

Topics : Circular Motion, Rigid Body Dynamics, Surface Tension, Geometrical Optics, Gravitation

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.4

(3 marks, 3 min.)

M.M., Min.

[12, 12]

Subjective Questions ('-1' negative marking) Q.5

(4 marks, 5 min.)

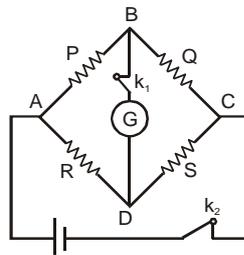
[4, 5]

Comprehension ('-1' negative marking) Q.6 to Q.8

(3 marks, 3 min.)

[9, 9]

1. With keys k_1 and k_2 closed : if $PS < QR$ then



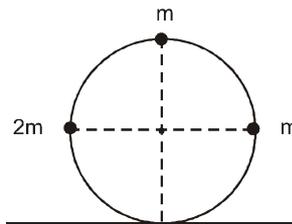
(A) $V_B = V_D$

(B) $V_B > V_D$

(C) $V_B < V_D$

(D) $V_B \begin{matrix} > \\ < \end{matrix} V_D$

2. A ring of mass m and radius R has three particles attached to the ring and it is rolling on the surface as shown in the figure. The ratio of kinetic energy of the system to the kinetic energy of the particle of mass $2m$ will be (speed of centre of ring is V_0 and slipping is absent) :



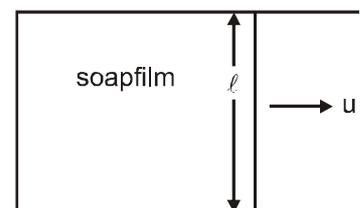
(A) $\frac{2}{5}$

(B) $\frac{1}{2}$

(C) $\frac{3}{1}$

(D) $\frac{2}{3}$

3. A soap film is created in a small wire frame as shown in the figure. The sliding wire of mass m is given a velocity u to the right and assume that u is small enough so that film does not break. Plane of the film is horizontal and surface tension is T . Then time to regain the original position of wire is equal to :



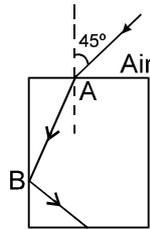
(A) $\frac{um}{Tl}$

(B) $\frac{Tl}{um}$

(C) $\frac{mu^2}{lT}$

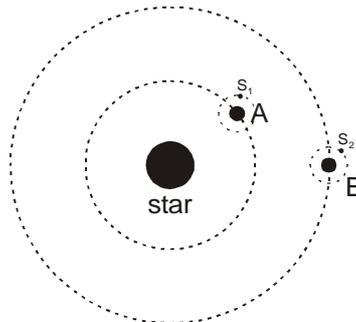
(D) It will never regain original position

4. A tube of length ℓ open at only one end is cut into two equal halves. The sixth overtone frequency of piece closed at one end equals to sixth overtone frequency of piece open at both ends. The radius of cross-section of tube is :
- (A) $\frac{\ell}{12}$ (B) $\frac{5\ell}{72}$ (C) $\frac{\ell}{24}$ (D) $\frac{5\ell}{24}$
- (E) $\frac{5\ell}{48}$
5. A ray of light is incident from air to a glass rod at point A the angle of incidence being 45° . The minimum value of refractive index of the material of the rod so that T.I.R. takes place at B is _____.



COMPREHENSION

Consider a star and two planet system. The star has mass M . The planet A and B have the same mass m , radius a and they revolve around the star in circular orbits of radius r and $4r$ respectively ($M \gg m$, $r \gg a$). Planet A has intelligent life, and the people of this planet have achieved a very high degree of technological advance. They wish to shift a geostationary satellite of their own planet to a geostationary orbit of planet B. They achieve this through a series of high precision maneuvers in which the satellite never has to apply brakes and not a single joule of energy is wasted. S_1 is a geostationary satellite of planet A and S_2 is a geostationary satellite of planet B. Neglect interaction between A and B, S_1 and S_2 , S_1 and B & S_2 and A.



6. If the time period of the satellite in geostationary orbit of planet A is T , then its time period in geostationary orbit of planet B is :
- (A) T (B) $4T$ (C) $8T$ (D) Data insufficient
7. If the radius of the geostationary orbit in planet A is given by $r_G = r \left(\frac{m}{M} \right)^{1/3}$, then the time in which the geostationary satellite will complete 1 revolution is
- I. 1 planet year = time in which planet revolves around the star
 II. 1 planet day = time in which planet revolves about its axis.
- (A) I (B) II (C) both I and II (D) neither I nor II.
8. If planet A and B, both complete one revolution about their own axis in the same time, then the energy needed to transfer the satellite of mass m_0 from planet A to planet B is.
- (A) $\frac{Gmm_0}{4r}$ (B) $\frac{GMm_0}{4r}$ (C) $\frac{3GMm_0}{8r}$ (D) Zero

Answers Key

1. (B) 2. (C) 3. (A) 4. (B)
 5. $\sqrt{3/2}$ 6. (D) 7. (C) 8. (C)

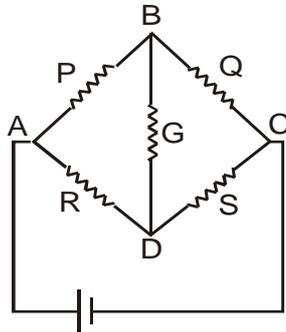
Hints & Solutions

1. (B) $V_B > V_D = I_n$ in a Wheatstone's bridge circuit shown if $PS = QR$, $V_B = V_D$. No current flows between B and D.

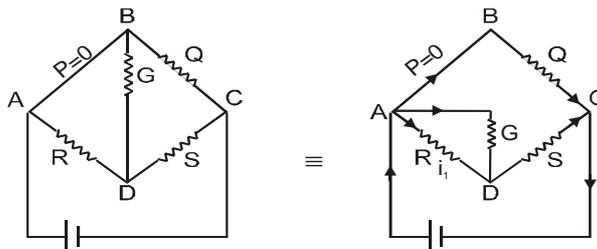
If $PS < QR$, $V_B > V_D$ current flows from B to D.

If $PS > QR$, $V_B < V_D$ current flows from D to B.

Alternate :

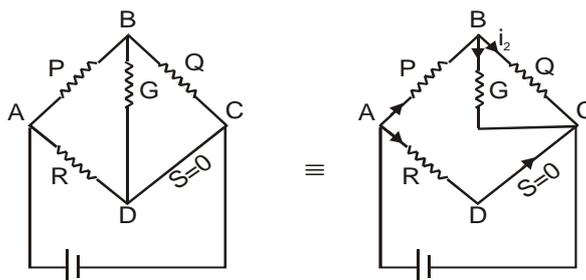


Let resistance $P = 0$ and all other resistances Q, R, S, G are non-zero then $PS < QR$ condition is satisfied.



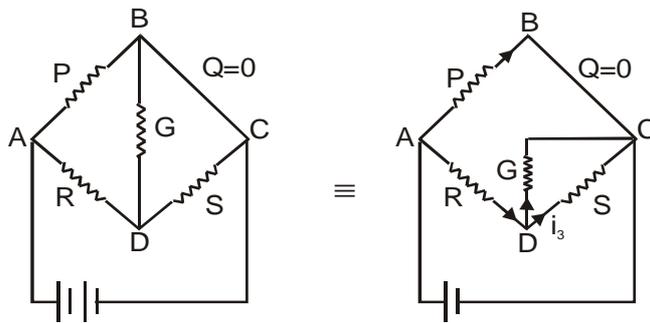
$$\begin{aligned} V_A &= V_B \\ \Rightarrow V_A - i_1 R &= V_D \\ \Rightarrow V_A - V_D &= i_1 R = \text{positive (or current flows from A to D through G, then } V_A > V_D) \\ \Rightarrow V_A &> V_D \\ \Rightarrow V_B &> V_D \end{aligned}$$

Let resistance $S = 0$ and all other resistances P, Q, R, G are non-zero then $PS < QR$ condition is also satisfied.



$$\begin{aligned}
 &V_D = V_C \\
 \Rightarrow &V_B - i_2 Q = V_C = V_D \\
 \Rightarrow &V_B - V_D = i_2 Q = \text{positive} \\
 \Rightarrow &V_B > V_D
 \end{aligned}$$

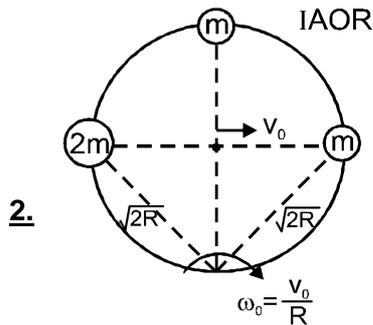
Suppose resistance Q is zero and all other resistances P,R,S,G are non-zero then $PS > QR$.



$$\begin{aligned}
 &V_B = V_C \\
 &V_D - V_C = i_3 S = \text{positive} \\
 \Rightarrow &V_D > V_C \\
 \Rightarrow &V_D > V_B
 \end{aligned}$$

If resistance R is zero and all other resistances are non-zero, then $PS > QR$

and similarly, we get $V_D > V_B$
Hence if $PS < QR$, $V_B > V_D$.



$$\text{K.E.}_{\text{system}} = \frac{1}{2} \times 2mR^2 \omega_0^2 + \frac{1}{2} m(2R)^2 \omega_0^2 +$$

$$\frac{1}{2} m(\sqrt{2}R)^2 \omega_0^2 + \frac{1}{2} 2m(\sqrt{2}R)^2 \omega_0^2$$

$$= 6mv_0^2$$

$$\text{K.E.}_{2m} = \frac{1}{2} \times 2m(\sqrt{2}R)^2 \omega_0^2 = 2mv_0^2.$$

3. $a = \frac{2T\ell}{m}$

$$v = u + at \Rightarrow 0 = u - at$$

$$t = \frac{u}{a} = \frac{um}{2T\ell}$$

$$\text{Total time } T = 2t = \frac{um}{T\ell}.$$

4. According to given condition,

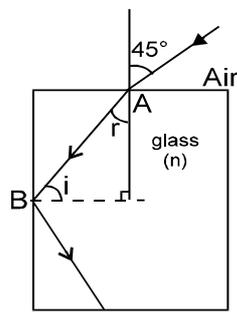
$$\frac{13v}{4\left(\frac{\ell}{2} + e\right)} = \frac{7v}{2\left(\frac{\ell}{2} + 2e\right)}$$

$$e = \frac{\ell}{24}$$

$$\text{So, } r = \frac{10e}{6}$$

$$r = \frac{5\ell}{72}$$

5. For TIR at B, the angle of incidence $i > c$



$$\& \quad r + i = 90 \Rightarrow i = 90 - r$$

by snell's law at pt A,

$$\sin 45^\circ = n \sin r = n \cos i$$

$$\text{Now } \because i > c \Rightarrow \sin i > \sin c$$

$$\Rightarrow \cos r > \frac{1}{n}$$

$$\Rightarrow n > \frac{1}{\cos r}$$

$$\Rightarrow n > \frac{1}{\sqrt{1 - \sin^2 r}}$$

$$\Rightarrow n > \frac{1}{\sqrt{1 - \frac{1}{2n^2}}}$$

$$\Rightarrow n > \frac{\sqrt{2} n}{\sqrt{2n^2 - 1}}$$

$$\Rightarrow 2n^2 - 1 > 2 \Rightarrow n > \sqrt{\frac{3}{2}}$$

6. The time in which the planet rotates about its axis is not given for either planet.

7. For geostationary satellite, time period = 1 planet day (by def.)

Let $T = 1$ planet day

$T_0 = 1$ planet year

$$\text{Now } T^2 = \frac{4\pi^2}{Gm} r_G^3 = \frac{4\pi^2}{Gm} r^3 \left(\frac{m}{M} \right)$$

$$= \frac{4\pi^2}{GM} r^3 = T_0^3 \Rightarrow T = T_0$$

8. The energy of any geostationary satellite is the sum of kinetic energy of satellite, interaction energy of satellite and its own planet and interaction energy of satellite and star. Both planets have same mass and same length of day. Geostationary satellite - planet system will have same interaction energy in either planet. Also kinetic energy of both satellites will be same. But the satellite-sun system will account for the energy difference.

$$U_i = -\frac{GMm_0}{2r} + U_{\text{satellite - planet}}$$

$$U_f = -\frac{GMm_0}{2(4r)} + U_{\text{satellite - planet}}$$

$$E_{\min} = U_f - U_i = \frac{3GMm_0}{8r}$$