HOTS (Higher Order Thinking Skills)

Que 1. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

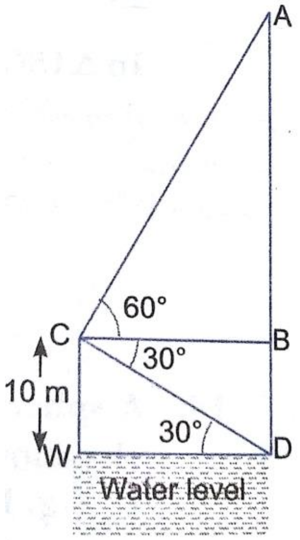

Fig. 11.51

Sol. In **Fig. 11.15,** let C represents the position of the man on the deck of the ship, A represents the top of hill and D its base.

Now in right-angled triangle CWD.

$$\tan 30° = \frac{10}{WD} \qquad \Rightarrow \quad WD = \frac{10}{\tan 30°}$$

$$\Rightarrow \qquad WD = \frac{10}{1} = 10\sqrt{3} = 17.3\ m$$

Also, in right-angled triangle ABC, we have,

$$\tan 60° = \frac{AB}{BC} or \frac{AB}{WD} \qquad \text{[From fig. BC = WD]}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AB}{10\sqrt{3}} \qquad \Rightarrow \quad AB = 10\sqrt{3} \times \sqrt{3} = 30\ m$$

Now, AD = AB + BD = 30 m + 10 m = 40 m.

Therefore, the distance of the hill from the ship = 17.3 m and height of the hill = 40 m

Que 2. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

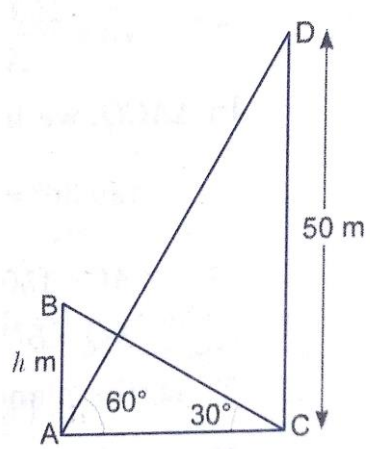

Fig. 11.52

Sol. Let AB be the building of height h m and CD be the tower of height 50 m. We have,

$$\angle ACB = 30° \; and \; \angle DAC = 60°$$

Now, in ΔACD, we have

$$\tan 60° = \frac{DC}{AC} \qquad \Rightarrow \qquad \sqrt{3} = \frac{50}{AC}$$

$$\Rightarrow \qquad AC = \frac{50}{\sqrt{3}} = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{50\sqrt{3}}{3}$$

$$\Rightarrow \qquad AC = \frac{50\sqrt{3}}{3} \qquad \text{....(i)}$$

Now in ΔABC, we have

$$\tan 30° = \frac{AB}{AC}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{AC} \qquad \Rightarrow \qquad AC = \sqrt{3}h$$

$$\therefore \qquad h = \frac{AC}{\sqrt{3}} = \frac{50\sqrt{3}}{\frac{3}{\sqrt{3}}} = \frac{50}{3} = 16\frac{2}{3}\; m \qquad \text{(From equation (i))}$$

Hence, the height of the building is $16\frac{2}{3}\; m$.

Que 3. From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ and ϕ respectively. Show that the height of the opposite house is h (1 + tan θ cotϕ).

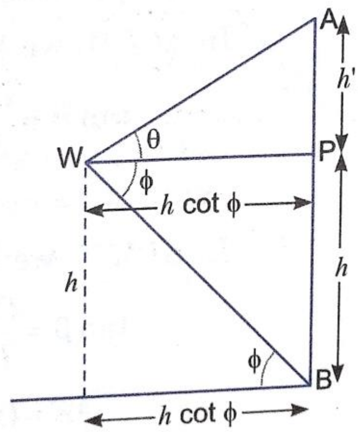

Fig. 11.53

Sol. Let W be the window and AB be the house on the opposite side.
Then, WP is the width of the street **(Fig. 11.53).**
Let AP = h' m

In ΔBPW, $\tan\phi = \frac{PB}{WP}$

$\Rightarrow \quad \frac{h}{WP} = \tan\phi \quad \Rightarrow \quad WP = h\cot\phi$ …(i)

Now, in ΔAWP, $\tan\theta = \frac{AP}{WP} = \frac{h\prime}{WP}$

$\Rightarrow \quad h' = WP\tan\theta \quad \Rightarrow \quad h' = h\cot\phi\tan\theta$

∴ Height of house = h' + h

= h cot ϕ tan θ + h (1 + tan θ cotϕ)

Que 4. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $1500\sqrt{3}$ m find the speed of the jet plane.

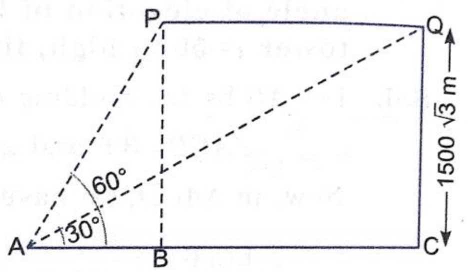

Fig. 11.54

Sol. Let P and Q be the two position of the plane and jet A be the point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from a point A are 60° and 30°, respectively.
Then, $\angle PAB = 60°, \angle QAB = 30°$

It is also given that PB = $1500\sqrt{3}$ metres

In ΔABP, we have

$$\tan 60° = \frac{BP}{AB} m$$

$$\Rightarrow \quad \sqrt{3} = \frac{1500\sqrt{3}}{AB} \quad \Rightarrow \quad AB = 1500$$

In ΔACQ, we have

$$\tan 30° = \frac{CQ}{AC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AC}$$

$$\Rightarrow \quad AC = 1500 \times 3 = 4500\ m$$

∴ PQ = BC = AC – AB = 4500 – 1500 = 3000 m

Thus, the plane travels 3000 m in 15 seconds.

Hence, the speed of plane = $\frac{3000}{15} = 200 = 200\ m/s$

$$= 200 \times \frac{3600}{1000} km/h = 200 \times \frac{18}{5} km/h = 720\ km/h.$$

Que 5. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $\frac{h(\tan\beta + \tan\alpha}{\tan\beta - \tan\alpha}$.

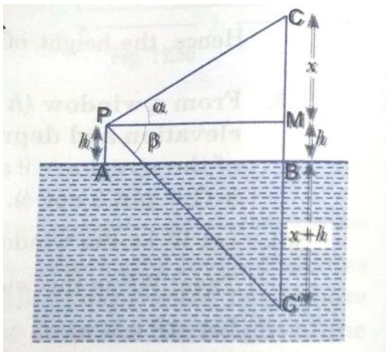

Fig. 11.55

Sol. Let AB be the surface of the lake and let P be a point of observation **(Fig. 11.55)** such that AP = h metres. Let C be the position of the cloud and C" be its reflection in the lake.

Then, CB = C" B. Let PM be perpendicular from P on CB. Then $\angle CPM = \alpha$ and $\angle MPC" = \beta$. Let CM = x.

Then, CB = CM + MB = CM + PA = x + h.

In ΔCPM, we have

$$\tan\alpha = \frac{C"M}{PM} \quad \Rightarrow \quad \tan\alpha = \frac{x}{AB} \quad [\because PM = AB]$$

$\Rightarrow$ AB = x cot α ....(i)

In $\Delta PMC"$, we have

$$\tan\beta = \frac{C'M}{PM} \quad \Rightarrow \quad \tan\beta = \frac{x+2h}{AB} \quad [\because\ C'M = C'B + BM = x + h + h]$$

$\Rightarrow$ AB = (x + 2h) cot β ....(ii)

From (i) and (ii), we have

$$x \cot\alpha = (x+2h)\cot\beta \quad \Rightarrow \quad x(\cot\alpha - \cot\beta) = 2h\cot\beta$$

$$\Rightarrow \quad x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta} \quad \Rightarrow \quad x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \quad x = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, the height CB of the cloud is given by

$$CB = x + h \quad \Rightarrow \quad CB = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} + h$$

$$\Rightarrow \quad CB = \frac{2h\tan\alpha + h\tan\beta - h\tan\alpha}{\tan\beta - \tan\alpha} = \frac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$$