Geometry and Mensuration

Geometry

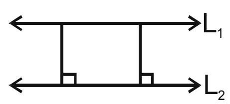
Let us start by revising a few basic geometrical figures that all of us would already be knowing. We would start with plane geometry and towards the end, include 3 dimensional figures. Plane geometry is about figures in a plane and a plane is any 2 dimensional surface such as a table top. Most basic figures that we are acquainted with such as triangle, square, rectangle, circle etc. can be drawn on any plane surface and there is no need of a third dimension i.e. a height or thickness.

(i) Lines

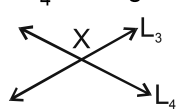
Line is the most elementary figure in geometry, it is a one dimensional figure which only has length. It does not has any height, thickness or width. A line consists of infinite number of points.

a. Types of lines

Parallel Lines: If the distance between two lines (Like L_1 and L_2) in a plane is constant they are said to be parallel. "Distance" between two lines means perpendicular distance.

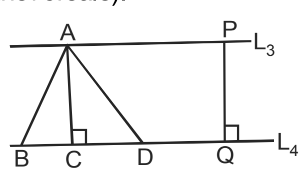


Intersecting Lines: Lines which share a common point (line L_3 and L_4 sharing a common point X).



Intersecting lines

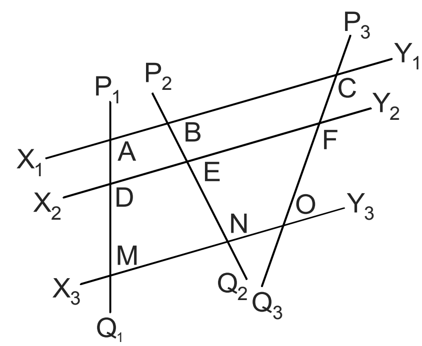
Transversals: A line which intersect two or more parallel lines at distinct points(AD, AC, PQ and AB are the transversals).



Parallel lines

Proportionality Theorem

This is a very important concept and is used extensively in similarity of triangles. It says, the intercepts made by two (or more) transversals on three (or more) parallel lines are proportional.



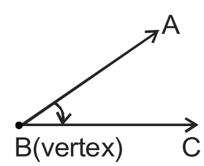
i.e. $\frac{AD}{DM} = \frac{BE}{EN} = \frac{CF}{FO}$

Note:

Here X_1Y_1, X_2Y_2 and X_3Y_3 are three parallel lines and P_1Q_1, P_2Q_2 and P_3Q_3 are the transversals.

b. Angles

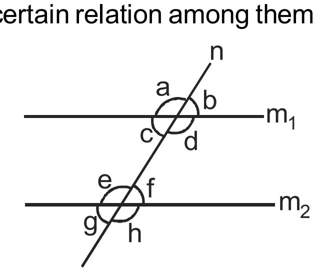
In geometry, angles are of utmost importance . When two rays have the same starting or end-point, they form an angle and the common end point is called VERTEX.



In the figure, rays BA and BC form an angle ABC.

Relation between the angles

When two parallel lines (m_1, m_2) are cut by another line, n (called TRANSVERSAL), the angles formed, share a certain relation among them.



I. **Corresponding angles** are equal.

a = e, c = g, b = f and d = h

II. **Alternate angles** are equal.

c = f, d = e

III. A pair of interior opposite angles are supplementry. (i.e. sum of interior angles is 180°).

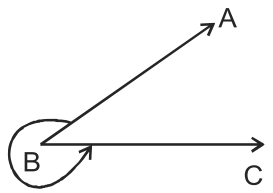
$e + c = 180°$, $d + f = 180°$

IV. Vertically opposite angles are equal.

$a = d$, $e = h$, $c = b$, $g = f$

Reflex Angle

A reflex angle is an angle which is more than 180º in measurement.



Reflex Angle

Reflex $\angle ABC = 360° - \angle ABC$

($\angle ABC$ is less than 180°)

Note:

Do not confuse it with obtuse angle, obtuse angle is in **between** 90° and 180°, whereas reflex is **more** than 180°.

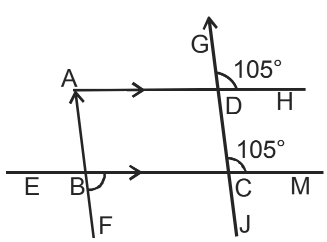
1. In the given figure, AF is parallel to GJ and AH is parallel to CE. If $\angle GDH = 105°$, then $\angle FBC$ =?

Solution :

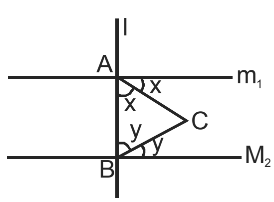
$\angle GDH = \angle DCM = 105°$ (Corresponding angles)

$\Rightarrow \angle BCD = 180° - \angle DCM = 180° - 105° = 75°$

$\Rightarrow \angle FBC = \angle BCD = 75°$ (Alternate angles)



2. In the given figure, m_1 is parallel to m_2, AC and BC are angle bisectors. Find the measure of $\angle ACB$.



Solution :

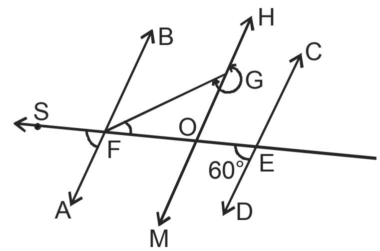
$2x = 180° - 2y$ (interior opposite angles)

$\Rightarrow 2x + 2y = 180°$ or $x + y = 90°$

In ΔABC, $\angle ACB + x + y = 180°$

$\Rightarrow \angle ACB = 180° - (x + y) = 90°$

3.



In the figure above, AB, CD and MH are parallel to each other and FG is the angle bisector of $\angle BFO$. If $\angle FED = 60°$, then reflex $\angle FGH$ =?

Solution :

$\angle FED = \angle BFE$ (Alternate angles)

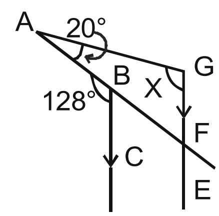
$\Rightarrow \angle GFE = \frac{1}{2}\angle BFE = 30°$

But $\angle BFG = \angle FGO$ (Alternate angles)

$\angle GFE = \angle BFG = \angle FGO = 30°$

$\Rightarrow$ Reflex $\angle FGH = 180° + \angle FGO = 210°$

4. Find the value of X in the figure given below.



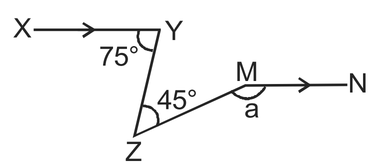
Solution :

$\angle ABC = \angle BFE = 128°$ [Corresponding angles]

$\angle AFG = 180° - 128° = 52°$

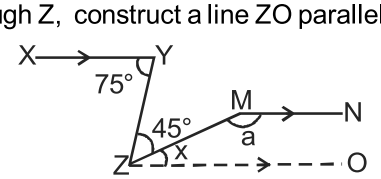
In ΔAGF, $a + 20° + X = 180° \Rightarrow X = 108°$

5. An ant moves in a straight line. After two turns of 75° and 45° as shown in figure, it moves parallel to the initial direction of motion. What is the value of $\angle a$?



Solution :

Through Z, construct a line ZO parallel to MN



As MN || XY

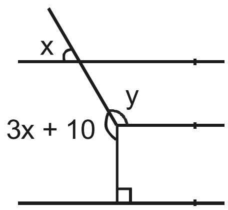
$\therefore$ ZO || XY

$x + 45° = 75°$ (Alternate angles) $\Rightarrow x = 30°$

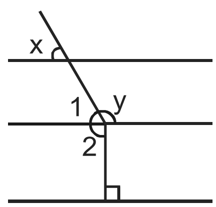
$x + a = 180°$ (Opposite interior angles)

$a = 180° - 30° \Rightarrow a = 150°$

6. Find the values of x, y in the figure given below.



Solution :



$\angle 2 = 90°$ (Alternate angles),

$\angle 1 = x$ (Corresponding angles)

$\angle 1 + \angle 2 = 3x + 10$

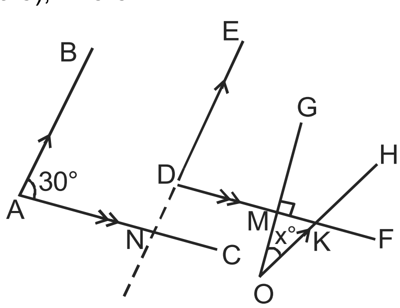
$x + 90° = 3x + 10$

$2x = 80°$ or $x = 40°$

$y = 180° - \angle 1$ (Adjacent angles)

$y = 180° - 40° = 140°$

7. Find the measure of angle x in the figure (not drawn to scale), where



AB || DE || OH and AC || DF.

Solution :

Let ED be extended and cut AC at N.

$\angle EDF = \angle DNC = \angle BAC = 30°$
(corresponding angles)

Again DE || OH.

$\therefore \angle EDF = \angle DKO = 30°$ (alternate angles)

Also $\angle KMO = 90°$

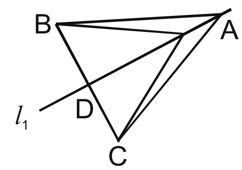
In ΔMKO

$\therefore x = 180° - 90° - 30° = 60°$

c. Specific Lines and Points in a Triangle

Perpendicular Bisector of a Line:

Consider the line segment BC. D is the mid-point and l_1 is the perpendicular bisector of BC.



Consider any point A on the perpendicular bisector. The distance of point A from the endpoints of the line BC is AB and AC.

In ΔADB and ΔADC,

BD = CD (D is midpoint of BC)

$\angle ADB = \angle ADC$ (right angles)

AD = AD (common side)

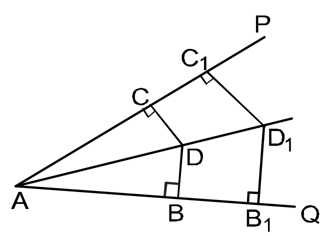
Thus by S – A – S, the two triangles are congruent.

Hence AB = AC i.e. the point A is equidistant from the endpoint of BC. This is true for any point A on the perpendicular bisector of the line. Thus,

All points on the perpendicular bisector of a line is equidistant from the endpoints of the line

Angle Bisector:

Consider the angle bisector AD of $\angle PAQ$ as shown in the figure below. Taking any point, say D on the angle bisector. The distance of D from the sides AC and AB are the perpendicular lengths DB and DC.



In ΔADB and ΔADC,

$\angle BAD = \angle CAD$ (angle bisector)

$\angle ABD = \angle ACD$ (right angles)

AD = AD (common side)

Thus by A – A – S, the two triangles are Congruent.

Thus DB = DC i.e. the point D is equidistant from the sides. This is same for any point on the angle bisector.

Thus,

All points on the angle bisector are equidistant from the sides forming the angle.

(ii) Triangles

The next step in the development of our geometric knowledge is to get acquainted with one of the simplest plane figures, that is **a triangle.**

Three lines in a plane, of which no two are parallel, either have to intersect at the same point, as shown in figure (i) or would enclose a triangle as shown in figure (ii).

Lines intersecting at the same points are called concurrent lines, [figure (i)].

The closed plane in figure - (ii), formed by the intersection of three lines, no two of which are parallel is a Triangle.

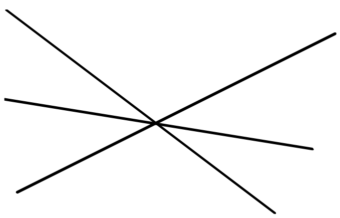


Figure (i)

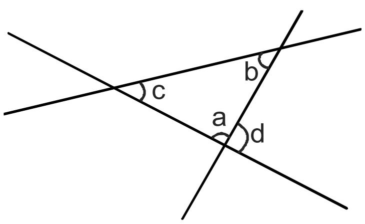


Figure (ii)

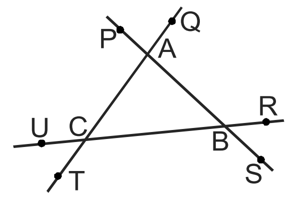
In other words, a **triangle** is a figure that consists of three points that do not lie on the same line and three segments that connect each pair of points. The points are called *vertices* and segments are called *sides* of a triangle.

The following would surely be known by everyone and hence one can go through it rapidly. All new concepts of triangles would be taken later in the chapter.

Classification:

Based on Angles	Based on Sides
Acute angle Triangle: All angles are acute i.e. < 90°	***Equilateral Triangle:*** All three sides of the triangle are equal in length.
Right angle triangle: One angle of the triangle is 90°	***Isosceles Triangle:*** Two sides of the triangle are equal in length
Obtuse angle triangle: One angle of the triangle is obtuse i.e. > 90°	***Scalene Triangle:*** No two sides of the triangle are equal in length

a. Important Properties/Observation in triangles:



In ΔABC ,

- **Sum of three interior angles is 180°**

  $\angle ABC + \angle ACB + \angle BAC = 180°$

- **Exterior angles = Sum of the two interior angles which are not adjacent to it (also called *interior opposite angles*)**

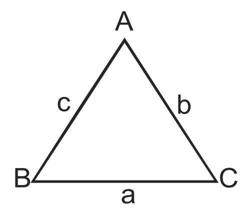
  $\angle ABR = \angle ACB + \angle BAC$

- **If two sides of the triangle are not equal, then the angle opposite to greater side is greater.**

i.e. if in ΔABC, if $AC > AB$, then $\angle ABC > \angle ACB$

Conversely, if two angles of a triangle are not equal, then side opposite to the greater angle is greater in length.

Sum *of any two sides is always greater than the third side.*



i.e. $a+b>c$, $b+c>a$ and $c+a>b$.

Difference *of any two sides of a triangle is always less than the third side.*

i.e. $a-b<c$, $b-c<a$ and $c-a<b$.

8. What will be the range of perimeter of a triangle with two sides as 4 and 7?

Solution :

For perimeter to be as large as possible, the third side should be as large as possible. But it has to be less than the sum of other two sides. Thus if x is the third side, x < (4 + 7) i.e. x < 11. In this case perimeter, P < 22

For perimeter to be least, x has to be as small as possible. Can x be 1? No, because in that case 1 + 4 is not greater than 7. Thus least value of x is such that (x + 4) > 7 i.e. x > 3. In this case perimeter, P > (3 + 4 + 7) i.e. P > 14.

Thus range of perimeter is 14 < P < 22

9. How many triangles can be formed such that the sides are natural numbers and the perimeter is equal to 10?

Solution :

Since sum of two sides has to be greater than the third side, the largest side has to be less than 5. Thus the possible triangles can be (4, 2, 4) and (4, 3, 3).

- ***Proportionality of sides and angles***

Sides do show a proportionality with angles opposite to them, but the precise relation is that sides are proportional to the sin of the angle opposite to it. The mentioned results is as follows:

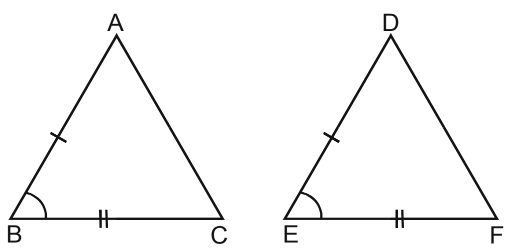
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

b. Congruency of triangles

Two figures are congruent if when placed one over the other, they completely overlap each other i.e. the two figures are identical. They would have the same shape and the same area.

Rules for two triangles to be congruent:

1. SAS condition

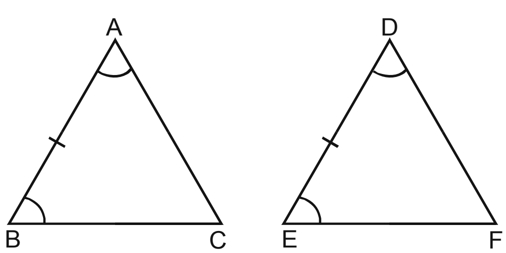


If two sides and the included angle of one triangle is equal to the corresponding sides and included angle of the other triangle, then both triangles are congruent.

AB = DE, BC = EF and $\angle B = \angle E$, then

$\Delta ABC \cong \Delta DEF$

2. ASA condition

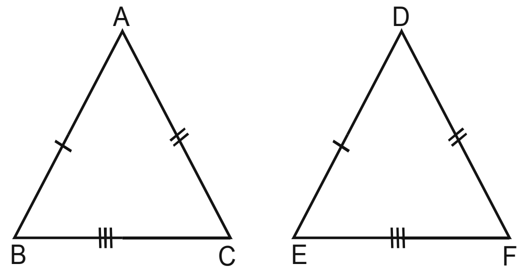


If two angles and the included side of one triangle is equal to the corresponding two angles and the included side of the other triangle, then both triangles are congruent.

If $\angle A = \angle D$, $\angle B = \angle E$ and AB = DE, then

$\Delta ABC \cong \Delta DEF$

3. SSS condition

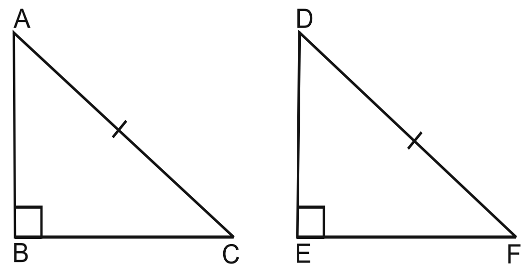


If each of the sides of one triangle is equal to the corresponding side of other triangle ,then both triangles are congruent.

If AB = DE, AC = DF and BC = EF, then

$\Delta ABC \cong \Delta DEF$

4. RHS condition



If the two triangles are right-angled triangle and hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of other triangle, then both triangles are congruent.

Here, $\angle B = \angle E = 90°$, AC = DF and AB = DE or BC = EF, then $\Delta ABC \cong \Delta DEF$

c. Similarity

Two geometrical figures are similar if they are similar in shape.

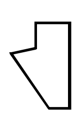
In similar figures, it is important to identify corresponding sides. Sides opposite to equal angles are called corresponding sides.

Why is similarity important? What if two figures are similar?

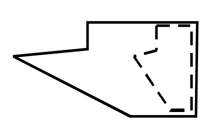
In similar figures, the ratios of corresponding sides are equal

The above should be thoroughly understood as this is what we shall be using after identifying two figures to be similar.

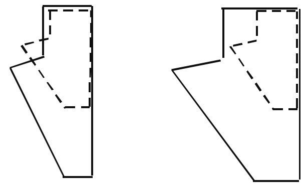
Consider the following shapes:



(a)



(i)



Which of the figures (i), (ii) and (iii) are similar to figure (a)?

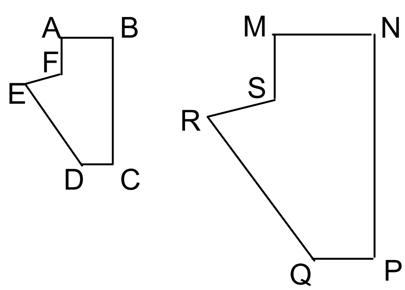
For your convenience the figure (a) is drawn along with each of the figure (i), (ii) and (iii) with dotted lines.

Loosely speaking all the four figures may have the same shape. But then, by using this logic all triangles should be similar because basically all have three vertices and three sides!

In figure (i), the shape has been stretched out horizontally but there is no corresponding change in the vertical dimension. Hence the figure gets distorted in one direction and the shape is not similar to figure (a).

Same is the case in figure (ii) viz. the change in the vertical dimension is disproportionate to the change in horizontal dimension and hence distortion of shape takes place.

In figure (iii), the shape has been enlarged and the enlargement is equal in all possible dimensions. Thus the basic shape remains same and only the size changes. Thus this figure is similar to figure (a).



For no distortion to take place, the change in the dimension along all directions should be the same i.e. the ratio $\frac{MN}{AB}$ should be equal to the ratio $\frac{NP}{BC}$.

In general,

$$\frac{MN}{AB} = \frac{NP}{BC} = \frac{PQ}{CD} = \frac{QR}{DE} = \frac{RS}{EF} = \frac{SM}{FA}$$

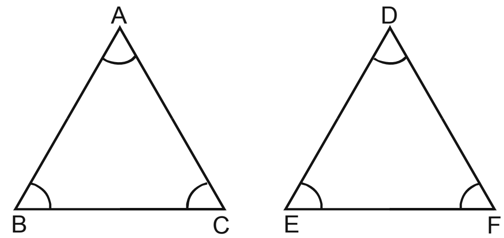
Also important to understand is that ratio of any corresponding linear dimension will be equal to the above ratio i.e. even $\frac{NQ}{DB} = \frac{SP}{FC} = \frac{MN}{AB}$

Similar triangles

If in two triangles corresponding angles are equal, then the two triangles are similar to each other. In similar triangles, ratios of corresponding sides are same.

Rules for two triangles to be similar:

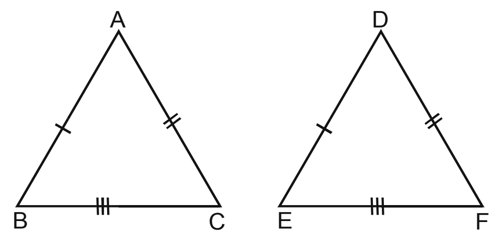
1. **AA condition**



If all the three angles of a triangle are equal to the corresponding three angles of the other triangle, then both the triangles are similar.

If, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$,
then $\Delta ABC \sim \Delta DEF$

2. **SSS condition**

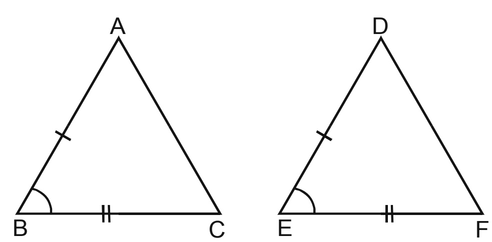


If all the three sides of a triangle are in proportion with the corresponding three sides of the other triangle, both the triangles are similar.

If, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$,
then $\Delta ABC \sim \Delta DEF$

3. **SAS condition**

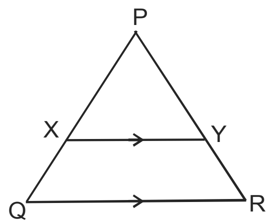
If two sides of a triangle are in proportion with the corresponding two sides of the other triangle and the included angle of one is equal to the included angle of the other, then both triangles are similar.



If $\frac{AB}{DE} = \frac{BC}{EF}$ and $\angle B = \angle E$, then
$\Delta ABC \sim \Delta DEF$

Basic Proportionality theorem

It says, *in a triangle, line drawn parallel to any side divides the other two sides in the same proportion.*



In ΔPQR, $XY||QR$, then according to Basic Proportionality theorem,

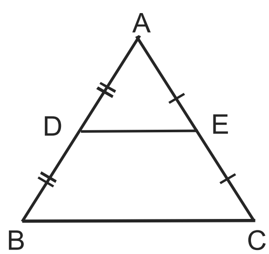
$$\frac{PX}{XQ}=\frac{PY}{YR} \quad \ldots (i)$$

$$\frac{PQ}{PX}=\frac{PR}{PY} \quad \ldots (ii)$$

$$\frac{PQ}{XQ}=\frac{PR}{YR} \quad \ldots (iii)$$

Mid Point Theorem

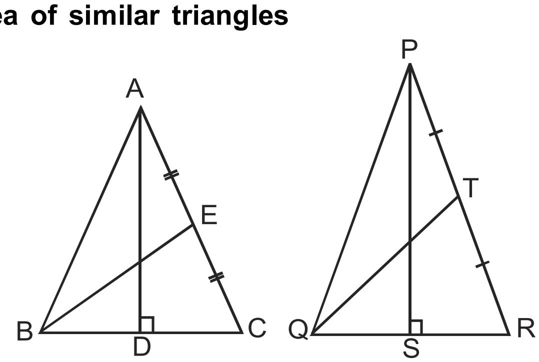
Extension of Basic Proportionality theorem is Mid - Point theorem.



If, D is mid point of AB and E is mid point of AC. Then, DE is parallel to BC and $DE=\frac{1}{2}BC$.

Line joining mid point of any two sides of a triangle is parallel to the third side and equal to half of it.

Area of similar triangles



If $\Delta \mathbf{ABC} \sim \Delta \mathbf{PQR}$, then

$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PS}=\frac{BE}{QT}=k \ (\text{say})$$

The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

i.e., if $\Delta ABC \sim \Delta PQR$, then

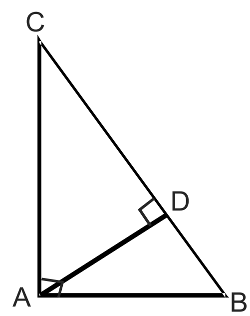
$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)}=\frac{\frac{1}{2}\times BC\times AD}{\frac{1}{2}\times QR\times PS}$$

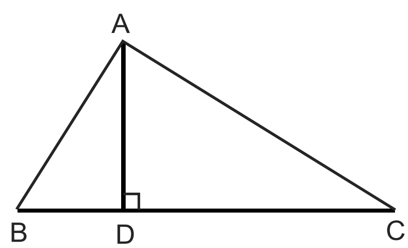
$$=\left(\frac{BC}{QR}\right)\times\left(\frac{AD}{PS}\right)=k^2$$

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)}=\frac{(AB)^2}{(PQ)^2}=\frac{(BC)^2}{(QR)^2}=\frac{(AC)^2}{(PR)^2}$$

Another specific case of use of similarity is involving a right-angled triangle as shown in the figure below.

The figure can be drawn in two ways but essentially only the orientation is different and the two figures are the same. Thus, whatever follows hold true for both the figures.

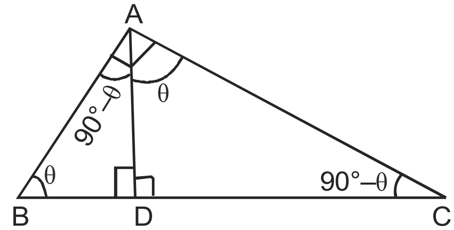




In the figure, all the three triangles are similar as all the three triangles have angles equal to θ, $90°$, $(90° - \theta)$. Thus, considering

$\angle ABC = \theta$ and $\angle ACB = (90° - \theta)$,

We get, $\angle CAD = \theta$, $\angle BAD = (90° - \theta)$



Now, $\Delta ADB \sim \Delta CDA$ (AA similarity)

Thus $\frac{AD}{DC} = \frac{BD}{AD}$. So $AD^2 = BD \times DC$

Also $\Delta ABD \sim \Delta CBA$ (AA similarity)

$\frac{BC}{AB} = \frac{AB}{BD} \Rightarrow AB^2 = BD \times BC$

Similarly,

$\Delta ADC \sim BAC$

$AC^2 = CD \times BC$

Thus the results

1. **$AD^2 = BD \times DC$**
2. **$AB^2 = BD \times BC$**
3. **$AC^2 = CD \times BC$**

In other words,

The triangles on each side of the altitude drawn from the vertex of the right angle (B) to the hypotenuse (AC) are similar to the original triangle and to each other. i.e. $\Delta ABC \sim \Delta ADB \sim \Delta BDC$

Also, the altitude from the vertex to the hypotenuse is the geometric mean of the segments into which hypotenuse is divided.

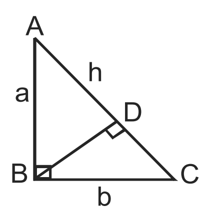
Note:

All the congruent triangles are similar but all similar triangles are not congruent.

Pythagoras Theorem

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the perpendicular sides.

Pythagorean triplets can be derived if one of the side 'n' given is integer. When 'n' is odd, then other two sides are $\frac{n^2 \pm 1}{2}$ and when 'n' is even, other two sides are $\frac{n^2}{4} \pm 1$.



Three sides which satisfy the relation

$$a^2 + b^2 = h^2$$

are called Pythagorean triplets and can form the sides of a right angled triangle. The most commonly used triplets are (3, 4, 5); (5, 12, 13); and (6, 8, 10). In most of the problems only these triplets are used and one need not do any calculation if one remembers these. Multiples of Pythagorean triplets are also Pythagorean triplets.

E.g. (6, 8, 10); (9, 12, 15); (12, 16, 20) are all multiples of the basic triplet (3, 4, 5). Few other triplets used occasionally are (8, 15, 17);

(7, 24, 25); (9, 40, 41); (11, 60, 61) etc.

Important Observation

If in a triangle,

$c^2 = a^2 + b^2$, it is a right angled triangle

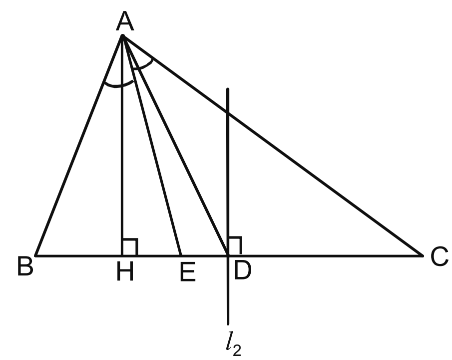
$c^2 < a^2 + b^2$, it is an acute angled triangle

$c^2 > a^2 + b^2$, it is an obtuse angled triangle

d. Important Lines in a Triangle

Median, Angle Bisector, Perpendicular Bisector of Sides and Altitudes of a Triangle:

With each vertex (or side) as reference, one can draw four specific lines in a triangle viz. median, angle bisector, perpendicular bisector of side and altitude.



Median joins the vertex to midpoint of opposite side. It need not be perpendicular to the side and may also not be the angle bisector. In the figure, AD is the median, where D is the mid-point of BC.

Angle bisector bisects an internal angle of a triangle. It need not pass through the midpoint of opposite side and need not be perpendicular to the opposite side. In the figure AE is the angle bisector.

Perpendicular bisector of side is the perpendicular line passing through the mid-point of the side. It need not pass through the vertex. In the figure, the line l_2 is the perpendicular bisector of the side BC.

Altitude is the line dropped from a vertex and perpendicular to the opposite side. It passes through the vertex but need not pass through the midpoint of the opposite side. In the figure, AH is the altitude from the vertex A.

For that matter the altitude may not even lie within the triangle (in case of an obtuse angle triangle)

In any triangle, the three medians are concurrent, so are the three angle bisectors and also the three perpendicular bisectors. The same is true for the three altitudes as well.

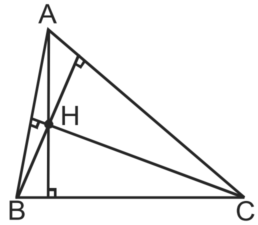
e. Important Points

In a triangle there are FOUR important points. They are Centroid, Orthocenter, Circumcenter and Incenter.

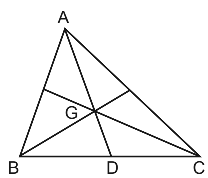
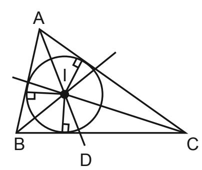
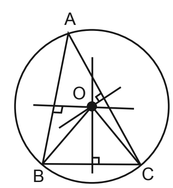
Orthocenter:

The point of concurrency of the altitudes is known as Orthocenter.

There is no ‘ortho-circle’ with center as orthocenter.



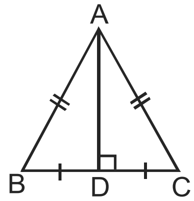
Property: $\angle BHC + \angle A = 180°$

Centroid	In-center	Circumcenter
The point of concurrency of the medians of a triangle.	The point of concurrency of the angle bisectors	The point of concurrency of the perpendicular bisector of sides
	Since perpendicular distance of in-center of a triangle from all its sides is equal, an ‘in-circle’ can be drawn which touches all three sides of the triangle.	Since circum-center of a triangle is equidistance from all the vertices of the triangle, a ‘circum-circle’ can be drawn which passes through the vertices of the circle.
	 Please note the in-radius is the perpendicular distance to the sides and not ID. This is a very common error made.	
Properties: 1. The median divides the triangle in two parts of equal areas (but need not be congruent). Area of ΔADB = Area of ΔADC 2. The centroid divides the median in the ratio 2 : 1 with the larger part towards the vertex. Thus AG : GD is 2 : 1	**Properties:** 1. $\frac{AB}{AC} = \frac{BD}{CD}$ 2. $\angle BIC = 90 + \frac{\angle A}{2}$	**Properties:** 1. $\angle BOC = 2 \times \angle A$ Thus, it can be visualized that for i. **Acute angled triangle →** Circum-center lies inside the triangle. ii. **Obtuse angled triangle →** Circum-center lies outside the triangle. iii. **Right angled triangle →** Circum-center lies on the mid point of the hypotenuse.

Observations

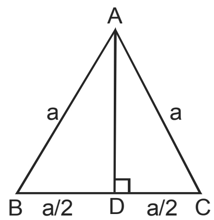
In an ***isosceles triangle***, ΔABC,

where, AB = AC, (see figure)



the angle bisector of $\angle BAC$, AD, is also the perpendicular bisector of the base BC as well as the median drawn from vertex A to side BC.

In *an* ***equilateral triangle*** Δ***ABC,***



AB = AC = BC = a

Height /altitude = $\dfrac{\sqrt{3}}{2} \times \text{side} = \dfrac{\sqrt{3}}{2}a$

Area = $\dfrac{1}{2} \times \left(\dfrac{\sqrt{3}}{2} \times \text{side}\right) \times \text{side}$

$= \dfrac{\sqrt{3}}{4}(\text{side})^2 = \dfrac{\sqrt{3}}{4}a^2$

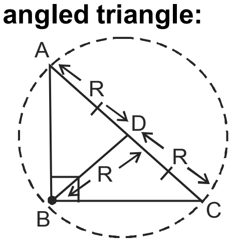
In radius = $\dfrac{1}{3} \times \text{height}(AD) = \dfrac{a}{2\sqrt{3}}$

Circumradius = $\dfrac{2}{3} \times \text{height}(AD) = \dfrac{a}{\sqrt{3}}$

Perimeter = 3 × side = 3a

- ***In an equilateral triangle, altitude, median, angle bisector, perpendicular bisector of each base are equal. Centroid, incentre, orthocentre and circumcentre coincide.***
- ***In case of triangles, given the perimeter as constant, an equilateral triangle has the maximum area.***

In **Right - angled triangle:**



Let D be the mid point of the hypotenuse AC.

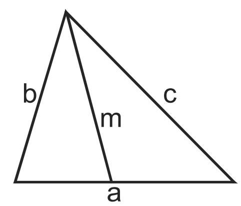
Median drawn from the right angular vertex to the hypotenuse in a right angle triangle is equal to the circumradius.

AD = BD = DC = R = Circumradius

Apollonius Theorem

Let a, b, c be the sides of a triangle and m is the length of the median to the side with length a.

Then, $b^2 + c^2 = 2\left[m^2 + \left(\dfrac{a}{2}\right)^2\right]$

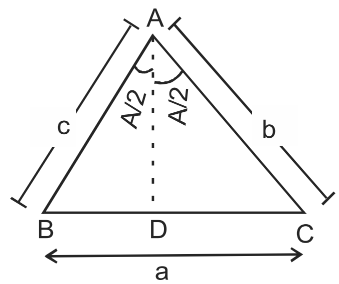


Special case:

If b = c (the triangle is isosceles), then we have

$2b^2 = 2\left[m^2 + \left(\dfrac{a}{2}\right)^2\right] \Rightarrow m^2 = \left(b^2 - \dfrac{a^2}{4}\right)$

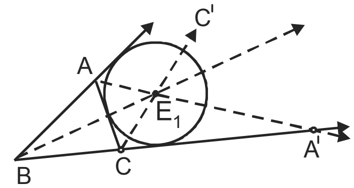
Angle – Bisector Theorem



Angle bisector divides the opposite side in the ratio of sides containing the angle. So $\dfrac{BD}{DC} = \dfrac{AB}{AC}$

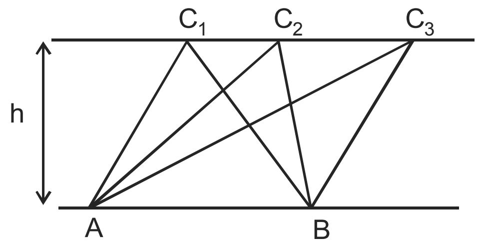
External Angle Bisector:

If we extend two sides of the triangle, we can bisect the exterior angles in the same way. In the figure the exterior angle at A is bisected by the ray AA′ and the exterior angle at C is bisected by the ray CC′.



The External bisectors of $\angle A$ and $\angle C$ and the internal bisector of $\angle B$ all intersect in a common point also, and this point is the center of a circle that is tangent to the three sides also. This is called an Excircle, and the center is the excenter E_1. Since there are three sides, this could be done in three ways and we can get three ex-centers. The radius of this circle is also related to the area and perimeter of the triangle.

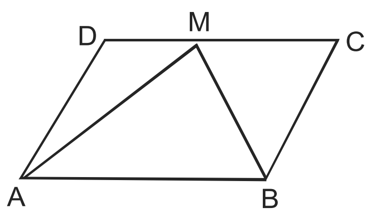
Triangles on the same base and between the same parallel lines are equal in area.



Area $(\Delta ABC_1) = \text{Area}(\Delta ABC_2)$

$= \text{Area}(\Delta ABC_3) = \frac{1}{2}(AB) \times h$

If a parallelogram and a triangle are drawn on the same base and between same parallel lines, area of parallelogram is twice the area of the triangle.



Area of ABCD = 2 × (Area of ΔABM)

Few other formulae related to a triangle:

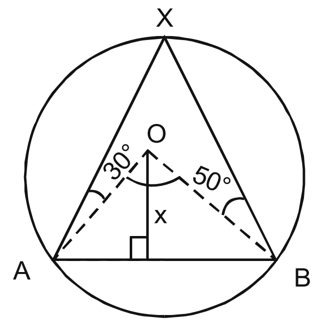
1. Area of a triangle = $\frac{1}{2} \times b \times h$, where b is base and h is height
2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b and c are the sides of the triangle and s is the semi-perimeter i.e. $s = \frac{a+b+c}{2}$.
   This formulae of area is known as Heron's formula.
3. Area of a triangle = $\frac{1}{2} ab \sin\theta$, where a and b are the sides of the triangle and θ is the included angle i.e. angle between sides of length a and b.
4. Area of a triangle = r × s, where r is the in-radius and s is the semi-perimeter
5. **Cosine rule:** If a, b and c are the three sides of a triangle and if θ is the included angle between the sides of length a and b, then

   $\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$ or $c^2 = a^2 + b^2 - 2ab\cos\theta$

Exercise

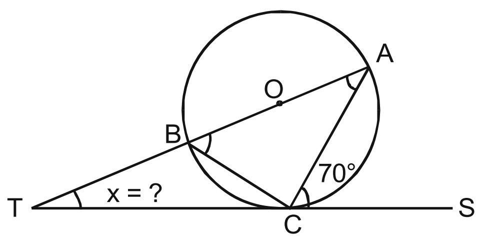
PRACTICE EXERCISE-1

1. In the adjoining figure, find $\angle x$.



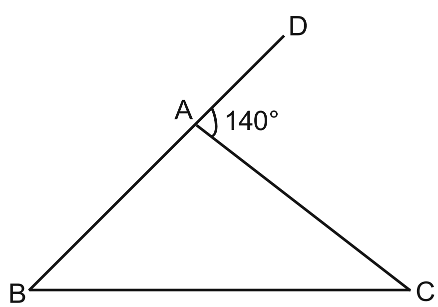
(a) 160° (b) 180°
(c) 80° (d) 50°

2. In the adjoining figure, find $\angle x$.



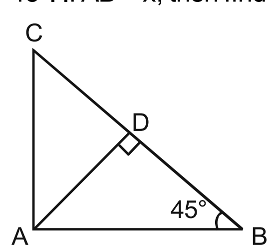
(a) 60° (b) 50°
(c) 70° (d) 80°

3. In the following diagram, if $\angle B : \angle C = 3 : 4$, then find $\angle B$.



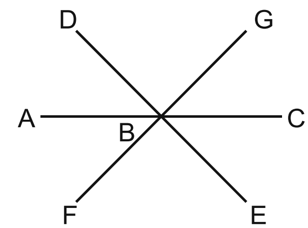
(a) 90° (b) 100°
(c) 80° (d) 60°

4. In ΔABC, $\angle A = 90°$, AD is perpendicular to BC and $\angle B = 45°$. If AB = x, then find AD in terms of x.



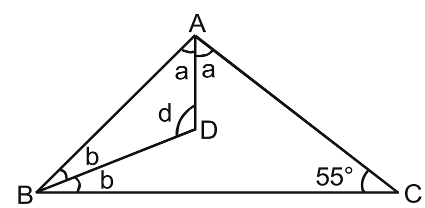
(a) 2x (b) 3x
(c) $x\sqrt{2}$ (d) $\frac{x}{\sqrt{2}}$

5. In the given figure, if $\angle DBG$ is 55° and $\angle CBF$ is 115°, then find $\angle EBC$.



(a) 70° (b) 30°
(c) 80° (d) 60°

6. What is the value of 'd' in the given figure?

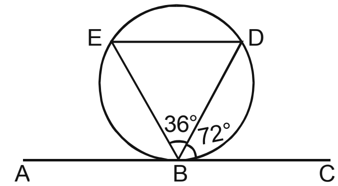


(a) 117.5° (b) 220°
(c) 100° (d) 120°

7. The diagonals AC and BD of a parallelogram ABCD intersect at O. If $\angle DAC = 30°$, $\angle AOB = 80°$ then $\angle DBC$ is
(a) 150° (b) 50°
(c) 120° (d) 100°

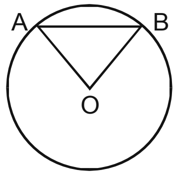
8. Triangles ABC and DEF are similar. Their areas are 64 cm^2 and 49 cm^2 respectively. If AB is 7 cm, then find the length of DE.
(a) 8 cm (b) $\frac{49}{8}$ cm
(c) $\frac{50}{4}$ cm (d) $\frac{25}{3}$ cm

9. In the given figure $\angle DBE = 36°$ and $\angle DBC = 72°$. Find value of $\angle BDE$.



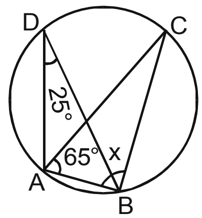
(a) 72° (b) 70°
(c) 36° (d) 18°

10. In the given circle, OA = 8 cm and $\angle AOB = 132°$. Find $\angle ABO$ if O is the center of the circle.



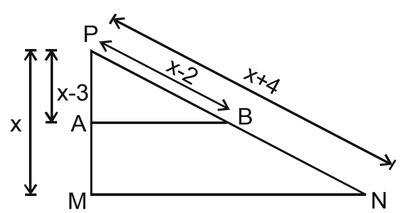
(a) 48° (b) 24°
(c) 20° (d) 30°

11. Find the value of 'x' in the given figure.



(a) 90° (b) 85°
(c) 80° (d) 75°

12. In the given figure, AB | | MN. If PA = x – 3, PM = x, and PB = x – 2, PN = x + 4, then find the value of x.



(a) 4 (b) 2
(c) 3 (d) 5

13. In ΔABC, DE | | BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, then find the value of x.

(a) 2 (b) 3
(c) 4 (d) 5

14. In ΔABC, AD is the bisector of $\angle BAC$ meeting BC at D. If AB = 16 and AC = 8, find BD : CD.

(a) 3 : 2 (b) 3 : 1
(c) 2 : 1 (d) 2 : 6

15. In ΔABC, AD is the bisector of $\angle BAC$. If BD = 2 cm, DC = 3 cm and AB = 5 cm, find AC.

(a) 7.5 cm (b) 8 cm
(c) 9 cm (d) 10 cm

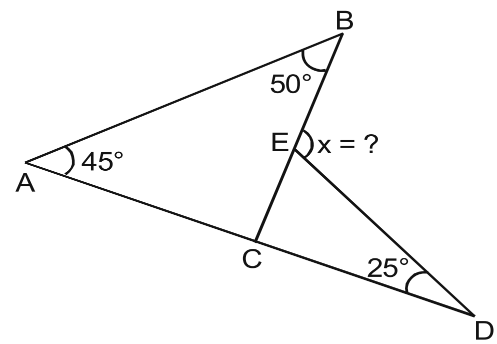
16. The difference between the interior and exterior angles of a regular polygon is 120°. The number of sides of that polygon is

(a) 18 (b) 12
(c) 8 (d) 10

17. Find the angle that is equal to one-fourth of its supplement.

(a) 36° (b) 40°
(c) 30° (d) 60°

18. Find the value of x in the given figure.



(a) 110° (b) 70°
(c) 100° (d) 120°

19. The angles of the quadrilateral are in the ratio of 2 : 4 : 5 : 7, then the smallest angle is

(a) 20° (b) 40°
(c) 10° (d) 30°

20. Find the perimeter of triangle ABC. If A = 90°, AB = 8 cm and AC = 6 cm.

(a) 24 cm (b) 22 cm
(c) 20 cm (d) 18 cm

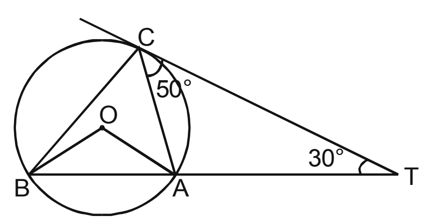
21. In an equilateral triangle, the circumcentre, orthocentre and incentre are

(a) coincident (b) concurrent
(c) collinear (d) None of these

22. Find the angle that is four times as large as its complement.

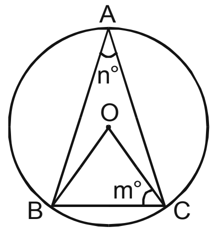
(a) 70° (b) 72°
(c) 80° (d) 90°

23. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If $\angle ATC = 30°$ and $\angle ACT = 50°$, then the angle $\angle BOA$ is



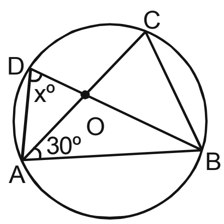
(a) 100° (b) 150°
(c) 80° (d) 60°

24. As shown in the figure, O is the centre of the circle. $\angle BCO = m°$ and $\angle BAC = n°$. Find out the relationship between m and n.



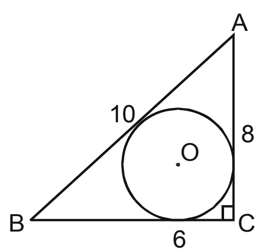
(a) m + n = 180° (b) 2m + n = 180°
(c) m + 2n = 180° (d) m + n = 90°

25. As shown in the figure, $\angle BAO = 30°$ and the radius of the circle is 4 cm. Find the value of 'x' if O is the center of the circle.



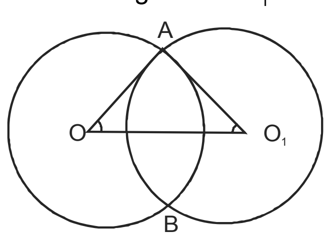
(a) 30° (b) 45°
(c) 55° (d) 60°

26. ABC is a right-angled triangle with BC = 6 cm and AC = 8 cm. A circle with centre O is inscribed in ΔACB. The radius of the circle is



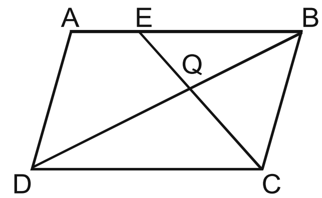
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm

27. In the following figure, the circles whose centres are O and O_1 intersect at two points A and B, where AB = 2 cm and $OO_1 = 2\sqrt{3}$. Find the measure of the angle $\angle OAO_1$.



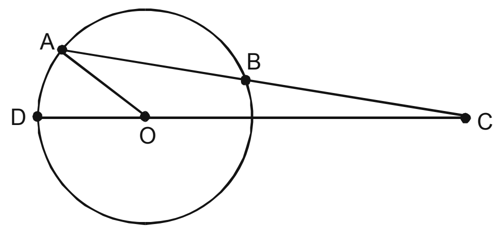
(a) 150° (b) 90°
(c) 60° (d) 120°

28. In the given parallelogram ABCD, if 3(BE) = 2(DC) and the area of ΔDQC is 36, then find out the area of ΔBQE.



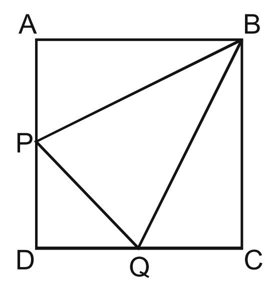
(a) 16 (b) 20
(c) 24 (d) 18

29. In the figure below, AB is the chord of a circle with center O. AB is extended to C such that BC = OB. The straight line CO is produced to meet the circle at D. If $\angle ACD = y°$ and $\angle AOD = x°$ such that x = ky, then the value of k is



(a) 3 (b) 2
(c) 1 (d) 4

30. ABCD is a square in which P and Q are mid-points of AD and DC respectively. The area of ΔBPQ constitutes what part of the whole area?



(a) 50% (b) 37.5%
(c) 66.66% (d) 40%

PRACTICE EXERCISE-2

1. If the diagonal of a square field is 60 m, find the area of the square.

(a) 800 m^2 (b) 1,200 m^2
(c) 1,600 m^2 (d) 1,800 m^2

2. The perimeter of a rectangular plot is 40 m and its length is 12 m. Calculate its area.

(a) 80 m^2 (b) 96 m^2
(c) 105 m^2 (d) 125 m^2

3. A rectangular plot is 510 sq. m in area. If its breadth is 17 m, find its perimeter.
   (a) 86 m (b) 88 m
   (c) 92 m (d) 94 m
4. The perimeter of a square is (8x + 40). Find its diagonal.
   (a) $\sqrt{2}\,(2x+10)$ (b) $\sqrt{2}x$
   (c) $\sqrt{2}x+40$ (d) 2x
5. Find the perimeter of a square, the sum of lengths of whose diagonals is 196 cm.
   (a) 196 cm (b) $196\sqrt{2}$ cm
   (c) 204 cm (d) 216 cm
6. The cost of levelling a square ground at the rate of Re. 0.80 per 100 m^2 is Rs. 28.80. Find the cost of fencing the same at Re.0. 60 per metre.
   (a) Rs. 144 (b) Rs. 196
   (c) Rs. 225 (d) Rs. 250
7. A chessboard contains 64 equal squares and the area of each square is 2.56 cm^2. Find the length of the side of the chessboard.
   (a) 3.5 cm (b) 4 cm
   (c) 12.8 cm (d) 8 cm
8. A lawn is in the shape of a rectangle of length 80 m and width 40 m. Outside the lawn there is a footpath of uniform width 3 m. Find the area of the path.
   (a) 756 m^2 (b) 800 m^2
   (c) 825 m^2 (d) 1,100 m^2
9. The area of a triangle is 32 cm^2. Its base is 8 cm. What is its altitude?
   (a) 6 cm (b) 10 cm
   (c) 12 cm (d) 8 cm
10. An isosceles right-angled triangle has an area 800 cm^2. What is the length of its hypotenuse?
    (a) $40\sqrt{2}$ cm (b) 44 cm
    (c) $30\sqrt{2}$ cm (d) 65 cm
11. Find the area of the triangle whose sides are 6 cm, 7 cm, and 11 cm.
    (a) 10 cm^2 (b) 14 cm^2
    (c) $6\sqrt{10}$ cm^2 (d) $9\sqrt{2}$ cm^2
12. The side of an equilateral triangle is 4a, find its altitude.
    (a) $\frac{a}{2}$ (b) $\sqrt{2}$ a
    (c) $\sqrt{3}$ a (d) $2\sqrt{3}$ a
13. The perimeter of a triangular park is 240 m and its sides are in the ratio 6 : 8 : 10. Find its area.
    (a) 220 m^2 (b) 2,400 m^2
    (c) 250 m^2 (d) 300 m^2
14. Find the area of a right-angled triangle if radius of its circumcircle is 3 cm and altitude drawn to hypotenuse is 2 cm.
    (a) 4 cm^2 (b) 4.5 cm^2
    (c) 5.2 cm^2 (d) 6 cm^2
15. Find the area of a rhombus whose diagonals are 10 cm and 8 cm.
    (a) 32 cm^2 (b) 39 cm^2
    (c) 40 cm^2 (d) 54 cm^2
16. The diagonals of a rhombus are 24 cm and 10 cm. Find its side.
    (a) 10 cm (b) 11 cm
    (c) 12 cm (d) 13 cm
17. The area of a rhombus is 90 cm^2. One of its diagonals is 18 cm, find the other diagonal.
    (a) 8 cm (b) 15 cm
    (c) 11 cm (d) 10 cm
18. A wire is in the form of a circle of radius 28 cm. It is bent into a square. Determine the side of the square.
    (a) 23 cm (b) 44 cm
    (c) 63 cm (d) 75 cm
19. The radius of a circle is 21 cm. Find the radius of another circle whose area is one-fourth the area of this circle.
    (a) 10.5 cm (b) 20 cm
    (c) 21.5 cm (d) 22 cm
20. A paper is in the form of rectangle ABCD, where AB = 30 cm and BC = 14 cm. A semicircular portion having diameter equal to the width of the rectangle is cut. Find the area of the remaining portion.
    (a) 250 cm (b) 343 cm
    (c) 243 cm (d) 543 cm
21. The dimension of a rectangular park is 150 m × 120 m. There is a circular lawn at the centre of the park. The area of the park excluding the lawn is 5,500 m^2. Find the radius of the circular lawn.
    (a) 70 m (b) 65.07 m
    (c) 60 m (d) 63.06 m
22. The radius of a circle is 14 cm. Find the length of the arc of the sector with central angle 18° and its area.
    (a) 4.4 cm, 30 cm^2 (b) 4.4 cm, 30.8 cm^2
    (c) 3 cm, 31 cm^2 (d) 3.2 cm, 32 cm^2

23. A circular disc of radius 16 cm is divided into 3 sectors with central angle 60°, 140°, 160°. Find the ratio of the areas of three sectors.
(a) 5 : 7 : 8 (b) 6 : 7 : 8
(c) 2 : 7 : 8 (d) 3 : 7 : 8

24. Find the length of the longest pole that can be placed in a room of size 20 m × 10 m × 5 m.
(a) $\sqrt{600}$ m (b) $\sqrt{525}$ m
(c) 30 m (d) 25 m

25. What is the volume of the cube whose largest diagonal is $8\sqrt{3}$ cm.
(a) 500 cm^3 (b) 508 cm^3
(c) 600 cm^3 (d) 512 cm^3

26. Find the radius of the sphere, given that when the volume of the sphere is divided by its surface area the answer is 5 cm.
(a) 15 cm (b) 6 cm
(c) 7 cm (d) 8 cm

27. If the height and radius of the base of a cylinder are both increased by 200%, then find the increase in volume of the cylinder.
(a) 25 times (b) 8 times
(c) 27 times (d) 26 times

28. A metal sphere of radius 10 cm is heated and drawn into wire of radius 0.1 cm. Find out the length of the wire.
(a) 140 cm (b) 150 cm
(c) 1.333 km (d) 133.33 cm

29. Find length of the side of a cube whose surface area is 150 cm^2.
(a) 10 cm (b) 8 cm
(c) 5 cm (d) 6 cm

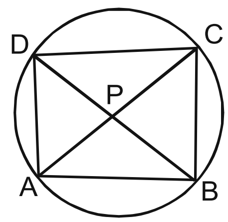
30. Find the quantity of water (in terms of height) that has fallen on a rectangular field of size 150 m × 25 m, if the volume of water accumulated is 2,250 m^3.
(a) 80 cm (b) 60 cm
(c) 100 cm (d) 120 cm

PRACTICE EXERCISE-3

1. Find the volume of the cube whose surface area is 216 cm^2.
(a) 216 cm^3 (b) 280 cm^3
(c) 300 cm^3 (d) 180 cm^3

2. A circular wire of radius 28 cm is bent to form a square, find out the ratio of areas of the square to that of the circle.
(a) 11 : 14 (b) 14 : 15
(c) 20 : 21 (d) 28 : 21

3. In the given circle A, C, B, D are four points on the circumference of the circle. Line AC and DB intersect at P which is not the centre of the circle. If $\angle ACB = 50°$ then what is the value of angle ADB?



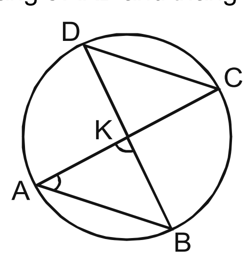
19

(a) 50° (b) 60°
(c) 130° (d) None of these

4. Two circles touch each other externally whose radii are 5 cm and 9 cm, find out the distance between their centres.
(a) 12 cm (b) 16 cm
(c) 14 cm (d) 18 cm

5. Find the area of the largest circle that can be inscribed in a rectangle of length 16 cm and breadth 8 cm.
(a) 16π (b) 20π
(c) 22π (d) 24π

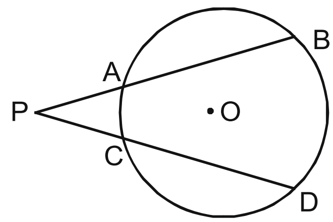
6. ABCD are four points on the circumference of the circle such that AB II CD. Two lines DB and AC intersect at point K. If the ratio of the length of the sides AB and CD is 1:2 then what is the ratio of area of triangle AKB and triangle DKC?



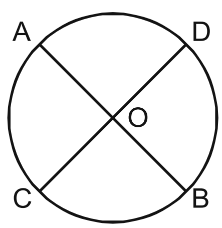
(a) 1 : 4 (b) 4 : 1
(c) 2 : 1 (d) None of these

7. Find the percentage increase in the volume of a cube whose side is increased by 40%.
(a) 200% (b) 350%
(c) 170% (d) 174.4%

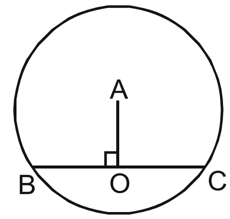
8. The length of a rectangle is increased by 60% and its width remains the same. What is the ratio of the new area to the old area of the rectangle?
   (a) 5 : 8 (b) 3 : 5
   (c) 5 : 3 (d) 8 : 5
9. In the given figure, if the length of AP is equal to 5 cm, PB is equal to 12 cm and PC is equal to 6 cm, then what is the length of CD?



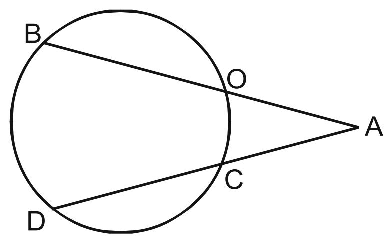
   (a) 4 cm (b) 10 cm
   (c) 8 cm (d) Cannot be determined
10. If both the length and the width of a rectangle are increased by 10% and 20% each, then by what percentage does the area of the resulting rectangle exceeds the original size?
   (a) 32% (b) 30%
   (c) 10% (d) 0%
11. Find the change in volume of a cylinder, if its radius and height are increased by 10% and 20%, respectively.
   (a) 45.2% (b) 36%
   (c) 40% (d) 30%
12. In the given figure, OD = 4 cm, OC = 6 cm, OA = 8 cm. O is the point of intersection of two chords. What is the length of OB?



   (a) 5 cm (b) 3 cm
   (c) 7 cm (d) 4.5 cm
13. Find the volume of a cube to that of a sphere, if the sphere of maximum volume is fitted inside the cube.
   (a) 5 : 3 (b) 6 : π
   (c) π : 6 (d) 3 : 5
14. The 3 sides of a triangle are three consecutive natural numbers. Which among the following cannot be the three sides of the triangle?
   (a) 1 cm, 2 cm, 3 cm (b) 2 cm, 3 cm, 4 cm
   (c) 3 cm, 4 cm, 5 cm (d) None of these
15. In the given figure, A is the centre of the circle and AO is perpendicular on chord BC. BC = 12 cm. What is the length of OB?

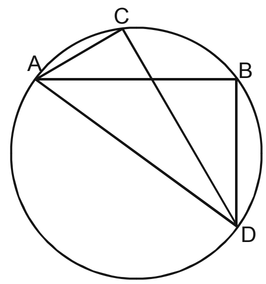


   (a) 4 cm (b) 5 cm
   (c) 6 cm (d) 7 cm
16. Two cones of equal volumes have their radii in the ratio 2 : 3. Find the ratio of their heights.
   (a) 3 : 2 (b) 2 : 3
   (c) 9 : 4 (d) 4 : 9
17. If each tree takes 7 cm^2 area, then how many trees can be planted in a circular bed whose circumference is 22 cm?
   (a) 8 (b) 6
   (c) 7 (d) 5
18. In the given figure, AB = 6 cm, AO = 2 cm, AC = 3 cm. What is the length of CD?



   (a) 1 cm (b) 1.5 cm
   (c) 2 cm (d) 2.5 cm
19. If the length of a field is 8 m and a man walked 10 m diagonally to cross the rectangular field, then find the area of the field.
   (a) 40 m^2 (b) 48 m^2
   (c) 60 m^2 (d) None of these
20. If 4 isosceles right-angled triangles of smaller sides 10 m each are removed from the corners of a rectangular plot of dimension 40 m × 20 m, then find area of the remaining portion.
   (a) 750 m^2 (b) 600 m^2
   (c) 525 m^2 (d) 7,500 m^2

21. In the given figure, AD is the diameter of the circle, what is the value $\angle ACD$ and $\angle ABD$?



(a) 45°, 90° (b) 60°, 30°
(c) 75°, 45° (d) 90°, 90°

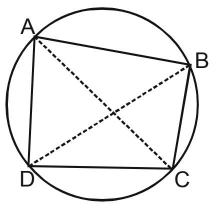
22. A rectangular lawn of dimension 40 m × 30 m has two roads, each with 5 m wide running in the middle of it; one is parallel to the length and the other is parallel to the breadth. Find the cost of gravelling them at 15 paise per sq. m.

(a) Rs. 44.25 (b) Rs. 46
(c) Rs. 48.75 (d) Rs. 50.25

23. The diagonal of a square is twice the side of an equilateral triangle. Find the ratio of the area of the triangle to the area of the square.

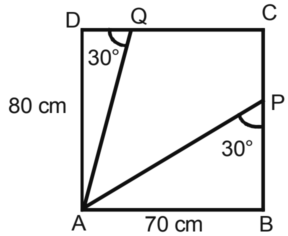
(a) $7\sqrt{3}:8$ (b) $7\sqrt{3}:13$
(c) $\sqrt{3}:8$ (d) None of these

24. ABCD is cyclic quadrilateral such that $\angle ADB = 40°$ and $\angle DCA = 70°$, what is the value of $\angle DAB$?



(a) 40° (b) 45°
(c) 60° (d) 70°

25. In the given figure, ABCD is a rectangle in which segments AP and AQ are drawn, find length of AP + AQ :



(a) 200 cm (b) 250 cm
(c) 300 cm (d) 350 cm

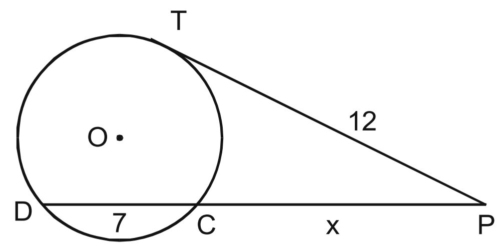
26. If a cone, a hemisphere and a cylinder stand on the same base and have the same height, then find the ratio of their volumes.

(a) 1 : 3 : 2 (b) 2 : 3 : 1
(c) 1 : 2 : 3 (d) 3 : 2 : 1

27. A triangle with sides 5 cm,12 cm and 13 cm is inscribed in a circle. The radius of the circle is:

(a) 2 cm (b) 3 cm
(c) 6.5 cm (d) 5 cm

28. In the adjoining figure, find (x = PC).

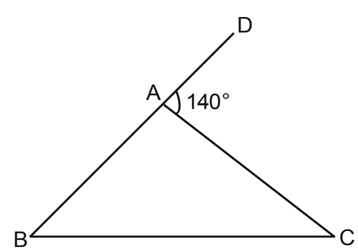


(a) 8 (b) 9
(c) 7 (d) 3

29. The dimensions of a room are 8 m × 6 m × 4.5 m. It has 2 doors each of size 2 m × 1 m and 1 almirah of size 3 m × 2 m. Find the cost of covering the walls by wallpaper which is 40 cm wide at Rs. 1.25 per metre.

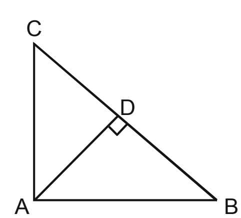
(a) Rs. 312.50 (b) Rs. 352.25
(c) Rs. 362.50 (d) Rs. 412

30. In the following diagram, $\angle B : \angle C = 3:4$ find $\angle B$.



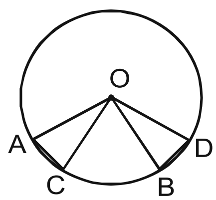
(a) 90° (b) 100°
(c) 80° (d) 60°

31. ABC is a triangle, right angled at A, and AD is perpendicular to BC, if $\angle DAB = \angle B$. Which of the following is correct.

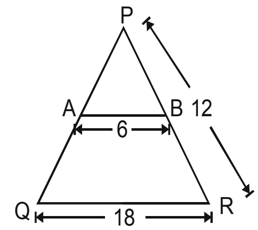


(a) AD = BD (b) AD = CD
(c) BD = CD (d) All of the above

32. A bicycle wheel makes 5,000 revolutions in travelling 11,000 m. Find the diameter of the wheel.
    (a) 25 cm (b) 35 cm
    (c) 3.5 m (d) 2.5 m
33. The side of a square is 14 cm. Find the area of inscribed circle.
    (a) 154 cm^2 (b) 204 cm^2
    (c) 15.4 cm^2 (d) 20.4 cm^2
34. In the given figure, chord AC = chord BD, $\angle AOB = 60°$ and $\angle BOC = 15°$ then, find the value of $\angle BOD$



    (a) 45° (b) 50°
    (c) 40° (d) 120°
35. In the given figure, AB || QR. Find the length of PB.



    (a) 1 (b) 6
    (c) 5 (d) 4

Answer Key

Practice Exercise 1

1. (a)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (b)
11. (a)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (a)
21. (a)	**22.** (b)	**23.** (a)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (b)

Practice Exercise 2

1. (d)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (a)
11. (c)	**12.** (d)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (b)
21. (d)	**22.** (b)	**23.** (d)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (b)

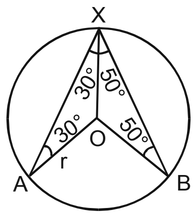
Practice Exercise 3

1. (a)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (a)
11. (a)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (b)
21. (d)	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (d)
31. (d)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (d)					

Explanations

Practice Exercise – 1

1. a OA = OB = OX (radii of a circle)



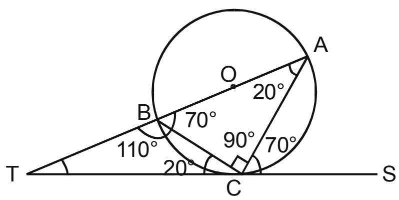
$\therefore$ ΔXOA and ΔXOB are isoceles triangles.

$\therefore$ $\angle$AXO = $\angle$OAX = 30° and $\angle$OBX = $\angle$OXB = 50°

$\therefore$ $\angle$AXB = 80° and

$\angle$AOB = 2 $\angle$AXB = 2 × 80° = 160°

2. b



$\angle$ACS = $\angle$ABC (By alternate segment theorem)

and $\angle$BCA = 90°

(The angle that diameter makes with circumference = 90°)

$\angle$TBC = 180 – $\angle$CBA = 180 – 70 = 110°

$\angle$BCT = 180 – $\angle$BCA – $\angle$ACS = 180° – 90° – 70° = 20°

$\therefore$ $\angle$BTC = 180 – $\angle$TBC – $\angle$BCT

= 180° – 110° – 20° = 50°.

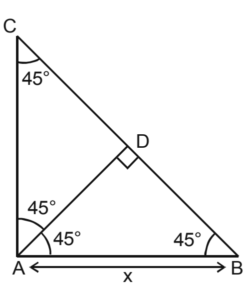
3. d Let $\angle$B and $\angle$C be 3x and 4x respectively.

Then, 3x + 4x = 140 or x = 20°

(Sum of opposite interior angles is equal to exterior angle)

$\therefore$ $\angle$B = 3x = 3 × 20 = 60°

4. d

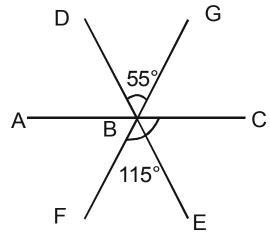


In ΔADB cos 45° = $\frac{AD}{AB}$

AD = x cos 45°

AD = $\frac{x}{\sqrt{2}}$

5. d $\angle$ABG = $\angle$FBC = 115º



(Vertically opposite angles are equal)

$\angle$ABG = $\angle$ABD + $\angle$DBG

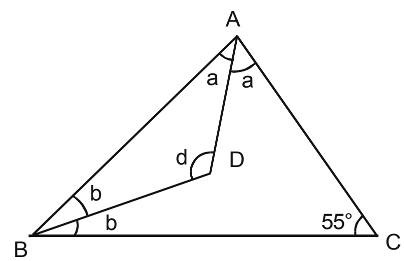
115º = $\angle$ABD + 55º

$\angle$ABD = 60°

$\angle$ABD = $\angle$EBC (vertically opposite angles)

$\therefore$ $\angle$EBC = 60°

6. a



2a + 2b + 55° = 180°

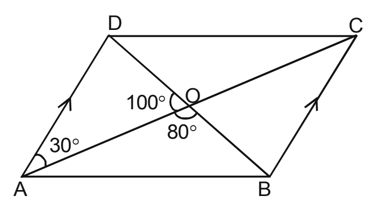
(Sum of all the angles of a triangle = 180°)

$\therefore$ a + b = 62.5°

$\angle$d = 180 – (a + b) = 180 – 62.5

$\angle$d = 117.5°

7. b



$\angle$AOB + $\angle$DOA = 180

$\angle$DOA = 180 – 80 = 100°

$\angle$ADO = 180 – $\angle$DAO – $\angle$DOA

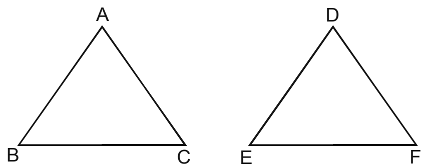
$\angle$ADO = 180 – 30 – 100 = 50°

$\angle$ADO = $\angle$DBC (As AD | | BC and DC | | AB)

$\therefore$ $\angle$DBC = 50°

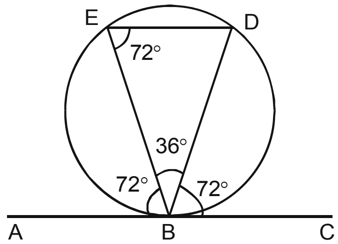
8. b $\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ DEF}} = \frac{\text{Square of side}}{\text{Square of side}} = \frac{AB^2}{DE^2}$

(When two triangles ABC and DEF are similar)



$\frac{AB^2}{DE^2}=\frac{64}{49}$; $\frac{49}{DE^2}=\frac{64}{49}$; $DE=\sqrt{\frac{49\times 49}{64}}$; $DE=\frac{49}{8}$ cm.

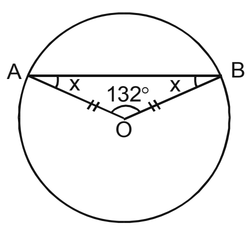
9. a



$\angle CBD = \angle BED$ and $\angle EBA = 180 - \angle CBD - \angle DBE = 180 - 36 - 72 = 72°$ (Both by alternate segment theorem)

$\therefore \angle BDE = 180 - \angle EBD - \angle DEB = 180 - 36 - 72$, or $\angle BDE = 72°$

10. b



OA = OB = radius = 8

($\therefore$ AOB becomes an isosceles triangle)

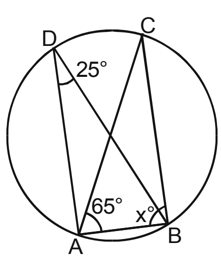
$\angle OAB + \angle ABO + \angle AOB = 180°$

(as $\angle OAB = \angle ABO$)

$x° + x° + 132 = 180$

$2x° = 48° \Rightarrow x = 24°$

11. a

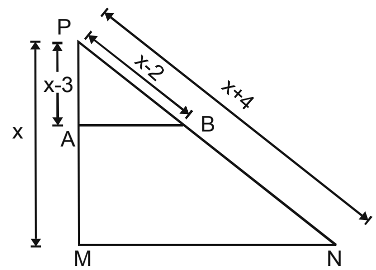


$\angle ADB = \angle ACB = 25°$ [Angles that a chord subtends with circumference are equal]

In ΔABC, $\angle CAB + \angle x + \angle ACB = 180°$

$65 + 25 + x° = 180°$; $x° = 90°$.

12. a



As AB || MN

$\therefore \Delta APB$ and ΔMPN are similar.

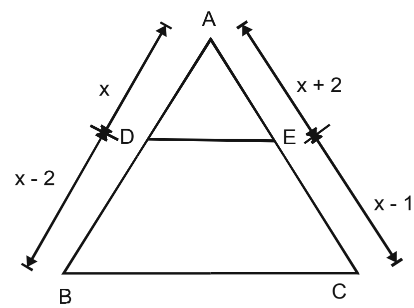
$\therefore \frac{PM}{PA}=\frac{PN}{PB} \Rightarrow \frac{x}{x-3}=\frac{x+4}{x-2}$

$x(x-2) = (x-3)(x+4)$

$x^2 - 2x = x^2 + 4x - 3x - 12$

$x = 4$.

13. c



DE || BC

ΔADE and ΔABC are similar

$\frac{AD}{AB}=\frac{AE}{AC}$

$\frac{AD}{AD+DB}=\frac{AE}{AE+EC}$

$\frac{x}{x+x-2}=\frac{x+2}{x+2+x-1} \Rightarrow \frac{x}{2x-2}=\frac{x+2}{2x+1}$

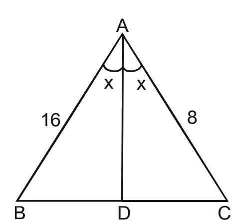
$(2x + 1)x = (2x - 2)(x + 2)$

$2x^2 + x = 2x^2 + 4x - 2x - 4$

$x - 4x + 2x + 4 = 0$

$-x + 4 = 0 \Rightarrow x = 4$

14. c

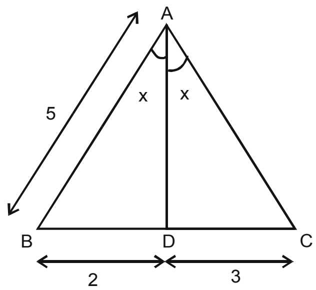


AD is the angle bisector

$\therefore \frac{AB}{AC}=\frac{BD}{DC}$ (internal angle bisector theorem)

$\frac{16}{8}=\frac{BD}{DC}=\frac{2}{1}$.

15. a AD is the angle bisector



$\therefore \frac{AB}{AC} = \frac{BD}{DC}$ (internal angle bisector theorem)

$\frac{5}{AC} = \frac{2}{3}$

$\therefore AC = \frac{15}{2} = 7.5$ cm.

16. b Number of sides of regular polygon = $\frac{360°}{\text{exterior angle}}$

Interior angle + exterior angle = 180° … (i)

It is given that interior angle – exterior angle

= 120° … (ii)

By solving (i) and (ii), exterior angle = 30°

$\therefore$ Number of sides = $\frac{360}{30} = 12.$

17. a Let A be the angle.

So $A = \frac{1}{4}(180° - A)$

4A = 180° – A

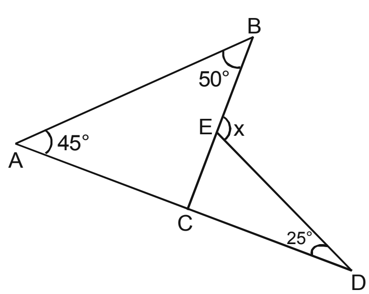
5A = 180°

A = 36°.

18. d We know exterior angle = sum of interior opposite angles

$\therefore \angle ECD = \angle CAB + \angle ABC = 45° + 50°$

and $\angle BED = \angle ECD + \angle EDC = 95° + 25° = 120°$.



19. b Let the angles of quadrilateral be 2x, 4x, 5x and 7x.

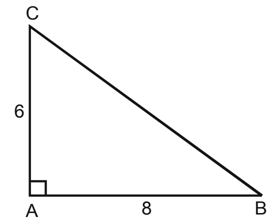
So, sum of all the angles of quadrilateral is 360°

$\therefore$ 2x + 4x + 5x + 7x = 360°

18x = 360° $\Rightarrow$ x = 20°

$\therefore$ smallest angle = 2x = 40°.

20. a



$BC = \sqrt{AC^2 + AB^2} = \sqrt{36 + 64} = 10$

perimeter of ΔABC

= AB + AC + BC = 8 + 6 + 10 = 24 cm.

21. a Incentre: the point at which all the angular bisectors coincide.

Orthocentre: the point at which the three altitudes coincide.

Circumcentre: the point at which the perpendicular bisector of sides coincide.

For an equilateral triangle, they are coincident.

22. b Let the angle be A.

So A = 4(90 – A)

A = 360 – 4A $\Rightarrow$ 5A = 360°

$\therefore$ A = 72°.

23. a $\angle BAC = \angle ACT + \angle ATC = 50 + 30 = 80°$

and $\angle ACT = \angle ABC$ (Angle in alternate segment)

So $\angle ABC = 50°$

$\angle BCA = 180 - (\angle ABC + \angle BAC)$

$= 180 - (50 + 80) = 50°$

Since $\angle BOA = 2\angle BCA = 2 \times 50 = 100°$

Alternative Method :

Join OC

$\angle OCT = 90°$ (TC is tangent to OC)

$\angle OCA = 90° - 50° = 40°$

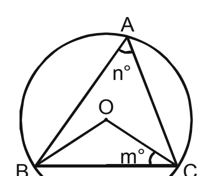
$\angle OAC = 40°$ (OA = OC = radius)

$\angle BAC = 50° + 30° = 80°$

$\angle OAB = 80° - 40° = 40° = \angle OBA$ (OA = OB = radius)

$\angle BOA = 180° - (\angle OBA + \angle OAB) = 100°$

24. d



$\because$ OB = OC [Radius of the same circle]

So $\angle OBC = \angle OCB = m°$

$\therefore \angle BOC = 180 - 2m$ … (i)

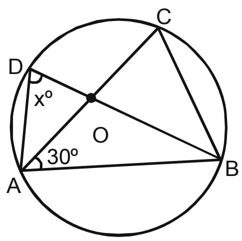
$\because \angle BAC = n°$

$\because \angle BOC = 2\angle BAC = 2n$ … (ii)

Equalising equation (i) and (ii)

180 – 2m = 2n, 2m + 2n = 180° or (m + n) = 90°

25. d $\angle OAB = \angle OBA$

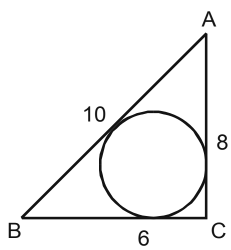


(OA = OB = radius)

$\angle AOB = 180 - 2 \times 30 = 120°$

or $\angle ADB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 120 = 60^{\circ}$

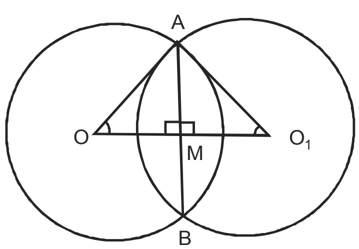
26. b Radius of the inner circle r = $\frac{A}{S}$



where $S = \frac{a+b+c}{2} = \frac{6+8+10}{2} = 12$

and $A = \frac{1}{2} \times 8 \times 6 = 24$ So $r = \frac{24}{12} = 2\text{cm}$

27. d



Length of OM = $\frac{OO_1}{2} = \sqrt{3}\,\text{cm}$

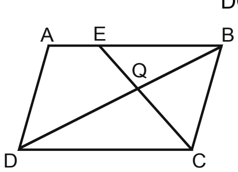
Length of AM = $\frac{AB}{2} = 1\,\text{cm}$

$\tan(\angle OAM) = \frac{\sqrt{3}}{1}$

$\Rightarrow \tan(\angle OAM) = 60°$

Hence, $\angle OAO_1 = 120°$

28. a It is given that 3BE = 2DC or $\frac{BE}{DC} = \frac{2}{3}$

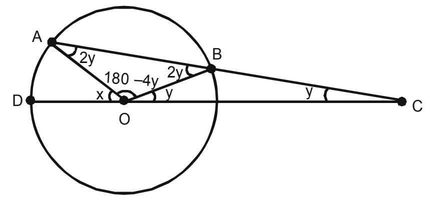


Since ΔDQC and ΔBQE are similar,

$\frac{Ar(\Delta BQE)}{Ar(\Delta DQC)} = \left(\frac{BE}{DC}\right)^2 \therefore \frac{Ar(\Delta BQE)}{36} = \left(\frac{2}{3}\right)^2$

or Ar(ΔBQE) = $36 \times \frac{4}{9} = 16$

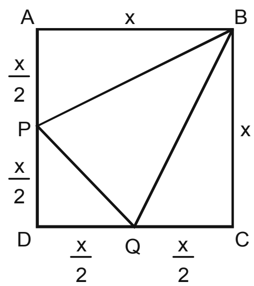
29. a



Therefore, x + 180 – 4y + y = 180

Therefore, x = 3y

30. b



Let us assume the side of the square is x.

So PD = DQ = $\frac{x}{2}$

Area of ΔBPQ = Area of the square – Area of (ΔAPB + ΔPDQ + ΔQBC)

∵ Area of ΔAPB = ΔBCQ = $\frac{x^2}{4}$

Area of ΔPDQ = $\frac{x^2}{8}$

So area of ΔBPQ

$= x^2 - \left(\frac{x^2}{4} + \frac{x^2}{4} + \frac{x^2}{8}\right) = x^2 - \frac{5x^2}{8} = \frac{3x^2}{8}$

So area of ΔBPQ = 37.5% of total area.

Practice Exercise – 2

1. d $a^2 + a^2 = d^2$

d = $a\sqrt{2}$

d = 60

a = $\frac{60}{\sqrt{2}}$

Area of a square is = a × a = $\frac{60 \times 60}{2} = 1800$ m^2.

2. b Perimeter of rectangle = 2 (l + b) = 40 m

l + b = 20

b = 20 – 12 = 8 m

l = length

b = breadth

Area = l × b = 8 × 12 = 96 sq. m.

3. d l = length

b = breadth

l × b = 510

$l = \frac{510}{17} = 30$ m.

Perimeter = 2 (l + b) = 2 × (30 + 17)

= 2 × 47 = 94 m

4. a Let the side of the square be a

$\therefore$ Perimeter of square = 4a = 8x + 40

$a = \frac{8x+40}{4} = 2x+10$

Diagonal $d = a\sqrt{2} = \sqrt{2}\,(2x+10)$

5. b Let length of side of the square = a

and diagonal = d

$\therefore d^2 = a^2 + a^2 \Rightarrow d = a\sqrt{2}$

Sum of lengths of two diagonals = $2a\sqrt{2}$

$\therefore 2a\sqrt{2} = 196 \Rightarrow a = \frac{98}{\sqrt{2}}$

Perimeter of square = 4a = $\frac{4 \times 98}{\sqrt{2}} = 196\sqrt{2}$ cm.

6. a Cost of 100 sq. m = 80 paise

Cost of 1 sq. m = Rs. $\frac{0.8}{100} = \frac{80}{10000}$

Area of square ground = $\frac{\text{Total Cost}}{\text{Cost per m}^2}$

$= \frac{28.8}{80} \times 10000 = 3600\text{ m}^2$

Length of its side = $\sqrt{\text{Area of sq.}} = \sqrt{3600} = 60$ m

Perimeter of square = 4 × side = 4 × 60 = 240 m

Cost of fencing = Rs. 0.6 × 240 = Rs. 144.

7. c Side of the small square $= \sqrt{2.56} = 1.6$ cm

So length = 8 × 1.6 = 12.8 cm.

8. a Length of rectangular lawn = 80 m & its width = 40 m

Area of lawn = 80 × 40 = 3,200 sq. m

Length of lawn including footpath = 80 + 3(2) = 86 m

Breadth of lawn including footpath = 40 + 3(2) = 46 m

Area of lawn including footpath = 86 × 46 = 3956 m^2.

Area of path = Area of lawn including footpath – Area of lawn = 86 × 46 – 3200 = 3956 – 3200 = 756 m^2.

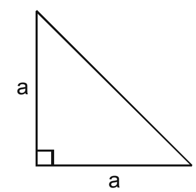
9. d Area of triangle = 32 cm^2.

Base = b = 8 cm.

Area = $\frac{1}{2}$base × height = $\frac{1}{2}$ × 8 × h = 32

h = 8 cm.

10. a Area of isosceles triangle



= $\frac{1}{2}$ × a × a = 800

$a^2 = 1600 \Rightarrow a = 40$ cm.

Hypotenuse = $\sqrt{a^2+a^2} = a\sqrt{2}$

$= 40\sqrt{2}$ cm.

11. c Let the side of triangles be, a, b, and c

So a = 6 cm, b = 7 cm and c = 11 cm

S = semi-perimeter = $\frac{a+b+c}{2} = 12$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-6)(12-7)(12-11)} = \sqrt{12 \times 6 \times 5 \times 1}$

$= \sqrt{3 \times 4 \times 3 \times 2 \times 5} = 3 \times 2\sqrt{10} = 6\sqrt{10}\text{ cm}^2$

12. d Altitude = $\frac{\sqrt{3}}{2} \times \text{side} = \frac{\sqrt{3}}{2} \times 4a = 2\sqrt{3}a$

13. b Let the sides of the trianglular park be

a = 6x, b = 8x and c = 10x

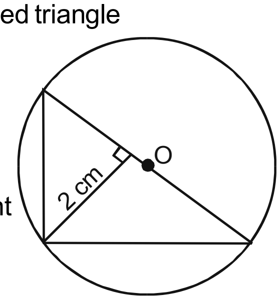
Perimeter of the park = 6x + 8x + 10x = 24x = 240 m

or x = 10

Since, (60, 80 and 100) is the pythagorus triplet

Hence, area = $\frac{1}{2} \times 80 \times 60 = 2400\text{ m}^2$

14. d Hypotenuse of right-angled triangle



= 2 × radius of circle

= 2 × 3 = 6 cm

Altitude = 2 cm

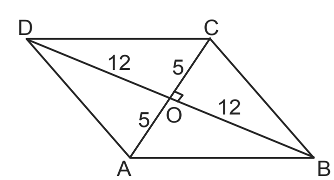
Area = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times 6 \times 2 = 6\text{ cm}^2$

15. c Area of a rhombus = $\frac{1}{2}$(Product of two diagonals).

$\frac{1}{2}$ × 10 × 8 = 40 sq. cm

16. d



Let d_1 and d_2 be lengths of diagonals of rhombus.

ΔAOD in figure is right-angled O.

$AD^2 = AO^2 + OD^2$

$AD^2 = 5^2 + 12^2$

$AD = \sqrt{169} = 13$ cm

17. d Let d_1 and d_2 be two diagonals of a rhombus then,

area of a rhombus = $\frac{1}{2} d_1 \times d_2$

$90 = \frac{1}{2} \times d_1 \times 18 \Rightarrow d_1 = \frac{180}{18} = 10$ cm

18. b Radius of the circle is 28 cm

$\therefore$ Length of wire = Circumference of circle, i.e. $2\pi r$.

$\frac{2 \times 22 \times 28}{7} = 176$ cm.

The circle is bent to form a square, so 176 cm is perimeter of square.

Assuming the side of the square as l,

we get perimeter of square = 4l

4l = 176

l = 44 cm.

19. a Radius of circle = 21 cm

Area = $\pi r^2 = \frac{22}{7} \times 21 \times 21 = 1{,}386\ cm^2$

= Area of new circle is $\frac{1}{4}$th of previous.

i.e. $\frac{1}{4} \times 1386 = 346.5\ cm^2$

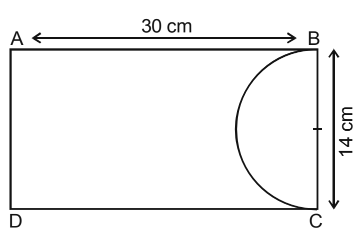
Let radius of new circle = r cm $\therefore$ Area = πr^2

$\pi r^2 = 346.5 \Rightarrow r = \sqrt{\frac{346.5 \times 7}{22}} = 10.5$ cm

Short cut:

Area is one-fourth so, radius should be half = 10.5 cm

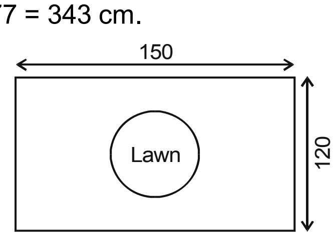
20. b



Area of remaining portion = Area of rectangle – Area of semicircle = $30 \times 14 - \frac{\pi}{2} \times 7 \times 7$

= 422 – 77 = 343 cm.

21. d



Area of park excluding lawn

= Area of rectangle – Area of lawn

$5500 = (120 \times 150) - \frac{22}{7} r^2$

[assuming *r* to be the radius of the circle]

$\Rightarrow \frac{22}{7} r^2 = 18000 - 5500 = 12500$

$\Rightarrow r^2 = \frac{7 \times 12500}{22} \Rightarrow r = 63.06$ m.

22. b Radius of circle = 14

Angle of sector = 18°

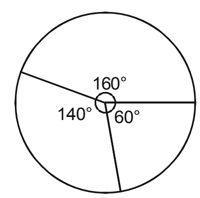
Length of arc of sector = $\frac{\theta}{360} \times 2\pi r$

$= \frac{18}{360} \times 2 \times \frac{22}{7} \times 14 = 4.4$ cm

Area of sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{18}{360} \times \frac{22}{7} \times 14 \times 14 = \frac{308}{10} = 30.8\ cm^2$.

23. d



Area of the sector with central angle 60°

$= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \pi r^2$

Area of sector with central angle 140° = $\frac{140}{360} \times \pi r^2$

Area of sector with central angle 160° = $\frac{160}{360} \times \pi r^2$

Ratio of area of three sectors

$= \frac{60}{360} \times \pi r^2 : \frac{140}{360} \times \pi r^2 : \frac{160}{360} \times \pi r^2$

= 6 : 14 : 16 = 3 : 7 : 8

24. b The longest pole will be the diagonal of the room.

Diagonal = $\sqrt{l^2 + b^2 + h^2} = \sqrt{20^2 + 10^2 + 5^2} = \sqrt{525}$ m

25. d Diagonal = $8\sqrt{3}$ cm

$\sqrt{3}$ × side = Diagonal = $8\sqrt{3}$

side = 8 cm

$\therefore$ Volume of cube = $(8)^3 = 512\ cm^3$

26. a Let 'R' be the radius of the sphere, then

Volume of sphere = $\frac{4}{3} \pi r^3$

and total surface area = $4\pi r^2$

$\Rightarrow \frac{\frac{4}{3} \times \pi r^3}{4\pi r^2} = 5 \Rightarrow r = 3 \times 5 = 15$ cm.

27. d Let ‘r’ and ‘h’ be the radius and height of cylinder

Original volume of cylinder = $\pi r^2 h$

Increased height = $h + \frac{200}{100}h = 3h$

Increased radius = $r + \frac{200}{100}r = 3r$

Changed volume = $\pi(3r)^2 \times 3h = 27\pi r^2 h$

increase = $\pi r^2 h(27-1) = 26\pi r^2 h$.

Hence, 26 times.

28. c Volume of metal sphere is equal to volume of wire formed.

Let ‘L’ be the length of wire, then

$\frac{4}{3}\pi \times 10^3 = \pi \times (0.1)^2 \times L$

$L = \frac{\frac{4}{3}\pi \times 1000}{\pi \times .01} = \frac{4}{3} \times 100000 = 133333.33 \text{ cm}$

= 1333.33 m = 1.33 km

29. c Surface area of cube = 6 × $(\text{side})^2$

6 × $(\text{side})^2$ = 150

$(\text{side})^2 = 25 \Rightarrow$ side = 5 cm

30. b Let the amount of water that has fallen be up to height ‘h’, then 150 × 25 × h = 2250

$h = \frac{2250}{150 \times 25} = 0.6 \text{ m} = 60 \text{ cm.}$

Practice Exercise – 3

1. a Surface area of cube = 6 × $(\text{side})^2$

6 × $(\text{side})^2$ = 216

$(\text{side})^2 = 36 \Rightarrow$ side = 6

Volume of cube = $(\text{side})^3 = 6^3 = 216 \text{ cm}^3$.

2. a Perimeter of circular wire = Perimeter of square

$2 \times \frac{22}{7} \times 28 = 4 \times \text{side}$

side = 44 cm

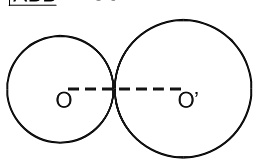
$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\frac{22}{7} \times 28 \times 28}{44 \times 44} = \frac{14}{11}.$

Hence, area of square : area of circle = 11 : 14

3. a $\angle ADB = \angle ACB$ (Angle in same segment is equal)

$\angle ADB$ = 50°

4. c



If the circle touches each other externally, then distance between their centres is equal to sum of their radius, i.e. 5 + 9 = 14 cm.

5. a The largest possible circle will have the breadth of rectangle as its diameter.

Radius of circle = $\frac{\text{diameter}}{2} = \frac{8}{2} = 4 \text{ cm}$

Area of circle = $\pi \times (4)^2 = 16\pi$.

6. a $\Delta DKC \sim \Delta AKB$

Hence $\frac{\text{Area } \Delta AKB}{\text{Area } \Delta DKC} = \frac{AB^2}{CD^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

7. d Let us assume that the initial side of cube be x

Volume = x^3

New side = 1.4x

Volume = $(1.4x)^3 = 2.744x^3$

Percentage increase in volume

$= \frac{2.744x^3 - x^3}{x^3} \times 100 = 174.4\%$

8. d Let ‘l’ and ‘b’ be original length and width of the rectangle, then

area = l × b

Increased length = $\frac{160}{100}l = \frac{8}{5}l$

New area = $\frac{8}{5}l \times b = \frac{8}{5}l \times b$

$\frac{\text{New area of rectangle}}{\text{Initial area of rectangle}} = \frac{\frac{8}{5}l \times b}{lb} = \frac{8}{5} = 8 : 5.$

9. a Let PD = x m

PA × PB = PC × PD (Chord theoram)

$\Rightarrow$ 5 × 12 = 6 × x $\Rightarrow$ x = 10

CD = 4 cm.

10. a Let the length and the width of a rectangle are increased by a% and b% respectively, then percentage increase in its area

$= a + b + \frac{a \times b}{100} = 10 + 20 + \frac{10 \times 20}{100} = 32\%$

11. a Let the radius and height of the cylinder increases by *a*% and *b*% respectively.

Volume = $\pi r^2 h$

Hence, there are two successive increase in radius followed by one successive increase in height. So, first of all we can calculate for radius and then the result with height.

Percentage increase in the base area

$a + b + \frac{ab}{100} = 10 + 10 + \frac{100}{100} = 21\%$

and then percentage increase in volume

$= 21 + 20 + \frac{20 \times 21}{100} = 45.2\%$

12. b By using chord formula
OB × OA = OC × OD
OB × 8 = 6 × 4, we get OB = 3 cm.

13. b If side of cube is x, then radius of the sphere which will fit exactly inside the cube is $\frac{x}{2}$
$\therefore$ ratio of their volumes
$= x^3 : \frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = 1 : \frac{4\pi}{24} = 1 : \frac{\pi}{6} = 6 : \pi$

14. a The sum of two sides of a triangle should be greater than the third side.

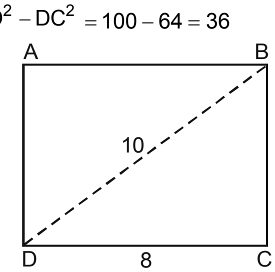
15. c Perpendicular from centre to chord bisects the chord.
BC = 12 cm, so BO = OC = 6 cm.

16. c Let r_1, r_2 and h_1, h_2 be radii and heights of the two cones respectively.
$\therefore \frac{r_1}{r_2} = \frac{2}{3}$ and $\pi(r_1)^2 \times h_1 = \pi(r_2)^2 \times h_2$
$\Rightarrow \left(\frac{2}{3}r_2\right)^2 \times h_1 = \pi(r_2)^2 \times h_2$
$\frac{h_1}{h_2} = \frac{9}{4}.$

17. d Circumference of circular bed = 22 cm
Let ‘r’ be the radius of bed,
then $2\pi r = 22$
$2 \times \frac{22}{7} \times r = 22$
r = 3.5
Area = $\frac{22}{7} \times (3.5)^2 = 38.5$
Number of plants = $\frac{38.5}{7} \approx 5.5$.
But as only integral number of trees can be planted
$\therefore$ Number of trees = 5.

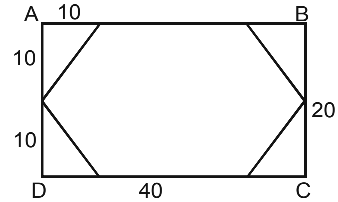
18. a We have
AB × AO = AD × AC
$\Rightarrow$ 6 × 2 = AD ×3
AD = $\frac{12}{3}$ = 4 cm
$\because$ CD = AD – AC = 4 – 3 = 1 cm.

19. b $BC^2 = BD^2 - DC^2 = 100 - 64 = 36$



$\Rightarrow BC = \sqrt{36} = 6$
Area = 6 × 8 = 48 m^2

20. b



Area of the remaining portion
= Area of rectangle – Area of 4 isosceles triangle.
= 40 × 20 – 4 × $\left(\frac{1}{2} \times 10 \times 10\right)$ = 800 – 200 = 600 m^2

21. d $\angle ACD = \angle ABD = 90°$

22. c Area of road parallel to length = 40 × 5 = 200 m^2
Area of road parallel to width = 30 × 5 = 150 m^2
Area of the common portion = 5 × 5 = 25 m^2
Cost of gravelling = $(350 - 25) \times \frac{15}{100}$
$= 325 \times \frac{15}{100} = \frac{195}{4} = \text{Rs. } 48.75$

23. c Suppose that each side of equilateral triangle is ‘a’.
So area of the triangle = $\frac{\sqrt{3}}{4}a^2$
Let side of a square = x
So diagonal = $\sqrt{2}x$ but given that diagonal = 2a
So $\sqrt{2}x = 2a$ and $x = \sqrt{2}a$
So area of a square = $\left(\sqrt{2}a\right)^2 = 2a^2$
So $\frac{\text{Area of triangle}}{\text{Area of square}} = \frac{\frac{\sqrt{3}}{4}a^2}{2a^2} = \sqrt{3} : 8$

24. d $\angle ADB = 40°$
$\angle DCA = 70°$
$\angle ABD = 70°$ {Angles in the same segment of circle are equal}
In Δ ADB
$\because \angle ADB + \angle A + \angle ABD = 180°$
40° + A + 70° = 180°
$\angle A$ = 180 –110 = 70°

25. c In Δ ADQ, Sin 30° = $\frac{AD}{AQ}$
AQ = AD × 2 = 160 cm
In Δ APB
Sin 30° = $\frac{AB}{AP}$
AP = 2 × AB, AP = 140 cm
$\therefore$ AP + AQ = 300 cm.

26. c Suppose that the diameters of the bases for all the three be 'a' units and height be 'b' units.

For cylinder, radius = $\frac{a}{2}$ units and height b = $\frac{a}{2}$ units.

For cones, radius = $\frac{a}{2}$ units and height b = $\frac{a}{2}$ units.

For hemisphere, radius = $\frac{a}{2}$ units and height b = $\frac{a}{2}$ units

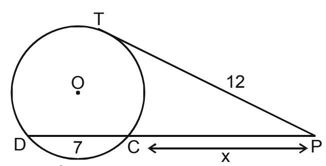
So ratio of their volumes

$= \frac{1}{3}\pi\frac{a^2}{4}\times\frac{a}{2}:\frac{2}{3}\pi\frac{a^3}{8}:\frac{a^2}{4}\times\frac{a}{2} = 1:\ 2\ :3$

27. c R = $\frac{abc}{4\Delta}$ (where Δ is area of triangle)

$= \frac{780}{4\times 30} = 6.5$ cm

28. b



PC × PD = PT^2

$\Rightarrow$ x × (x + 7) = 12^2

$\Rightarrow$ x^2 + 7x – 144 = 0

$\Rightarrow$ (x + 16) (x – 9) = 0

$\Rightarrow$ x = 9

29. c Area of four walls =2(l+b)×h = 2(8+6) × 4.5 = 126 m^2

Wallpapered area = 126 – 2 (2 × 1) – (3 × 2)

126 – 4 – 6 = 116 = 116 m^2

Length of the paper = $\frac{116}{0.4} = 290$ m

Cost = 290 × 1.25 = Rs. 362.50.

30. d Let $\angle$B and $\angle$C be 3x and 4x

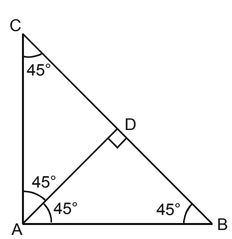
Then 3x + 4x = 140

x = 20°

(Sum of opposite interior angles is equal to exterior angle)

$\therefore$ $\angle$B = 3x = 3 × 20 = 60° and $\angle$C = 4x = 4 × 20 = 80°.

31. d



As $\angle$DAB = $\angle$B

$\therefore$ ΔADB is an isosceles triangle, i.e. BD = AD

$\angle$DAC = $\angle$ACD = 45°, i.e. AD = CD

$\therefore$ AD = BD = CD.

32. b In 5000 revolutions = 11000 m

In 1 revolution = $\frac{11000}{5000} = 2.2$ m

1 revolution = $2\pi r$

$2\times\frac{22}{7}\times r = 2.2$ m

r = $\frac{2.2\times 7}{22\times 2} = \frac{7}{20}$ m $= \frac{7}{20}\times 100 = 35$ cm.

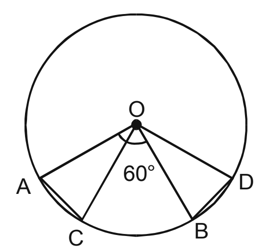
33. a The diameter of the inscribed circle is equal to the length of the side of the square.

So the radius of the inscribed circle = 7 cm

So the area of the inscribed circle = πr^2

= $\frac{22}{7}\times 7\times 7 = 154$ cm^2.

34. a AC = BD



Equal chords make equal angles with the centre.

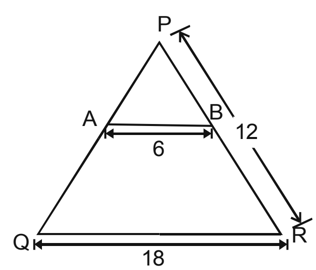
$\angle$AOB = 60

$\angle$BOC = 15

$\therefore$ $\angle$AOC = 45°

$\angle$AOC = $\angle$BOD = 45°

35. d AB || QR



$\therefore$ ΔAPB and ΔQPR are similar triangles

$\therefore \frac{AB}{QR} = \frac{PB}{PR}$; $\frac{6}{18} = \frac{PB}{12}$

PB = $\frac{12\times 6}{18}$ = 4.