280 and 420 (iv) 1014 and 654
3. Do the given problems by repeated subtraction method and verify the result.

(i) 56 and 12




(ii) 320, 120 and 95



4. Kalai wants to cut identical squares as big as she can, from a piece of paper measuring 168mm and by 196mm. What is the length of the side of the biggest square? (To find HCF using repeated subtraction method)

## Objective Type Questions

5. What is the eleventh Fibonacci number?
   (a) 55 (b) 77 (c) 89 (d) 144

6. If F(n) is a Fibonacci number and n =8, which of the following is true?
   (a) F(8) = F(9)+F(10) (b) F(8) = F(7)+F(6) (c) F(8) = F(10)×F(9) (d) F(8) = F(7)–F(6)

7. Every 3rd number of the Fibonacci sequence is a multiple of _____
   (a) 2 (b) 3 (c) 5 (d) 8

8. Every _____ number of the Fibonacci sequence is a multiple of 8
   (a) $2^{nd}$ (b) $4^{th}$ (c) $6^{th}$ (d) $8^{th}$

9. The difference between the $18^{th}$ and $17^{th}$ Fibonacci number is
   (a) 233 (b) 377 (c) 610 (d) 987

10. Common prime factors of 30 and 250 are
    (a) 2 x 5 (b) 3 x 5 (c) 2 x 3 x 5 (d) 5 x 5

11. Common prime factors of 36, 60 and 72 are
    (a) 2 x 2 (b) 2 x 3 (c) 3 x 3 (d) 3 x 2 x 2

12. Two numbers are said to be co-prime numbers if their HCF is
    (a) 2 (b) 3 (c) 0 (d) 1

## 7.7 Cryptology

In today's world, security in information is a fundamental necessity not only for military and political departments but also for private communication. Today's world of communication has increased the importance of financial data exchange, image processing, biometrics and e-commerce transaction which in turn has made data security an important issue. Cryptology is defined as the science which is concerned with communication in secured form.

### 7.7.1 Cryptology – Some technical details


**Plain text:** The original message is called plain text.

**Cipher text or Cipher number:** The encrypted output (converted message into code) is called Cipher text or Cipher number. Cipher text is written in capital letters, while plain text is usually written in lowercase. A secret key is to use something to generate the Cipher text from the plain text.

**Encryption and Decryption:** The process of converting the plain text to the Cipher text is called encryption and the vice versa is called decryption.

Let us try to create some Cipher text that we use in the form of coded message at some point in our real life.

### 7.7.2 Examples of Cipher Code

#### 1. Shifting Cypher Text

#### Ceasar Cipher

The Ceasar Cipher is one of the earliest known and simplest ciphers. It is a type of substitution cipher in which each letter in the text is **"shifted"** a certain number of places down the alphabets.

To pass an encrypted message from one person to another, it is first necessary that both parties have the **'key'** for the cipher, so that the sender may encrypt it and the receiver may decrypt it. For the Caesar Cipher, the **"key"** is the number of characters to shift the cipher alphabet. So, we have to know how big the switch is to break the code.

Let us know about more Ciphers from the following examples and situations.

**Example 7.8**

Use Ceasar Cipher table set **+4** and try to solve the given secret sentence.

**fvieo mr gshiw ger fi xvmgoc**

***Solution:***

Let us make Ceasar Cipher table first. Here, we have to set to **+4** table.

For that, we have to start letter **e** to set as **A**, **f** as **B** … likewise **d** as **Z.**

Now, the **+4** Ceasar Cipher table looks like

| Plain Text | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Text | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |

The given plain text is

**fvieo mr gshiw ger fi xvmgoc**

To crack this secret code, follow the steps given below.

**Step 1:** Using Ceasar Cipher table, let us first match the most repeated letters. This will help us to progress faster.

**fvieo mr gshiw ger fi xvmgoc**

| Plain Text | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Text | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |

f v i e o m r g s h i w g e r f i x v m g o c

B R E A K I N C E C A N B E R I C K

**Step 2:** Then, let us find remaining letters to complete the code.

| Plain Text | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Text | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V |

f v i e o m r g s h i w g e r f i x v m g o c

B R E A K I N C O D E S C A N B E T R I C K Y

Thus, the secret sentence is decoded as, **BREAK IN CODES CAN BE TRICKY**

## 2. Substituting Cypher Text

Each letter in a text is "substituted" by certain pictures and symbols, a key message, group of words, letters or a combination of these can be used to encode or decode the information. Let us learn more from the following situation.

***Situation:***

The teacher divides the class into two teams and displays a worksheet given in the figure below for students to complete. Play a game, teams take turns suggesting letter substitutions and someone entering your suggestions in the table. Teams earn points for correct letter guesses.

### Worksheet 1 :- Additive Cipher [key = 5]

**Convert a given plain text into Additive Cipher text (number) code.**

**" mathematics is a unique symbolic language in which the whole world works and acts accordingly."**

### Tips for cracking additive ciphers:

- Especially for additive Ciphers, you only need **key number**. You can complete the cipher table by filling in the rest of the numbers in order.
- If you can, find and make **frequency table for alphabets**, it is help you to get started.
- Match the most **repeated letters** first and fill the Cipher number.
- Look for familiar one-letter words like **a** or **i**. common two and three letter words like **of, to, in, it, at, the, and, for, you …..**
- Look for **consecutive numbers** in the Cipher text and match them with possible **consecutive letters** in the plain text.

Now, to start converting a plain text into Cipher number, first we have to make a cipher table as shown below. Here there **key is 5**. As per key number, we have to start and fill **a = 05, b = 06 …….. z = 04** respectively.

| **Plain Text** | **a** | **b** | **c** | **d** | **e** | **f** | **g** | **h** | **i** | **j** | **k** | **l** | **m** | **n** | **o** | **p** | **q** | **r** | **s** | **t** | **u** | **v** | **w** | **x** | **y** | **z** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Numbers** | **00** | **01** | **02** | **03** | **04** | **05** | **06** | **07** | **08** | **09** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| **Cipher Numbers** | **05** | **06** | **07** | **08** | **09** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **00** | **01** | **02** | **03** | **04** |

To start encoding the text, let us count the frequency of alphabets and frame frequency table as shown below.

| Plain Text | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency of alphabets in the plain text | 8 | 1 | 6 | 3 | 5 | 0 | 3 | 5 | 7 | 0 | 1 | 5 | 3 | 5 | 5 | 0 | 1 | 3 | 5 | 4 | 3 | 0 | 4 | 0 | 2 | 0 |

With the help of Cipher table and frequency table, let us fill the Cipher numbers for the plain text. Let us first match the most repeated letters (5 times and above), then 2 to 4 times repeated letters and finally remaining letters step by step as shown in given figure.

**Step 1:- Cipher number for most repeated letters(5 times and above)**

| Plain text | m | a | t | h | e | m | a | t | i | c | s | | i | s | | a | | u | n | i | q | u | e | | s | y | m | b | o | l | i | c |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | | 05 | | 12 | 09 | | 05 | | 13 | 07 | 23 | | 13 | 23 | | 05 | | | 18 | 13 | | | 09 | | 23 | | | | 19 | 16 | 13 | 07 |

| Plain text | l | a | n | g | u | a | g | e | | i | n | | w | h | i | c | h | | t | h | e | | w | h | o | l | e |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | 16 | 05 | 18 | | | 05 | | 09 | | 13 | 18 | | | 12 | 13 | 07 | 12 | | | 12 | 09 | | | 12 | 19 | 16 | 09 |

| Plain text | w | o | r | l | d | | w | o | r | k | s | | a | n | d | | a | c | t | s | | a | c | c | o | r | d | i | n | g | l | y |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | | 19 | | 16 | | | | 19 | | | 23 | | 05 | 18 | | | 05 | 07 | | 23 | | 05 | 07 | 07 | 19 | | | 13 | 18 | | 16 | |

**Step 2: Cipher number for most repeated letters (2 times and above)**

| Plain text | m | a | t | h | e | m | a | t | i | c | s | | i | s | | a | | u | n | i | q | u | e | | s | y | m | b | o | l | i | c |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | 17 | 05 | 24 | 12 | 09 | 17 | 05 | 24 | 13 | 07 | 23 | | 13 | 23 | | 05 | | 25 | 18 | 13 | | 25 | 09 | | 23 | 03 | 17 | | 19 | 16 | 13 | 07 |

| Plain text | l | a | n | g | u | a | g | e | | i | n | | w | h | i | c | h | | t | h | e | | w | h | o | l | e |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | 16 | 05 | 18 | 11 | 24 | 05 | 11 | 09 | | 13 | 18 | | 01 | 12 | 13 | 07 | 12 | | 24 | 12 | 09 | | 01 | 12 | 19 | 16 | 09 |

| Plain text | w | o | r | l | d | | w | o | r | k | s | | a | n | d | | a | c | t | s | | a | c | c | o | r | d | i | n | g | l | y |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | 01 | 19 | 22 | 16 | 08 | | 01 | 19 | 22 | | 23 | | 05 | 18 | 08 | | 05 | 07 | 24 | 23 | | 05 | 07 | 07 | 19 | 22 | 08 | 13 | 18 | 11 | 16 | 03 |

**Step 3: Cipher numbers for non-repeated letters**

| Plain text | m | a | t | h | e | m | a | t | i | c | s | | i | s | | a | | u | n | i | q | u | e | | s | y | m | b | o | l | i | c |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | 17 | 05 | 24 | 12 | 09 | 17 | 05 | 24 | 13 | 07 | 23 | | 13 | 23 | | 05 | | 25 | 18 | 13 | 21 | 25 | 09 | | 23 | 03 | 17 | 06 | 19 | 16 | 13 | 07 |

| Plain text | l | a | n | g | u | a | g | e | | i | n | | w | h | i | c | h | | t | h | e | | w | h | o | l | e |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | 16 | 05 | 18 | 11 | 25 | 05 | 11 | 09 | | 13 | 18 | | 01 | 12 | 13 | 07 | 12 | | 24 | 12 | 09 | | 01 | 12 | 19 | 16 | 09 |

| Plain text | w | o | r | l | d | | w | o | r | k | s | | a | n | d | | a | c | t | s | | a | c | c | o | r | d | i | n | g | l | y |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Numbers | 01 | 19 | 22 | 16 | 08 | | 01 | 19 | 22 | 15 | 23 | | 05 | 18 | 08 | | 05 | 07 | 24 | 23 | | 05 | 07 | 07 | 19 | 22 | 08 | 13 | 18 | 11 | 16 | 03 |

Thus, the Additive Cipher text for plain text is as follows:

**"17 05 24 12 09 17 05 24 13 07 23 13 23 05 25 18 13 21 25 09 23 03 17 06 19 16 13 07 16 05 18 11 25 05 11 09 13 08 01 12 13 07 12 24 12 09 01 12 19 16 09 01 19 22 16 08 01 19 22 15 23 05 18 08 05 07 24 23 05 07 07 19 22 08 13 18 11 16 03"**

Activity

## TREASURE HUNT

# Treasure in the Mathematics Club room

The teacher divides the students into four groups and gives each group a code and a clue and then asks them to cracking the clues to find

(i) the identity of the treasure (ii) the place of the treasure

(iii) the room in which the treasure is present.

You may take notes on this piece of paper as you proceed through the search

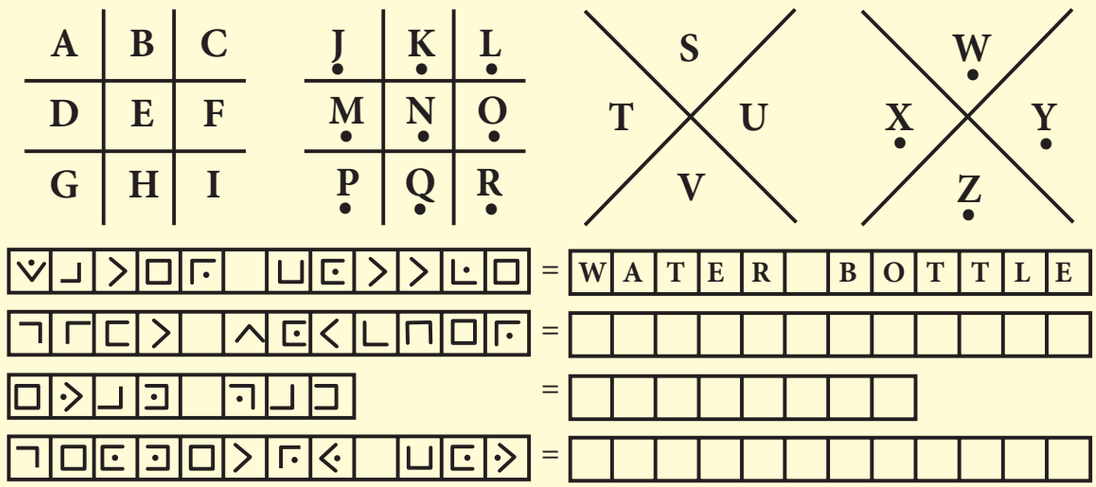
## Code 1: Pigpen

**Message:-** If you decode the clue here you can get the four expected treasure names

### *I. Fill in the blank boxes and decode*

The **Pigpen code** looks like meaningless writing, but it is quite easy to catch on to. Each letter is represented by the part of the "Pigpen" that surrounds it.

The first code uses the following **key**. To complete the code, you need to work out how to use the key to decode the message.




## Code 2: Polybius Square Cipher

**Message:-** If you decode this you can get the clue to identify the name of the treasure name

### *II. Fill in the blanks*

A Polybius Square is a table that allows someone to convert letters into numbers. Use the Polybius square rows and column values to find the code

| | | | | | |
|---|---|---|---|---|---|
| 5 | A | B | C | D | E |
| 4 | F | G | H | I/J | K |
| 3 | L | M | N | O | P |
| 2 | Q | R | S | T | U |
| 1 | V | W | X | Y | Z |
| | 1 | 2 | 3 | 4 | 5 |

(4,2) (3,4) (5,5) (3,3) (1,5) (2,3) (5,5) (4,3) (1,4)

T H E _____ O F

(4,2) (2,2) (5,5) (1,5) (3,2) (5,2) (2,2) (5,5) (4,5) (4,3) (5,5) (3,2)

_____ D O E S

(3,3) (4,3) (4,2) (3,4) (1,5) (1,1) (5,5) "(2,5)" (1,5) (3,3) (4,5) "(4,5)"

_____ _____ _____ A N D _____

## Code 3: Atbash Cipher

**Message:-** If you decode this you can get the clue room number where the treasure is present (It may be 24 or 25 or 26 or 27 or 28 or 29 or 30)

***III. Find the code using the key as shown in given figure:***

Atbash cipher is a substitution cipher with just one specific key where all the letters are reversed that is **A to Z** and **Z to A**.

| Plain Text | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cipher Text | Z | Y | X | W | V | U | T | S | R | Q | P | O | N | M | L | K | J | I | H | G | F | E | D | C | B | A |

**GSV ILLN MFNYVI RH Z NFOGRKOV LU ULFI ZMW HVEVM**

**THE** ____ ____ _____ __ _ _____ __ ____ ____ _____.

## Code 4: Using a Key – Reflection Table

**Message:-** If you decode this clue you can get the possible place where the treasure is located (It used to sit)

Use the reflection table which is given below and find the correct word by using a reflected alphabet.

| A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ |
| N | O | P | Q | R | S | T | U | V | W | X | Y | Z |

**JVAQBJ** - _____

**PBZCHGRE GNOYR** - _____

**PUNVE** - _____

**PHOOBNEQ** - _____

After finding the codes, the teacher then asks students to rearrange the clues one by one

**CLUES**

i. _____

ii. _____

iii. _____

iv. _____

**RESULT**

(i) The room in which the treasure took place :- _____.

(ii) The place of the treasure :- _____.

(iii) The identity of the treasure :- _____.

(Hint:- If you answered the question number 6 in Exercise 4.3, you can compare and verify your results) The gift voucher contains (20 full marks awarded).

## Try these

1. Use Pigpen Cipher code and write the code for your name _____and your chapter names

   (i) LIFE MATHEMATICS (ii) ALGEBRA

   (iii) GEOMETRY (iv) INFORMATION PROCESSING

2. Decode the following Shifting and Substituting secret codes given below. Which one is easier for you?

   (i) Shifting method:- M N S G H M F H R H L O N R R H A K D

   (ii) Substituting method:-

3. Write a short message to a friend and get him to decipher it.(use either shifting or substituting code method)

## Exercise 7.3

1. Fill in the blanks **(Use Atbash Cipher that is given in code 3)**

   (i) G Z N R O = _____

   (ii) V M T O R H S = _____

   (iii) N Z G S V N Z G R X H = _____

   (iv) H X R V M X V = _____

   (v) H L X R Z O H X R V M X V = _____

2. Match the following **( a = 00..... Z= 25).**

   (i) mathematics - (a) 18 20 01 19 17 00 02 19 08 14 13

   (ii) addition - (b) 03 08 21 08 18 08 14 13

   (iii) subtraction - (c) 12 00 19 07 04 12 0019 08 02 18

   (iv) multiplication - (d) 00 03 03 08 19 08 14 13

   (v) division - (e) 12 20 11 19 08 15 11 15 02 00 19 08 14 13

   G8W5R5

3. Frame Additive cipher table **(key = 4).**

4. A message like **"Good Morning"** written in reverse would instead be **"Doog Gninrom".** In the same way decode the sentence given below:

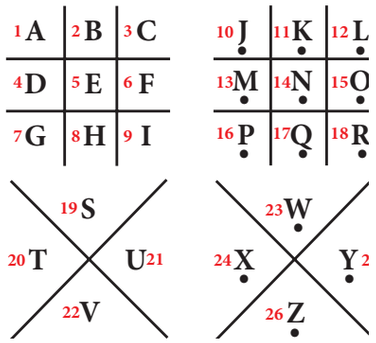
   **"Ot dnatsrednu taht scitamehtam nac eb decneirepxe erehwreve ni erutan dna laer efil."**

5. Decode the given **Pigpen Cipher text** and compare your answer to get the **Activity 3 result**.

I. The room number in which the treasure took place:

II. Place of the treasure :

III. The name of the treasure :




6. Praveen recently got the registration number for his new two-wheeler. Here, the number is given in the form of mirror-image. Encode the image and find the correct registration number of praveen's two-wheeler.


(a) 


(b) 


(c) 


(d) 


## Objective Type Questions

7. In questions **(i)** and **(ii)**, there are four groups of letters in each set. Three of these sets are alike in some way while one is different. Find the one which is different.

**(i)**. (A) C R D T (B) A P B Q (C) E U F V (D) G W H X

**(ii)**. (A) H K N Q (B) I L O R (C) J M P S (D) A D G J

8. A group of letters are given. A numerical code has been given to each letter. These letters have to be unscrambled into a meaningful word. Find out the code for the word so formed from the 4 answers given.

| L | I | N | C | P | E |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |

(A) 2 3 4 1 5 6 (B) 5 6 3 4 2 1 (C) 6 1 3 5 2 4 (D) 4 2 1 3 5 6

9. Questions **(iii)** and **(iv)** are based on code language. Find the correct answer from the four alternatives given.

**(iii)** In a certain code, '**M E D I C I N E**' is coded as '**E O J D J E F M**', then how is '**C O M P U T E R**' written in the same code ?

(A)C N P R V U F Q (B) C M N Q T U D R

(C) R F U V Q N P C (D) R N V F T U D Q

**(iv)** If the word '**P H O N E**' is coded as '**S K R Q H**', how will '**R A D I O**' be coded ?

(A) S C G N H (B) V R G N G (C) U D G L R (D) S D H K Q

## 7.8 Shopping Comparison

Students, I hope you all have an experience in shopping. can you share a few of your experiences? I would like to raise some questions on your shopping experience. How will you shop the things you need? By (i) attractive colour or (ii) best price or (iii) big in size or (iv) on seeing. Whatever it may be, among all these things, one important point should be noted. What is that? Yes, that is the expiry date. Have you ever noticed the expiry date on all the packed goods? It is very important to see that and another way of shopping is to compare goods by means of its price, quality, quantity, offers, discount and other considerable things.

Before spending your money to shop any item from a market or a departmental store, consider the best prices, the best quality and other reliable things. This is what we call as wise shopping.

Here, we will learn how to be a wise consumer before shopping a product from the following situation.

***Situation:***

Imagine that the teacher maker you and your friend to be incharge of the fruit section of your school canteen for a week. She also instructs the following steps and she can help you when needed.

**Shopping list**

1. 20 kg apples
2. 20 kg of guavas
3. 30 boxes of strawberries
4. 20 dozens of bananas

- Now you have to buy fruits for 2 days as per your the shopping list.
- One of you should go to the market and the other should go to the departmental store to know the cost of the fruits before shopping.
- Estimate yourselves about which place will give you the best deal.

After that,

- Check your shopping list to see how much fruits you require.
- Compare the weight and price for each item from both places.
- Select the best deal for all items in only one place .
- Discuss and compare the price list so that you decide where to buy the required list of fruits.

For example, the collected model price list from both shops is given in the table below:

| S. No | Fruit name | Departmental store | | Market price | |
|---|---|---|---|---|---|
| | | Quantity | Price (₹) | Quantity | Price (₹) |
| 1 | Apple | 1 kg | 120 | 1 kg | 110 |
| 2 | Guava | 1 kg | 50 | 1 kg | 40 |
| 3 | Strawberry | 1 box | 80 | 1 box | 85 |
| 4 | Banana | 1 dozen | 60 | 1 kg | 50 |

Now, we will calculate the total price of the required and quantity of fruits from both the departmental store and market.

### Calculating the total price from the Departmental store:

| Fruit Name | Cost of the required fruits | Total Price (₹) |
|---|---|---|
| Apple  | Cost of 1 kg of apples = ₹120<br>Cost of 20 kg of apples = 20 × 120 = ₹2400 | 2400 |
| Guava  | Cost of 1 kg guavas = ₹50<br>Cost of 20 kg guavas = 20 × 50 = ₹1000 | 1000 |
| Strawberry  | Cost of 1 box of strawberries = ₹80<br>Cost of 30 boxes of strawberries = 30 × 80= ₹2400 | 2400 |
| Banana  | Cost of 1dozen of bananas = ₹60<br>Cost of 20 kg of bananas = 20 × 60 = ₹1200 | 1200 |

### Calculating the total price from the Market Price:

| Fruit Name | Cost of the required fruits | Total Price (₹) |
|---|---|---|
| Apple  | Cost of 1 kg of apples = ₹110<br>Cost of 20 kg of apples = 20 × 110 = ₹2200 | 2200 |
| Guava  | Cost of 1 kg of guavas = ₹40<br>Cost of 20 kg of guavas = 20 × 40 = ₹800 | 800 |
| Strawberry  | Cost of 1 box of strawberries = ₹85<br>Cost of 30 boxes of strawberries = 30 × 85 = ₹2550 | 2550 |
| Banana  | Cost of 1 kg of bananas = ₹50<br>Cost of 20 kg of bananas = 20 × 50 = ₹1000 | 1000 |

Now, let us compare the shopping price of the Departmental store to that of the Market shop.

| Fruits | Cost of items as per your requirement (₹) | |
|---|---|---|
| | Departmental Store | Market |
| 20 kg of Apples | 2400 | 2200 |
| 20 kg of Guavas | 1000 | 800 |
| 30 boxes of Strawberries | 2400 | 2550 |
| 20 dozens of Bananas | 1200 | 1000 |
| **Total cost of shopping** | **7000** | **6550** |

From the above comparison, we find that shopping made at the Market shop is the best deal quantity wise as well as in price wise and hence it is wise to shop in the Market.

## Activity

Consider that you are going to a store with your total budget of ₹220 to buy things without changing the quantity of the items given in the list below with the following conditions.


**Conditions:**

(i) First you have to complete the price list given.

(ii) You have to buy three items as per the given price list but within your budget **₹220**.

(iii) You won't carry exceeding **5kg** because you have to walk home carrying them, so they cannot be bulky.

**Now, answer the following questions:**

1. In how many ways can you buy your items? Complete the price lists given below. One is done for you.
2. Which one is the best purchase price list and why?

**Price List**

| S.No. | Description | Price / 1 kg (₹) | Quantity kg | Amount (₹) |
|---|---|---|---|---|
| 1 | Rice | 37.50 | 2.50 | |
| 2 | Toor Dal | 62.00 | 1.00 | |
| 3 | Sugar | 32.50 | 1.50 | |
| 4 | Wheat | 26.50 | 1.00 | |
| Total Bill Amount | | | | |

**Price List**

| S.No. | Description | Price / 1 kg (₹) | Quantity kg | Amount (₹) |
|---|---|---|---|---|
| 1 | Rice | 37.50 | 2.50 | 93.75 |
| 2 | Toor Dal | 62.00 | 1.00 | 62.00 |
| 3 | Wheat | 26.50 | 1.00 | 26.50 |
| Total Bill Amount | | | | 182.25 |

**Price List**

| S. No. | Description | Price / 1 kg (₹) | Quantity kg | Amount (₹) |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| Total Bill Amount | | | | |

**Price List**

| S. No. | Description | Price / 1 kg (₹) | Quantity kg | Amount (₹) |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| Total Bill Amount | | | | |

**Price List**

| S. No. | Description | Price / 1 kg (₹) | Quantity kg | Amount (₹) |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| Total Bill Amount | | | | |

### 7.8.1 Comparing containers of different size

Many times, items are packed in different sized of containers.

- Sometimes, shoppers save money by selecting a larger container of the same item. For example, 1 litre of milk often costs less than 5 units of 200ml pack of milk.
- Sometimes, a store has two prices for the same item. One price is for buying a single item, while the other price is for buying more than one of the same item. For example, groundnut oil may cost ₹135 for 1 litre bottle and ₹240 for 2 litre bottles. In this case, if you buy two 1 litre bottles, you will pay more. Sometimes, buying in quantity also saves money.
- Sometimes, the consumer may not be able to use up the larger size of an item before it becomes stale or outdated. To find out which size container is the best to buy, you will need to know the price of single pack of the contents.

**Activity**

Consider that you want to buy 12 litres of the same quality of edible oil at your budget price of ₹250 per litre. In a supermarket, there are a lot of offers on various oil brands. Some of the offers are given below. Complete the table and find which one is the best offer for you and how much you will save for your total purchase.

**Which one is the best deal?**

| Product (Edible oil) | Size (in litres) | Regular Price (₹) | Offer | Special Price (₹) | Saving Price (₹) | Cost of 1 litre (₹) | Cost of 12 litres(₹) |
|---|---|---|---|---|---|---|---|
|  | 1 | 293 | ₹ 50 off | 243 | | 243 | |
|  | 2 | 850 | 1 *l* +1 *l* combo | 499 | 351 (850-499) | 249.50 | |
|  | 5+1 = 6 | 2000 | Buy 5 *l* get 1 *l* free | 1500 | | | 3000 |
|  | 2+2 = 4 | 1486 | Buy 1 get 1 free | 743 | | 185.75 | |
|  | 1+1= 2 | 850 | Spl. offer 1 *l* pack of 2 ₹ 390 | 390 | | 195 | |
|  | 12 (1) = 12 | 5100 | 1 *l* pack of 12 | 1650 | 3450 | | |

Best offer price for you _____

Amount that you saved for your total purchase _____

## Try these

The teacher divides the class into four groups and sets up a mock market in the class room and ask the students to involve in role play as two groups of businessmen and two groups of consumers. Consumers have to buy products at different shops and prepare a price list.

The two supermarkets in which the two groups can buy are Star Food Mart and Super Provisions. This week they each have got a special deal on some products. At Star Food Mart, you can buy items at discount prices. At Super Provisions, there are some "**BUY ONE GET ONE**" deals. Have a look at their deal:

### Star Food Mart

Chocolate Biscuits worth ₹114 per packet at ₹30 off


Peanut candies worth ₹90 now at ₹20 off


Protein milk worth ₹60 now at ₹20 off

Badam nuts worth ₹450 now at ₹150 off

### Super Provisions

Chocolate Biscuits worth ₹180 per packet Buy one get one free


Peanut candies worth ₹150 Buy one get one free


Protein milk worth ₹80 Buy one get one free

Badam nuts worth ₹580 Buy one get one free

Now, answer the following questions.

**I. Here is your shopping list:**

**(i) 4 bottles of Protein Milk (200 ml size)** **(ii) 2 packets of Peanut candies(200 gm)**

**(iii) 1 packet of Chocolate biscuits** **(iv) 1 packet of Badam nuts (500 gm)**

(i) If you buy all the items in one shop, where will you get the best price?

(ii) If you buy the items from the two shops, how will you do it to spend the least amount of money?

**II. You have ₹1000/- to spend to buy the following shopping list:**

**(i) 6 bottles of Protein Milk (200 ml size)** **(ii) 3 packets of Peanut candies(200 gm)**

**(iii) 3 packets of Chocolate biscuits** **(iv) 1 packet of Badam nuts (250 gm).**

(i) How can you do this so that you don't go over your budget amount ₹1000?

(ii) Which shop offers you the best value for money on each item?

(iii) Is the "**BUY ONE GET ONE**" deal at Super Provisions the same as "**50% off**" deal?

## 7.9 Packing


When we are packing something in a box or a suitcase or a cupboard, first we have to decide how we are going to pack. How many items can be fitted into that fixed space? A good example of this is, before you go to school, you try to pack everything you need (like your books, notebooks, geometry box, sports equipment, food and water bottle) into your school bag. At that time, you are very clear that your books should not be damaged and you are able to carry everything yourself. Think! The same rules apply to posting a package to a friend or family member or others.

Apart from these, the packaging methods are used in many cases such as cutting of sheets, glass, paper, wood, cloth or other materials and room allotment, seating arrangement in the particular space provided, parking vehicles with proper lanes and saving data in hard disk, CD, pen drive and so on.

Using some of the packing methods from the following situations and examples, let us try to understand how best to fit the items into the space in the containers or in rooms or in boxes etc.,

### 7.9.1 Packing Approaches - Fractional method

Here, when we fill items in bags or in containers, we determine the weight, value and number of each item with the condition that the total weight of the container is less than or equal to a given limit and the total value is as large as possible. Fractional method uses the technique of buying things fractionally and admits buying of more items within a given budget. Let us learn more about this approach from the following situation.

U1Q3M1

***Situation 1:***

Suppose that you want to buy some vegetables and fruits that are given in the Fig. 7.20 with their weights and price and that you have a bag that capacity of carrying 15 kg. The objective is to buy more items within your budget of ₹550 and also that weight should not exceed 15 kg.

Fig. 7.20

You cannot buy all the items because if you calculate the total weight of all the items, then it would be greater than 15 kg (maximum capacity of your bag is 15 kg). So, let us try some approaches, to find how you can select more items so that you will buy them with maximum price within your budget of ₹550. For that let us tabulate the items with their weights and price you want to buy.

| Items |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
| Weight (kg) | 1 | 3 | 5 | 4 | 1 | 3 | 2 |
| Price (₹) | 60 | 105 | 150 | 70 | 80 | 90 | 40 |

## Approach I - Selecting items with Maximum Price

In this approach, we select the items according to the maximum price. Here the maximum price in the table is ₹150/-. Now, let us tabulate to find the total price and how much can we buy vegetables and fruits within our budget and not exceeding 15kg.

| Items | Weight (kg) | Remaining weight to buy | Price (₹) |
|---|---|---|---|
|  | 5 | 15−5=10 | 150.00 |
|  | 3 | 10−3=7 | 105.00 |

| | | | |
|---|---|---|---|
|  | 3 | 7−3=4 | 90.00 |
|  | 1 | 4−1=3 | 80.00 |
|  | 3 | 3−3=0 | $70 \times \frac{3}{4} = 52.50$ |
| **Total price** | **15 kg** | | **472.50** |

Here, 3 kg of papaya is enough as the total weight reaches 15 kg. So, 3 kg of papaya costs ₹52.50. Hence, in this approach we will spend maximum ₹472.50 to buy 15 kg of vegetables and fruits.

### Approach II - Selecting items with Minimum Weight

In this approach, we select the items according to the minimum weight. Here, we can select more and more items. Now, let us tabulate to find the total price and how much can we buy vegetables and fruits within our budget and not exceeding 15kg.

| Items | Weight(kg) | Remaining weight to buy | Price (₹) |
|---|---|---|---|
|  | 1 | 15−1=14 | 60.00 |
|  | 1 | 14−1=13 | 80.00 |
|  | 2 | 13−2=11 | 40.00 |
|  | 3 | 11−3=8 | 105.00 |
|  | 3 | 8−3=5 | 90.00 |

| | | | |
|---|---|---|---|
|  | 4 | 5−4=1 | 70.00 |
|  | **1** | 1−1=0 | **$150 \times \frac{1}{5}$ =30.00** |
| **Total price** | **15 kg** | | **475.00** |

Here, 1 kg of Sapotta is enough to complete 15 kg with minimum price of ₹30 per kg. Hence in this approach, we will spend maximum ₹475 to buy 15 kg of vegetables and fruits.

### Approach III - Finding the Maximum price to Weight Ratio.

In this approach, we select the items according to the maximum price to weight ratio (find the rate of 1kg). Now, let us tabulate to find the total price and how much we can buy vegetables and fruits within our budget and not exceeding 15 kg.

| Items | Weight (kg) | Price of 1kg | Remaining weight to buy | Price (₹) |
|---|---|---|---|---|
|  | 1 | 80.00 | 15−1=14 | 80.00 |
|  | 1 | 60.00 | 14−1=13 | 60.00 |
|  | 3 | 35.00 | 13−3=10 | 105.00 |
|  | 5 | 30.00 | 10−5=5 | 150.00 |
|  | 3 | 30.00 | 5−3=2 | 90.00 |
|  | 2 | 20.00 | 3−2=0 | 40.00 |
| **Total price** | **15 kg** | | | **520.00** |

In this approach, we can buy all vegetables and fruits except papaya as we need with maximum price within our budget and not exceeding 15 kg. Comparatively, in the II approach we can buy more items but spend minimum amount only. So, we can say third approach is best one. Isn't it?

## Exercise 7.4

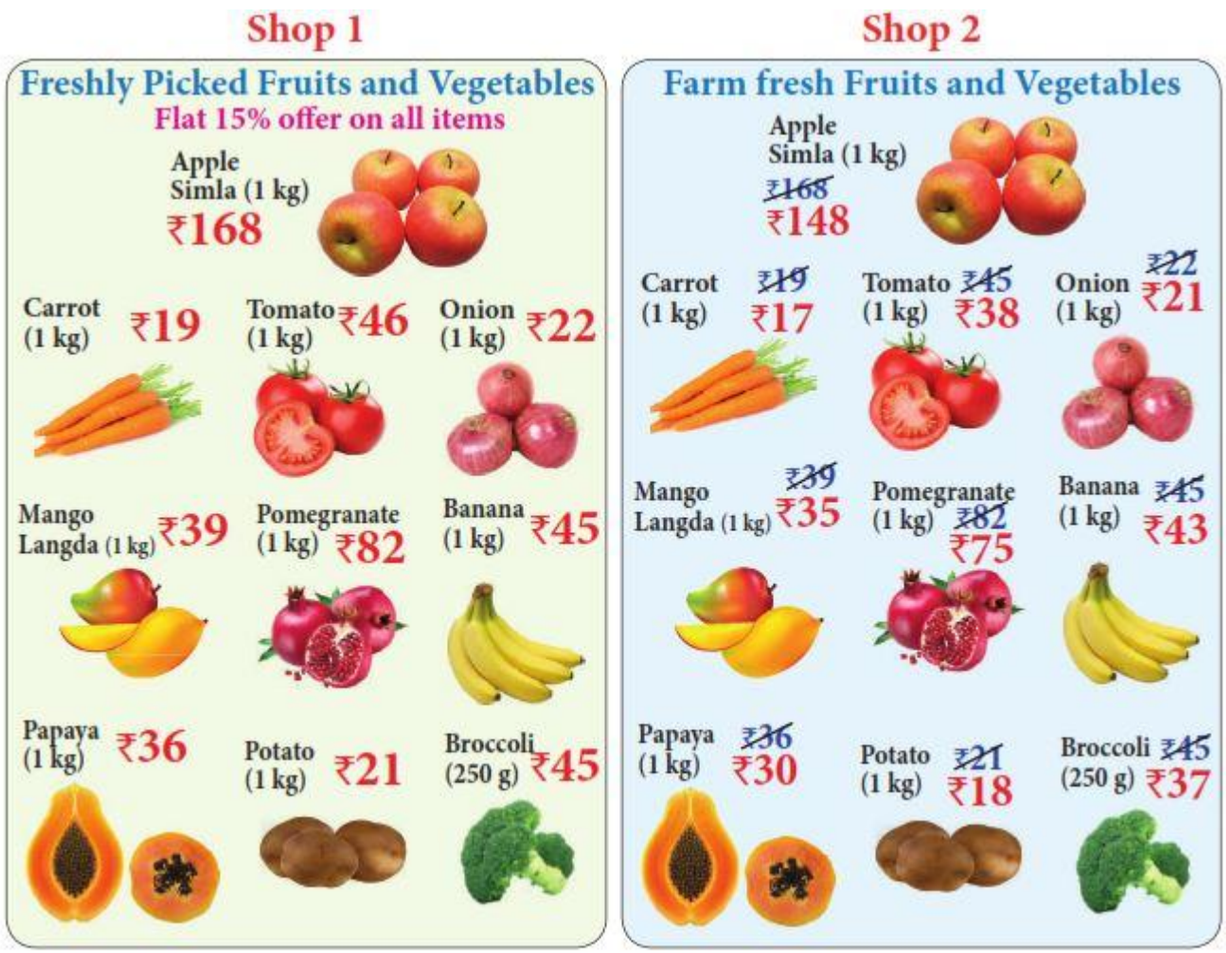
P8J7K2

1. Find the best buy of the following purchases:
   (i) A pack of 5 chocolate bars for ₹175 or 3 chocolate bars for ₹114?
   (ii) Basker buy 1½ dozen of eggs for ₹81 and Aruna buy 15 eggs for ₹64.50?

2. Using the given picture find the total special offer price of fresh sweets and bakery products to buy ½ kg laddu, 1 kg cake, 6 pockets of bread.


3. Using the given picture prepare a price list.

Suppose you plan to buy 1½ kg of apple, 2 kg of pomegranate, 2 kg of banana, 3 kg of mango, ½ kg of papaya, 3 kg of onion, 1½ kg of tomato, and 1 kg of carrot in shop 1, how much will you save compared to shop 2.



4. You want to buy some grocery items as per your shopping list that are given in the picture with their price. Also you have a bag that capacity of carrying 7 kg. Using weight ratio approach tabulate to find the total price and how much can you buy more grocery items within your budget of ₹1000 and not exceeding 7 kg.

**Shopping list**

1. 2 kg of red chilli
2. 2 kg of coriander
3. 1 kg of garlic
4. 1 kg of tamarind
5. 2 kg of toor dal

| Red chilli | Coriander | Garlic | Tamarind | Toor dal |
|---|---|---|---|---|
|  |  |  |  |  |
| ₹145/1 kg. | ₹130/1 kg. | ₹82/1 kg. | ₹99/1 kg. | ₹78/1 kg. |

## Objective Type Questions

5. Online or television advertisements influence people on spending decisions by
   (a) using special music
   (b) making them think that they need the item
   (c) using attractive pictures
   (d) all the above

6. When I go shopping, I will buy
   (a) something that looks attractive
   (b) something my friend has
   (c) something that I need to purchase
   (d) the first thing I see in the store

7. The best shopping choice is to
   (a) shop at brand name stores always buy
   (b) compare the choices before buying
   (c) the same thing my friends bought
   (d) buy at a regular shop always