# MATRICES

CHAPTER – 3

## MATRICES

**MATRIX (Definition):** A matrix is defined as a rectangular array (arrangement) of numbers or functions.

**Example**

$$A=\begin{bmatrix}-2 & 5\\ 0 & \sqrt{5}\\ 3 & 6\end{bmatrix}$$

**ELEMENTS:** The numbers or functions in a matrix are called its elements.

**Example**

Let matrix $A=\begin{bmatrix}-2 & 5\\ 0 & \sqrt{5}\\ 3 & 6\end{bmatrix}$

∴ Elements of matrix A = { -2 , 5, 0, $\sqrt{5}$ , 3, 6}.

**ROW:** The elements lying in a horizontal line form a row.

**Example**

Let matrix $A=\begin{bmatrix}1 & 4\\ 5 & 7\end{bmatrix}$ ,then rows are $[1 \quad 4]$ , $[5 \quad 7]$

**COLUMNS:** The elements lying in a vertical line form a column.

**Example**

Let matrix $A=\begin{bmatrix}1 & 4\\ 5 & 7\end{bmatrix}$ , then columns are $\begin{bmatrix}1\\ 5\end{bmatrix}$, $\begin{bmatrix}4\\ 7\end{bmatrix}$

**ORDER OF MATRIX:** A matrix of order m × n, i.e., consisting of m rows and n columns is denoted by $A = [a_{ij}]_{m\times n}$.

$$\text{i.e., } A=[a_{ij}]_{m\times n}=\begin{bmatrix}a_{11} & a_{12} & a_{13} & \cdots & a_{1j} & \cdots & a_{1n}\\ a_{21} & a_{22} & a_{23} & \cdots & a_{2j} & \cdots & a_{2n}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{ij} & \cdots & a_{in}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mj} & \cdots & a_{mn}\end{bmatrix}$$

A number of elements in the matrix $[a_{ij}]_{m\times n}$ are m × n and nth element = $a_{ij}$ is that element that lies in the $i^{th}$ row and $j^{th}$ column.

**Example**

Let $A=\begin{bmatrix}1 & -2 & 3\\ 1 & 0 & -2\end{bmatrix}$, then the order of matrix A is 2×3

**Example**

Construct a 3×3 matrix A = $[a_{ij}]$ whose elements are given by $a_{ij} = i + j$

**Solution:** Here , $a_{11} = 1 + 1 = 2$ , $a_{12} = 1 + 2 = 3$ , $a_{13} = 1 + 3 = 4$ , $a_{21} = 2 + 1 = 3$ , $a_{22} = 2 + 2 = 4$ , $a_{23} = 2 + 3 = 5$ , $a_{31} = 3 + 1 = 4$ , $a_{32} = 3 + 2 = 5$ , $a_{33} = 3 + 3 = 6$

∴ Matrix A = $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

## TYPES OF MATRICES

### ROW MATRIX

A matrix having only one row is called a **row matrix**. Thus A = $[a_{ij}]_{mxn}$ is a row matrix if m = 1. So, a row matrix can be represented as **A = $[a_{ij}]_{1\times n}$**. It is called so because it has only one row and the order of a row matrix will hence be 1 × n.

**Example**

A = [1 2 4 5] is row matrix of order 1 x 4.

### COLUMN MATRIX

A matrix having only one column is called a **column matrix**. Thus, A = $[a_{ij}]_{mxn}$ is a column matrix if n = 1. Thus, the value of for a column matrix will be 1. Hence, the order is m × 1.

**Example**

Let A = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is column matrix of order 3 x 1.

### ZERO OR NULL MATRIX

If in a matrix all the elements are zero then it is called a **zero matrix** and it is generally denoted by 0. Thus, A = $[a_{ij}]_{mxn}$ is a zero-matrix if $a_{ij}$ = 0 for all i and j;

**Example**

A= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a zero matrix of order 3 x 3.

### SINGLETON MATRIX

If in a matrix there is only element then it is called **singleton matrix**. Thus, A = $[a_{ij}]_{mxn}$ is a singleton matrix if m = n = 1.

**Example**

[2], [3], [a], are singleton matrices.

### HORIZONTAL MATRIX

A matrix of order m x n is a **horizontal matrix** if n > m.

**Example**

A= $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ of order 2×3

### VERTICAL MATRIX

A matrix of order m x n is a **vertical matrix** if m > n.

**Example**

A= $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ of order 3×1

### SQUARE MATRIX

If the number of rows and the number of columns in a matrix are equal, then it is called a **square matrix**.

Thus, A = $[a_{ij}]_{mxn}$ is a square matrix if m = n.

**Example**

A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a square matrix of order 3 × 3.

### UPPER TRIANGULAR MATRIX

A square matrix A = $[a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$ .

Thus , in an upper triangular matrix ,all elements below the ,main diagonal are zero.

**Example**

A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is an upper triangular matrix.

### LOWER TRIANGULAR MATRIX

A square matrix A = $[a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ for all $i < j$.

Thus ,in a lower triangular matrix ,all elements above the main diagonal are zero.

**Example**

A = $\begin{bmatrix}1 & 0 & 0\\3 & 4 & 0\\4 & 5 & 2\end{bmatrix}$ is a lower triangular matrix of order 3.

**Note**

A triangular matrix A = $[a_{ij}]$ n× n is called a strictly triangular iff $a_{ij} = 0$ for all i = 1 ,2,..., n

### DIAGONAL MATRIX

If all the elements, except the principal diagonal, in a square matrix, are zero, it is called a **diagonal matrix**. Thus, a square matrix A = $[a_{ij}]$ is a diagonal matrix if $a_{ij}$ = 0, when i≠j

**Example**

A= $\begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}$ is a diagonal matrix of order 3 x 3, which can also be denoted by diagonal [2 3 4].

The special thing is, all the non-diagonal elements of this matrix are zero. That means only the diagonal has non-zero elements. There are two important things to note here which are

**(i)** A diagonal matrix is always a square matrix

**(ii)** The diagonal elements are characterized by this general form: $a_{ij}$ where i = j. This means that a matrix can have only one diagonal.

### SCALAR MATRIX

If all the elements in the diagonal of a diagonal matrix are equal, it is called a **scalar matrix**. Thus, a square matrix A= $[a_{ij}]$ of order n×n is a scalar matrix if

$a_{ij} = \begin{cases}0\,;\, i \neq j\\ k\,;\, i = j\end{cases}$ where k is a constant.

**Example**

A= $\begin{bmatrix}-7 & 0 & 0\\0 & -7 & 0\\0 & 0 & -7\end{bmatrix}$ is a scalar Matrix.

### UNIT MATRIX OR IDENTITY MATRIX

If all the elements of a principal diagonal in a diagonal matrix are 1, then it is called a **unit matrix**. A unit matrix of order n is denoted by $I_n$. Thus, a square matrix A = $[a_{ij}]_{m\times n}$ is an identity matrix if $a_{ij} = \begin{cases}0\,;\, i \neq j\\ 1\,;\, i = j\end{cases}$

**Example**

A = $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

### EQUALITY OF MATRICES

Two matrices A and B are said to be equal, if

(i) order of A and B are same.

(ii) corresponding elements of A and B are same i.e. $a_{ij}$ = $b_{ij}$, ∀ i and j.

**Example**

$\begin{bmatrix}2 & 0\\1 & 3\end{bmatrix}$ and $\begin{bmatrix}2 & 0\\1 & 3\end{bmatrix}$ are equal matrices, but $\begin{bmatrix}3 & 0\\1 & 2\end{bmatrix}$ and $\begin{bmatrix}1 & 0\\2 & 3\end{bmatrix}$ are not equal matrices.

### ADDITION OF MATRICES

Let A , B be two matrices ,each of order $m \times n$.Then their sum A + B is a matrix of order $m \times n$ and is obtained by adding the corresponding elements of A and B.

Thus , if A = $[a_{ij}]_{m\times n}$ and B =$[b_{ij}]_{m\times n}$ are two matrices of the same order ,their sum A + B is defined to be the matrix of order $m \times n$ such that

$(A+B)_{ij} = a_{ij} + b_{ij}$ for I = 1 , 2, ...,m and j= 1 ,2, ...,n

**Note**

The sum of two matrices is defined only when they are of the same order.

**Example**

If A = $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$ , B = $\begin{bmatrix}6 & 5 & 4\\3 & 2 & 1\end{bmatrix}$ , then A + B = $\begin{bmatrix}1+6 & 2+5 & 3+4\\4+3 & 5+2 & 6+1\end{bmatrix} = \begin{bmatrix}7 & 7 & 7\\7 & 7 & 7\end{bmatrix}$

### PROPERTIES OF MATRIX ADDITION

**THEOREM 1** (Commutativity) If A and B are two $m \times n$ matrices, then A + B = B + A . i.e., matrix addition is commutative.

**THEOREM 2** ( Associativity ) If A ,B,C are three matrices of the same order , then (A+B)+C=A+(B+C) i.e., matrix addition is associative.

**THEOREM 3** (Existence of Identity ) The null matrix is the identity element for matrix addition i.e., A + O = A = O+A

**THEOREM 4** (Existence of Inverse ) For any matrix A = $[a_{ij}]_{m\times n}$ there exists a matrix $[-a_{ij}]_{m\times n}$

Denoted by – A, such that A + (-A) = O =(-A) +A

**THEOREM 5** (Cancellation laws) If A ,B ,C are matrices of the same order , then A + B = A+C ⇒ B = C (Left cancellation law )

And , B + A = C + A ⇒ B = C (Right cancellation law )

## MULTIPLICATION OF A MATRIX BY A SCLAR

Let A = $[a_{ij}]$ be an $m \times n$ matrix and k be any number called a scalar .Then the matrix obtained by multiplying every element of A by k is called the scalar multiple by A by k and denoted by k A.

**Example**

If A = $\begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$ ,then 3 A = $\begin{bmatrix} 9 & 18 \\ 3 & 12 \end{bmatrix}$

**THEOREM** If A = $[a_{ij}]_{m\times n}$ , B = $[b_{ij}]_{m\times n}$ are two matrices and k , l are scalar , the

$$k(A+B) = k\,A + kB$$

$$(k+l)A = k\,A + l\,A$$

$$(k\,l\,)A = k\,(l\,A\,) = l(k\,A\,)$$

$$(-k)A = -(k\,A\,) = k\,(-A)$$

1 .A = A

(-1) A = - A

**Example**

Find a matrix X such that 2 A + B + X = O ,where A = $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and , B = $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

**Solution:** We have,

2 A + B + X = O ⇒ X = -2 A -B

$\Rightarrow X = -2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$

## MULTIPLICATION OF MATRICES

Two matrices A and B are conformable for the product AB if the number of columns in A is same as the number of rows in B.

Thus , if A = $[a_{ij}]_{m\times n}$ and B = $[b_{ij}]_{n\times p}$ are two matrices of orders $m \times n$ and $n \times p$ respectively ,then their product AB is of order $m \times p$ and is defined as

$(AB)_{ij} = (i^{th}\,row\,of\,A\,)(j^{th}\,column\,of\,B\,)$ for all I = 1, 2, ..., m and j = 1, 2, ..., p

$$\Rightarrow [a_{i1} \quad a_{i2} \quad \cdots \quad a_{in}]\begin{bmatrix} b_{1j} \\ \vdots \\ b_{nj} \end{bmatrix}$$

$$\Rightarrow (AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} = \sum_{r=1}^{n} a_{ir}b_{rj}$$

**Note**

If A and B are two matrices such that AB exists , then BA may or may not exist.

**Example**

If A = $\begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 4 & -3 \end{bmatrix}$ then , AB = $\begin{bmatrix} 16 & -12 \\ 3 & -11 \\ 3 & -1 \end{bmatrix}$

**Note**

In this case BA does not exist , because the number of columns in B is not same as the number of rows in A.

## PROPERTIES OF MATRIX MULTIPLICATION

**THEOREM 1** Matrix multiplication is not commutative in general.

**THEOREM 2** Matrix multiplication is associative i.e., (AB)C =A(BC) , whenever both sides are defined.

**THEOREM 3** Matrix multiplication is distributive over matrix addition i.e.,

$$A(B+C) = AB + AC$$

$(A+B)C = AC + BC$ , whenever both sides of equality are defined.

**THEOREM 4** If A is an $m \times n$ matrix , then $I_m A = A = AI_n$

**Remark**

The product of two matrices can be the null matrix while neither of them is the null matrix.

**Example**

If A = $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ , then AB = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ while neither A nor B is the null matrix.

**THEOREM 5** If A is $m \times n$ matrix and O is a null matrix , then

$$A_{m\times n}O_{n\times p} = O_{n\times p}$$
$$O_{p\times m}A_{m\times n} = O_{p\times n}$$

i.e., the product of the matrix with a null matrix is always a null matrix.

**Remark**

In the case of matrix multiplication if AB = O, then it does not necessarily imply that BA = O.

**Example**

If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ .Then , AB = O. But , BA = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \neq$ O.

Thus, AB = O while BA≠O.

## TRANSPOSE OF A MATRIX

If A = [$a_{ij}$] be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A and is denoted by A′ or ($A^T$ ).

In other words, if A = $[a_{ij}]_{m\times n}$ , then A′ =$[a_{ij}]_{n\times m}$ .

**Example**

Matrix A = $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 4 \end{bmatrix}$ Then transpose of A = $A^T$ = $\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$

## PROPERTIES OF TRANSPOSE

**THEOREM 1** For any matrix A , $(A^T)^T = A$

**THEOREM 2** For any two matrices A and B of the same order , $(A + B)^T = A^T + B^T$

**THEOREM 3** If A is a matrix and k is a scalar ,then $(kA)^T = k(A)^T$

**THEOREM 1** If A and B are two matrices such that AB is defined , then $(AB)^T = B^T A^T$.

**Remark**

The above law is called the reversal law for transpose i.e., the transpose of the product is the product of the transpose taken in the reverse order.

**Example**

If A = $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$ ,verify that $(AB)^T = B^T A^T$

**Solution:** We have ,

A = $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$

∴ AB = $\begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$.........(i)

Also , $B^T A^T$ = $\begin{bmatrix} -2 & -1 & -4 \end{bmatrix}^T \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}^T$ = $\begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$.......(ii)

From (i) and (ii) , we observe that $(AB)^T = B^T A^T$

## SYMMETRIC MATRIX

A square matrix A = [$a_{ij}$] is said to be symmetric if the transpose of A is equal to A, that is, [$a_{ij}$] = [$a_{ji}$] for all possible values of i and j.

**Example**

A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

## SKEW -SYMMETRIC MATRIX

A square matrix A = [$a_{ij}$] is a skew-symmetric matrix if A′ = – A, that is $a_{ji}$ = – $a_{ij}$ for all possible values of i and j. Also, if we substitute i = j, we have $a_{ii}$ = – $a_{ii}$ and thus, 2$a_{ii}$ = 0 or $a_{ii}$ = 0 for all i's. Therefore, all the diagonal elements of a skew symmetric matrix are zero.

**Example**

A = $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

## SOME IMPORTANT POINTS

If A is any square matrix, then A + $A^T$ is a symmetric matrix and A −$A^T$ is a skew-symmetric matrix.

Every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix. A= $\frac{1}{2}$(A+$A^T$) + $\frac{1}{2}$(A-$A^T$) = $\frac{1}{2}$(B+C) where B is symmetric and C is a skew symmetric matric.

If A and B are symmetric matrices, then AB is symmetric AB = BA, i.e. A & B commute.

The matrix $B^T$AB is symmetric or skew-symmetric in correspondence if A is symmetric or skew-symmetric.

All positive integral powers of a symmetric matrix are symmetric.

Positive odd integral powers of a skew-symmetric matrix are skew-symmetric and positive even integral powers of a skew-symmetric matrix are symmetric.

## SPECIAL TYPES OF MATRICES

### (a) IDEMPOTENT MATRIX

A square matrix is idempotent, provided $A^2$ = A.

**Example**

A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

### (b) NILPOTENT MATRIX

A nilpotent matrix is said to be nilpotent of index p, i.e. $A^p$ = 0 , $A^{p-1} \neq 0$.

**Example**

A = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent matrix of index 3.

### (c) PERIODIC MATRIX

A square matrix which satisfies the relation $A^{k+1}$= A for some positive integer k, then A is periodic with period k

**Example**

A = $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ has the period 1

**Note**

(i) Period of a square null matrix is not defined.

(ii) Period of an idempotent matrix is 1.

### (d) INVOLUNTARY MATRIX

If $A^2$ = I the matrix is said to be an involuntary matrix. An involuntary matrix its own inverse.

**Example**

A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

# QUESTIONS

## MCQ

**Q1.** Find the value of x if

$[1\ x\ 1]\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0.$

(a) $x = -2, -15$
(a) $x = 2, -15$
(b) $x = -2, -14$
(c) $x = 2, -14$

**Q2.** Find the matrix $A$ such that $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A = \begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$.

(a) $A = \begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$
(b) $A = \begin{bmatrix}1 & 2 & -5\\ 3 & 4 & 1\end{bmatrix}$
(c) $A = \begin{bmatrix}1 & 2 & 5\\ 3 & 4 & 0\end{bmatrix}$
(d) $A = \begin{bmatrix}-1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$

**Q3.** If $A \neq 0, B \neq 0$ $then\ AB \neq 0$. Is the given statement true?
(a) No (b) Yes
(c) Cannot say (d) None

**Q4.** If $A = [3\ 5], B = [7\ 3]$, then find a non-zero matrix C such that AC = BC.
(a) $\begin{bmatrix}x\\ 2x\end{bmatrix}$ (b) $\begin{bmatrix}-x\\ -2x\end{bmatrix}$
(c) $\begin{bmatrix}-x\\ 2x\end{bmatrix}$ (d) $\begin{bmatrix}x\\ -2x\end{bmatrix}$

**Q5.** *If A,B and C are three matrices such that AB=AC, where A is non-zero* matrix, but B ≠ C. Is the given statement true?
(a) No (b) Yes
(c) Cannot say (d) None

**Q6.** If $A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, then $A^n$ =?
(a) $\begin{bmatrix}1 & -n\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & n\\ 0 & 0\end{bmatrix}$
(c) $\begin{bmatrix}0 & n\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

**Q7.** If $A = \begin{bmatrix}\lambda & 1\\ -1 & -\lambda\end{bmatrix}$, then for what value of $\lambda, A^2 = O$.
(a) $\lambda = 1$ (b) $\lambda = \pm 1$
(c) $\lambda = 0$ (d) $\lambda = -1$

**Q8.** If A and B are symmetric matrices of the same order and X = AB + BA and Y= AB - BA, then $(XY)^{\top}$ is equal to
(a) XY (b) $YX$
(c) $-YX$ (d) None

**Q9.** If A is a skew-symmetric matrix of order 3, then the matrix $A^4$ is
(a) Skew-symmetric (b) Symmetric
(c) Diagonal (d) None

**Q10.** Let A and B be two 2 × 2 matrices. Consider the statements"
(i) $AB = 0 \Rightarrow A = 0\ or\ B = 0$
(ii) $AB = I \Rightarrow A = B^{-1}$
(iii) $(A + B)^2 = A^2 + 2AB + B^2$
(a) (i) is false, (ii) and (iii) are true
(b) (i) and (iii) are false, (ii) is true
(c) (i) and (iii) are false, (iii) is true
(d) (ii) and (iii) are false ,(i) is true

**Q11.** If $A = \begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$ then value of α for which $A^2 = B$ is
(a) 1 (b) 2
(c) 4 (d) No real values

**Q12.** Which of the given values of x and y make the following pair of matrices equal
$\begin{bmatrix}3x + 7 & 5\\ y + 1 & 2 - 3x\end{bmatrix} = \begin{bmatrix}0 & y - 2\\ 8 & 4\end{bmatrix}$
(a) $x = \frac{-1}{3}, y = 7$ (b) Cannot determined
(c) $y = 7, x = \frac{-2}{3}$ (d) $x = \frac{-1}{3}, y = \frac{-2}{3}$

**Q13.** Given, $3\begin{bmatrix}x & y\\ z & w\end{bmatrix} = \begin{bmatrix}x & 6\\ -1 & 2w\end{bmatrix} + \begin{bmatrix}4 & x + y\\ z + w & 3\end{bmatrix}$, determine the values of $x, y, z\ and\ w$.
(a) $x = 2, y = 4, z = 1\ w = 3$.
(b) $x = 2, y = -4, z = 1\ w = -3$.
(c) $x = -2, y = 4, z = -1\ w = 3$.
(d) $x = -2, y = -4, z = 1\ w = 3$.

**Q14.** If A=$\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, and $A + A' = I$, then the value of $\alpha$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\pi$ (d) $\frac{3\pi}{2}$

**Q15.** If a matrix has 24 elements, what are the possible orders it can have.
(a) 7 (b) 6
(c) 4 (d) 8

**Q16.** The no. of all possible matrices of order 3×3 with each entry 0 or 1 is
(a) 512 (b) 256
(c) 128 (d) 64

**Q17.** Find the trace of A = $\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$
(a) 6 (b) 1
(c) 5 (d) 4

**Q18.** What can be said about the diagonal entries of a skew-symmetric matrix

(a) All entries are 0.

(b) All entries are 1

(c) There is no restriction on choosing

(d) None of the above

**Q19.** Given a matrix A, how can we write it as a sum of symmetric and skew symmetric matrix respectively,

(a) $\frac{1}{2}(A+A')-\frac{1}{2}(A-A')$

(b) $\frac{1}{2}(A+A')+\frac{1}{2}(A'-A)$

(c) $\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$

(d) $\frac{1}{2}(A-A')-\frac{1}{2}(A+A')$

**Q20.** If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ and $\mathbf{A}^2=\mathbf{I}$ then which of the following is true if A is a skews symmetric, then A will be:

(a) $\alpha^2+\beta\gamma-1=0$

(b) $\alpha^2-\beta\gamma-1=0$

(c) $\alpha^2+\beta\gamma+1=0$

(d) $-\alpha^2+\beta\gamma-1=0$

**Q21.** If the matrix A is both symmetric and skews symmetric, then A will be:

(a) Identity matrix (b) Zero matrix

(c) Both (a) and (b) (d) Doesn't exist

**Q22.** If A,B are symmetric matrices of same order, them AB-BA is a

(a) Symmetric matrix

(b) Skew symmetric matrix

(c) Diagonal matrix

(d) Zero matrix

**Q23.** $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ Then which of the following is true:

(a) $f(x)+f(y)=f(x+y)$

(b) $f(x)-f(y)=f(x-y)$

(c) $f(x).f(y)=f(x+y)$

(d) $f(x).f(y)=f(xy)$

**Q24.** $A=\begin{bmatrix}3 & -4\\ 1 & 1\end{bmatrix}$, then

(a) $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$

(b) $A^n=\begin{bmatrix}1-2n & -4n\\ n & 1-2n\end{bmatrix}$

(c) $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1+2n\end{bmatrix}$

(d) $A^n=\begin{bmatrix}1+2n & -4n\\ -n & 1-2n\end{bmatrix}$

**Q25.** Find the $P^{-1}$, if it exists, given P = $\begin{bmatrix}10 & -2\\ -5 & 1\end{bmatrix}$

(a) $\begin{bmatrix}\frac{1}{10} & 0\\ \frac{1}{2} & 1\end{bmatrix}$ (b) $\begin{bmatrix}\frac{1}{5} & 1\\ 1 & 10\end{bmatrix}$

(c) $\begin{bmatrix}\frac{1}{2} & 2\\ 1 & 3\end{bmatrix}$ (d) None of these

**Q26.** If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$,

(a) $A^2+5A-7I=0$ (b) $A^2+5A-7I=0$

(c) $A^2-5A+7I=0$ (d) $-A^2+5A+7I=0$

**Q27.** If A and B are symmetric matrices of the same order, then $(AB'-BA')$ is a

(a) Skew symmetric matrix

(b) Null matrix

(c) Symmetric matrix

(d) None

**Q28.** If A and B are square matrices of the same order, then (A+B)(A-B) is equal to"

(a) $A^2-B^2$ (b) $A^2-BA-AB-B^2$

(c) $A^2-B^2+BA-AB$ (d) $A^2-BA+B^2+AB$

**Q29.** The matrix $\begin{bmatrix}0 & 5 & -7\\ -5 & 0 & 11\\ 7 & -11 & 0\end{bmatrix}$ is

(a) a skew-symmetric matrix

(b) a symmetric matrix

(c) a diagonal matrix

(d) an upper triangular matrix

**Q30.** If A is 2x3 matrix and B is a 3x3 matrix and CA=B then possible order of C is ?

(a) 1 x 3 (b) 3 x 2

(c) 2 x 3 (d) 3 x 3

## SUBJECTIVE QUESTIONS

**Q1.** Show that BAB′ is symmetric or skew-symmetric according as A is symmetric or skew symmetric (where B is any square matrix whose order is same as that of A).

**Q2.** Let A = $\begin{bmatrix}\sin\theta & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \cos\theta\\ \cos\theta & \tan\theta\end{bmatrix}$ & B = $\begin{bmatrix}\frac{1}{\sqrt{2}} & \sin\theta\\ \cos\theta & \cos\theta\\ \cos\theta & -1\end{bmatrix}$ Find θ so that A = B.

**Q3.** If A = $\begin{bmatrix}\lambda & 1\\ -1 & 2\end{bmatrix}$ and $A^2=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$ then λ is equal to

**Q4.** Let P and Q be 3 × 3 matrices P ≠ Q. If $P^3=Q^3$ and $P^2Q=Q^2P$, then determinant of $(P^2+Q^2)$ is equal to :

**Q5.** If A = $\begin{bmatrix} 1 & a \\ b & -1 \end{bmatrix}$ is a matrix such that AA' = A'A then the relationship between a and b.

## NUMERICAL TYPE QUESTIONS

**Q1.** If $\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$ , then the value of x -y ____.

**Q2.** If A is a square matrix such that $A^2 = I$ , then $A^3 + (A+I)^2 - 9A - I^2 - A^2 = kA$ .Find the value of k ___________.

**Q3.** If A = [$a_{ij}$] is a square matrix of order 2 such that $a_{ij}$ = 1, when i ≠ j and $a_{ij}$ = 0, when i = j, then the value of $a_{21}$ in $A^2$ is ____.

**Q4.** Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is _____.

**Q5.** If A = $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ be a matrix and $A^8 = \lambda A + \mu I$, λ, μ∈I then λ+ μ __________.

## TRUE AND FALSE

**Q1.** A square matrix A = $[a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$

**Q2.** A unit matrix of order n is denoted by $I_n$. Thus, a square matrix A = $[a_{ij}]_{m\times n}$ is an identity matrix if $a_{ij} = \begin{cases} 1 \,; i \neq j \\ 0 \,; i = j \end{cases}$

**Q3.** An identity matrix is always idempotent matrix.

**Q4.** Matrix multiplication is always commutative.

**Q5.** Every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

## ASSERTION AND REASONING

The following questions consist of two statements, one labelled as "Assertion [A] and the other labelled as Reason [R]". You are to examine these two statements carefully and decide if Assertion [A] and Reason [R] are individually true and if so, whether the Reason [R] is the correct explanation for the given Assertion [A]. Select your answer from following options.

(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true and R is not the correct explanation of A.
(c) 'A' is true but 'R' is false
(d) 'A' is false but 'R' is true

**Q1.** **Assertion (A):** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.
**Reason (R) :** A matrix A = $[a_{ij}]$| is an identity matrix if $a_{ij} = \begin{cases} 1 \,, if\ i = j \\ 0 \,, if\ i \neq j \end{cases}$

**Q2.** **Assertion [A]:** Matrix $\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ is a column matrix.
**Reason [R]:** A matrix of order m×1 is called a column matrix.

**Q3.** **Assertion [A]:** Transpose of the matrix A= $[2 \quad 5 \quad -1]$, is column matrix.
**Reason [R]:** Transpose of a matrix of order m × n is a matrix of same order.

**Q4.** **Assertion [A]:** Matrix $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ is a symmetric matrix.
**Reason [R]:** A matrix A is symmetric if A'=−A

**Q5.** **Assertion [A]:** For two matrices $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$ then ,
$(A+B)^2 = A^2 + 2AB + B^2$
**Reason [R]:** For given two matrices A and B, AB = BA.

# HOMEWORK

## MCQ

**Q1.** A 3 × 2 matrix whose elements are given by $a_{ij} = 2i - j$ is

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 3 & -2 \\ 5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$

**Q2.** If $\begin{bmatrix} x-y & 1 & z \\ 2x-y & 0 & w \end{bmatrix} = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$, then x + y + z + w =

(a) 10 (b) 8
(c) 9 (d) 12

**Q3.** $\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$ then x is equal to –

(a) – 1 (b) 0
(c) 1 (d) No value of x

**Q4.** If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = O$ then x =

(a) 2 (b) –2
(c) $\frac{5}{2}$ (d) $-\frac{9}{8}$

**Q5.** If A and B are square matrices of order 2, then $(A + B)^2$ =

(a) $A^2 + 2AB + B^2$ (b) $A^2 + AB + BA + B^2$
(c) $A^2 + 2BA + B^2$ (d) $A^2 + B^2$

**Q6.** Which relation is true for $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$

(a) $(A + B)^2 = A^2 + 2AB + B^2$
(b) $(A - B)^2 = A^2 - 2AB + B^2$
(c) AB = BA
(d) AB ≠ BA

**Q7.** $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ then $B^TA^T$ is

(a) a null matrix
(b) an identity matrix
© scalar, but not an identity matrix
(d) such that Tr $(B^T A^T)$ = 4

**Q8.** Let $A = [a_{ij}]_{n \times n}$ where $a_{ij} = i^2 - j^2$. Then A is:

(a) skew-symmetric matrix
(b) symmetric matrix
© null matrix
(d) unit matrix

**Q9.** Which one of the following is wrong?

(a) The elements on the main diagonal of a symmetric matrix are all zero
(b) The elements on the main diagonal of a skew – symmetric matrix are all zero
© For any square matrix A, $\frac{1}{2}$ (A + A′) is symmetric matrix
(d) For any square matrix A, $\frac{1}{2}$ (A – A′) is skew – symmetric matrix

**Q10.** Matrix $A = \begin{bmatrix} 5 & -3 & 6 \\ -3 & 7 & -4 \\ 6 & -4 & 8 \end{bmatrix}$ is a

(a) diagonal matrix
(b) skew symmetric matrix
©symmetric matrix
(d) lower triangular matrix

## SUBJECTIVE QUESTIONS

**Q1.** If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $A^2 - kA - I_2 = 0$, find the value of k.

**Q2.** What is the order of the product $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

**Q3.** If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, find the value of $A^{100}$

**Q4.** If $A = \begin{bmatrix} 4 & 11 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}$, then find the value of $A^T - B^T$.

**Q5.** The matrix $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then the value of $(aI + bA)^n$ .

## NUMERICAL TYPE QUESTIONS

**Q1.** If $\begin{bmatrix} 2p+q & p-2q \\ 5r-s & 4r+3s \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then the value of p + q -r + 2s is _________.

**Q2.** If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $KA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the value of k – a + b is _______.

**Q3.** If A = $\begin{bmatrix}-1 & 2 & 0\\ 3 & -3 & 4\end{bmatrix}$ then the value $a_{21}+a_{23}$ is ________.

**Q4.** If A = $\begin{bmatrix}1 & 3 & 5\\ a & 2 & -4\\ 0 & 0 & 3\end{bmatrix}$ is upper triangular matrix then value of a is _____.

**Q5.** If A = $\begin{bmatrix}1 & 0 & x-5\\ a & 2 & 0\\ -1 & 2 & 3\end{bmatrix}$ is lower triangular matrix then the value of x ___.

## TRUE AND FALSE

**Q1.** $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 4 & -5\\ -3 & -5 & 4\end{bmatrix}$ is a skew symmetric matrix.

**Q2.** $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ is nilpotent matrix.

**Q3.** If all the elements in the diagonal of a diagonal matrix are equal, it is called a **scalar matrix**.

**Q4.** If A = $[a_{ij}]_{m\times n}$ and B = $[b_{ij}]_{n\times p}$ are two matrices of orders $m\times n$ and $n\times p$ respectively ,then their product AB is of order $m\times p$

**Q5.** If A and B are two matrices such that AB exists , then BA may or may not exist.

## ASSERTION AND REASONING

The following questions consist of two statements, one labelled as "Assertion [A] and the other labelled as Reason [R]". You are to examine these two statements carefully and decide if Assertion [A] and Reason [R] are individually true and if so, whether the Reason [R] is the correct explanation for the given Assertion [A]. Select your answer from following options.

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(b) Both A and R are individually true and R is not the correct explanation of A.
(c) 'A' is true but 'R' is false
(d) 'A' is false but 'R' is true

**Q1.** **Assertion (A):** If A = $\begin{bmatrix}\cos\alpha & \sin\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$ and B = $\begin{bmatrix}\cos\alpha & \cos\alpha\\ \sin\alpha & \sin\alpha\end{bmatrix}$ ,then AB $\neq I$
**Reason (R) :** The product of two matrices can never be equal to an identity matrix.

**Q2. Assertion [A]:** A = $\begin{bmatrix}1 & 2\\ 5 & 9\end{bmatrix}$ is neither symmetric nor anti – symmetric.
**Reason [R]:** The matrix A cannot be expressed as a sum of symmetric and anti – symmetric matrices.

**Q3.** **Assertion [A]:** [2] is a singleton matrix
**Reason [R]:** If in a matrix A there is only element then it is called **singleton matrix.**

**Q4.** **Assertion [A]:** The product of two matrices may be a zero matrix when none of them is a zero matrix.
**Reason [R]:** If A = $\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ and B = $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ , then AB = $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

**Q5.** **Assertion [A]:** A = $\begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 1\\ 0 & 0 & 2\end{bmatrix}$ is an upper triangular matrix.
**Reason [R]:** A square matrix A = $[a_{ij}]$ is called a lower triangular matrix if $a_{ij}=0$ for all $i<j$.

# SOLUTIONS

## MCQ

**S1.** **(c)** Given,

$$[1\ x\ 1]\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$$

$$\Rightarrow [2x+16\quad 5x+6\quad x+4]\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$$

$[16+2x+10x+12+x^2+4x]=0$
$[x^2+16x+28]=0$
$x^2+16x+28=0$
$x+2)(x+14)=0$
Therefore x=-2, -14

**S2.** **(a)** Let A=$\begin{bmatrix}x\ y\ z\\ a\ b\ c\end{bmatrix}$," then the given matrix equation becomes

$$\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}\begin{bmatrix}x\ y\ z\\ a\ b\ c\end{bmatrix}=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2x-a & 2y-b & 2z-c\\ x & y & z\\ -3x+4a & -3y+4b & -3z+4c\end{bmatrix}=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$$

$\&\Rightarrow 2x-a=-1, 2y-b=-8, 2z-c=-10, x=1, y=-2,$
$\&\Rightarrow z=-5, -3x+4a=9, -3y+4b=22, -3z+4c=15$

$\therefore x = 1, y = -2, z = -5, a = 3, b = 4, c = 0$

$\therefore \mathrm{A} = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

**S3.** **(a)** Let A=$\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \neq O$ and $B = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \neq O$

So, the product AB=$\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

**S4.** **(a)** *Given, A*=$[3\ 5]_{1\times 2}$ and $B = [7\ 3]_{1\times 2}$

For AC=BC

We have order of $C$ is $2 \times 1$

Let $C = \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore AC = [3\ 5]\begin{bmatrix} x \\ y \end{bmatrix} = [3x + 5y]$

And $BC = [7\ 3]\begin{bmatrix} x \\ y \end{bmatrix} = [7x + 3y]$

For $AC = BC$,

$[3x + 5y] = [7x + 3y]$

$3x + 5y = 7x + 3y$

$4x = 2y$

$x = \frac{1}{2}y$

$y = 2x$

Hence, $C = \begin{bmatrix} x \\ 2x \end{bmatrix}$

**S5.** **(b)** *Let A*=$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ $[\because B \neq C]$

$\therefore AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ ...(i)

and $AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$...(ii)

From (i) and (ii),

We have $AB = AC$ but $B \neq C$.

**S6.** **(d)**

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, and

$A^3 = A^2.A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$A^n = A^{n-1} \cdot A = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

**S7.** **(b)** $A^2 = A.A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix} =$

$\begin{bmatrix} \lambda^2 - 1 & 0 \\ 0 & -1 + \lambda^2 \end{bmatrix} = 0$

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

**S8.** **(c)** Given that $A$ and $B$ are symmetric.

$\& A^\top = A, B^\top = B$

$\& X = AB + BA$

$\& \Rightarrow X^\top = (AB)^\top + (BA)^\top$

$\& = B^\top A^\top + A^\top B^\top = BA + AB =$

$X, (Since\ (AB)^\top = B^\top A^\top)$

$Y = AB - BA$

$\Rightarrow Y^\top = (AB)^\top - (BA)^\top$

$= B^\top A^\top - A^\top B^\top = BA - AB =$

$-Y, (Since\ (AB)^\top = B^\top A^\top)$

Now $(XY)^\top = Y^\top X^\top = -YX$

**S9.** **(b)** We have $A^\top = -A$

$(A^4)^\top = (\mathrm{A} \cdot \mathrm{A} \cdot \mathrm{A} \cdot \mathrm{A}.)^\top = \mathrm{A}^\top \mathrm{A}^\top \mathrm{A}^\top \mathrm{A}^\top$

$\Rightarrow (-\mathrm{A})(-\mathrm{A})(-\mathrm{A})(-\mathrm{A})$

$= (-1)^4 \mathrm{A}^4 = \mathrm{A}^4$

**S10.** **(b)** (i) is false:

If $A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then $\mathrm{AB} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Thus, AB = 0 but neither A = 0 nor B = 0

**(iii)** is true as the product AB is an identity matrix, if and only B is inverse of the matrix A.

**(iv)** is false since matrix multiplication is not commutative.

$(A + B)^2 = A^2 + AB + BA + B^2$

**S11.** **(d)** Given, $A = A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$

Also, given, $A^2 = B$

$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 = 1$ and $\alpha + 1 = 5$

Which is not possible at the same time.

$\therefore$ No real values of $\alpha$ exists.

**S12.** **(b)** "Since the given equations are equal, we can" "equate the corresponding elements. Thus, we get"

$3x + 7 = 0 \Rightarrow x = -7/3$

$y - 2 = 5 \Rightarrow y = 7$

$y + 1 = 8 \Rightarrow y = 7$

$2 - 3x = 4 \Rightarrow x = -2/3$

*Since, x and y cannot have two values, then values of x and y are impossible to find.*

**S13.** **(a)** Given that,

$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x + y \\ z + w & 3 \end{bmatrix}$

$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x + 4 & 6 + x + y \\ -1 + z + w & 2w + 3 \end{bmatrix}$

To find the values of x, y, z and w, equate the corresponding elements, and we get

$3x = x + 4 \Rightarrow x = 2$

$3z = -1 + z + w$ ... ... (1)

$3y = 6 + x + y$ ... ... (2)

$3w = 2w + 3 \Rightarrow w = 3$

Substitute the value of $w$ in equation (1),

we get the value of $z$.

$3z = -1 + z + 3 \Rightarrow 2z = 2 \Rightarrow z = 1$

Similarly, substitute the value of x in equation (2),we get the value of y.

$3y = 6 + 2 + y \Rightarrow y = 4$

Hence, the values are: $x = 2, y = 4, z = 1$, and $w = 3$

**S14.** **(b)** $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To find the value of α, equate the corresponding terms, we get

$2\cos\alpha = 1$

$\Rightarrow \cos\alpha = \cos\frac{\pi}{3}$

$\Rightarrow \alpha = \frac{\pi}{3}$

**S15. (d)** The possible sizes are 1x24,2x12,3x8,4x6,6x4,8x3,12x2,24x1

**S16. (a)** For each entry we have 2 choices and the number of entries in 3x3 matrix is 9

∴The possible number of matrices are $512 = 2^9$

**S17. (a)** Let A be the matrix of order n ,then trace of A = sum of diagonal elements of matrix A.

**S18. (a)** The diagonal elements of a skew-symmetric matrix is always 0.

**S19. (c)** Any matrix A can be written as:

$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Where $\frac{1}{2}(A + A')$ is symmetric and $\frac{1}{2}(A - A')$ is skew symmetric.

**S20. (a)** $A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix}$

ATQ. $\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\alpha^2 + \beta\gamma = 1$

$\alpha^2 + \beta\gamma - 1 = 0$

**S21. (b)** Given, $A' = A$ and

$A' = -A$

$\Rightarrow A = -A$

$\Rightarrow 2A = 0$

$\Rightarrow A = 0$

**S22. (b)** $P = AB - BA$

$P' = (AB - BA)'$

$P' = (AB)' - (BA)'$

$= B'A' - A'B' = \begin{bmatrix} \because A' = A \\ B' = B \end{bmatrix}$

$= BA - AB$

$= -(AB - BA)$

$= -P$

**S23. (c)** L.H.S $= f(x).f(y)$

$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos x\cos y - \sin x \cdot \sin y + 0 & -\sin y\cos x - \sin x\cos y + 0 & 0+0+0 \\ \sin x\cos y + \cos x \cdot \sin y + 0 & -\sin x \cdot \sin y + \cos x \cdot \cos y + 0 & 0+0+0 \\ 0+0+0 & 0+0+0+0 & 0+0+1 \end{bmatrix}$

$\begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x+y) = R.H.S$

**S24. (a)** For n $= 1$

$\therefore A^1 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

Hence result is true for $n = 1$

Let result is true for $n = k$

$A^K = \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix} \dots (i)$

now, we prove their result is true for $n = k + 1$

$A^{K+1} = A.A^K$

$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix}$

$= \begin{bmatrix} 2K+3 & -4K-4 \\ K+1 & -2K-1 \end{bmatrix}$

$= \begin{bmatrix} 2(k+1)+1 & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$

∴P(K+1)" is true Hence " P(n)" is true."

**S25. (a)** We have P = I.P , i.e., $= \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} P$

$= \begin{bmatrix} 1 & \frac{-1}{5} \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & 0 \\ 0 & 1 \end{bmatrix} P$ (applying $R_1 \rightarrow \frac{1}{10} R_1$)

$= \begin{bmatrix} 1 & \frac{-1}{5} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & 0 \\ \frac{1}{2} & 1 \end{bmatrix} P$ (applying $R_2 \rightarrow R_2 + 5R_1$)

**S26. (c)** L.H.S $= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} =$ R.HS

**S27. (a)** Given that A and B are symmetric matrices of the same order.

Let's find the transpose of $(AB' - BA')$.

$(AB' - BA')' = (AB')' - (BA')'$

$= (BA' - AB')$

$= -(AB' - BA')$

As $(AB' - BA')' = -(AB' - BA')$, the matrix $(AB' - BA')$ is skew symmetric.

**S28. (c)** $(A + B)(A - B) = A^2 - AB + BA - B^2$

**S29. (c)** Since $A^T = -A$ .It is a skew-symmetric matrix.

**S30. (b)** We know if A is matrix of order nxp and B is matrix of order rxs then AB is possible only when p=r And the resultant matrix is of order nxs.

## SUBJECTIVE QUESTIONS

**S1.** **Case - I** A is symmetric

$\Rightarrow$ A′ = A

(BAB′)′ = (B′)′A′B′ = BAB′

$\Rightarrow$ BAB′ is symmetric.

**Case - II** A is skew-symmetric

$\Rightarrow$ A′ = – A

(BAB′)′ = (B′)′A′B′ = B ( – A) B′ = – (BAB′)

$\Rightarrow$ BAB′ is skew-symmetric

**S2.** By definition A & B are equal if they have the same order and all the corresponding elements are equal. Thus we have $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$ & $\tan\theta = -1$

$\Rightarrow \theta = (2n + 1)\pi - \frac{\pi}{4}$ .

**S3.** If $A = \begin{bmatrix} \lambda & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ then λ is equal to

$A^2 = \begin{bmatrix} \lambda & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} \lambda & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \lambda^2-1 & \lambda+2 \\ -\lambda-2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

Then $\lambda = 3$

**S4.** On subtracting, we get

$P^3\text{-}P^2Q = Q^3\text{-}Q^2P$

$\Rightarrow P^2(P\text{-}Q) + Q^2(P\text{-}Q) = 0$

$\Rightarrow (P^2+Q^2)(P\text{-}Q) = 0$

If $|P^2+Q^2| = 0$; then $P^2+Q^2$ is invertible

$\Rightarrow P\text{-}Q = 0$, contradiction

$\therefore |P^2+Q^2| = 0$

**S5.** If $A = \begin{bmatrix} 1 & a \\ b & -1 \end{bmatrix}$ is a matrix such that AA’ = A’A then the relationship between a and b.

$A' = \begin{bmatrix} 1 & b \\ a & -1 \end{bmatrix}$

$A.A' = \begin{bmatrix} 1 & a \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & b \\ a & -1 \end{bmatrix} = \begin{bmatrix} 1+a^2 & b-a \\ b-a & b^2+1 \end{bmatrix}$

$A'A = \begin{bmatrix} 1 & b \\ a & -1 \end{bmatrix}\begin{bmatrix} 1 & a \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1+b^2 & a-b \\ a-b & a^2+1 \end{bmatrix}$

Since AA’ = A’A

$\begin{bmatrix} 1+a^2 & b-a \\ b-a & b^2+1 \end{bmatrix} = \begin{bmatrix} 1+b^2 & a-b \\ a-b & a^2+1 \end{bmatrix}$

$\Rightarrow a = b$

## NUMERICAL TYPE QUESTIONS

**S1.** **(0)** If $\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

Comparing the elements we have ,

$\Rightarrow 2x + 1 = x + 3$ .......(1)

$\Rightarrow 3y = y^2 + 2$ ....(2)

$\Rightarrow y^2 - 5y = -6$ ............(3)

$\Rightarrow x = 2$ [ from eq.(1) ]

$\Rightarrow y = 1 \text{ or } 2$ [ from eq. (2) ]

$\Rightarrow y = 2 \text{ or } 3$ [ from eq. (3) ]

$\Rightarrow x = 2 \text{ and } y = 2$

Then , x – y = 0

**S2.** **(-6)**

Given , $A^2 = I$

Now , $A^3 + (A+I)^2 - 9A - I^2 - A^2 = kA$

$\Rightarrow A^2.A + A^2 + I + 2AI - 9A - I - A^2 = kA$

$\Rightarrow I.A + I + I + 2AI - 9A - I - I = kA$

$\Rightarrow -6A = kA$

$\Rightarrow k = -6$

**S3.** **(0)** Given , A = $[a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = 1$ , when $i \neq j$ and $a_{ij} = 0$ , when $i = j$

So , $a_{11} = 0, a_{12} = 1, a_{21} = 1 \text{ and } a_{22} = 0$

Thus , A = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

∴ The value of $a_{21} = 0$

**S4.** **(512)**

We know that a matrix 3× 3 contains 9 elements.

Given that each entry of this 3× 3 matrix is either 0 or 2.

Thus, by simple counting principle, we can calculate the total number of possible matrices as:

Total number of possible matrices = Total number of ways in which 9 elements can take possible values

= $2^9$

= 512

**S5.** **(1)**

Given, $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$

Now,

$3A - 2I = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3-2 & 0 \\ 3-0 & 6-2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$

Hence, $A^2 = 3A - 2I$

Now,

$A^4 = (A^2)^2 = [3A - 2I]^2$

$= 9A^2 - 12AI + 4I^2$

$= 9A^2 - 12A + 4I$

$= 3(3A - 2I) - 12A + 4I$

$= 15A - 14I$

$A^8 = (A^4)^2 = (15A - 14I)^2$

$= 225A^2 - 420AI + 196I^2$

$= 255A - 254I$

From questions

$A^8 = \lambda A + \mu I$

Here,

$\lambda = 225$

$and\ \ \mu = -254$

$\Rightarrow \lambda + \mu = 255 + (-254)$

$= 255 - 254$

$= 1$

## TRUE AND FALSE

**S1. (True)** By definition of upper triangular matrix, A square matrix A = $[a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$

**S2. (False)** A unit matrix of order n is denoted by $I_n$. Thus, a square matrix A = $[a_{ij}]_{m\times n}$ is an <u>identity matrix</u> if $a_{ij} = \begin{cases} 0\,; i \neq j \\ 1\,; i = j \end{cases}$

**S3. (True)** A square matrix is idempotent, provided $A^2$ = A.

A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is idempotent matrix.

**S4. (False)** Matrix multiplication is not commutative in general.

Example: A=$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

AB = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

And BA = $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

A and B is not commutative.

**S5. (True)** Every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

A= $\frac{1}{2}(A+A^T)$ + $\frac{1}{2}(A-A^T)$ = $\frac{1}{2}$(B+C) where B is symmetric and C is a skew symmetric matric.

## ASSERTION AND REASON ING

**S1. (d)** We know that , A = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is an identity matrix

∴ Given assertion (A) is false , we know that for identity matrix $a_{ij} = 1$ , if $i = j$ and $a_{ij} = 0$ , if $i \neq j$

∴ Given reason (R) is true .

**S2. (a)** We know that order of column matrix is always m×1

∴ $\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ is column matrix.

⇒ Assertion [A] is true Also Reason (R) is true and is correct explanation of A.

**S3. (c)** Given A=$[2 \quad 5 \quad -1]$1×3 and $A' = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$ which is column matrix

∴ Given Assertion [A] is true , We know that transpose of matrix of order m× n is of order n× m

∴ Given Reason (R) is false

**S4. (c)** Let A = $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ , $A'$= $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ = A

⇒ A is a symmetric matrix

∴ Given assertion (A) is true , we know that A matrix is symmetric if $A' = A$.

⇒ Given reason is false A is true but R is false.

**S5. (a)** Given $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}$

⇒AB=BA.

Now $(A+B)^2$ = $(A+B)(A+B)$= $A^2$ +AB + BA + $B^2$ =$A^2$ + AB + AB + $B^2$

{∵AB=BA}

= $A^2$+ 2AB+$B^2$

∴ Given Assertion [A] is true Also AB=BA

⇒ Reason (R) is true and is correct explanation of A .

# HOMEWORK

## MCQ

**S1. (d)** A 3 × 2 matrix whose elements are given by $a_{ij}$ = 2i − j is

$a_{11} = 2 - 1 = 1$ , $a_{12} = 2 - 2 = 0$ , $a_{21} = 2 \times 2 - 1 = 3$ , $a_{22} = 2 \times 2 - 2 = 2$ , $a_{31} = 2 \times 3 - 1 = 5$ , $a_{32} = 2 \times 3 - 2 = 4$

$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$

**S2. (d)** If $\begin{bmatrix} x-y & 1 & z \\ 2x-y & 0 & w \end{bmatrix} = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$,

then x + y + z + w =

$x - y = -1$ , z = 4 , 2x − y = 0 , w = 5

$x = 1, y = 2, z = 4, w = 5$

Then x + y + z + w = 1 + 2 + 4 + 5 = 12

**S3. (a)** $\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2+x & x-1 \\ 3-x+1 & 2+x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2+x & x-1 \\ 4-x & 2+x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow x^2 + x = 0$ ... ... ... ... $(1) x - 1 = -2 \Rightarrow x = -2 + 1 = -14 - x = 5 \Rightarrow x = 4 - 5 = -12 + x = 1 \Rightarrow x = 1 - 2 = -1$ And substituting $x = -1$ in (1), $(-1)^2 + (-1) \Rightarrow 1 - 1 = 0 \therefore x = -1$.

**S4. (d)** Given ,

$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

$\Rightarrow [1 \times 1 + x \times 4 + 1 \times 3 1 \times 2 + x \times 5 + 1 \times 2 1 \times 3 + x \times 6 + 1 \times 5] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

$\Rightarrow [4 + 4x 4 + 5x 8 + 6x] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

$\Rightarrow 4 + 4x + 8 + 10x + 24 + 18x = 0$

$\Rightarrow 4 + 4x + 8 + 10x + 24 + 18x = 0$

$\Rightarrow 36 + 32x = 0 \Rightarrow 32x = -36$

$\therefore x = \frac{-36}{32} = \frac{-9}{8}$

**S5. (b)** $(A + B)^2$ = (A+B)(A+B) = $A$(A+B)+B(A+B) = $A^2$+AB+BA+$B^2$

**S6.** Given that,

$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} and B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 7 \\ -3 & -2 \end{bmatrix}, \therefore BA = \begin{bmatrix} -2 & 7 \\ -3 & 3 \end{bmatrix}$

$\therefore A + B = \begin{bmatrix} 3 & 3 \\ -2 & 3 \end{bmatrix}, (A + B)^2 = \begin{bmatrix} 3 & 3 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 18 \\ -12 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} and B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 7 \\ -3 & -2 \end{bmatrix}, \therefore BA = \begin{bmatrix} -2 & 7 \\ -3 & 3 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & 3 \\ -2 & 3 \end{bmatrix}, (A + B)^2 = \begin{bmatrix} 3 & 3 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ -2 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}, B^2 = \begin{bmatrix} -3 & 8 \\ -2 & 3 \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} 2 & 4 \\ -6 & 2 \end{bmatrix}$

$[A - B] = \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}, [A - B]^2 = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$

$\therefore (A - B)^2 = A^2 + B^2 - 2AB = \begin{bmatrix} -4 & 10 \\ 0 & 2 \end{bmatrix}$

$AB \neq BA$

**S7. (b)**

$A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$

$A^T = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}, B^T = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ i.e., an identity matrix.

**S8. (a)** Given, $A = [a_{ij}]_{n \times n}$ where $a_{ij} = i^2 - j^2$.

Now, $a_{ji} = j^2 - i^2 = -(i^2 - j^2) = -a_{ij}, \forall i, j$.

Also $a_{ii} = 0, \forall i$.

So $A^T$=-A. [According to the definition of skew symmetric matrix]

**S9. (a)** Every square matrix can be expressed as the sum of symmetric and skew symmetric matrix.

i.e.

$$A = \left(\frac{A + A^T}{2}\right) + \left(\frac{A - A^T}{2}\right)$$

↓ ↓

*symmetric* *skew symmetric*

The elements on the diagonal elements of a skew symmetric matrix are all zero. And the elements on the diagonals element of a symmetric matrix are non-zero.

**S10. (c)** Matrix A = $\begin{bmatrix} 5 & -3 & 6 \\ -3 & 7 & -4 \\ 6 & -4 & 8 \end{bmatrix}$

$A^T = \begin{bmatrix} 5 & -3 & 6 \\ -3 & 7 & -4 \\ 6 & -4 & 8 \end{bmatrix}$

$\therefore\ A = A^T$

Hence , A is symmetric matrix.

## SUBJECTIVE QUESTIONS

**S1.** We have , A = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$\Rightarrow\ A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$

$\therefore\ A^2 - kA - I_2 = 0$

$\Rightarrow\ \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - k\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\Rightarrow\ \begin{bmatrix} 4 & 8 \\ 8 & 12 \end{bmatrix} - k\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = 0$

$\Rightarrow k = 4$

**S2.** The product $\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Order of matrix = 1 × 3 ; 3 × 3 ; 3 × 1 = 1 × 1

**S3.** Let A = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\therefore\ A^2 = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$

$\therefore\ A^4 = 2A.2A = 4A^2 = 8A = 2^3A$

Similarly , $A^{100} = 2^{99}A$

**S4.** If A = $\begin{bmatrix} 4 & 11 \\ 4 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}$ ,then find the value of $A^T - B^T$.

$A^T = \begin{bmatrix} 4 & 4 \\ 11 & 5 \end{bmatrix}$ and $B^T = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix}$

Then, the value of $A^T - B^T$ = $\begin{bmatrix} 4 & 4 \\ 11 & 5 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 8 & 3 \end{bmatrix}$

**S5.** Given that , A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$aI + bA = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

Now, $(aI + bA)^2 = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix} = \begin{bmatrix} a^2 & 0 \\ 0 & a^2 \end{bmatrix} + \begin{bmatrix} 0 & 2ab \\ 0 & 0 \end{bmatrix}$

$= a^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2ab\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = a^2I$+2abA

Similarly , $(aI + bA)^3 = a^3I$+$3a^2$bA

Continue in this process , we get

$(aI + bA)^n = a^nI$+$3a^{n-1}$bA

## NUMERICAL TYPE QUESTIONS

**S1. (8)** If $\begin{bmatrix} 2p+q & p-2q \\ 5r-s & 4r+3s \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$ , then the value of p + q -r + 2s is

$\Rightarrow 2p + q = 4$ ...........(1)

$\Rightarrow p - 2q = -3$ ..........(2)

$\Rightarrow 5r - s = 11$ .......(3)

$\Rightarrow 4r + 3s = 24$ .......(4)

$\Rightarrow p = 1 , q = 2$ [ from eq. (1) and ( 2) ]

$\Rightarrow r = 3 , s = 4$

⇒ p + q -r + 2s = 1 + 2 - 3 + 2× 4 = 3 – 3 + 8 = 8

**S2. (-11)**

If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ ,

$\Rightarrow\ \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow\ -4k = 24$

$\Rightarrow k = -6$

$\Rightarrow 3a = -12$

$\Rightarrow a = -4$

$\Rightarrow 2b = -18$

$\Rightarrow b = -9$

Then the value of k – a + b = -6 +4 -9 =-11

**S3. (7)** If A = $\begin{bmatrix} -1 & 2 & 0 \\ 3 & -3 & 4 \end{bmatrix}$ then the value $a_{21} + a_{23}$ = 3 + 4 = 7

**S4. (0)** A square matrix A = $[a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$ .

If A = $\begin{bmatrix} 1 & 3 & 5 \\ a & 2 & -4 \\ 0 & 0 & 3 \end{bmatrix}$ is upper triangular matrix then value of a is 0.

**S5. (5)** A square matrix A = $[a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ for all $i < j$.

If A = $\begin{bmatrix} 1 & 0 & x-5 \\ a & 2 & 0 \\ -1 & 2 & 3 \end{bmatrix}$ is lower triangular matrix then the value of x = 5

## TRUE AND FALSE

**S1. (False)**

$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -5 \\ -3 & -5 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -5 \\ -3 & -5 & 4 \end{bmatrix}$

$\therefore\ A = A^T$ ,Hence A is symmetric matrix.

**S2. (False)** A nilpotent matrix is said to be nilpotent of index p, i.e. $A^p$ = 0 , $A^{p-1}$≠0.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is not nilpotent matrix.

**S3. (True)** If all the elements in the diagonal of a diagonal matrix are equal, it is called a **scalar matrix**. Thus, a square matrix A= $[a_{ij}]$ of order n×n is a scalar matrix if $a_{ij} = \begin{cases} 0\ ; i \neq j \\ k\ ; i = j \end{cases}$ where k is a constant.

**S4. (True)** As we know that ,
If A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{n \times p}$ are two matrices of orders $m \times n$ and $n \times p$ respectively ,then their product AB is of order $m \times p$

**S5. (True)** If A and B are two matrices such that AB exists , then BA may or may not exist.

**Example**

If A = $\begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 4 & -3 \end{bmatrix}$ then ,
AB = $\begin{bmatrix} 16 & -12 \\ 3 & -11 \\ 3 & -1 \end{bmatrix}$ but BA does not exist.

## ASSERTION AND REASONING

**S1. (c)** Since , A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$ and B = $\begin{bmatrix} \cos\alpha & \cos\alpha \\ \sin\alpha & \sin\alpha \end{bmatrix}$
$\therefore\ AB = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \cos\alpha \\ \sin\alpha & \sin\alpha \end{bmatrix} =$ $\begin{bmatrix} cos^2\alpha + sin^2\alpha & cos^2\alpha + sin^2\alpha \\ cos^2\alpha + sin^2\alpha & cos^2\alpha + sin^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ [ since $cos^2\alpha + sin^2\alpha = 1$ ]
$\Rightarrow AB \neq I$
Hence , A is true but R is false.

**S2. (c)** Sol. We have ,
A = $\begin{bmatrix} 1 & 2 \\ 5 & 9 \end{bmatrix}$
$\therefore\ A' = \begin{bmatrix} 1 & 5 \\ 2 & 9 \end{bmatrix}$
Here , $A \neq A'$ and $A \neq -A'$
∴ It is neither symmetric nor anti-symmetric matrices.
∴ It is true .
(R) We know that the matrix A cannot be expressed as a sum of symmetric and anti -symmetric matrices. It is false. Hence , A is true but R is false.

**S3. (a)** **Assertion [A]:** [2] is a singleton matrix
**Reason [R]:** If in a matrix A there is only element then it is called **singleton matrix.**
Hence , both A and R are True.

**S4. (a)** The product of two matrices may be a zero matrix when none of them is a zero matrix.
**Example**
If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ , then AB = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
while neither A nor B is a null matrix.
Hence , A and R are true.

**S5. (b)** **Assertion [A]:** A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is an upper triangular matrix.
**Reason [R]:** A square matrix A = $[a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ for all $i < j$.

Both A and R are individually true and R is not the correct explanation of A.