Quadratic Equations

Important Concepts

1. If p(x) is a quadratic polynomial, then p(x) = 0 is called a **quadratic equation**.
   The general form of a quadratic equation, in the variable x, is $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

2. The value of x that satisfies an equation is called the **zeroes** or **roots** of the equation.
   A real number α is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$

   if $a\alpha^2 + b\alpha + c = 0$.

3. A quadratic equation has **at most two zeros**.

Nature of the roots of a quadratic equation:

The nature of the roots of a quadratic equation depends upon the value of discriminant $b^2 - 4ac$.

i. If $\mathbf{b^2 - 4ac > 0}$, the roots are **real** and **unequal**
ii. If $\mathbf{b^2 - 4ac = 0}$, the roots are **real** and **equal**
iii. If $\mathbf{b^2 - 4ac < 0}$, the roots are **imaginary (not real)**

If $ax^2 + bx + c$, $a \neq 0$, can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

Solution of Quadratic Equation by Factroisation:

Example,

Solve the equation $\frac{9}{2}x = 5 + x^2$ by factorization:

Step 1: Clear all fractions and brackets, if necessary

$$9x = 2(5 + x^2)$$

Step 2: Transpose all the terms to the left hand side to get an equation in the form $ax^2 + bx + c = 0$

$$9x = 2x^2 + 10$$
$$\Rightarrow 2x^2 - 9x + 10 = 0$$

Step 3: Factorise the expression on the left hand side.

$$2x^2 - 9x + 10 = 0$$
$$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$$
$$\Rightarrow x(2x - 5) - 2(2x - 5) = 0$$
$$\Rightarrow (x - 2)(2x - 5) = 0$$

Step 4: Put each factor equal to zero and solve

$$(x-2)(2x-5)=0$$
$$\Rightarrow x-2=0 \qquad 2x-5=0$$
$$\Rightarrow x=2; \qquad 2x=5$$
$$\Rightarrow x=2; \qquad x=\frac{5}{2}$$
$$\text{Thus, we have, } x=2 \text{ or } x=\frac{5}{2}$$

Solution of Quadratic Equation by Quadratic Formula:

The roots of a quadratic equation $ax^2+bx+c=0$ $(a\neq 0)$ can be calculated by using **quadratic formula:**

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}, \text{ where } b^2-4ac\geq 0$$

Equations Reducible to Quadratic Form

There are many equations which are not in the quadratic form but can be reduced to quadratic form by simplifications.

Let us solve the equation $x^4-2x^2-3=0$

It is clear that the above equation is not a quadratic equation.

Now assume that $x^2=y$

Then rewrite the given quadratic equation as, $\left(x^2\right)^2-2\left(x^2\right)-3=0$

Substituting $x^2=y$ in the above equation, we have $y^2-2y-3=0$

This is a quadratic equation in y.

Let us solve the quadratic equation through factorization.

$$y^2-2y-3=0$$
$$\Rightarrow y^2-3y+y-3=0$$
$$\Rightarrow y(y-3)+(y-3)=0$$
$$\Rightarrow (y+1)(y-3)=0$$
$$\Rightarrow y+1=0 \text{ or } y-3=0$$
$$\Rightarrow y=-1 \text{ or } y=3$$

Applications of quadratic equation in solving real life problems

Following points can be helpful in solving word problems:

i. Every two digit number ‘xy’ where x is a ten’s place and y is a unit’s place can be expressed as $xy = 10x + y$

ii. Downstream: It means that the boat is running in the direction of the stream
Upstream: It means that the boat is running in the opposite direction of the stream
Thus, if

Speed of boat in still water is x km/h

And the speed of stream is y km/h

Then the speed of boat downstream will be $(x + y)$ km/h and in upstream it will be $(x - y)$ km/h.

iii. If a person takes x days to finish a work, then his one day's work = $\frac{1}{x}$