27. Straight Line in Space

Exercise 27A

1. Question

A line passes through the point (3, 4, 5) and is parallel to the vector $\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$. Find the equations of the line in the vector as well as Cartesian forms.

Answer

Given: line passes through point (3, 4, 5) and is parallel to $2\hat{i}+2\hat{j}-3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}-3\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{i}+4\hat{j}+5\hat{k}+\lambda\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$$

Cartesian form:

$$\frac{x-3}{2}=\frac{y-4}{2}=\frac{z-5}{-3}$$

2. Question

A line passes through the point (2, 1, -3) and is parallel to the vector $\left(\hat{i}-2\hat{j}+3\hat{k}\right)$. Find the equations of the line in vector and Cartesian forms.

Answer

Given: line passes through (2, 1, -3) and is parallel to $\hat{i}-2\hat{j}+3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+3}{3}$$

3. Question

Find the vector equation of the line passing through the point with position vector $\left(2\hat{i} + \hat{j} - 5\hat{k}\right)$ and parallel to the vector $\left(\hat{i} + 3\hat{j} - \hat{k}\right)$. Deduce the Cartesian equations of the line.

Answer

Given: line passes through $2\hat{i} + \hat{j} - 5\hat{k}$ and is parallel to $\hat{i} + 3\hat{j} - \hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a} = 2\hat{i} + \hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} + \hat{j} - 5\hat{k} + \lambda(\hat{i} + 3\hat{j} - \hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{3} = \frac{z+5}{-1}$$

4. Question

A line is drawn in the direction of $(\hat{i} + \hat{j} - 2\hat{k})$ and it passes through a point with position vector $(2\hat{i} - \hat{j} - 4\hat{k})$. Find the equations of the line in the vector as well as Cartesian forms.

Answer

Given: line passes through $2\hat{i} - \hat{j} - 4\hat{k}$ and is drawn in the direction of $\hat{i} + \hat{j} - 2\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since line is drawn in the direction of $(\widehat{i} + \hat{j} - \widehat{2k})$, it is parallel to $(\hat{i} + \hat{j} - 2\hat{k})$

Here, $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} - \hat{j} - 4\hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z+4}{-2}$$

5. Question

The Cartesian equations of a line are $\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$. Find the vector equation of the line.

Answer

Given: Cartesian equation of line

$$\frac{x-3}{2}=\frac{y+2}{-5}=\frac{z-6}{4}$$

To find: equation of line in vector form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

From the Cartesian equation of the line, we can find $\vec{a}$ and $\vec{b}$

Here, $\vec{a}=3\hat{i}-\hat{2}j+6\hat{k}$ and $\vec{b}=\hat{2}\imath-5\hat{j}+4\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+\lambda\left(2\hat{i}-5\hat{j}+4\hat{k}\right)$$

6. Question

The Cartesian equations of a line are 3x + 1 = 6y − 2 = 1 − z. Find the fixed point through which it passes, its direction ratios and also its vector equation.

Answer

Given: Cartesian equation of line are 3x + 1 = 6y − 2 = 1 - z

To find: fixed point through which the line passes through, its direction ratios and the vector equation.

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line and also its direction ratio.

Explanation:

The Cartesian form of the line can be rewritten as:

$$\frac{x+\frac{1}{3}}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1} = \lambda$$

$$\Rightarrow \frac{x+\frac{1}{3}}{2} = \frac{y-\frac{1}{3}}{1} = \frac{z-1}{-6} = \lambda$$

Therefore, $\vec{a} = \frac{-1}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 6\hat{k}$

So, the line passes through $\left(\frac{-1}{3}, \frac{1}{3}, 1\right)$ and direction ratios of the line are (2, 1, -6) and vector form is:

$$\vec{r} = \frac{-1}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath} + \hat{k} + \lambda(2\hat{\imath} + \hat{\jmath} - 6\hat{k})$$

7. Question

Find the Cartesian equations of the line which passes through the point (1, 3, -2) and is parallel to the line given by $\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$. Also, find the vector form of the equations so obtained.

Answer

Given: line passes through (1, 3, -2) and is parallel to the line

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

Since the equation of L_2 is

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

$$\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \lambda(3\hat{i} + 5\hat{j} - 6\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y-3}{5} = \frac{z+2}{-6}$$

8. Question

Find the equations of the line passing through the point (1, -2, 3) and parallel to the line $\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$. Also find the vector form of this equation so obtained.

Answer

Given: line passes through (1, -2, 3) and is parallel to the line

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

Since the equation of L_2 is

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

$$\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} - 4\hat{j} + 5\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3}=\frac{y+2}{-4}=\frac{z-3}{5}$$

9. Question

Find the Cartesian and vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$.

Answer

Given: line passes through (1, 2, 3) and is parallel to the line

$$\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

Equation of L_2 can be rewritten as:

$$\frac{x+2}{-1}=\frac{y+3}{7}=\frac{z-3}{\frac{3}{2}}$$

$$\Rightarrow \frac{x+2}{-2}=\frac{y+3}{14}=\frac{z-3}{3}$$

$$\vec{b}=-2\hat{i}+14\hat{j}+3\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(-2\hat{i}+14\hat{j}+3\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{-2}=\frac{y-2}{14}=\frac{z-3}{3}$$

10. Question

Find the equations of the line passing through the point (-1, 3, -2) and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.

Answer

Given: line passes through (-1, 3, -2) and is perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

Here, $\vec{a}=-\hat{i}+3\hat{j}-2\hat{k}$

Let the direction ratios of the line be $b_1:b_2:b_3$

Direction ratios of the other two lines are 1 : 2 : 3 and -3 : 2 : 5

Since the other two line are perpendicular to the given line, we have

$b_1 + 2b_2 + 3b_3 = 0$

$-3b_1 + 2b_2 + 5b_3 = 0$

Solving,

$$\frac{b_1}{\begin{vmatrix}2 & 3\\2 & 5\end{vmatrix}}=\frac{-b_2}{\begin{vmatrix}1 & 3\\-3 & 5\end{vmatrix}}=\frac{b_3}{\begin{vmatrix}1 & 2\\-3 & 2\end{vmatrix}}$$

$$\Rightarrow\frac{b_1}{4}=\frac{b_2}{-14}=\frac{b_3}{8}$$

$$\Rightarrow\frac{b_1}{2}=\frac{b_2}{-7}=\frac{b_3}{4}$$

$\vec{b} = 2\hat{i} - 7\hat{j} + 4\hat{k}$

Therefore,

Vector form of the line is:

$$\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

Cartesian form of the line is:

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

11. Question

Find the Cartesian and vector equations of the line passing through the point (1, 2, -4) and perpendicular to each of the lines $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$.

Answer

Given: line passes through (1, 2, -4) and is perpendicular to each of the lines $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

Here, $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

Let the direction ratios of the line be $b_1:b_2:b_3$

Direction ratios of other two lines are 8 : -16 : 7 and 3 : 8 : -5

Since the other two line are perpendicular to the given line, we have

$8b_1 - 16b_2 + 7b_3 = 0$

$3b_1 + 8b_2 - 5b_3 = 0$

Solving,

$$\frac{b_1}{\begin{vmatrix}-16 & 7\\ 8 & -5\end{vmatrix}} = \frac{-b_2}{\begin{vmatrix}8 & 7\\ 3 & -5\end{vmatrix}} = \frac{b_3}{\begin{vmatrix}8 & -16\\ 3 & 8\end{vmatrix}}$$

$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{61} = \frac{b_3}{112}$$

$$\vec{b} = 24\hat{i} + 61\hat{j} + 112\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(24\hat{i} + 61\hat{j} + 112\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112}$$

12. Question

Prove that the lines $\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect each other and find the point of their intersection.

Answer

Given: The equations of the two lines are

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} \text{ and } \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} = \lambda_1$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda_2$$

So a point on the first line is $(\lambda_1 + 4, 4\lambda_1 - 3, 7\lambda_1 - 1)$

A point on the second line is $(2\lambda_2 + 1, -3\lambda_2 - 1, 8\lambda_2 - 10)$

If they intersect they should have a common point.

$\lambda_1 + 4 = 2\lambda_2 + 1 \Rightarrow \lambda_1 - 2\lambda_2 = -3$ … (1)

$4\lambda_1 - 3 = -3\lambda_2 - 1 \Rightarrow 4\lambda_1 + 3\lambda_2 = 2$ … (2)

Solving (1) and (2),

$11\lambda_2 = 14$

$\lambda_2 = \frac{14}{11}$

Therefore, $\lambda_1 = \frac{-5}{11}$

Substituting for the z coordinate, we get

$7\lambda_1 - 1 = \frac{-46}{11}$ and $8\lambda_2 - 10 = \frac{2}{11}$

So, the lines do not intersect.

13. Question

Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect each other. Also, find the point of their intersection.

Answer

Given: The equations of the two lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-4}{5} = \frac{y-1}{2} = z$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda_1$$

$$\frac{x-4}{5}=\frac{y-1}{2}=z=\lambda_2$$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 + 2, 4\lambda_1 + 3)$

A point on the second line is $(5\lambda_2 + 4, 2\lambda_2 + 1, \lambda_2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 + 4 \Rightarrow 2\lambda_1 - 5\lambda_2 = 3$ ... (1)

$3\lambda_1 + 2 = 2\lambda_2 + 1 \Rightarrow 3\lambda_1 - 2\lambda_2 = -1$ ... (2)

Solving (1) and (2),

$-11\lambda_2 = 11$

$\lambda_2 = -1$

Therefore, $\lambda_1 = -1$

Substituting for the z coordinate, we get

$4\lambda_1 + 3 = -1$ and $\lambda_2 = -1$

So, the lines intersect and their point of intersection is (-1, -1, -1)

14. Question

Show that the lines $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}, z=2$ do not intersect each other.

Answer

Given: The equations of the two lines are

$$\frac{x-1}{2}=\frac{y+1}{3}=z \text{ and } \frac{x+1}{5}=\frac{y-2}{1}, z=2$$

To Prove: the lines do not intersect each other.

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2}=\frac{y+1}{3}=z=\lambda_1$$

$$\frac{x+1}{5} = \frac{y-2}{1} = \lambda_2, z = 2$$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 - 1, \lambda_1)$

A point on the second line is $(5\lambda_2 - 1, \lambda_2 + 1, 2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 - 1 \Rightarrow 2\lambda_1 - 5\lambda_2 = -2$ … (1)

$3\lambda_1 - 1 = \lambda_2 + 1 \Rightarrow 3\lambda_1 - \lambda_2 = 2$ … (2)

Solving (1) and (2),

$-13\lambda_2 = -10$

$$\lambda_2 = \frac{10}{13}$$

Therefore, $\lambda_1 = \frac{33}{65}$

Substituting for the z coordinate, we get

$\lambda_1 = \frac{33}{65}$ and z = 2

So, the lines do not intersect.

15. Question

Find the coordinates of the foot of the perpendicular drawn from the point (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the length of the perpendicular from the given point to the line.

Answer

Given: Equation of line is $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

To find: coordinates of foot of the perpendicular from (1, 2, 3) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Explanation:

Let

$$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=\lambda$$

So the foot of the perpendicular is (3λ + 6, 2λ + 7, -2λ + 7)

Direction ratio of the line is 3 : 2 : -2

Direction ratio of the perpendicular is

⇒ (3λ + 6 - 1) : (2λ + 7 - 2) : (-2λ + 7 - 3)

⇒ (3λ + 5) : (2λ + 5) : (-2λ + 4)

Since this is perpendicular to the line,

3(3λ + 5) + 2(2λ + 5) − 2(-2λ + 4) = 0

⇒ 9λ + 15 + 4λ + 10 + 4λ − 8 = 0

⇒ 17λ = -17

⇒ λ = -1

So the foot of the perpendicular is (3, 5, 9)

Distance $= \sqrt{(3-1)^2+(5-2)^2+(9-3)^2}$

$= \sqrt{4+9+36}$

= 7 units

Therefore, the foot of the perpendicular is (3, 5, 9) and length of perpendicular is 7 units.

16. Question

Find the length and the foot of the perpendicular drawn from the point (2, -1, 5) to the line

$$\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}.$$

Answer

Given: Equation of line is $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

To find: coordinates of foot of the perpendicular from (2, -1, 5) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Explanation:

Let

$$\frac{x - 11}{10} = \frac{y + 2}{-4} = \frac{z + 8}{-11} = \lambda$$

So the foot of the perpendicular is (10λ + 11, -4λ - 2, -11λ - 8)

Direction ratio of the line is 10 : -4 : -11

Direction ratio of the perpendicular is

⇒ (10λ + 11 - 2) : (-4λ - 2 + 1) : (-11λ - 8 - 5)

⇒ (10λ + 9) : (-4λ - 1) : (-11λ - 13)

Since this is perpendicular to the line,

10(10λ + 9) - 4(-4λ - 1) - 11(-11λ - 13) = 0

⇒ 100λ + 90 + 16λ + 4 + 121λ + 143 = 0

⇒ 237λ = -237

⇒ λ = -1

So the foot of the perpendicular is (1, 2, 3)

Distance $= \sqrt{(1 - 2)^2 + (2 + 1)^2 + (3 - 5)^2}$

$= \sqrt{1 + 9 + 4}$

= √14 units

Therefore, the foot of the perpendicular is (1, 2, 3) and length of perpendicular is √14 units.

17. Question

Find the vector and Cartesian equations of the line passing through the points A(3, 4, -6) and B(5, -2, 7).

Answer

Given: line passes through the points (3, 4, -6) and (5, -2, 7)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a}=3\hat{\imath}+4\hat{\jmath}-6\hat{k}$

The direction ratios of the line are (3 - 5) : (4 + 2) : (-6 - 7)

⇒ -2 : 6 : -13

⇒ 2 : -6 : 13

So, $\vec{b}=2\hat{\imath}-6\hat{\jmath}+13\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{\imath}+4\hat{\jmath}-6\hat{k}+\lambda\left(2\hat{\imath}-6\hat{\jmath}+13\hat{k}\right)$$

Cartesian form:

$$\frac{x-3}{2}=\frac{y-4}{-6}=\frac{z+6}{13}$$

18. Question

Find the vector and Cartesian equations of the line passing through the points A(2, -3, 0) and B(-2, 4, 3).

Answer

Given: line passes through the points (2, -3, 0) and (-2, 4, 3)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a}=2\hat{\imath}-3\hat{\jmath}$

The direction ratios of the line are (2 + 2) : (-3 - 4) : (0 - 3)

⇒ 4 : -7 : -3

⇒ -4 : 7 : 3

So, $\vec{b} = -4\hat{i} + 7\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} - 3\hat{j} + \lambda(-4\hat{i} + 7\hat{j} + 3\hat{k})$$

Cartesian form:

$$\frac{x-2}{-4} = \frac{y+3}{7} = \frac{z}{3}$$

19. Question

Find the vector and Cartesian equations of the line joining the points whose position vectors are $(\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 3\hat{j} - 2\hat{k})$.

Answer

Given: line passes through the points whose position vectors are $(\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 3\hat{j} - 2\hat{k})$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

The direction ratios of the line are (1 - 1) : (-2 - 3) : (1 + 2)

⇒ 0 : -5 : 3

⇒ 0 : 5 : -3

So, $\vec{b} = -5\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{i} - 2\hat{j} + \hat{k} + \lambda(5\hat{j} - 3\hat{k})$$

Cartesian form:

$$\frac{x-1}{0}=\frac{y+2}{5}=\frac{z-1}{-3}$$

20. Question

Find the vector equation of a line passing through the point A(3, -2, 1) and parallel to the line joining the points B(-2, 4, 2) and C(2, 3, 3). Also, find the Cartesian equations of the line.

Answer

Given: line passes through the point (3, -2, 1) and is parallel to the line joining points B(-2, 4, 2) and C(2, 3, 3).

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$

The direction ratios of the line are (-2 - 2) : (4 - 3) : (2 - 3)

⇒ -4 : 1 : -1

⇒ 4 : -1 : 1

So, $\vec{b}=4\hat{i}-\hat{j}+\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{i}-2\hat{j}+\hat{k}+\lambda\left(4\hat{i}-\hat{j}+\hat{k}\right)$$

Cartesian form:

$$\frac{x-3}{4}=\frac{y+2}{-1}=\frac{z-1}{1}$$

21. Question

Find the vector equation of a line passing through the point having the position vector $\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and parallel to the line joining the points with position vectors $\left(\hat{i}-\hat{j}+5\hat{k}\right)$ and $\left(2\hat{i}+3\hat{j}-4\hat{k}\right)$.

Also, find the Cartesian equivalents of this equation.

Answer

Given: line passes through the point with position vector $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{\imath} - \hat{\jmath} + 5\hat{k}$ and $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

The direction ratios of the line are (1 - 2) : (-1 - 3) : (5 + 4)

⇒ -1 : -4 : 9

⇒ 1 : 4 : -9

So, $\vec{b} = \hat{\imath} + 4\hat{\jmath} - 9\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k} + \lambda(\hat{\imath} + 4\hat{\jmath} - 9\hat{k})$$

Cartesian form:

$$\frac{x-1}{1} = \frac{y-2}{4} = \frac{z+3}{-9}$$

22. Question

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining the points B(1, 4, 6) and C(5, 4, 4).

Answer

Given: perpendicular drawn from point A (1, 2, 1) to line joining points B (1, 4, 6) and C (5, 4, 4)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (1, 4, 6) is a point on the line.

Therefore, $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Also direction ratios of the line are (1 - 5) : (4 – 4) : (6 – 4)

⇒ -4 : 0 : 2

⇒ -2 : 0 : 1

So, equation of the line in Cartesian form is

$$\frac{x-1}{-2} = \frac{y-4}{0} = \frac{z-6}{1} = \lambda$$

Any point on the line will be of the form (-2λ + 1, 4, λ + 6)

So the foot of the perpendicular is of the form (-2λ + 1, 4, λ + 6)

The direction ratios of the perpendicular is

(-2λ + 1 – 1) : (4 - 2) : (λ + 6 - 1)

⇒ (-2λ) : 2 : (λ + 5)

From the direction ratio of the line and the direction ratio of its perpendicular, we have

-2(-2λ) + 0 + λ + 5 = 0

⇒ 4λ + λ = -5

⇒ λ = -1

So, the foot of the perpendicular is (3, 4, 5)

23. Question

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8, 4) to the line joining the points B(0, -1, 3) and C(2, -3, -1).

Answer

Given: perpendicular drawn from point A (1, 8, 4) to line joining points B (0, -1, 3) and C (2, -3, -1)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (0, -1, 3) is a point on the line.

Therefore, $\vec{a}=-\hat{j}+3\hat{k}$

Also direction ratios of the line are (0 - 2) : (-1 + 3) : (3 + 1)

⇒ -2 : 2 : 4

⇒ -1 : 1 : 2

So, equation of the line in Cartesian form is

$$\frac{x}{-1}=\frac{y+1}{1}=\frac{z-3}{2}=\lambda$$

Any point on the line will be of the form $(-\lambda, \lambda - 1, 2\lambda + 3)$

So the foot of the perpendicular is of the form $(-\lambda, \lambda - 1, 2\lambda + 3)$

The direction ratios of the perpendicular is

$(-\lambda - 1) : (\lambda - 1 - 8) : (2\lambda + 3 - 4)$

⇒ $(-\lambda - 1) : (\lambda - 9) : (2\lambda - 1)$

From the direction ratio of the line and the direction ratio of its perpendicular, we have

$-1(-\lambda - 1) + \lambda - 9 + 2(2\lambda - 1) = 0$

⇒ $\lambda + 1 + \lambda - 9 + 4\lambda - 2 = 0$

⇒ $6\lambda = 10$

⇒ $\lambda=\frac{5}{3}$

So, the foot of the perpendicular is $\left(\frac{-5}{3},\frac{2}{3},\frac{19}{3}\right)$

24. Question

Find the image of the point (0, 2, 3) in the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

Answer

Given: Equation of line is $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

To find: image of point (0, 2, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

Explanation:

Let

$$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$$

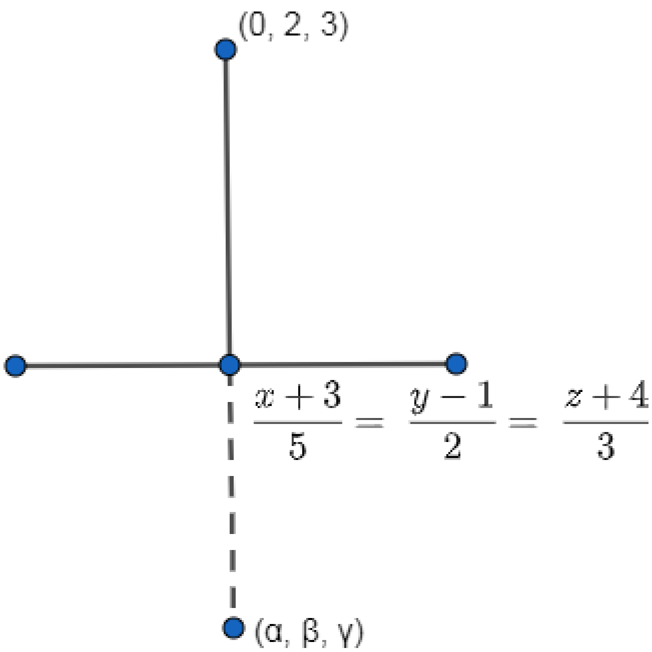
So the foot of the perpendicular is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

The direction ratios of the perpendicular is

$(5\lambda - 3 - 0) : (2\lambda + 1 - 2) : (3\lambda - 4 - 3)$

$\Rightarrow (5\lambda - 3) : (2\lambda - 1) : (3\lambda - 7)$

Direction ratio of the line is 5 : 2 : 3



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (2, 3, -1)

The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (α, β, γ)

So, we have

$$\frac{\alpha + 0}{2} = 2 \Rightarrow \alpha = 4$$

$$\frac{\beta + 2}{2} = 3 \Rightarrow \beta = 4$$

$$\frac{\gamma + 3}{2} = -1 \Rightarrow \gamma = -5$$

So, the image is (4, 4, -5)

25. Question

Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2} = \frac{y=2}{3} = \frac{z-3}{4}$.

Answer

Given: Equation of line is $\frac{x-1}{2}=\frac{y=2}{3}=\frac{z-3}{4}$.

To find: image of point (5, 9, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

Explanation:

Let

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$$

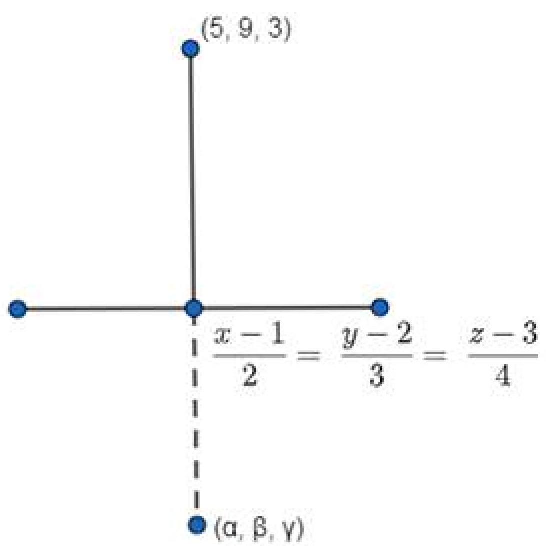
So the foot of the perpendicular is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

The direction ratios of the perpendicular is

$(2\lambda + 1 - 5) : (3\lambda + 2 - 9) : (4\lambda + 3 - 3)$

$\Rightarrow (2\lambda - 4) : (3\lambda - 7) : (4\lambda)$

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$

$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$

$\Rightarrow 29\lambda = 29$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α, β, γ)

So, we have

$$\frac{\alpha + 5}{2} = 3 \Rightarrow \alpha = 1$$

$$\frac{\beta + 9}{2} = 5 \Rightarrow \beta = 1$$

$$\frac{\gamma + 3}{2} = 7 \Rightarrow \gamma = 11$$

So, the image is (1, 1, 11)

26. Question

Find the image of the point (2, -1, 5) in the line

$$\vec{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$$

Answer

Given: Point (2, -1, 5)

Equation of line $= \left(11\,\hat{\imath} - 2\,\hat{\jmath} - 8\hat{k}\right) + \lambda\left(10\hat{\imath} - 4\,\hat{\jmath} - 11\hat{k}\right)$

The equation of line can be re-arranged as $\frac{x-11}{10} = \frac{x+2}{-4} = \frac{x+8}{-11} = r$

The general point on this line is

(10r + 11, -4r – 2, -11r – 8)

Let N be the foot of the perpendicular drawn from the point P(2, 1, -5) on the given line.

Then, this point is N(10r + 11, -4r – 2, -11r – 8) for some fixed value of r.

D.r.'s of PN are (10r + 9, -4r - 3, -11r - 3)

D.r.'s of the given line is 10, -4, -11.

Since, PN is perpendicular to the given line, we have,

10(10r + 9) – 4(-4r – 3) – 11(-11r – 3) = 0

100r + 90 + 16r + 12 + 121r + 33 = 0

237r = 135

$r = \frac{135}{237}$

Then, the image of the point is

$$\frac{\alpha - 11}{-11} = 0, \frac{\beta + 2}{7} = 1, \frac{\gamma + 8}{9} = 1$$

Therefore, the image is (0, 5, 1).

Exercise 27B

1. Question

Show that the points A(2, 1, 3), B(5, 0, 5) and C(-4, 3, -1) are collinear.

Answer

Given -

A = (2,1,3)

B = (5,0,5)

C = (-4,3,-1)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((5-2),(0-1),(5-3))

=(3,-1,-2)

Similarly, the direction ratios of the line BC can be given by

((-4-5),(3-0),(-1-5))

=(-9,3,-6)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(3,-1,-2)

=(-1/3)X(-9,3,-6)

=(-1/3)Xd.r. of BC

Hence, **A, B and C are collinear**

2. Question

Show that the points A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11) are collinear.

Answer

Given -

A = (2,3,-4)

B = (1,-2,3)

C = (3,8,-11)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(-2-3),(3+4))

=(-1,-5,7)

Similarly, the direction ratios of the line BC can be given by

((3-1),(8+2),(-11-3))

=(2,10,-14)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-5,7)

=(-1/2)X(2,10,-14)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

3. Question

Find the value of λ for which the points A(2, 5, 1), B(1, 2, -1) and C(3, λ, 3) are collinear.

Answer

Given -

A = (2,5,1)

B = (1,2,-1)

C = (3,λ,3)

To find – The value of λ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(2-5),(-1-1))

=(-1,-3,-2)

Similarly, the direction ratios of the line BC can be given by

((3-1),(λ-2),(3+1))

=(2,λ-2,4)

Tip – If it is shown that direction ratios of AB=ɑ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-3,-2)

=(-1/2)X(2,λ-2,4)

=(-1/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{1}{2}(\lambda - 2) = -3$

⇨ $\lambda - 2 = 6$

⇨ $\boldsymbol{\lambda = 8}$

4. Question

Find the values of λ and μ so that the points A(3, 2, -4), B(9, 8, -10) and C(λ, μ -6) are collinear.

Answer

Given -

A = (3,2,-4)

B = (9,8,-10)

C = (λ,μ,-6)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((9-3),(8-2),(-10+4))

=(6,6,-6)

Similarly, the direction ratios of the line BC can be given by

((λ-9),(μ-8),(-6+10))

=(λ-9,μ-8,4)

Tip – If it is shown that direction ratios of AB=ɑ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(6,6,-6)

=(-6/4)X(-4,-4,4)

=(-3/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(\lambda-9)=6$

$\Rightarrow \lambda-9=-4$

$\Rightarrow \boldsymbol{\lambda=5}$

And,

$\therefore -\frac{3}{2}(\mu-8)=6$

$\Rightarrow \mu-8=-4$

$\Rightarrow \boldsymbol{\lambda=4}$

5. Question

Find the values of λ and μ so that the points A(-1, 4, -2), B(λ, μ 1) and C(0, 2, -1) are collinear.

Answer

Given -

A = (-1,4,-2)

B = (λ,μ,1)

C = (0,2,-1)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((λ+1),(μ-4),(1+2))

=(λ+1,μ-4,3)

Similarly, the direction ratios of the line BC can be given by

((0-λ),(2-μ),(-1-1))

=(-λ,2-μ,-2)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(λ+1,μ-4,3)

Say, α be an arbitrary constant such that d.r. of AB = α X d.r. of BC

So, 3 = α X (-2)

i.e. α = -3/2

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(-\lambda) = \lambda + 1$

⇨ $3\lambda = 2\lambda + 2$

⇨ $\boldsymbol{\lambda = 2}$

And,

$\therefore -\frac{3}{2}(2 - \mu) = \mu - 4$

⇨ $-6 + 3\mu = 2\mu - 8$

⇨ $\boldsymbol{\mu = -2}$

6. Question

The position vectors of three points A, B and C are $\left(-4\hat{i}+2\hat{j}-3\hat{k}\right),\left(\hat{i}+3\hat{j}-2\hat{k}\right)$ and $\left(-9\hat{i}+\hat{j}-4\hat{k}\right)$ respectively. show that the points A, B and C are collinear.

Answer

Given -

$\vec{A} = -4\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{C} = -9\hat{i} + \hat{j} - 4\hat{k}$

It can thus be written as:

A = (-4,2,-3)

B = (1,3,-2)

C = (-9,1,-4)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1+4),(3-2),(-2+3))

=(5,1,1)

Similarly, the direction ratios of the line BC can be given by

((-9-1),(1-3),(-4+2))

=(-10,-2,-2)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(5,1,1)

=(-1/2)X(-10,-2,-2)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

Exercise 27C

1. Question

Find the angle between each of the following pairs of lines:

$\vec{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

Answer

Given – $\overrightarrow{L_1} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$

& $\overrightarrow{L_2} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,-1,-2)

Direction ratios of L_2 = (3,-5,-4)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3+(-1)\times(-5)+(-2)\times(-4)}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right)$$

$$= \cos^{-1}\left(\frac{16}{5\sqrt{6}\sqrt{2}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$

2. Question

Find the angle between each of the following pairs of lines:

$\vec{r} = \left(3\hat{i}-4\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+3\hat{k}\right)$ and $\vec{r} = 5\hat{i}+\mu\left(-\hat{i}+\hat{j}+\hat{k}\right)$

Answer

Given – $\overrightarrow{L_1} = (3\hat{i}-4\hat{j}+2\hat{k})+\lambda(\hat{i}+3\hat{k})$

& $\overrightarrow{L_2} = (5\hat{i})+\mu(-\hat{i}+\hat{j}+\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,0,3)

Direction ratios of L_2 = (-1,1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times(-1)+0\times 1+3\times 1}{\sqrt{1^2+0^2+3^2}\sqrt{1^2+1^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{-1+3}{\sqrt{10}\sqrt{3}}\right)$$

$= \cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$

$= \cos^{-1}\left(\frac{\sqrt{30}}{15}\right)$

3. Question

Find the angle between each of the following pairs of lines:

$\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = 3\hat{k} + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

Answer

Given – $\overrightarrow{L_1} = (\hat{\imath} - 2\hat{\jmath}) + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$

& $\overrightarrow{L_2} = (3\hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,-2,1)

Direction ratios of L_2 = (1,2,-2)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{2\times 1 + (-2)\times 2 + 1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+2^2+2^2}}\right)$

$= \cos^{-1}\left(\frac{2-4-2}{3\times 3}\right)$

$= \cos^{-1}\left(-\frac{4}{9}\right)$

4. Question

Find the angle between each of the following pairs of lines:

$\frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ and $\frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

Answer

Given – $\overrightarrow{L_1} = \frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

& $\overrightarrow{L_2} = \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,1,2)

Direction ratios of L_2 = (3,5,4)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3 + 1\times 5 + 2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\times\sqrt{50}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$

5. Question

Find the angle between each of the following pairs of lines:

$\frac{x-4}{4} = \frac{y+1}{4} = \frac{z-6}{5}$ and $\frac{x-5}{1} = \frac{2y+5}{-2} = \frac{z-3}{1}$

Answer

Given – $\overrightarrow{L_1} = \frac{x-4}{4} = \frac{y+1}{3} = \frac{z-6}{5}$

& $\overrightarrow{L_2} = \frac{x-5}{1} = \frac{y+5/2}{-1} = \frac{z-3}{1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (4,3,5)

Direction ratios of L_2 = (1,-1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{4\times1+3\times(-1)+5\times1}{\sqrt{4^2+3^2+5^2}\sqrt{1^2+1^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-3+5}{5\sqrt{2}\times\sqrt{3}}\right)$$

$$= \cos^{-1}\left(\frac{6}{5\sqrt{6}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{2\sqrt{6}}{15}\right)}$$

6. Question

Find the angle between each of the following pairs of lines:

$$\frac{3-x}{-2}=\frac{y+5}{1}=\frac{1-z}{3} \text{ and } \frac{x}{3}=\frac{1-y}{-2}=\frac{z+2}{-1}$$

Answer

Given – $\overrightarrow{L_1} = \frac{x-3}{2}=\frac{y+5}{1}=\frac{z-1}{-3}$

& $\overrightarrow{L_2} = \frac{x}{3}=\frac{y-1}{2}=\frac{z+2}{-1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,1,-3)

Direction ratios of L_2 = (3,2,-1)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times3+1\times2+(-3)\times(-1)}{\sqrt{2^2+1^2+3^2}\sqrt{3^2+2^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{6+2+3}{\sqrt{14}\times\sqrt{14}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{11}{14}\right)}$$

7. Question

Find the angle between each of the following pairs of lines:

$\frac{x}{1} = \frac{z}{-1}, y = 0$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

Answer

Given – $\overrightarrow{L_1} = \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$

& $\overrightarrow{L_2} = \frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,0,-1)

Direction ratios of L_2 = (3,4,5)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3 + 0\times 4 + (-1)\times 5}{\sqrt{1^2+0^2+1^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{3-5}{5\sqrt{2}\times\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{1}{5}\right)$$

8. Question

Find the angle between each of the following pairs of lines:

$\frac{5-x}{3} = \frac{y+3}{-2}, z = 5$ and $\frac{x-1}{1} = \frac{1-y}{3} = \frac{z-5}{2}$

Answer

Given – $\overrightarrow{L_1} = \frac{x-5}{-3} = \frac{y+3}{-2} = \frac{z-5}{0}$

& $\overrightarrow{L_2} = \frac{x-1}{1} = \frac{y-1}{-3} = \frac{z-5}{2}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (-3,-2,0)

Direction ratios of L_2 = (1,-3,2)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{(-3)\times 1+(-2)\times(-3)+0\times 2}{\sqrt{3^2+2^2+0^2}\sqrt{1^2+3^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{-3+6}{\sqrt{13}\times\sqrt{14}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{3}{\sqrt{182}}\right)}$$

9. Question

Show that the lines $\frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$ and $\frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$ are perpendicular to each other.

Answer

Given – $\overrightarrow{L_1} = \frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$

& $\overrightarrow{L_2} = \frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$

To prove – The lines are perpendicular to each other

Direction ratios of L_1 = (2,-3,4)

Direction ratios of L_2 = (2,4,2)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 2+(-3)\times 4+4\times 2}{\sqrt{2^2+3^2+4^2}\sqrt{2^2+4^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-12+8}{\sqrt{29}\times\sqrt{24}}\right)$$

$$= \cos^{-1}(0)$$

$= \frac{\pi}{2}$

Hence, **the lines are perpendicular to each other.**

10. Question

If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{5}$ are perpendicular to each other then find the value of λ.

Answer

Given – $\overrightarrow{L_1} = \frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$

& $\overrightarrow{L_2} = \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$

To find – The value of λ

Direction ratios of L_1 = (-3,2λ,2)

Direction ratios of L_2 = (3λ,1,-5)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2} \times \sqrt{a'^2+b'^2+c'^2}}\right)$

Since the lines are perpendicular to each other,

The angle between the lines

⇨ $\cos^{-1}\left(\frac{(-3)\times 3\lambda + 2\lambda \times 1 + 2\times(-5)}{\sqrt{3^2+(2\lambda)^2+2^2}\sqrt{(3\lambda)^2+1^2+5^2}}\right) = \frac{\pi}{2}$

⇨ $\cos^{-1}\left(\frac{-9\lambda + 2\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \frac{\pi}{2}$

⇨ $\cos^{-1}\left(\frac{-7\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \frac{\pi}{2}$

⇨ $\left(\frac{-7\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \cos\frac{\pi}{2} = 0$

⇨ $-7\lambda - 10 = 0$

⇨ $\boldsymbol{\lambda = -\frac{10}{7}}$

11. Question

Show that the lines x = - y = 2z and x + 2 = 2y − 1 = - z + 1 are perpendicular to each other.

HINT: The given lines are $\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$ and $\frac{x+2}{1}=\frac{y-\frac{1}{2}}{1}=\frac{z-1}{-2}$.

Answer

Given – $\overrightarrow{L_1} = \frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$

& $\overrightarrow{L_2} = \frac{x+2}{2}=\frac{y-1/2}{1}=\frac{z-1}{-2}$

To prove – The lines are perpendicular to each other

Direction ratios of L_1 = (2,-2,1)

Direction ratios of L_2 = (2,1,-2)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{2\times 2+(-2)\times 1+1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+1^2+2^2}}\right)$$

$$=\cos^{-1}\left(\frac{4-2-2}{\sqrt{29}\times\sqrt{24}}\right)$$

$$=\cos^{-1}(0)$$

$$=\frac{\pi}{2}$$

Hence, **the lines are perpendicular to each other.**

12. Question

Find the angle between two lines whose direction ratios are

i. 2, 1, 2 and 4, 8, 1

ii. 5, -12, 13 and -3, 4, 5

iii. 1, 1, 2 and $(\sqrt{3}-1),(-\sqrt{3}-1),4$

iv. a, b, c and (b – c), (c – a), (a – b)

Answer

(i): Given – Direction ratios of L_1 = (2,1,2) & Direction ratios of L_2 = (4,8,1)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 4+1\times 8+2\times 1}{\sqrt{2^2+1^2+2^2}\sqrt{4^2+8^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{8+8+2}{3\times 9}\right)$$

$$= \cos^{-1}\left(\frac{18}{27}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{2}{3}\right)}$$

(ii): Given – Direction ratios of L_1 = (5,-12,13) & Direction ratios of L_2 = (-3,4,5)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{5\times(-3)+(-12)\times 4+13\times 5}{\sqrt{5^2+12^2+13^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{-15-48+65}{13\sqrt{2}\times 5\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{2}{130}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{1}{65}\right)}$$

(iii) Given – Direction ratios of L_1 = (1,1,2) & Direction ratios of L_2 = (√3-1,-√3-1,4)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times(\sqrt{3}-1)+1\times(-\sqrt{3}-1)+2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{24}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

(iv) Given – Direction ratios of L_1 = (a,b,c) & Direction ratios of L_2 = ((b-c),(c-a),(a-b))

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{a\times(b-c)+b\times(c-a)+c\times(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}\left(\frac{0}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

13. Question

If A(1, 2, 3), B(4, 5, 7), C(-4, 3, -6) and D(2, 9, 2) are four given points then find the angle between the lines AB and CD.

Answer

Given -

A = (1,2,3)

B = (4,5,7)

C = (-4,3,-6)

D = (2,9,2)

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((4-1),(5-2),(7-3))

=(3,3,4)

Similarly, the direction ratios of the line CD can be given by

((2+4),(9-3),(2+6))

=(6,6,8)

To find – Angle between the two pair of lines AB and CD

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{3\times 6+3\times 6+4\times 8}{\sqrt{3^2+3^2+4^2}\sqrt{6^2+6^2+8^2}}\right)$$

$$= \cos^{-1}\left(\frac{18+18+32}{\sqrt{34}\times 2\sqrt{34}}\right)$$

$$= \cos^{-1}\left(\frac{68}{2\times 34}\right)$$

$$= \cos^{-1} 1$$

$$= \mathbf{0}$$

Exercise 27D

1. Question

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i}+\hat{j}\right)+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right),$$

$$\vec{r} = \left(2\hat{i}+\hat{j}-\hat{k}\right)+\mu\left(3\hat{i}-5\hat{j}+2\hat{k}\right).$$

Answer

<u>Given equations</u> :

$$\bar{r} = (\hat{i}+\hat{j})+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$$

$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

For given lines,

$\bar{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = \hat{i} + \hat{j}$

$\overline{b_1} = 2\hat{i} - \hat{j} + \hat{k}$

$\overline{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\overline{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} - \hat{j} - 7\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{3^2 + (-1)^2 + (-7)^2}$

$= \sqrt{9+1+49}$

$= \sqrt{59}$

$\overline{a_2} - \overline{a_1} = (2-1)\hat{i} + (1-1)\hat{j} + (-1-0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} - \hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (3\hat{i} - \hat{j} - 7\hat{k}).(\hat{i} + 0\hat{j} - \hat{k})$

$= (3 \times 1) + ((-1) \times 0) + ((-7) \times (-1))$

= 3 + 0 + 7

= 10

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{10}{\sqrt{59}}\right|$$

2. Question

Find the shortest distance between the given lines.

$$\vec{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right),$$

$$\vec{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

Answer

Given equations :

$$\bar{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$$

$$\bar{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

Answer :

For given lines,

$$\bar{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$$

$$\bar{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

Here,

$\overline{a_1} = -4\hat{i} + 4\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} + \hat{j} - \hat{k}$

$\overline{a_2} = -3\hat{i} - 8\hat{j} - 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & 3 \end{vmatrix}$$

$= \hat{i}(3+3) - \hat{j}(3+2) + \hat{k}(3-2)$

$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{i} - 5\hat{j} + \hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{6^2 + (-5)^2 + 1^2}$

$= \sqrt{36 + 25 + 1}$

$= \sqrt{62}$

$\overline{a_2} - \overline{a_1} = (-3+4)\hat{i} + (-8-4)\hat{j} + (-3-1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 12\hat{j} - 4\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (6\hat{i} - 5\hat{j} + \hat{k}).(\hat{i} - 12\hat{j} - 4\hat{k})$

$= (6 \times 1) + ((-5) \times (-12)) + (1 \times (-4))$

= 6 + 60 – 4

= 62

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{62}{\sqrt{62}} \right|$$

$d = \sqrt{62}\ units$

3. Question

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right),$$

$$\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right).$$

Answer

<u>Given equations</u> :

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$$

$$\bar{r} = (4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$$

<u>To Find</u> : d

<u>Formula</u> :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

<u>Answer</u> :

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\bar{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\overline{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 - 6) - \hat{j}(1 - 4) + \hat{k}(3 + 6)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-9)^2 + 3^2 + 9^2}$$

$$= \sqrt{81 + 9 + 81}$$

$$= \sqrt{171}$$

$$\overline{a_2} - \overline{a_1} = (4 - 1)\hat{i} + (5 - 2)\hat{j} + (6 - 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Now,

$$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-9\hat{i} + 3\hat{j} + 9\hat{k}).(3\hat{i} + 3\hat{j} + 3\hat{k})$$

$$= ((-9) \times 3) + (3 \times 3) + (9 \times 3)$$

= - 27 + 9 + 27

= 9

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{9}{\sqrt{171}} \right|$$

$$\therefore d = \frac{9}{\sqrt{19}.\sqrt{9}}$$

$$\therefore d = \frac{3}{\sqrt{19}}$$

$$\therefore d = \frac{3\sqrt{19}}{19}$$

4. Question

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right),$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right).$$

Answer

Given equations :

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$$

$$\bar{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>**Answer**</u> **:**

For given lines,

$\bar{r} = \left(\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) + \lambda\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right)$

$\bar{r} = \left(2\hat{\imath} - \hat{\jmath} - \hat{k}\right) + \mu\left(2\hat{\imath} + \hat{\jmath} + 2\hat{k}\right)$

Here,

$\overline{a_1} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\overline{b_1} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\overline{a_2} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\overline{b_2} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= \hat{\imath}(-2 - 1) - \hat{\jmath}(2 - 2) + \hat{k}(1 + 2)$

$\therefore \overline{b_1} \times \overline{b_2} = -3\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-3)^2 + 0^2 + 3^2}$

$= \sqrt{9 + 0 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{\imath} + (-1 - 2)\hat{\jmath} + (-1 - 1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{\imath} - 3\hat{\jmath} - 2\hat{k}$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-3\hat{i} + 0\hat{j} + 3\hat{k}\right).\left(\hat{i} - 3\hat{j} - 2\hat{k}\right)$$

$$= ((-3) \times 1) + (0 \times (-3)) + (3 \times (-2))$$

= - 3 + 0 – 6

= - 9

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-9}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{3}{\sqrt{2}}$$

$$\therefore d = \frac{3\sqrt{2}}{2}$$

5. Question

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right),$$

$$\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(-2\hat{i} + 3\hat{j} + 8\hat{k}\right).$$

Answer

<u>Given equations</u> :

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(-2\hat{i} + 3\hat{j} + 8\hat{k}\right)$$

<u>To Find</u> : d

<u>Formula</u> :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\overline{b_2} = -2\hat{i} + 3\hat{j} + 8\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$$= \hat{i}(24 - 18) - \hat{j}(16 + 12) + \hat{k}(6 - 6)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{i} - 28\hat{j} + 0\hat{k}$$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\overline{a_2} - \overline{a_1} = (3-1)\hat{i} + (3-2)\hat{j} + (-5+4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (6\hat{i} - 28\hat{j} + 0\hat{k}).(2\hat{i} + \hat{j} - \hat{k})$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 – 28 + 0

= - 16

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{-16}{\sqrt{820}}\right|$$

$$d = \frac{16}{\sqrt{820}}\ units$$

6. Question

Find the shortest distance between the given lines.

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k}),$$

$$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k}).$$

Answer

<u>Given equations</u> :

$\bar{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

$\bar{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

<u>To Find</u> : d

<u>Formula</u> :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>Answer</u> :

For given lines,

$$\bar{r} = \left(6\hat{i} + 3\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$$

$$\bar{r} = \left(-9\hat{i} + \hat{j} - 10\hat{k}\right) + \mu\left(4\hat{i} + \hat{j} + 6\hat{k}\right)$$

Here,

$$\overline{a_1} = 6\hat{i} + 3\hat{k}$$

$$\overline{b_1} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\overline{a_2} = -9\hat{i} + \hat{j} - 10\hat{k}$$

$$\overline{b_2} = 4\hat{i} + \hat{j} + 6\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$$= \hat{i}(-6-4) - \hat{j}(12-16) + \hat{k}(2+4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-10)^2 + 4^2 + 6^2}$$

$$= \sqrt{100 + 16 + 36}$$

$$= \sqrt{152}$$

$$\overline{a_2} - \overline{a_1} = (-9-6)\hat{i} + (1-0)\hat{j} + (6-3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-10\hat{i} + 4\hat{j} + 6\hat{k}\right).\left(-15\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$$

= 150 + 4 + 18

= 172

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{172}{\sqrt{152}}\right|$$

$$\therefore d = \frac{172}{2\sqrt{38}}$$

$$\therefore d = \frac{86}{\sqrt{38}}$$

$$d = \frac{86}{\sqrt{38}} \text{ units}$$

7. Question

Find the shortest distance between the given lines.

$$\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k},$$

$$\vec{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}.$$

Answer

Given equations :

$$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

$$\bar{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}$$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

Answer :

Given lines,

$$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

$\bar{r} = (1 + s)\hat{i} + (3s - 7)\hat{j} + (2s - 2)\hat{k}$

Above equations can be written as

$\bar{r} = (3\hat{i} + 4\hat{j} - 2\hat{k}) + t(-\hat{i} + 2\hat{j} + \hat{k})$

$\bar{r} = (\hat{i} - 7\hat{j} - 2\hat{k}) + s(\hat{i} + 3\hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

$\overline{b_1} = -\hat{i} + 2\hat{j} + \hat{k}$

$\overline{a_2} = \hat{i} - 7\hat{j} - 2\hat{k}$

$\overline{b_2} = \hat{i} + 3\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$= \hat{i}(4 - 3) - \hat{j}(-2 - 1) + \hat{k}(-3 - 2)$

$\therefore \overline{b_1} \times \overline{b_2} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{1^2 + 3^2 + (-5)^2}$

$= \sqrt{1 + 9 + 25}$

$= \sqrt{35}$

$\overline{a_2} - \overline{a_1} = (1 - 3)\hat{i} + (-7 - 4)\hat{j} + (-2 + 2)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -2\hat{i} - 11\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (\hat{i} + 3\hat{j} - 5\hat{k}).(-2\hat{i} - 11\hat{j} + 0\hat{k})$

$= (1 \times (-2)) + (3 \times (-11)) + ((-5) \times 0)$

= - 2 – 33 + 0

= - 35

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$\therefore \text{d} = \left|\frac{-35}{\sqrt{35}}\right|$

$\therefore \text{d} = \sqrt{35}$

$d = \sqrt{35}\, units$

8. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k},$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}.$$

Answer

<u>**Given equations**</u> **:**

$\bar{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$

$\bar{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

<u>**To Find**</u> **:** d

<u>**Formula**</u> **:**

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

<u>Answer</u> :

Given lines,

$$\bar{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$$

$$\bar{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

Above equations can be written as

$$\bar{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$$\bar{a_1} = -\hat{i} + \hat{j} - \hat{k}$$

$$\bar{b_1} = \hat{i} + \hat{j} - \hat{k}$$

$$\bar{a_2} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\bar{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1 + 2) - \hat{j}(1 - 1) + \hat{k}(2 + 1)$$

$$\therefore \bar{b_1} \times \bar{b_2} = 3\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\therefore |\bar{b_1} \times \bar{b_2}| = \sqrt{3^2 + 0^2 + 3^2}$$

$$= \sqrt{9 + 0 + 9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$\bar{a_2} - \bar{a_1} = (1 + 1)\hat{i} + (-1 - 1)\hat{j} + (2 + 1)\hat{k}$$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} - 2\hat{j} + 3\hat{k}$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(3\hat{i} - 0\hat{j} + 3\hat{k}\right).\left(2\hat{i} - 2\hat{j} + 3\hat{k}\right)$$

$$= (3 \times 2) + (0 \times (-2)) + (3 \times 3)$$

= 6 + 0 + 9

= 15

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{5}{\sqrt{2}}$$

$$\therefore d = \frac{5\sqrt{2}}{2}$$

$$d = \frac{5\sqrt{2}}{2}\ units$$

9. Question

Compute the shortest distance between the lines $\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} - \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$. Determine whether these lines intersect or not.

Answer

Given equations :

$$\bar{r} = (\hat{i} - \hat{j}) + \lambda\left(2\hat{i} - \hat{k}\right)$$

$$\bar{r} = (2\hat{i} - \hat{j}) + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \bar{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>Answer</u> :

For given lines,

$$\bar{r} = (\hat{i} - \hat{j}) + \lambda\left(2\hat{i} - \hat{k}\right)$$

$$\bar{r} = (2\hat{i} - \hat{j}) + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$$

Here,

$$\bar{a_1} = \hat{i} - \hat{j}$$

$$\overline{b_1} = 2\hat{i} - \hat{k}$$

$$\overline{a_2} = 2\hat{i} - \hat{j}$$

$$\overline{b_2} = \hat{i} - \hat{j} - \hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{i}(0 - 1) - \hat{j}(-2 + 1) + \hat{k}(-2 - 0)$$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 1^2 + (-2)^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{\imath} + (-1 + 1)\hat{\jmath} + (0 - 0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-\hat{\imath} + \hat{\jmath} - 2\hat{k}).(\hat{\imath} + 0\hat{\jmath} + 0\hat{k})$

$= ((-1) \times 1) + (1 \times 0) + ((-2) \times 0)$

= - 1 + 0 + 0

= - 1

Therefore, the shortest distance between the given lines is

$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$

$\therefore d = \left|\frac{-1}{\sqrt{6}}\right|$

$\therefore d = \frac{1}{\sqrt{6}}$

$\therefore d = \frac{\sqrt{6}}{6}$

$d = \frac{\sqrt{6}}{6}\ units$

As d ≠ 0

Hence, the given lines do not intersect.

10. Question

Show that the lines $\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda\ (2\hat{i} - 7\hat{j} + 5\hat{k})$, and $\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu$ $(2\hat{i} + \hat{j} - 3\hat{k})$ do not intersect.

Answer

Given equations :

$$\bar{r} = \left(3\hat{i} - 15\hat{j} + 9\hat{k}\right) + \lambda\left(2\hat{i} - 7\hat{j} + 5\hat{k}\right)$$

$$\bar{r} = \left(-\hat{i} + \hat{j} + 9\hat{k}\right) + \mu\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$$

<u>To Find</u> : d

<u>Formula</u> :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>Answer</u> :

For given lines,

$$\bar{r} = \left(3\hat{i} - 15\hat{j} + 9\hat{k}\right) + \lambda\left(2\hat{i} - 7\hat{j} + 5\hat{k}\right)$$

$$\bar{r} = \left(-\hat{i} + \hat{j} + 9\hat{k}\right) + \mu\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$$

Here,

$$\overline{a_1} = 3\hat{i} - 15\hat{j} + 9\hat{k}$$

$\overline{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\overline{a_2} = -\hat{i} + \hat{j} + 9\hat{k}$

$\overline{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$

$\therefore \overline{b_1} \times \overline{b_2} = 17\hat{i} + 16\hat{j} + 16\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{17^2 + 16^2 + 17^2}$

$= \sqrt{289 + 256 + 289}$

$= \sqrt{834}$

$\overline{a_2} - \overline{a_1} = (-1 - 3)\hat{i} + (1 + 15)\hat{j} + (9 - 9)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -4\hat{i} + 16\hat{j} + 0\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(17\hat{i} + 16\hat{j} + 16\hat{k}\right).\left(-4\hat{i} + 16\hat{j} + 0\hat{k}\right)$

$= (17 \times (-4)) + (16 \times 16) + (16 \times 0)$

= - 68 + 256 + 0

= 188

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{188}{\sqrt{834}}\right|$$

$$\therefore d = \frac{188}{\sqrt{834}} \text{units}$$

As d ≠ 0

Hence, the given lines do not intersect.

11. Question

Show that the lines $\vec{r} = \left(2\hat{i} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu$ $\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ intersect.

Also, find their point of intersection.

Answer

Given equations :

$$\bar{r} = (2\hat{\imath} - 3\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$$

$$\bar{r} = (2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) \, . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

For given lines,

$$\bar{r} = \left(2\hat{i} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\bar{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

Here,

$$\overline{a_1} = 2\hat{i} - 3\hat{k}$$

$$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(12 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{3^2 + 2^2 + (-1)^2}$$

$$= \sqrt{9 + 4 + 1}$$

$$= \sqrt{14}$$

$$\overline{a_2} - \overline{a_1} = (2 - 2)\hat{i} + (6 - 0)\hat{j} + (3 + 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 0\hat{i} + 6\hat{j} + 6\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(3\hat{i} + 2\hat{j} - \hat{k}\right).\left(0\hat{i} + 6\hat{j} + 6\hat{k}\right)$$

$$= (3 \times 0) + (2 \times 6) + ((-1) \times 6)$$

= 0 + 12 – 6

= 6

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$\therefore d = \left|\frac{6}{\sqrt{14}}\right|$

$\therefore d = \frac{6}{\sqrt{14}}$ units

As d ≠ 0

Hence, the given lines do not intersect.

12. Question

Show that the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\ (2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu\ (5\hat{i} + 2\hat{j} + \hat{k})$ intersect.

Also, find their point of intersection.

Answer

Given equations :

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\bar{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

<u>Answer</u> :

For given lines,

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

$$\bar{r} = (4\hat{\imath} + \hat{\jmath}) + \mu(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$$

Here,

$$\bar{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\bar{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$$

$$\bar{a_2} = 4\hat{\imath} + \hat{\jmath}$$

$$\bar{b_2} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{\imath}(3 - 8) - \hat{\jmath}(2 - 20) + \hat{k}(4 - 15)$$

$$\therefore \bar{b_1} \times \bar{b_2} = -5\hat{\imath} + 18\hat{\jmath} - 11\hat{k}$$

$$\therefore |\bar{b_1} \times \bar{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$$

$$= \sqrt{25 + 324 + 121}$$

$$= \sqrt{470}$$

$$\bar{a_2} - \bar{a_1} = (4 - 1)\hat{\imath} + (1 - 2)\hat{\jmath} + (0 - 3)\hat{k}$$

$$\therefore \bar{a_2} - \bar{a_1} = 3\hat{\imath} - \hat{\jmath} - 3\hat{k}$$

Now,

$$(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1}) = (-5\hat{\imath} + 18\hat{\jmath} - 11\hat{k}).(3\hat{\imath} - \hat{\jmath} - 3\hat{k})$$

$$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$$

= - 15 – 18 + 33

= 0

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{0}{\sqrt{470}}\right|$$

$$\therefore d = 0 \text{ units}$$

As d = 0

Hence, the given lines not intersect each other.

Now, to find point of intersection, let us convert given vector equations into Cartesian equations.

For that substituting $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$$\therefore L1:\ x\hat{i} + y\hat{j} + z\hat{k} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\therefore L2:\ x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

$$\therefore L1: (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\therefore L2: (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\therefore L1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore L2: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$x_1 = 2\lambda+1$, $y_1 = 3\lambda+2$, $z_1 = 4\lambda+3$

let, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$$

$$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$$

⇒ $4\lambda - 6 = 15\lambda + 5$

⇒ $11\lambda = -11$

⇒ $\lambda = -1$

Therefore, $x_1 = 2(-1)+1$, $y_1 = 3(-1)+2$, $z_1 = 4(-1)+3$

$\Rightarrow x_1 = -1$, $y_1 = -1$, $z_1 = -1$

Hence point of intersection of given lines is (-1, -1, -1).

13. Question

Find the shortest distance between the lines L_1 and L_2 whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

HINT: The given lines are parallel.

Answer

<u>Given equations</u> :

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

<u>To Find</u> : d

<u>Formula</u> :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{|\bar{b}|}\right|$$

Answer :

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

As $\overline{b_1} = \overline{b_2} = \bar{b}\ (\text{say})$, given lines are parallel to each other.

Therefore,

$$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49}$$

= 7

$$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$$

$$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$$

$$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\therefore \left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right| = \sqrt{9^2 + (-14)^2 + 4^2}$$

$$= \sqrt{81 + 196 + 16}$$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\overline{a_2} - \overline{a_1}) \times \overline{b}|}{|\overline{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{293}}{7}\right|$$

$$d = \frac{\sqrt{293}}{7}\ units$$

14. Question

Find the distance between the parallel lines L_1 and L_2 whose vector equations are $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$.

Answer

Given equations :

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

To Find : d

Formula :

1. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \overline{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{\left|\bar{b}\right|}\right|$$

Answer :

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\bar{b} = \hat{i} - \hat{j} + \hat{k}$

$\therefore \left|\bar{b}\right| = \sqrt{1^2 + (-1)^2 + 1^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 3\hat{j} - 4\hat{k}$

$$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(-3 - 4) - \hat{j}(1 + 4) + \hat{k}(-1 + 3)$

$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$

$\therefore \left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right| = \sqrt{(-7)^2 + (-5)^2 + 2^2}$

$= \sqrt{49 + 25 + 4}$

$= \sqrt{78}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{78}}{\sqrt{3}}\right|$$

$$\therefore d = \sqrt{26}$$

$$d = \sqrt{26}\, units$$

15. Question

Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$. Also, find the distance between these lines.

HINT: The given line is

$$L_1 : \vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).$$

The required line is

$$L_2 : \vec{r} = \left(2\hat{i} + 3\hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).$$

Now, find the distance between the parallel lines L_1 and L_2.

Answer

<u>Given</u> : point A ≡ (2, 3, 2)

Equation of line : $\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

<u>To Find</u> : i) equation of line

ii) distance d

<u>Formulae</u> :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and parallel to vector $\bar{b} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \overline{a_1} + \lambda\bar{b}$ and $\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left| \frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{\left|\bar{b}\right|} \right|$$

<u>**Answer**</u> **:**

As the required line is parallel to the line

$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

Therefore, the vector parallel to the required line is

$\bar{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Given point A ≡ (2, 3, 2)

$\therefore \bar{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Therefore, equation of line passing through A and parallel to $\bar{b}$ is

$\bar{r} = \bar{a} + \mu\bar{b}$

$\therefore \bar{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

Now, to calculate distance between above line and given line,

$\bar{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

Here,

$\bar{a_1} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\bar{a_2} = -2\hat{i} + 3\hat{j}$

$\bar{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\bar{a_2} - \bar{a_1} = (-2 - 2)\hat{i} + (3 - 3)\hat{j} + (0 - 2)\hat{k}$

$\therefore \bar{a_2} - \bar{a_1} = -4\hat{i} + 0\hat{j} - 2\hat{k}$

$$(\bar{a_2} - \bar{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & -2 \\ 2 & -3 & 6 \end{vmatrix}$$

$= \hat{i}(0 - 6) - \hat{j}(-24 + 4) + \hat{k}(12 - 0)$

$\therefore (\bar{a_2} - \bar{a_1}) \times \bar{b} = -6\hat{i} + 20\hat{j} + 12\hat{k}$

$\therefore |(\bar{a_2} - \bar{a_1}) \times \bar{b}| = \sqrt{(-6)^2 + 20^2 + 12^2}$

$= \sqrt{36 + 400 + 144}$

$= \sqrt{580}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{580}}{7}\right|$$

$$\therefore d = \frac{\sqrt{580}}{7}$$

$$d = \frac{\sqrt{580}}{7}\ units$$

16. Question

Write the vector equation of each of the following lines and hence determine the distance between them :

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6} \text{ and } \frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}.$$

HINT: The given lines are

$$L_1 : \vec{r}=\left(-2\hat{i}+3\hat{j}\right)+\lambda\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$$

$$L_2 : \vec{r}=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+2\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

Now, find the distance between the parallel lines L_1 and L_2.

Answer

<u>Given</u> : Cartesian equations of lines

$$L1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$$

$$L2 : \frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$$

<u>To Find</u> : i) vector equations of given lines

ii) distance d

<u>Formulae</u> :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

And $\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

4. Shortest distance between two parallel lines :

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\bar{a_2} - \bar{a_1}) \times \bar{b}\right|}{\left|\bar{b}\right|}\right|$$

Answer :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

Line L1 is passing through point (1, 2, -4) and has direction ratios (2, 3, 6)

Therefore, vector equation of line L1 is

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

And

$$L2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Line L2 is passing through point (3, 3, -5) and has direction ratios (4, 6, 12)

Therefore, vector equation of line L2 is

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(4\hat{i} + 6\hat{j} + 12\hat{k}\right)$$

$$\therefore \bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

Now, to calculate distance between the lines,

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

Here,

$\bar{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

As $\overline{b_1} = \overline{b_2} = \bar{b}$ (say) , given lines are parallel to each other.

Therefore,

$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\overline{a_2} - \bar{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$

$\therefore \overline{a_2} - \bar{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

$$(\overline{a_2} - \bar{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$

$\therefore (\overline{a_2} - \bar{a_1}) \times \bar{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\therefore |(\overline{a_2} - \bar{a_1}) \times \bar{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$

$= \sqrt{81 + 196 + 16}$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\overline{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{293}}{7}\right|$$

$$d = \frac{\sqrt{293}}{7}\ units$$

17. Question

Write the vector equation of the following lines and hence find the shortest distance between them :

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}$.

Answer

Given : Cartesian equations of lines

$$L1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

$$L2 : \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}$$

To Find : i) vector equations of given lines

ii) distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

And $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

<u>Answer</u> :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

Line L1 is passing through point (1, 2, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

And

$$L2 : \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

Line L2 is passing through point (2, 3, 5) and has direction ratios (3, 4, 5)

Therefore, vector equation of line L2 is

$$\bar{r} = (3\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

$$\bar{r} = (3\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$$

Here,

$$\overline{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\overline{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$$

$$\overline{a_2} = 3\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$$

$$\overline{b_2} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \sqrt{1 + 4 + 1}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (5 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} + 2\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(-\hat{i} + 2\hat{j} - \hat{k}\right).\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

$= ((-1) \times 2) + (2 \times 1) + ((-1) \times 2)$

= - 2 + 2 - 2

= -2

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-2}{\sqrt{6}}\right|$$

$$\therefore d = \frac{2}{\sqrt{3}.\sqrt{2}}$$

$$\therefore d = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore d = \sqrt{\frac{2}{3}}$$

$$d = \sqrt{\frac{2}{3}}\ units$$

18. Question

Find the shortest distance between the lines given below:

$$\frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-2} \text{ and } \frac{x-1}{2}=\frac{y+1}{2}\ \frac{z+1}{-2}.$$

Answer

<u>Given</u> : Cartesian equations of lines

$$L1 : \frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-2}$$

$$L2 : \frac{x-1}{2}=\frac{y+1}{2}=\frac{z+1}{-2}$$

<u>To Find</u> : distance d

<u>Formulae</u> :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

And $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

then,

$$\bar{a}.\bar{b}=(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)$$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda \bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda \bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

Answer :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$$

Line L1 is passing through point (1, -2, 3) and has direction ratios (-1, 1, -2)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

And

$$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$$

Line L2 is passing through point (1, -1, -1) and has direction ratios (2, 2, -2)

Therefore, vector equation of line L2 is

$$\bar{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

$$\bar{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$$

Here,

$$\bar{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\bar{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\bar{a_2} = \hat{i} - \hat{j} - \hat{k}$$

$$\bar{b_2} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 2 & 2 & -2 \end{vmatrix}$$

$= \hat{\imath}(-2+4) - \hat{\jmath}(2+4) + \hat{k}(-2-2)$

$\therefore \overline{b_1} \times \overline{b_2} = 2\hat{\imath} - 6\hat{\jmath} - 4\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{2^2 + (-6)^2 + (-4)^2}$

$= \sqrt{4 + 36 + 16}$

$= \sqrt{56}$

$\overline{a_2} - \overline{a_1} = (1-1)\hat{\imath} + (-1+2)\hat{\jmath} + (-1-3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 0\hat{\imath} + \hat{\jmath} - 4\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (2\hat{\imath} - 6\hat{\jmath} - 4\hat{k}) . (0\hat{\imath} + \hat{\jmath} - 4\hat{k})$

$= (2 \times 0) + ((-6) \times 1) + ((-4) \times (-4))$

= 0 - 6 + 16

= 10

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{10}{\sqrt{56}} \right|$$

$$\therefore d = \frac{10}{\sqrt{56}}$$

$$d = \frac{10}{\sqrt{56}} \; units$$

19. Question

Find the shortest distance between the lines given below:

$$\frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2} \text{ and } \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-25}{3}.$$

HINT: Change the given equations in vector form.

Answer

Given : Cartesian equations of lines

$$L1 : \frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$$

$$L2: \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

To Find : distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

Answer :

Given Cartesian equations of lines

$$L1 : \frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$$

Line L1 is passing through point (12, 1, 5) and has direction ratios (-9, 4, 2)

Therefore, vector equation of line L1 is

$$\bar{r} = \left(12\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(-9\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

And

$$L2 : \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

Line L2 is passing through point (23, 10, 23) and has direction ratios (-6, -4, 3)

Therefore, vector equation of line L2 is

$$\bar{r} = \left(23\hat{i} + 10\hat{j} + 23\hat{k}\right) + \mu\left(-6\hat{i} - 4\hat{j} + 3\hat{k}\right)$$

Now, to calculate distance between the lines,

$$\bar{r} = \left(12\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(-9\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

$$\bar{r} = \left(23\hat{i} + 10\hat{j} + 23\hat{k}\right) + \mu\left(-6\hat{i} - 4\hat{j} + 3\hat{k}\right)$$

Here,

$$\overline{a_1} = 12\hat{i} + \hat{j} + 5\hat{k}$$

$$\overline{b_1} = -9\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\overline{a_2} = 23\hat{i} + 10\hat{j} + 23\hat{k}$$

$$\overline{b_2} = -6\hat{i} - 4\hat{j} + 3\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 4 & 2 \\ -6 & -4 & 3 \end{vmatrix}$$

$$= \hat{i}(12+8) - \hat{j}(-27+12) + \hat{k}(36+24)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 20\hat{i} + 15\hat{j} + 60\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{20^2 + 15^2 + 60^2}$$

$$= \sqrt{400 + 225 + 3600}$$

$$= \sqrt{4225}$$

= 65

$$\overline{a_2} - \overline{a_1} = (23 - 12)\hat{i} + (10 - 1)\hat{j} + (23 - 5)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 11\hat{i} + 9\hat{j} + 18\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(20\hat{i} + 15\hat{j} + 60\hat{k}\right).\left(11\hat{i} + 9\hat{j} + 18\hat{k}\right)$$

$$= (20 \times 11) + (15 \times 9) + (60 \times 18)$$

= 220 + 135 + 1080

= 1435

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{1435}{65}\right|$$

$$\therefore d = \frac{287}{13}$$

$$d = \frac{287}{13}\ units$$

Exercise 27E

1. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x-3}{3} = \frac{y-8}{-1} = z - 3 \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$

Answer

<u>Given</u> : Cartesian equations of lines

$$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$L2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

<u>Formulae</u> :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

2. Distance formula :

Distance between two points A≡(a_1, a_2, a_3) and B≡(b_1, b_2, b_3) is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

3. Equation of line :

Equation of line passing through points A≡(x_1, y_1, z_1) and B≡(x_2, y_2, z_2) is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

Answer :

Given equations of lines

$$L1 : \frac{x - 3}{3} = \frac{y - 8}{-1} = \frac{z - 3}{1}$$

$$L2 : \frac{x + 3}{-3} = \frac{y + 7}{2} = \frac{z - 6}{4}$$

Direction ratios of L1 and L2 are (3, -1, 1) and (-3, 2, 4) respectively.

Let, general point on line L1 is P≡(x_1, y_1, z_1)

x_1 = 3s+3 , y_1 = -s+8 , z_1 = s+3

and let, general point on line L2 is Q≡(x_2, y_2, z_2)

x_2 = -3t – 3 , y_2 = 2t – 7 , z_2 = 4t + 6

$$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)\hat{j} + (4t + 6 - s - 3)\hat{k}$$

$$\therefore \overline{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)\hat{j} + (4t - s + 3)\hat{k}$$

Direction ratios of $\overline{PQ}$ are ((-3t – 3s - 6), (2t + s - 15), (4t – s + 3))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

3(-3t – 3s - 6) – 1(2t + s - 15) + 1(4t – s + 3) = 0 and

-3(-3t – 3s - 6) + 2(2t + s - 15) + 4(4t – s + 3) = 0

⇒ -9t – 9s – 18 – 2t – s + 15 + 4t – s + 3 = 0 and

9t + 9s + 18 + 4t + 2s – 30 + 16t – 4s + 12 = 0

⇒ -7t – 11s = 0 and

29t + 7s = 0

Solving above two equations, we get,

t = 0 and s = 0

therefore,

P ≡ (3, 8, 3) and Q ≡ (-3, -7, 6)

Now, distance between points P and Q is

$$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$$

$$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$$

$$= \sqrt{36 + 225 + 9}$$

$$= \sqrt{270}$$

$$= 3\sqrt{30}$$

Therefore, the shortest distance between two given lines is

$$d = 3\sqrt{30}\ units$$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x - 3}{3 + 3} = \frac{y - 8}{8 + 7} = \frac{z - 3}{3 - 6}$$

$$\therefore \frac{x - 3}{6} = \frac{y - 8}{15} = \frac{z - 3}{-3}$$

$$\therefore \frac{x - 3}{2} = \frac{y - 8}{5} = \frac{z - 3}{-1}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x - 3}{2} = \frac{y - 8}{5} = \frac{z - 3}{-1}$$

2. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$\frac{x - 3}{-1} = \frac{y - 4}{2} = \frac{z + 2}{1}$ and $\frac{x - 1}{1} = \frac{y + 7}{3} = \frac{z + 2}{2}$.

Answer

Given : Cartesian equations of lines

$$L1 : \frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$$

$$L2 : \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$$

Formulae :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

2. Distance formula :

Distance between two points A≡(a_1, a_2, a_3) and B≡(b_1, b_2, b_3) is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

3. Equation of line :

Equation of line passing through points A≡(x_1, y_1, z_1) and B≡(x_2, y_2, z_2) is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

Answer :

Given equations of lines

$$L1 : \frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$$

$$L2 : \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$$

Direction ratios of L1 and L2 are (-1, 2, 1) and (1, 3, 2) respectively.

Let, general point on line L1 is P≡(x_1, y_1, z_1)

x_1 = -s+3 , y_1 = 2s+4 , z_1 = s-2

and let, general point on line L2 is Q≡(x_2, y_2, z_2)

x_2 = t+1 , y_2 = 3t – 7 , z_2 = 2t - 2

$$\therefore \overline{PQ} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

$$= (t + 1 + s - 3)\hat{\imath} + (3t - 7 - 2s - 4)\hat{\jmath} + (2t - 2 - s + 2)\hat{k}$$

$\therefore \overline{PQ} = (t + s - 2)\hat{i} + (3t - 2s - 11)\hat{j} + (2t - s)\hat{k}$

Direction ratios of $\overline{PQ}$ are ((t + s − 2), (3t − 2s − 11), (2t − s))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

-1(t + s − 2) + 2(3t − 2s − 11) + 1(2t − s) = 0 and

1(t + s − 2) + 3(3t − 2s − 11) + 2(2t − s) = 0

⇒ - t − s + 2 + 6t − 4s − 22 + 2t − s = 0 and

t + s − 2 + 9t − 6s − 33 + 4t − 2s = 0

⇒ 7t − 6s = 20 and

14t - 7s = 35

Solving above two equations, we get,

t = 2 and s = -1

therefore,

P ≡ (4, 2, -3) and Q ≡ (3, -1, 2)

Now, distance between points P and Q is

$d = \sqrt{(4-3)^2 + (2+1)^2 + (-3-2)^2}$

$= \sqrt{(1)^2 + (3)^2 + (-5)^2}$

$= \sqrt{1 + 9 + 25}$

$= \sqrt{35}$

Therefore, the shortest distance between two given lines is

$d = \sqrt{35}\ units$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-4}{4-3} = \frac{y-2}{2+1} = \frac{z+3}{-3-2}$$

$$\therefore \frac{x-4}{1} = \frac{y-2}{3} = \frac{z+3}{-5}$$

$$\therefore \frac{x-4}{-1} = \frac{y-2}{-3} = \frac{z+3}{5}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-4}{-1}=\frac{y-2}{-3}=\frac{z+3}{5}$$

3. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3} \text{ and } \frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}.$$

Answer

Given : Cartesian equations of lines

$$L1:\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

$$L2:\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$$

Formulae :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1)+(a_2 \times b_2)+(a_3 \times b_3)=0$$

2. Distance formula :

Distance between two points A≡(a_1, a_2, a_3) and B≡(b_1, b_2, b_3) is given by,

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

3. Equation of line :

Equation of line passing through points A≡(x_1, y_1, z_1) and B≡(x_2, y_2, z_2) is given by,

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

Answer :

Given equations of lines

$$L1:\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

$$L2:\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$$

Direction ratios of L1 and L2 are (2, 1, -3) and (2, -7, 5) respectively.

Let, general point on line L1 is P≡(x_1, y_1, z_1)

x_1 = 2s-1 , y_1 = s+1 , z_1 = -3s+9

and let, general point on line L2 is Q≡(x_2, y_2, z_2)

x_2 = 2t+3 , y_2 = -7t – 15 , z_2 = 5t + 9

$$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (5t + 9 - 2s + 1)\hat{i} + (-7t - 15 - s - 1)\hat{j} + (5t + 9 + 3s - 9)\hat{k}$$

$$\therefore \overline{PQ} = (5t - 2s + 10)\hat{i} + (-7t - s - 16)\hat{j} + (5t + 3s)\hat{k}$$

Direction ratios of $\overline{PQ}$ are ((5t - 2s + 10), (-7t – s – 16), (5t + 3s))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

2(5t - 2s + 10) + 1(-7t – s – 16) - 3(5t + 3s) = 0 and

2(5t - 2s + 10) – 7(-7t – s – 16) + 5(5t + 3s) = 0

⇒ 10t – 4s + 20 - 7t – s - 16 - 15t – 9s = 0 and

10t - 4s + 20 + 49t + 7s + 112 + 25t + 15s = 0

⇒ -12t – 14s = -4 and

84t + 18s = -132

Solving above two equations, we get,

t = -2 and s = 2

therefore,

P ≡ (3, 3, 3) and Q ≡ (-1, -1, -1)

Now, distance between points P and Q is

$$d = \sqrt{(3+1)^2 + (3+1)^2 + (3+1)^2}$$

$$= \sqrt{(4)^2 + (4)^2 + (4)^2}$$

$$= \sqrt{16 + 16 + 16}$$

$$= \sqrt{48}$$

$$= 4\sqrt{3}$$

Therefore, the shortest distance between two given lines is

$$d = 4\sqrt{3}\ units$$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x - 3}{3 + 1} = \frac{y - 3}{3 + 1} = \frac{z - 3}{3 + 1}$$

$$\therefore \frac{x - 3}{4} = \frac{y - 3}{4} = \frac{z - 3}{4}$$

$$\therefore x - 3 = y - 3 = z - 3$$

$$\Rightarrow x = y = z$$

Therefore, equation of line of shortest distance between two given lines is

x = y = z

4. Question

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1} \text{ and } \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}.$$

Answer

<u>Given</u> : Cartesian equations of lines

$$L1 : \frac{x - 6}{3} = \frac{y - 7}{-1} = \frac{z - 4}{1}$$

$$L2 : \frac{x}{-3} = \frac{y + 9}{2} = \frac{z - 2}{4}$$

<u>Formulae</u> :

1. Condition for perpendicularity :

If line L1 has direction ratios (a_1, a_2, a_3) and that of line L2 are (b_1, b_2, b_3) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

2. Distance formula :

Distance between two points A≡(a_1, a_2, a_3) and B≡(b_1, b_2, b_3) is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

3. Equation of line :

Equation of line passing through points A≡(x_1, y_1, z_1) and B≡(x_2, y_2, z_2) is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

Answer :

Given equations of lines

$$L1 : \frac{x - 6}{3} = \frac{y - 7}{-1} = \frac{z - 4}{1}$$

$$L2 : \frac{x}{-3} = \frac{y + 9}{2} = \frac{z - 2}{4}$$

Direction ratios of L1 and L2 are (3, -1, 1) and (-3, 2, 4) respectively.

Let, general point on line L1 is P≡(x_1, y_1, z_1)

x_1 = 3s+6 , y_1 = -s+7 , z_1 = s+4

and let, general point on line L2 is Q≡(x_2, y_2, z_2)

x_2 = -3t , y_2 = 2t – 9 , z_2 = 4t + 2

$$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (-3t - 3s - 6)\hat{i} + (2t - 9 + s - 7)\hat{j} + (4t + 2 - s - 4)\hat{k}$$

$$\therefore \overline{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 16)\hat{j} + (4t - s - 2)\hat{k}$$

Direction ratios of $\overline{PQ}$ are ((-3t - 3s - 6), (2t + s – 16), (4t – s – 2))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

3(-3t - 3s - 6) - 1(2t + s – 16) + 1(4t – s – 2) = 0 and

-3(-3t - 3s - 6) + 2(2t + s – 16) + 4(4t – s – 2) = 0

⇒ -9t - 9s - 18 - 2t – s + 16 + 4t – s – 2 = 0 and

9t + 9s + 18 + 4t + 2s – 32 + 16t – 4s – 8 = 0

⇒ -7t – 11s = 4 and

29t + 7s = -22

Solving above two equations, we get,

t = 1 and s = -1

therefore,

P ≡ (3, 8, 3) and Q ≡ (-3, -7, 6)

Now, distance between points P and Q is

$$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$$

$$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$$

$$= \sqrt{36 + 225 + 9}$$

$$= \sqrt{270}$$

$$= 3\sqrt{30}$$

Therefore, the shortest distance between two given lines is

$$d = 3\sqrt{30}\ units$$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$$

$$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$$

$$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

5. Question

Show that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ intersect and find their point of intersection.

Answer

Given : Cartesian equations of lines

$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

$$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$$

To Find : distance d

Formulae :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|} \right|$$

<u>**Answer**</u> **:**

Given Cartesian equations of lines

$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

Line L1 is passing through point (0, 2, -3) and has direction ratios (1, 2, 3)

Therefore, vector equation of line L1 is

$$\bar{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

And

$$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$$

Line L2 is passing through point (2, 6, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L2 is

$$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$$

Here,

$$\overline{a_1} = 0\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(8-9) - \hat{j}(4-6) + \hat{k}(3-4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$= \sqrt{6}$$

$$\overline{a_2} - \overline{a_1} = (2-0)\hat{i} + (6-2)\hat{j} + (3+3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

Now,

$$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (-\hat{i} + 2\hat{j} - \hat{k}) . (2\hat{i} + 4\hat{j} + 6\hat{k})$$

$$= ((-1) \times 2) + (2 \times 4) + ((-1) \times 6)$$

$= - 2 + 8 - 6$

$= 0$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{0}{\sqrt{14}}\right|$$

$$\therefore d = 0 \text{ units}$$

As d = 0

Hence, given lines intersect each other.

Now, general point on L1 is

$x_1 = \lambda$, $y_1 = 2\lambda+2$, $z_1 = 3\lambda-3$

let, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda + 2 - 6}{3} = \frac{3\lambda - 3 - 3}{4}$$

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda - 4}{3}$$

$\Rightarrow 3\lambda - 6 = 4\lambda - 8$

$\Rightarrow \lambda = 2$

Therefore, $x_1 = 2$, $y_1 = 2(2)+2$, $z_1 = 3(2)-3$

$\Rightarrow x_1 = 2$, $y_1 = 6$, $z_1 = 3$

Hence point of intersection of given lines is (2, 6, 3).

6. Question

Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ do not intersect each other.

Answer

<u>Given</u> : Cartesian equations of lines

$$L1 : \frac{x - 1}{3} = \frac{y + 1}{2} = \frac{z - 1}{5}$$

$$L2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$$

<u>To Find</u> : distance d

<u>Formulae</u> :

1. Equation of line :

Equation of line passing through point A (a_1, a_2, a_3) and having direction ratios (b_1, b_2, b_3) is

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

And $\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

2. Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3. Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

4. Shortest distance between two lines :

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>Answer</u> :

Given Cartesian equations of lines

$$L1 : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Therefore, vector equation of line L1 is

$$\bar{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 5\hat{k}\right)$$

And

$$L2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$$

Line L2 is passing through point (2, 1, -1) and has direction ratios (2, 3, -2)

Therefore, vector equation of line L2 is

$$\bar{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$$

Now, to calculate distance between the lines,

$$\bar{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 5\hat{k}\right)$$

$$\bar{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$$

Here,

$$\overline{a_1} = \hat{i} - \hat{j} + \hat{k}$$

$$\overline{b_1} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\overline{a_2} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} - 2\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}(-4 - 15) - \hat{j}(-6 - 10) + \hat{k}(9 - 4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -19\hat{i} + 16\hat{j} + 5\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-19)^2 + 16^2 + 5^2}$$

$$= \sqrt{361 + 256 + 25}$$

$$= \sqrt{642}$$

$\overline{a_2} - \overline{a_1} = (2-1)\hat{i} + (1+1)\hat{j} + (-1-1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 2\hat{j} - 2\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-19\hat{i} + 16\hat{j} + 5\hat{k}\right).\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$

= - 19 + 32 – 10

= 3

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{3}{\sqrt{642}}\right|$$

$$\therefore d = \frac{3}{\sqrt{642}} \text{ units}$$

As d ≠ 0

Hence, given lines do not intersect each other.

Exercise 27F

1. Question

If a line has direction ratios 2, -1, -2 then what are its direction cosines?

Answer

Given : A line has direction ratios 2, -1, -2

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by

$$\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$$

Here l = 2 , m = -1 , n = -2

Direction cosines of the line with direction ratios 2, -1, -2 is

$$\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

$$= \frac{2}{\sqrt{4+1+4}}, \frac{-1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

Direction cosines of the line with direction ratios 2, -1, -2 is $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

2. Question

Find the direction cosines of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

Answer

Given : A line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

To find : Direction cosines of the line

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-4}{-2}=\frac{y-0}{6}=\frac{z-1}{-3}$

Here l = -2 , m = 6 , n = -3

Direction cosines of the line $\frac{x-4}{-2}=\frac{y-0}{6}=\frac{z-1}{-3}$ is

$\frac{-2}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{6}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{-3}{\sqrt{(-2)^2+(6)^2+(-3)^2}}$

$= \frac{-2}{\sqrt{4+36+9}}, \frac{6}{\sqrt{4+36+9}}, \frac{-3}{\sqrt{4+36+9}} = \frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}, \frac{-3}{\sqrt{49}}$

$= \frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$

Direction cosines of the line $\frac{x-4}{-2}=\frac{y-0}{6}=\frac{z-1}{-3}$ is $\frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$

3. Question

If the equations of a line are $\frac{3-x}{-3}=\frac{y+2}{-2}=\frac{z+2}{6}$, find the direction cosines of a line parallel to the given line.

Answer

Given : A line $\frac{3-x}{-3}=\frac{y+2}{-2}=\frac{z+2}{6}$,

To find : Direction cosines of the line parallel to $\frac{3-x}{-3}=\frac{y+2}{-2}=\frac{z+2}{6}$, Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$, $\frac{m}{\sqrt{l^2+m^2+n^2}}$, $\frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-3}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$

Parallel lines have same direction ratios and direction cosines

Here l = 3 , m = -2 , n = 6

Direction cosines of the line $\frac{x-3}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$ is

$\frac{3}{\sqrt{(3)^2+(-2)^2+(6)^2}}$, $\frac{-2}{\sqrt{(3)^2+(-2)^2+(6)^2}}$, $\frac{6}{\sqrt{(3)^2+(-2)^2+(6)^2}}$

$=\frac{3}{\sqrt{9+4+36}}, \frac{-2}{\sqrt{9+4+36}}, \frac{6}{\sqrt{9+4+36}}=\frac{3}{\sqrt{49}}, \frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}$

$=\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$

Direction cosines of the line parallel to the line $\frac{x-3}{-3}=\frac{y+2}{-2}=\frac{z+2}{6}$ is

$\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$

4. Question

Write the equations of a line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through the point (1, -2, 3).

Answer

Given : A line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$

To find : equations of a line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through the point (1, -2, 3).

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then equation of parallel line passing through the point (p,q,r) is given by $\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

Here l = -3 , m = 2 , n = 6 and p = 1 , q = -2 , r = 3

The line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through the point (1,-2,3)

is given by

$\frac{x-1}{-3} = \frac{y-(-2)}{2} = \frac{z-3}{6}$

$\frac{x-1}{-3} = \frac{y+2}{2} = \frac{z-3}{6}$

The line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point

(1,-2,3) is given by $\frac{x-1}{-3} = \frac{y+2}{2} = \frac{z-3}{6}$

5. Question

Find the Cartesian equations of the line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

Answer

Given : A line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

To find : equations of a line parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

The given line is $\frac{x+3}{3} = \frac{y-4}{-5} = \frac{z+8}{6}$

Here l = 3 , m = -5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$\frac{x-(-2)}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

$\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

The line parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

6. Question

Write the vector equation of a line whose Cartesian equations are $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

Answer

Given : A line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

To find : vector equation of a line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$

Here a = 5 , b = -4 , c = 6 and l = 3 , m = 7 , n = -2

Substituting the above values,we get

$\vec{r} = 5\vec{i} - 4\vec{j} + 6\vec{k} + \lambda (3\vec{i} + 7\vec{j} - 2\vec{k})$

The vector equation of a line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$. is given by

$\vec{r} = 5\vec{i} - 4\vec{j} + 6\vec{k} + \lambda (3\vec{i} + 7\vec{j} - 2\vec{k})$

7. Question

The Cartesian equations of a line are $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$. Write the vector equation of the line.

Answer

Given : A line $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$.

To find : vector equation of a line $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-3}{-5}=\frac{y+4}{7}=\frac{z-3}{2}$

Here a = 3 , b = -4 , c = 3 and l = -5 , m = 7 , n = 2

Substituting the above values,we get

$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda\,(-5\vec{i} + 7\vec{j} + 2\vec{k})$

The vector equation of a line is given by $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda\,(-5\vec{i} + 7\vec{j} + 2\vec{k})$

8. Question

Write the vector equation of a line passing through the point (1, -1, 2) and parallel to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

Answer

Given : A line $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

To find : vector equation of a line passing through the point (1, -1, 2) and parallel

to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

Formula used : If a line is parallel to $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ and passing through the point (p,q,r) then vector equation of the line is given by $\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$

Here p = 1 , q = -1 , c = 2 and l = 1 , m = 2 , n = 2

Substituting the above values,we get

$\vec{r} = 1\vec{i} - 1\vec{j} + 2\vec{k} + \lambda\,(1\vec{i} + 2\vec{j} + 2\vec{k})$

The vector equation of a line passing through the point (1, -1, 2) and

parallel to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$. is given by

$\vec{r} = \vec{i} - \vec{j} + 2\vec{k} + \lambda\,(\vec{i} + 2\vec{j} + 2\vec{k})$

9. Question

If P(1, 5, 4) and Q(4, 1, -2) be two given points, find the direction ratios of PQ.

Answer

Given : P(1, 5, 4) and Q(4, 1, -2) be two given points

To find : direction ratios of PQ

Formula used : if $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two given points then direction ratios of PQ is given by $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

$x_1 = 1, y_1 = 5$, $z_1 = 4$ and $x_2 = 4, y_2 = 1$, $z_2 = -2$

Direction ratios of PQ is given by $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

Direction ratios of PQ is given by $4 - 1, 1 - 5, -2 - 4$

Direction ratios of PQ is given by $3, -4, -6$

Direction ratios of PQ is given by $3, -4, -6$

10. Question

The equations of a line are $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$. Find the direction cosines of a line parallel to this line.

Answer

Given : A line $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

To find : Direction cosines of the line parallel to $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$

Parallel lines have same direction ratios and direction cosines

Here $l = -2$, $m = 2$, $n = 1$

Direction cosines of the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$$\frac{-2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{1}{\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$$= \frac{-2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{1}{\sqrt{4+4+1}} = \frac{-2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}$$

$$= \frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$$

Direction cosines of the line parallel to the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$

11. Question

The Cartesian equations of a line are $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$. Find its vector equation.

Answer

Given : A line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$.

To find : vector equation of a line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda(l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-5}{-1}$

Here a = 1 , b = -2 , c = 5 and l = 2 , m = 3 , n = -1

Substituting the above values,we get

$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda(2\vec{i} + 3\vec{j} - 1\vec{k})$

The vector equation of a line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$. is given by

$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda(2\vec{i} + 3\vec{j} - 1\vec{k})$

12. Question

Find the vector equation of a line passing through the point (1, 2, 3) and parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

Answer

Given : A vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

To find : vector equation of a line passing through the point (1, 2, 3) and parallel

to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

Formula used : If a line is parallel to the vector $(l\vec{i} + m\vec{j} + n\vec{k})$

and passing through the point (p,q,r) then vector equation of the line is given by

$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda(l\vec{i} + m\vec{j} + n\vec{k})$

Here p = 1 , q = 2 , c = 3 and l = 3 , m = 2 , n = -2

Substituting the above values,we get

$\vec{r} = 1\vec{i} + 2\vec{j} + 3\vec{k} + \lambda (3\vec{i} + 2\vec{j} - 2\vec{k})$

The vector equation of a line passing through the point (1, 2, 3) and

parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.is $\vec{r} = \vec{i} + 2\vec{j} + 3\vec{k} + \lambda (3\vec{i} + 2\vec{j} - 2\vec{k})$

13. Question

The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$. Find its Cartesian equation.

Answer

Given : The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$.

To find : Cartesian equation of the line

Formula used : If the vector equation of the line is given by

$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda (l\vec{i} + m\vec{j} + n\vec{k})$ then its Cartesian equation is given by

$\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$.

Here p = 2, q = 1, r = -4 and l = 1,m = -1,n = -1

Cartesian equation is given by

$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-(-4)}{-1}$

$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$

Cartesian equation of the line is given by $\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$

14. Question

Find the Cartesian equation of a line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Answer

Given : A line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

To find : cartesian equations of a line parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

The given line is $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

Here l = 3 , m = 5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$\frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6}$

$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

The line parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

15. Question

Find the Cartesian equation of a line which passes through the point having position vector $\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i} + 2\hat{j} - \hat{k}\right)$.

Answer

Given : A line which passes through the point having position vector $\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i} + 2\hat{j} - \hat{k}\right)$.

To find : cartesian equations of a line

Formula used : If a line which passes through the point having position vector

$p\vec{\imath} + q\vec{\jmath} + r\vec{k}$ and is in the direction of the vector $l\vec{\imath} + m\vec{\jmath} + n\vec{k}$ then its Cartesian

equation is given by

$$\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$$

A line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

Here l = 1 , m = 2 , n = -1 and p = 2 , q = -1 , r = 4

$$\frac{x-2}{1}=\frac{y-(-1)}{2}=\frac{z-4}{-1}$$

$$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$$

The Cartesian equation of a line which passes through the point having

position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$. is

$$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$$

16. Question

Find the angle between the lines $\vec{r}=\left(2\hat{i}-5\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=\left(7\hat{i}-6\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.

Answer

Given : the lines $\vec{r}=\left(2\hat{i}-5\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=\left(7\hat{i}-6\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.

To find : angle between the lines

Formula used : If the lines are $a\vec{i}+b\vec{j}+c\vec{k}+\lambda(p\vec{i}+q\vec{j}+r\vec{k})$ and $d\vec{i}+e\vec{j}+f\vec{k}+$

$\lambda(l\vec{i}+m\vec{j}+n\vec{k})$ then the angle between the lines ‘θ’ is given by

$$\theta=\cos^{-1}\frac{pl+qm+rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$$

the lines $\vec{r}=\left(2\hat{i}-5\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=\left(7\hat{i}-6\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.

Here p = 3 , q = 2 , r = 6 and l = 1 , m = 2 , n = 2

$$\theta=\cos^{-1}\frac{3(1)+2(2)+6(2)}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}=\cos^{-1}\frac{3+4+12}{\sqrt{9+4+36}\sqrt{1+4+4}}$$

$$\theta=\cos^{-1}\frac{3+4+12}{\sqrt{49}\sqrt{9}}=\cos^{-1}\frac{19}{7\times3}=\cos^{-1}\frac{19}{21}$$

$\theta = \cos^{-1}\frac{19}{21}$

The angle between the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$. is $\cos^{-1}\frac{19}{21}$

17. Question

Find the angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

Answer

Given : the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

To find : angle between the lines

Formula used : If the lines are $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$ and $\frac{x-c}{l} = \frac{y-d}{m} = \frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$$\theta = \cos^{-1}\frac{pl + qm + rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$$

The lines are $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

Here p = 3 , q = 5 , r = 4 and l = 1 , m = 1 , n = 2

$$\theta = \cos^{-1}\frac{3(1)+5(1)+4(2)}{\sqrt{3^2+5^2+4^2}\sqrt{1^2+1^2+2^2}} = \cos^{-1}\frac{3+5+8}{\sqrt{9+25+16}\sqrt{1+1+4}}$$

$$\theta = \cos^{-1}\frac{3+5+8}{\sqrt{50}\sqrt{6}} = \cos^{-1}\frac{16}{10\sqrt{3}} = \cos^{-1}\frac{8}{5\sqrt{3}}$$

$$\theta = \cos^{-1}\frac{8\sqrt{3}}{15}$$

The angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

is $\cos^{-1}\frac{8\sqrt{3}}{15}$

18. Question

Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are at right angles.

Answer

Given : the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

To prove : the lines are at right angles.

Formula used : If the lines are $\frac{x-a}{p}=\frac{y-b}{q}=\frac{z-c}{r}$ and $\frac{x-c}{l}=\frac{y-d}{m}=\frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$\theta = \cos^{-1}\frac{pl+qm+rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$

The lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

Here p = 7 , q = -5 , r = 1 and l = 1 , m = 2 , n = 3

$\theta = \cos^{-1}\frac{7(1)+(-5)(2)+1(3)}{\sqrt{7^2+(-5)^2+1}\sqrt{1^2+2^2+3^2}} = \cos^{-1}\frac{7-10+3}{\sqrt{49+25+1}\sqrt{1+4+9}}$

$\theta = \cos^{-1}\frac{0}{\sqrt{75}\sqrt{14}} = \cos^{-1}0 = 90°$

θ = 90°

The Lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are at right angles.

19. Question

The direction ratios of a line are 2, 6, -9. What are its direction cosines?

Answer

Given : A line has direction ratios 2, 6 , -9

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = 6 , n = -9

Direction cosines of the line with direction ratios 2, 6 , -9 is

$\frac{2}{\sqrt{2^2+6^2+(-9)^2}}, \frac{6}{\sqrt{2^2+6^2+(-9)^2}}, \frac{-9}{\sqrt{2^2+6^2+(-9)^2}}$

$= \frac{2}{\sqrt{4+36+81}}, \frac{6}{\sqrt{4+36+81}}, \frac{-9}{\sqrt{4+36+81}} = \frac{2}{\sqrt{121}}, \frac{6}{\sqrt{121}}, \frac{-9}{\sqrt{121}}$

$= \frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

Direction cosines of the line with direction ratios 2, 6, -9 is $\frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

20. Question

A line makes angles 90^o, 135^o and 45^o with the positive directions of x-axis, y-axis and z-axis respectively. what are the direction cosines of the line?

Answer

Given : A line makes angles 90^o, 135^o and 45^o with the positive directions of x-axis, y-axis and z-axis respectively.

To find : Direction cosines of the line

Formula used : If a line makes angles α^o, β^o and γ^o with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$, $\cos(180° - \beta)$, $\cos(180° - \gamma)$

$\alpha = 90^o$, $\beta = 135^o$ and $\gamma = 45^o$

Direction cosines of the line is

$\cos 90°$, $\cos(180° - 135°)$, $\cos(180° - 45°)$

$\cos 90°$, $\cos 45°$, $\cos(135°)$

$0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Direction cosines of the line is $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

21. Question

What are the direction cosines of the y-axis?

Answer

To find : Direction cosines of the y- axis

Formula used : If a line makes angles α^o, β^o and γ^o with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$, $\cos\beta$, $\cos\gamma$

y-axis makes 90 ^{o} with the x and z axes

$\alpha = 90^o$, $\beta = 0\ ^o$ and $\gamma = 90\ ^o$

Direction cosines of the line is

$\cos 90°$, $\cos 0°$, $\cos 90°$

0, 1, 0

Direction cosines of the line is 0, 1 , 0

22. Question

What are the direction cosines of the vector $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$?

Answer

Given : A vector $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$?

To find : Direction cosines of the vector

Formula used : If a vector is $l\vec{i}$ + $m\vec{j}$ + $n\vec{k}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$, $\frac{m}{\sqrt{l^2+m^2+n^2}}$, $\frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = 1 , n = -2

Direction cosines of the line with direction ratios 2, 1, -2 is

$$\frac{2}{\sqrt{2^2+(1)^2+(-2)^2}}, \frac{1}{\sqrt{2^2+(1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(1)^2+(-2)^2}}$$

$$= \frac{2}{\sqrt{4+1+4}}, \frac{1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$$

Direction cosines of the vector is $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

23. Question

What is the angle between the vector $\vec{r}=\left(4\hat{i}+8\hat{j}+\hat{k}\right)$ and the x-axis?

Answer

Given : the vector $\vec{r}=\left(4\hat{i}+8\hat{j}+\hat{k}\right)$

To find : angle between the vector and the x-axis

Formula used : If the vector $l\vec{i}$ + $m\vec{j}$ + $n\vec{k}$ and x-axis then the angle between the

lines ‘θ’ is given by

$$\theta = \cos^{-1}\frac{l}{\sqrt{l^2+m^2+n^2}}$$

Here l = 4 , m = 8 , n = 1

$$\theta = \cos^{-1}\frac{4}{\sqrt{4^2+8^2+1^2}} = \cos^{-1}\frac{4}{\sqrt{16+64+1}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{81}} = \cos^{-1}\frac{4}{9}$$

$\theta = \cos^{-1}\frac{4}{9}$

The angle between the vector and the x-axis is $\cos^{-1}\frac{4}{9}$

Objective Questions

1. Question

The direction ratios of two lines are 3, 2, -6 and 1, 2, 2, respectively. The acute angle between these lines is

A. $\cos^{-1}\left(\frac{5}{18}\right)$

B. $\cos^{-1}\left(\frac{3}{20}\right)$

C. $\cos^{-1}\left(\frac{5}{21}\right)$

D. $\cos^{-1}\left(\frac{8}{21}\right)$

Answer

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = 3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$ and second parallel vector be $|\vec{b}| = \hat{\imath} + 2\,\hat{\jmath} + 2\hat{k}$.

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{3^2 + 2^2 + (-6)^2}$

$= 7$

$|\vec{b}| = \sqrt{1 + 2^2 + 2^2}$

$= 3$

$\cos\alpha = \frac{(3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}).(\hat{\imath} + 2\,\hat{\jmath} + 2\hat{k})}{7 \times 3}$

$\cos\alpha = \frac{3 + 4 - 12}{21}$

$\cos\alpha = \frac{-5}{21}$

$\alpha = \cos^{-1}\left(-\frac{5}{21}\right)$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$\alpha = \cos^{-1}\left(\frac{5}{21}\right)$

2. Question

The direction ratios of two lines are a, b, c and (b − c), (c − a), (a − b) respectively. The angle between these lines is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

Answer

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = a\hat{i} + b\hat{j} + c\hat{k}$ and second parallel vector be $|\vec{b}| = (b-c)\hat{i} + (c-a)\,\hat{j} + (a-b)\hat{k}.$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$

$= \sqrt{a^2 + b^2 + c^2}$

$|\vec{b}| = \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

$= \sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)}$

$\cos\alpha = \frac{(a\hat{i} + b\hat{j} + c\hat{k}).((b-c)\hat{i} + (c-a)\,\hat{j} + (a-b)\hat{k})}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$

$$\cos\alpha = \frac{ab - ac + bc - ba + ca - cb}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\cos\alpha = \frac{0}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\alpha = \cos^{-1}(0)$$

$$\alpha = \frac{\pi}{2}$$

3. Question

The angle between the lines $\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\cos^{-1}\left(\frac{3}{8}\right)$

Answer

Direction ratio are given implies that we can write the parallel vector towards those line, lets consider first parallel vector to be $|\vec{a}| = 2\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$ and second parallel vector be $|\vec{b}| = -\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$.

For the angle we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$

For that we need to find magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (7)^2}$$

$$= \sqrt{62}$$

$$|\vec{b}| = \sqrt{1 + 2^2 + 4^2}$$

$$= \sqrt{21}$$

$$\cos\alpha = \frac{(2\hat{i} + 7\hat{j} - 3\hat{k}).(-\hat{i} + 2\,\hat{j} + 4\hat{k})}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{-2 + 14 - 12}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{0}{\sqrt{21} \times \sqrt{62}}$$

$$\alpha = \cos^{-1} 0$$

Negative sign does not affect anything in cosine as cosine is positive in fourth quadrant

$$\alpha = \frac{\pi}{2}$$

4. Question

If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = ?

A. $\frac{-5}{7}$

B. $\frac{5}{7}$

C. $\frac{10}{7}$

D. $\frac{-10}{7}$

Answer

If the lines are perpendicular to each other then the angle between these lines will be $\frac{\pi}{2}$, me the cosine will be 0

$$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$$

$$|\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$$

$$= \sqrt{13 + 4k^2}$$

$$\vec{b} = 3k\hat{i} + \hat{j} - 5\hat{k}$$

$|\vec{b}| = \sqrt{(3k)^2 + 1 + 5^2}$

$= \sqrt{9k^2 + 26}$

$$\cos\left(\frac{\pi}{2}\right) = \frac{(3k\hat{i} + \hat{j} - 5\hat{k}).(-3\hat{i} + 2k\hat{j} + 2\hat{k})}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$0 = \frac{-9k + 2k - 10}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$k = -\frac{10}{7}$$

5. Question

A line passes through the points A(2, -1, 4) and B(1, 2, -2). The equations of the line AB are

A. $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$

B. $\frac{x+2}{-1} = \frac{y+1}{2} = \frac{z-4}{6}$

C. $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{6}$

D. none of these

Answer

To write the equation of a line we need a parallel vector and a fixed point through which the line is passing

Parallel vector=$((2-1)\hat{i} + (-1-2)\hat{j} + (4+2)\hat{k})$

$= \hat{i} - 3\hat{j} + 6\hat{k}$

Or = $-(\hat{i} - 3\hat{j} + 6\hat{k})$

Fixed point is 2î -ĵ +4k^

Equation

$$\frac{x-2}{1} = \frac{y-(-1)}{-3} = \frac{z-4}{6}$$

$$\frac{x-2}{1} = \frac{y+1}{-3} = \frac{z-4}{6}$$

Or

$$\frac{x-2}{-1}=\frac{y+1}{3}=\frac{z-4}{-6}$$

6. Question

The angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ is

A. $\cos^{-1}\left(\frac{3}{4}\right)$

B. $\cos^{-1}\left(\frac{5}{6}\right)$

C. $\cos^{-1}\left(\frac{2}{3}\right)$

D. $\frac{\pi}{3}$

Answer

Direction cosine of the lines are given 2î +2ĵ +k^ and 4î +ĵ +8k^

$\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$

$|\vec{a}|=\sqrt{2^2+2^2+1}$

$|\vec{a}|=3$

$\vec{b}=4\hat{i}+\hat{j}+8\hat{k}$

$|\vec{b}|=\sqrt{4^2+1+8^2}$

$=9$

$$\cos\alpha=\frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$$

$$\cos\alpha=\frac{(2\hat{i}+2\hat{j}+\hat{k}).(4\hat{i}+\hat{j}+8\hat{k})}{3\times 9}$$

$$\cos\alpha=\frac{8+8+2}{27}$$

$$\cos\alpha=\frac{2}{3}$$

7. Question

The angle between the lines $\vec{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$ is

A. $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

B. $\cos^{-1}\left(\frac{6\sqrt{2}}{5}\right)$

C. $\cos^{-1}\left(\frac{5\sqrt{3}}{8}\right)$

D. $\cos^{-1}\left(\frac{5\sqrt{2}}{6}\right)$

Answer

Let $\vec{a} = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} - 4\hat{k}$ and $|\vec{a}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$

$|\vec{b}| = \sqrt{3^2 + 5^2 + 4^2} = 5\sqrt{2}$

$$\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos\alpha = \frac{(3\hat{i} - 5\hat{j} - 4\hat{k}).(\hat{i} - \hat{j} - 2\hat{k})}{5\sqrt{2} \times \sqrt{6}}$$

$$\cos\alpha = \frac{3 + 5 + 8}{5\sqrt{12}}$$

$$\cos\alpha = \frac{8\sqrt{3}}{15}$$

8. Question

A line is perpendicular to two lines having direction ratios 1, -2, -2 and 0, 2, 1. The direction cosines of the line are

A. $\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}$

B. $\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}$

C. $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$

D. none of these

Answer

If a line is perpendicular to two given lines we can find out the parallel vector by cross product of the given two vectors.

$\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{b} = 2\hat{j} + \hat{k}$

$\vec{a} \times \vec{b} = (\hat{i} - 2\hat{j} - 2\hat{k}) \times (2\hat{j} + \hat{k})$

$= 2\hat{i} - \hat{j} + 2\hat{k}$

So the direction cosines are

$\hat{n} = \frac{1}{\sqrt{2^2 + 1 + 2^2}}$

$\hat{n} = \frac{1}{3}$

Direction cosine

$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

9. Question

A line passes through the point A(5, -2, 4) and it is parallel to the vector $\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$. The vector equation of the line is

A. $\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$

B. $\vec{r} = \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

C. $\vec{r} \cdot \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) = \sqrt{14}$

D. none of these

Answer

Fixed point is $5\hat{i} - 2\hat{j} + 4\hat{k}$ and parallel vector is $2\hat{i} - \hat{j} + 3\hat{k}$

Equation $5\hat{i} - 2\hat{j} + 4\hat{k} + \alpha(2\hat{i} - \hat{j} + 3\hat{k})$

10. Question

The Cartesian equations of a line are $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1}$. Its vector equation is

A. $\vec{r} = \left(-\hat{i} + 2\hat{j} - 5\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

B. $\vec{r} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 5\hat{k}\right)$

C. $\vec{r} = \left(\hat{i} - 2\hat{j} + 5\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)$

D. none of these

Answer

Dixed point (1,-2,5) and the parallel vector is $2\hat{i} + 3\hat{j} - \hat{k}$

Equation (î -2ĵ +5k^)+α $(2\hat{i} + 3\hat{j} - \hat{k})$

11. Question

A line passes through the pointA(-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. The vector equation of the line is

A. $\vec{r} = \left(-3\hat{i} + 4\hat{j} - 8\hat{k}\right) + \lambda\left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right)$

B. $r = \left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

C. $\vec{r} = \left(3\hat{i} + 5\hat{j} + 6\hat{k}\right) + \lambda\left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right)$

D. none of these

Answer

Fixed point is $-2\hat{i} + 4\hat{j} - 5\hat{k}$ and the parallel vector is $3\hat{i} + 5\hat{j} + 6\hat{k}$

Equation is $r = \left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

12. Question

The coordinates of the point where the line through the points A(5, 1, 6) and B(3, 4, 1) crosses the yz-plane is

A. (0, 17, -13)

B. $\left(0, \frac{-17}{2}, \frac{13}{2}\right)$

C. $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

D. none of these

Answer

We first need to find the equation of a line passing through the two given points

taking fixed point as $5\hat{i} + \hat{j} + 6\hat{k}$

and the parallel vector will be $(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

equation of the line in cartesian form

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5}$$

Assume above equation to be equal to k, a constant

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = k$$

And y-z plane have x-coordinate as zero we may get

$$\frac{0-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = k$$

$$k = -\frac{5}{2}$$

Now we can find y and z

$$\frac{y-1}{-3} = -\frac{5}{2}$$

$$y - 1 = \frac{15}{2}$$

$$y = \frac{17}{2}$$

$$\frac{z-6}{5} = -\frac{5}{2}$$

$$z - 6 = -\frac{25}{2}$$

$z = -\frac{13}{2}$

The coordinate where the line meets y-z plane is $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$

13. Question

The vector equation of the x-axis is given by

A. $\vec{r} = \hat{i}$

B. $\vec{r} = \hat{j} + \hat{k}$

C. $\vec{r} = \lambda \hat{i}$

D. none of these

Answer

Vector equation need a fixed point and a parallel vector

For x-axis fixed point can be anything ranging from negative to positive including origin

And parallel vector is î

Equation would be λ î

14. Question

The Cartesian equations of a lines are $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$. What is its vector equation?

A. $\vec{r} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

B. $\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

C. $\vec{r} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

D. none of these

Answer

Fixed point is $2\hat{i} - \hat{j} + 3\hat{k}$ and the vector is $2\hat{i} + 3\hat{j} - 2\hat{k}$

Equation $\left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

15. Question

The angle between two lines having direction ratios 1, 1, 2 and $\left(\sqrt{3} - 1\right)$, $\left(-\sqrt{3} - 1\right)$, 4 is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = (\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}$

$|\vec{a}| = \sqrt{6}\ |\vec{b}| = \sqrt{(4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16} = 2\sqrt{6}$

$$\cos\alpha = \frac{(\hat{i} + \hat{j} + 2\hat{k}).\left((\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}\right)}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos\alpha = \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{12}$$

$$\cos\alpha = \frac{1}{2}$$

$\alpha = 60°$

16. Question

The straight line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$ is

A. parallel to the x-axis

B. parallel to the y-axis

C. parallel to the z-axis

D. perpendicular to the z-axis

Answer

It is perpendicular to z-axis because cos90° is 0 which implies that it makes 90° with z-axis

17. Question

If a line makes angles α, β and γ with the x-axis, y-axis and z-axis respectively then $(\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma) = ?$

A. 1

B. 3

C. 2

D. $\frac{3}{2}$

Answer

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1 - \cos^2\alpha + 1 - \cos\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$ is the square of the direction ratios of all three axes which is always equal to 1

$= 3 - 1$

$= 2$

18. Question

If (a_1, b_1, c_1) and (a_2, b_2, c_2) be the direction ratios of two parallel lines then

A. $a_1 = a_2, b_1 = b_2, c_1 = c_2$

B. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

C. $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

D. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Answer

We know that if there is two parallel lines then their direction ratios must have a relation

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

19. Question

If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is

A. 5

B. 7

C. 8

D. 10

Answer

Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$$

$$10\lambda - 10 - 30 - 60 = 0$$

$$\lambda = 10$$