Factorization-2

Solution 1:

1. $36 - x^2 = (6 + x)(6 - x)$
   $36 - x^2 = (6)^2 - (x)^2$
   $= (6 + x)(6 - x)$
2. $a^2 - b^2c^2 = (a - bc)(a + bc)$
   $a^2 - b^2c^2 = (a)^2 - (bc)^2$
   $= (a - bc)(a + bc)$
3. $x^3 - 49x = x(x + 7)(x - 7)$
   $x^3 - 49x = x(x^2 - 49)$
   $= x(x^2 - 7^2)$
   $= x(x + 7)(x - 7)$
4. $4x^2 - 25 = (2x + 5)(2x - 5)$
   $4x^2 - 25 = (2x)^2 - (5)^2$
   $= (2x + 5)(2x - 5)$
5. $x^2 + 5x + 6 = (x + 3)(x + 2)$
   $x^2 + 5x + 6 = x^2 + 2x + 3x + 6$
   $= x(x + 2) + 3(x + 2)$
   $= (x + 3)(x + 2)$
6. $x^2 - x - 12 = (x + 3)(x - 4)$
   $x^2 - x - 12 = x^2 - 4x + 3x - 12$
   $= x(x - 4) + 3(x - 4)$
   $= (x + 3)(x - 4)$
7. $a^3 - 1 = (a - 1)(a^2 + a + 1)$
   $a^3 - 1 = a^3 - 1^3$
   $= (a - 1)[a^2 + (a)(1) + 1^2]$
   $= (a - 1)(a^2 + a + 1)$
8. $m^3 + 125 = (m + 5)(m^2 - 5m + 25)$
   $m^3 + 125 = m^3 + 5^3$
   $= (m + 5)[m^2 - (m)(5) + 5^2]$
   $= (m + 5)(m^2 - 5m + 25)$

Solution 2(1):

$16a^2b^2 - 36$

$= 4(4a^2b^2 - 9)$

$= 4[(2ab)^2 - (3)^2]$

$= 4(2ab - 3)(2ab + 3)$

Solution 2(2):

$625 - 64x^2$

$= (25)^2 - (8x)^2$

$= (25 + 8x)(25 - 8x)$

Solution 2(3):

$4x^5 - 64x$

$= 4x(x^4 - 16)$

$= 4x[(x^2)^2 - (4)^2]$

$= 4x(x^2 + 4)(x^2 - 4)$

$= 4x(x^2 + 4)(x + 2)(x - 2)$

Solution 2(4):

$(4a - 5b)^2 - 16c^2$

$= (4a - 5b)^2 - (4c)^2$

$= (m)^2 - (4c)^2$ (Taking $4a - 5b = m$)

$= (m + 4c)(m - 4c)$

$= [(4a - 5b) + 4c][(4a - 5b) - 4c]$ (Putting back $m = 4a - 5b$)

$= (4a - 5b + 4c)(4a - 5b - 4c)$

Solution 2(5):

$25 - (ab - 3x)^2$

$= (5)^2 - (ab - 3x)^2$

$= (5)^2 - (m)^2$ (Taking $ab - 3x = m$)

$= (5 + m)(5 - m)$

$= [5 + (ab - 3x)][5 - (ab - 3x)]$ (Putting back $m = ab - 3x$)

$= (5 + ab - 3x)(5 - ab + 3x)$

Solution 2(6):

$(x + 8)^2 - (2x - 3)^2$

$= (m)^2 - (n)^2$ (Taking $x + 8 = m$ and $2x - 3 = n$)

$= (m + n)(m - n)$

$= [(x + 8) + (2x - 3)][(x + 8) - (2x - 3)]$

(Putting back $m = x + 8$ and $n = 2x - 3$)

$= (x + 8 + 2x - 3)(x + 8 - 2x + 3)$

$= (3x + 5)(-x + 11)$

Solution 2(7):

$121x^2 - 22x + 1 - 9a^2 - 24ab - 16b^2$

$= (121x^2 - 22x + 1) - (9a^2 + 24ab + 16b^2)$

$= (11x - 1)^2 - (3a + 4b)^2$

$= (m)^2 - (n)^2$ (Taking $11x - 1 = m$ and $3a + 4b = n$)

$= (m + n)(m - n)$

$= [(11x - 1) + (3a + 4b)][(11x - 1) - (3a + 4b)]$

(Putting back $m = 11x - 1$ and $n = 3a + 4b$)

$= (11x - 1 + 3a + 4b)(11x - 1 - 3a - 4b)$

= (11x + 3a + 4b − 1)(11x − 3a − 4b − 1)

Solution 2(8):

$$\text{Middle term} = \pm 2 \times \sqrt{x^4} \times \sqrt{4y^4}$$
$$= \pm 2 \times x^2 \times 2y^2$$
$$= \pm 4x^2 y^2$$

$$x^4 + 4y^4 = x^4 + 4x^2 y^2 + 4y^4 - 4x^2 y^2$$
$$= \left(x^2 + 2y^2\right)^2 - (2xy)^2$$
$$= \left(x^2 + 2y^2 + 2xy\right)\left(x^2 + 2y^2 - 2xy\right)$$
$$= \left(x^2 + 2xy + 2y^2\right)\left(x^2 - 2xy + 2y^2\right)$$

Solution 2(9):

$$\text{Middle term} = \pm 2 \times \sqrt{x^4} \times \sqrt{9}$$
$$= \pm 2 \times x^2 \times 3$$
$$= \pm 6x^2$$

$$x^4 - 3x^2 + 9 = x^4 + 6x^2 + 9 - 9x^2 \quad \text{(Putting in the difference of two squares)}$$
$$= \left(x^2 + 3\right)^2 - (3x)^2$$
$$= \left(x^2 + 3 + 3x\right)\left(x^2 + 3 - 3x\right)$$
$$= \left(x^2 + 3x + 3\right)\left(x^2 - 3x + 3\right)$$

Solution 2(10):

$x^4 - 8x^2 - 65$
$= x^4 - 13x^2 + 5x^2 - 65$
$= x^2(x^2 - 13) + 5(x^2 - 13)$
$= (x^2 - 13)(x^2 + 5)$

Solution 2(11):

$x^6 - 27$
$= (x^2)^3 - (3)^3$
$= (x^2 - 3)[(x^2)^2 + (x^2)(3) + (3)^2]$
$= (x^2 - 3)(x^4 + 3x^2 + 9)$

Solution 2(12):

$64x^3 + 125y^3$
$= (4x)^3 + (5y)^3$
$= (4x + 5y)[(4x)^2 - (4x)(5y) + (5y)^2]$
$= (4x + 5y)(16x^2 - 20xy + 25y^2)$

Practice 1

Solution 1:

$m^2 - 16$
$= (m)^2 - (4)^2$
$= (m + 4)(m - 4)$

Solution 2:

$16x^2 - 49y^2$
$= (4x)^2 - (7y)^2$

= (4x + 7y)(4x – 7y)

Solution 3:
$4a^2b^2 – 1$
$= (2ab)^2 – (1)^2$
$= (2ab + 1)(2ab – 1)$

Solution 4:
$49 – 25x^2$
$= (7)^2 – (5x)^2$
$= (7 + 5x)(7 – 5x)$

Solution 5:
$16x^4 – 81y^4$
$= (4x^2)^2 – (9y^2)^2$
$= (4x^2 + 9y^2)(4x^2 – 9y^2)$
$= (4x^2 + 9y^2) [(2x)^2 – (3y)^2]$
$= (4x^2 + 9y^2)(2x + 3y)(2x – 3y)$

Solution 6:
$a^4b^4 – 1$
$= (a^2b^2)^2 – (1)^2$
$= (a^2b^2 + 1)(a^2b^2 – 1)$
$= (a^2b^2 + 1) [(ab)^2 – (1)^2]$
$= (a^2b^2 + 1)(ab + 1)(ab – 1)$

Solution 7:
$x^3 – 49x$
$= x(x^2 – 49)$
$= x[(x)^2 – (7)^2]$
$= x(x + 7)(x – 7)$

Solution 8:
$18x^3y^3 – 2xy$
$= 2xy(9x^2y^2 – 1)$
$= 2xy [(3xy)^2 – (1)^2]$
$= 2xy(3xy + 1)(3xy – 1)$

Practice 2

Solution 1(1):
$(4x + 3y)^2 – 49z^2$
$= (m)^2 – (7z)^2$ (Taking 4x + 3y = m)
$= (m + 7z)(m – 7z)$
$= [(4x + 3y) + 7z][(4x + 3y) – 7z]$ (Putting back m = 4x + 3y)
$= (4x + 3y + 7z)(4x + 3y – 7z)$

Solution 1(2):
$(ab – 1)^2 – 64x^2$
$= (m)^2 – (8x)^2$ (Taking ab – 1 = m)
$= (m + 8x)(m – 8x)$
$= [(ab – 1) + 8x][(ab – 1) – 8x]$ (Putting back m = ab – 1)

= (ab – 1 + 8x)(ab – 1 – 8x)

= (ab + 8x – 1)(ab – 8x – 1)

Solution 1(3):

$81 – (5x – 3y)^2$

$= (9)^2 – (m)^2$ (Taking 5x – 3y = m)

= (9 + m)(9 – m)

= [(9 + (5x – 3y)][9 – (5x – 3y)] (Putting back m = 5x – 3y)

= (9 + 5x – 3y)(9 – 5x + 3y)

= (5x – 3y + 9)(-5x + 3y + 9)

Solution 1(4):

$36z^2 – (x + 2y)^2$

$= (6z)^2 – (m)^2$ (Taking x + 2y = m)

= (6z + m)(6z – m)

= [6z + (x + 2y)][6z – (x + 2y)] (Putting back m = x + 2y)

= (6z + x + 2y)(6z – x – 2y)

Solution 2(1):

$(a + 8)^2 – (b – 3)^2$

$= (m)^2 – (n)^2$ (Taking a + 8 = m and b – 3 = n)

= (m + n)(m – n)

= [(a + 8) + (b – 3)][(a + 8) – (b – 3)] (Putting back m = a + 8 and n = b – 3)

= (a + 8 + b – 3)(a + 8 – b + 3)

= (a + b + 5)(a – b + 11)

Solution 2(2):

$(3x – 2y)^2 – (5a – 3b)^2$

$= (m)^2 – (n)^2$ (Taking 3x – 2y = m and 5a – 3b = n)

= (m + n)(m – n)

= [(3x – 2y) + (5a – 3b)][(3x – 2y) – (5a – 3b)]

(Putting back m = 3x – 2y and n = 5a – 3b)

= (3x – 2y + 5a – 3b)(3x – 2y – 5a + 3b)

Solution 2(3):

$(4x + 5)^2 – (2y + 3)^2$

$= (m)^2 – (n)^2$ (Taking 4x + 5 = m and 2y + 3 = n)

= (m + n)(m – n)

= [(4x + 5) + (2y + 3)][(4x + 5) – (2y + 3)] (Putting back m = 4x + 5 and n = 2y + 3)

= (4x + 5 + 2y + 3)(4x + 5 – 2y – 3)

= (4x + 2y + 8)(4x – 2y + 2)

= [2(2x + y + 4)][2(2x – y + 1)]

= 4(2x + y + 4)(2x – y + 1)

Solution 2(4):

$(ab + 6)^2 – (mn – 7)^2$

$= (x)^2 – (y)^2$ (Taking ab + 6 = x and mn – 7 = y)

= (x + y)(x – y)

= [(ab + 6) + (mn – 7)][(ab + 6) – (mn – 7)] (Putting back x = ab + 6 and y = mn – 7)

= (ab + 6 + mn – 7)(ab + 6 – mn + 7)

= (ab + mn – 1)(ab – mn + 13)

Solution 3(1):

$36a^2 - 12a + 1 - 4b^2$

$= (36a^2 - 12a + 1) - (4b^2)$

$= (6a - 1)^2 - (2b)^2$

$= (m)^2 - (2b)^2$ (Taking 6a – 1 = m)

$= (m + 2b)(m - 2b)$

$= [(6a - 1) + 2b][(6a - 1) - 2b]$ (Putting back m = 6a-1)

$= (6a - 1 + 2b)(6a - 1 - 2b)$

$= (6a + 2b - 1)(6a - 2b - 1)$

Solution 3(2):

$64 - x^2 - 10x - 25$

$= (64) - (x^2 + 10x + 25)$

$= (8)^2 - (x + 5)^2$

$= (8)^2 - (m)^2$ (Taking x + 5 = m)

$= (8 + m)(8 - m)$

$= [8 + (x + 5)][8 - (x + 5)]$ (Putting back m = x + 5)

$= (8 + x + 5)(8 - x - 5)$

$= (x + 13)(-x + 3)$

Solution 3(3):

$m^2n^2 - 4mn + 4 - x^2$

$= (m^2n^2 - 4mn + 4) - (x^2)$

$= (mn - 2)^2 - (x)^2$

$= (a)^2 - (x)^2$ (Taking mn – 2 = a)

$= (a + x)(a - x)$

$= [(mn - 2) + x][(mn - 2) - x]$

(Putting back a = mn – 2)

$= (mn - 2 + x)(mn - 2 - x)$

$= (mn + x - 2)(mn - x - 2)$

Solution 3(4):

$9m^2 - 25x^2 + 20xy - 4y^2$

$= (9m^2) - (25x^2 - 20xy + 4y^2)$

$= (3m)^2 - (5x - 2y)^2$

$= (3m)^2 - (a)^2$ (Taking 5x – 2y = a)

$= (3m + a)(3m - a)$

$= [3m + (5x - 2y)][3m - (5x - 2y)]$ (Putting back a = 5x – 2y)

$= (3m + 5x - 2y)(3m - 5x + 2y)$

Solution 4(1):

$9a^2 + 6a + 1 - x^2 - 2xy - y^2$

$= (9a^2 + 6a + 1) - (x^2 + 2xy + y^2)$

$= (3a + 1)^2 - (x + y)^2$

$= (m)^2 - (n)^2$ (Taking 3a + 1 = m and x + y = n)

$= (m + n)(m - n)$

$= [(3a + 1) + (x + y)][(3a + 1) - (x + y)]$ (Putting back m = 3a + 1 and n = x + y)

$= (3a + 1 + x + y)(3a + 1 - x - y)$

Solution 4(2):

$49x^2 - 14x + 1 - 64a^2 + 16ab - b^2$

$= (49x^2 - 14x + 1) - (64a^2 - 16ab + b^2)$

= (7x – 1)² – (8a – b)²

= (m)² – (n)² (Taking 7x – 1 = m and 8a – b = n)

= (m + n)(m – n)

= [(7x – 1) + (8a – b)][(7x – 1) – (8a – b)] (Putting back m = 7x – 1 and n = 8a – b)

= (7x – 1 + 8a – b)(7x – 1 – 8a + b)

= (7x + 8a – b – 1)(7x – 8a + b – 1)

Solution 4(3):

4a² + 12a + 9 – m² + 2mn – n²

= (4a² + 12a + 9) – (m² – 2mn + n²)

= (2a + 3)² – (m – n)²

= (x)² – (y)² (Taking 2a + 3 = x and m – n = y)

= (x + y)(x – y)

= [(2a + 3) + (m – n)][(2a + 3) – (m – n)]

(Putting back x = 2a + 3 and y = m – n)

= (2a + 3 + m – n)(2a + 3 – m + n)

Solution 4(4):

x² – 14x + 49 – a² – 12a – 36

= (x² – 14x + 49) – (a² +12a + 36)

= (x – 7)² – (a + 6)²

= (m)² – (n)² (Taking x – 7 = m and a + 6 = n)

= (m + n)(m – n)

= [(x – 7) + (a + 6)][(x – 7) – (a + 6)]

(Putting back m = x – 7 and n = a + 6)

= (x – 7 + a + 6)(x – 7 – a – 6)

= (x + a – 1)(x – a – 13)

Practice 3

Solution 1:

$$\text{Middle term} = \pm 2 \times \sqrt{4x^4} \times \sqrt{y^4}$$
$$= \pm 2 \times 2x^2 \times y^2$$
$$= \pm 4x^2 y^2$$

Now,
$$4x^4 + y^4 = 4x^4 + 4x^2 y^2 + y^4 - 4x^2 y^2$$
$$= \left(4x^4 + 4x^2 y^2 + y^4\right) - \left(4x^2 y^2\right)$$
$$= \left(2x^2 + y^2\right)^2 - (2xy)^2$$
$$= (m)^2 - (2xy)^2 \quad \left(\text{Taking } 2x^2 + y^2 = m\right)$$
$$= (m + 2xy)(m - 2xy)$$
$$= \left[\left(2x^2 + y^2\right) + 2xy\right]\left[\left(2x^2 + y^2\right) - 2xy\right] \left(\text{Putting back } m = 2x^2 + y^2\right)$$
$$= \left(2x^2 + y^2 + 2xy\right)\left(2x^2 + y^2 - 2xy\right)$$

Solution 2:

Middle term $= \pm 2 \times \sqrt{64a^4} \times \sqrt{b^4}$

$\qquad\qquad = \pm 2 \times 8a^2 \times b^2$

$\qquad\qquad = \pm 16a^2b^2$

Now,

$64a^4 + b^4 = 64a^4 + 16a^2b^2 + b^4 - 16a^2b^2$

$\qquad\qquad = \left(64a^4 + 16a^2b^2 + b^4\right) - \left(16a^2b^2\right)$

$\qquad\qquad = \left(8a^2 + b^2\right)^2 - (4ab)^2$

$\qquad\qquad = (m)^2 - (4ab)^2 \quad \left(\text{Taking } 8a^2 + b^2 = m\right)$

$\qquad\qquad = (m + 4ab)(m - 4ab)$

$\qquad\qquad = \left[\left(8a^2 + b^2\right) + 4ab\right]\left[\left(8a^2 + b^2\right) - 4ab\right] \quad \left(\text{Putting back } m = 8a^2 + b^2\right)$

$\qquad\qquad = \left(8a^2 + b^2 + 4ab\right)\left(8a^2 + b^2 - 4ab\right)$

$\qquad\qquad = \left(8a^2 + 4ab + b^2\right)\left(8a^2 - 4ab + b^2\right)$

Solution 3:

Middle term $= \pm 2 \times \sqrt{81a^4} \times \sqrt{4}$

$\qquad\qquad = \pm 2 \times 9a^2 \times 2$

$\qquad\qquad = \pm 36a^2$

Now,

$81a^4 + 4 = 81a^4 + 36a^2 + 4 - 36a^2$

$\qquad\qquad = \left(81a^4 + 36a^2 + 4\right) - \left(36a^2\right)$

$\qquad\qquad = \left(9a^2 + 2\right)^2 - (6a)^2$

$\qquad\qquad = (m)^2 - (6a)^2 \quad \left(\text{Taking } 9a^2 + 2 = m\right)$

$\qquad\qquad = (m + 6a)(m - 6a)$

$\qquad\qquad = \left[\left(9a^2 + 2\right) + 6a\right]\left[\left(9a^2 + 2\right) - 6a\right] \quad \left(\text{Putting back } m = 9a^2 + 2\right)$

$\qquad\qquad = \left(9a^2 + 2 + 6a\right)\left(9a^2 + 2 - 6a\right)$

$\qquad\qquad = \left(9a^2 + 6a + 2\right)\left(9a^2 - 6a + 2\right)$

Solution 4:

Middle term $= \pm 2 \times \sqrt{4m^4} \times \sqrt{625}$

$\qquad\qquad = \pm 2 \times 2m^2 \times 25$

$\qquad\qquad = \pm 100m^2$

Now,

$4m^4 + 625 = 4m^4 + 100m^2 + 625 - 100m^2$

$\qquad\qquad = \left(4m^4 + 100m^2 + 625\right) - \left(100m^2\right)$

$\qquad\qquad = \left(2m^2 + 25\right)^2 - (10m)^2$

$\qquad\qquad = (a)^2 - (10m)^2 \quad \left(\text{Taking } 2m^2 + 25 = a\right)$

$\qquad\qquad = (a + 10m)(a - 10m)$

$\qquad\qquad = \left[\left(2m^2 + 25\right) + 10m\right]\left[\left(2m^2 + 25\right) - 10m\right] \quad \left(\text{Putting back } a = 2m^2 + 25\right)$

$\qquad\qquad = \left(2m^2 + 25 + 10m\right)\left(2m^2 + 25 - 10m\right)$

$\qquad\qquad = \left(2m^2 + 10m + 25\right)\left(2m^2 - 10m + 25\right)$

Practice 4

Solution 1:

Middle term $= \pm 2 \times \sqrt{a^4} \times \sqrt{25}$

$\qquad\qquad = \pm 2 \times a^2 \times 5$

$\qquad\qquad = \pm 10a^2$

Now,

$a^4 + 6a^2 + 25 = a^4 + 10a^2 + 25 - 4a^2$ (Putting in the difference of two squares)

$\qquad\qquad = \left(a^2 + 5\right)^2 - (2a)^2$

$\qquad\qquad = (m)^2 - (2a)^2$ (Taking $a^2 + 5 = m$)

$\qquad\qquad = (m + 2a)(m - 2a)$

$\qquad\qquad = \left[\left(a^2 + 5\right) + 2a\right]\left[\left(a^2 + 5\right) - 2a\right]$ (Putting back $m = a^2 + 5$)

$\qquad\qquad = \left(a^2 + 5 + 2a\right)\left(a^2 + 5 - 2a\right)$

$\qquad\qquad = \left(a^2 + 2a + 5\right)\left(a^2 - 2a + 5\right)$

Solution 2:

Middle term $= \pm 2 \times \sqrt{m^4} \times \sqrt{9}$

$\qquad\qquad = \pm 2 \times m^2 \times 3$

$\qquad\qquad = \pm 6m^2$

Now,

$m^4 - 7m^2 + 9 = m^4 - 6m^2 + 9 - m^2$ (Putting in the difference of two squares)

$\qquad\qquad = \left(m^2 - 6m^2 + 9\right) - \left(m^2\right)$

$\qquad\qquad = \left(m^2 - 3\right)^2 - \left(m^2\right)$

$\qquad\qquad = (x)^2 - (m)^2$ (Taking $m^2 - 3 = x$)

$\qquad\qquad = (x + m)(x - m)$

$\qquad\qquad = \left[\left(m^2 - 3\right) + m\right]\left[\left(m^2 - 3\right) - m\right]$ (Putting back $x = m^2 - 3$)

$\qquad\qquad = \left(m^2 - 3 + m\right)\left(m^2 - 3 - m\right)$

$\qquad\qquad = \left(m^2 + m - 3\right)\left(m^2 - m - 3\right)$

Solution 3:

Middle term $= \pm 2 \times \sqrt{4x^4} \times \sqrt{25}$

$\qquad\qquad = \pm 2 \times 2x^2 \times 5$

$\qquad\qquad = \pm 20x^2$

Now,

$4x^4 - 21x^2 + 25 = 4x^4 - 20x^2 + 25 - x^2$ (Putting in the difference of two squares)

$\qquad\qquad = \left(4x^4 - 20x^2 + 25\right) - \left(x^2\right)$

$\qquad\qquad = \left(2x^2 - 5\right)^2 - \left(x^2\right)$

$\qquad\qquad = (m)^2 - (x)^2$ (Taking $2x^2 - 5 = m$)

$\qquad\qquad = (m + x)(m - x)$

$\qquad\qquad = \left[\left(2x^2 - 5\right) + x\right]\left[\left(2x^2 - 5\right) - x\right]$ (Putting back $m = 2x^2 - 5$)

$\qquad\qquad = \left(2x^2 - 5 + x\right)\left(2x^2 - 5 - x\right)$

$\qquad\qquad = \left(2x^2 + x - 5\right)\left(2x^2 - x - 5\right)$

Solution 4:

Middle term $= \pm 2 \times \sqrt{a^4} \times \sqrt{4b^4}$

$\qquad\qquad\quad = \pm 2 \times a^2 \times 2b^2$

$\qquad\qquad\quad = \pm 4a^2b^2$

Now,

$a^4 - 8a^2b^2 + 4b^4 = a^4 - 4a^2b^2 + 4b^4 - 4a^2b^2$

$\qquad\qquad\qquad\qquad\qquad$ (Putting in the difference of two squares)

$\qquad\qquad = \left(a^4 - 4a^2b^2 + 4b^4\right) - \left(4a^2b^2\right)$

$\qquad\qquad = \left(a^2 - 2b^2\right)^2 - (2ab)^2$

$\qquad\qquad = (m)^2 - (2ab)^2 \quad \left(\text{Taking } a^2 - 2b^2 = m\right)$

$\qquad\qquad = (m + 2ab)(m - 2ab)$

$\qquad\qquad = \left[\left(a^2 - 2b^2\right) + 2ab\right]\left[\left(a^2 - 2b^2\right) - 2ab\right] \left(\text{Putting back } m = a^2 - 2b^2\right)$

$\qquad\qquad = \left(a^2 - 2b^2 + 2ab\right)\left(a^2 - 2b^2 - 2ab\right)$

$\qquad\qquad = \left(a^2 + 2ab - 2b^2\right)\left(a^2 - 2ab - 2b^2\right)$

Solution 5:

Middle term $= \pm 2 \times \sqrt{y^4} \times \sqrt{4}$

$\qquad\qquad\quad = \pm 2 \times y^2 \times 2$

$\qquad\qquad\quad = \pm 4y^2$

Now,

$y^4 + 3y^2 + 4 = y^4 + 4y^2 + 4 - y^2$ (Putting in the difference of two squares)

$\qquad\qquad = \left(y^4 + 4y^2 + 4\right) - \left(y^2\right)$

$\qquad\qquad = \left(y^2 + 2\right)^2 - (y)^2$

$\qquad\qquad = (m)^2 - (y)^2 \quad \left(\text{Taking } y^2 + 2 = m\right)$

$\qquad\qquad = (m + y)(m - y)$

$\qquad\qquad = \left[\left(y^2 + 2\right) + y\right]\left[\left(y^2 + 2\right) - y\right] \quad \left(\text{Putting back } m = y^2 + 2\right)$

$\qquad\qquad = \left(y^2 + 2 + y\right)\left(y^2 + 2 - y\right)$

Solution 6:

Middle term $= \pm 2 \times \sqrt{m^4} \times \sqrt{64}$

$\qquad\qquad\quad = \pm 2 \times m^2 \times 8$

$\qquad\qquad\quad = \pm 16m^2$

Now,

$m^4 + 12m^2 + 64 = m^4 + 16m^2 + 64 - 4m^2$ (Putting in the difference of two squares)

$\qquad\qquad = \left(m^4 + 16m^2 + 64\right) - \left(4m^2\right)$

$\qquad\qquad = \left(m^2 + 8\right)^2 - (2m)^2$

$\qquad\qquad = (x)^2 - (2m)^2 \quad \left(\text{Taking } m^2 + 8 = x\right)$

$\qquad\qquad = (x + 2m)(x - 2m)$

$\qquad\qquad = \left[\left(m^2 + 8\right) + 2m\right]\left[\left(m^2 + 8\right) - 2m\right] \quad \left(\text{Putting back } x = m^2 + 8\right)$

$\qquad\qquad = \left(m^2 + 8 + 2m\right)\left(m^2 + 8 - 2m\right)$

$\qquad\qquad = \left(m^2 + 2m + 8\right)\left(m^2 - 2m + 8\right)$

Practice 5

Solution 1(1):

$x^3 + 27$

$= (x)^3 + (3)^3$

$= (x + 3)[(x)^2 - (x)(3) + (3)^2]$

$= (x + 3)(x^2 - 3x + 9)$

Solution 1(2):

$a^3 + 125b^3$

$= (a)^3 + (5b)^3$

$= (a + 5b)[(a)^2 - (a)(5b) + (5b)^2]$

$= (a + 5b)(a^2 - 5ab + 25b^2)$

Solution 1(3):

$8a^3b^3 - 27$

$= (2ab)^3 - (3)^3$

$= (2ab - 3)[(2ab)^2 + (2ab)(3) + (3)^2]$

$= (2ab - 3)(4a^2b^2 + 6ab + 9)$

Solution 1(4):

$8x^3 - 125$

$= (2x)^3 - (5)^3$

$= (2x - 5)[(2x)^2 + (2x)(5) + (5)^2]$

$= (2x - 5)(4x^2 + 10x + 25)$

Solution 2(1):

$a + b = 5$

$\therefore (a + b)^2 = (5)^2$

$\therefore a^2 + 2ab + b^2 = 25$

$\therefore a^2 + b^2 + 2(6) = 25$ …..(Putting ab = 6)

$\therefore a^2 + b^2 + 12 = 25$

$\therefore a^2 + b^2 = 25 - 12$

$\therefore a^2 + b^2 = 13$

Now, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (a + b)(a^2 + b^2 - ab)$

Putting a + b = 5, ab = 6 and $a^2 + b^2 = 13$, we have

$a^3 + b^3 = (5)(13 - 6)$

$= (5)(7)$

$= 35$

Solution 2(2):

$a + b = 8$, ab = 15 and $a^2 + b^2 = 34$

Now,

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (a + b)(\underline{a^2 + b^2} - ab)$

$= (8)(34 - 15)$

$= (8)(19)$

$= 152$

Solution 2(3):

$a - b = 2$, ab = 24 and $a^2 + b^2 = 52$

Now,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (a - b)(\underline{a^2 + b^2} + ab)$

= (2)(52 + 24)

= (2)(76)

= 152

Solution 3(1):

$(11)^3 + (9)^3$

$= (11 + 9)[(11)^2 - (11)(9) + (9)^2]$

= (20)(121 − 99 + 81)

= (20)(103)

= 2060

Solution 3(2):

$(23)^3 + (7)^3$

$= (23 + 7)[(23)^2 - (23)(7) + (7)^2]$

= (30)(529 − 161 + 49)

= (30)(417)

= 12510

Solution 3(3):

$(45)^3 - (25)^3$

$= (45 - 25)[(45)^2 + (45)(25) + (25)^2]$

= (20)(2025 + 1125 + 625)

= (20)(3775)

= 75500

Practice 6

Solution 1:

$x^2 + 5x + 6$

$= x^2 + 3x + 2x + 6$

= x(x + 3) + 2(x + 3)

= (x + 3)(x + 2)

Solution 2:

$x^2 + 15x + 50$

$= x^2 + 10x + 5x + 50$

= x(x + 10) + 5(x + 10)

= (x + 10)(x + 5)

Solution 3:

$x^2 - 11x + 24$

$= x^2 - 3x - 8x + 24$

= x(x − 3) − 8(x − 3)

= (x − 3)(x − 8)

Solution 4:

$x^2 - 7x + 12$

$= x^2 - 4x - 3x + 12$

= x(x − 4) − 3(x − 4)

= (x − 4)(x − 3)

Solution 5:

$x^2 + 6x - 2$

$= x^2 + 9x - 3x - 27$

$= x(x + 9) - 3(x + 9)$

$= (x + 9)(x - 3)$

Solution 6:

$a^2 + 4a - 21$

$= a^2 + 7a - 3a - 21$

$= a(a + 7) - 3(a + 7)$

$= (a + 7)(a - 3)$

Solution 7:

$m^2 - 2m - 8$

$= m^2 - 4m + 2m - 8$

$= m(m - 4) + 2(m - 4)$

$= (m - 4)(m + 2)$

Solution 8:

$n^2 - 4n - 45$

$= n^2 - 9n + 5n - 45$

$= n(n - 9) + 5(n - 9)$

$= (n - 9)(n + 5)$

Solution 9:

$4x^2 + 12x + 5$

$= 4x^2 + 10x + 2x + 5$

$= 2x(2x + 5) + 1(2x + 5)$

$= (2x + 5)(2x + 1)$

Solution 10:

$9y^4 - 13y^2 + 4$

$= 9y^4 - 9y^2 - 4y^2 + 4$

$= 9y^2(y^2 - 1) - 4(y^2 - 1)$

$= (y^2 - 1)(9y^2 - 4)$

$= (y^2 - 1^2)[(3y)^2 - (2)^2]$

$= (y + 1)(y - 1)(3y + 2)(3y - 2)$

Solution 11:

$2a^2 - 19a - 21$

$= 2a^2 + 2a - 21a - 21$

$= 2a(a + 1) - 21(a + 1)$

$= (a + 1)(2a - 21)$

Solution 12:

$6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$