03

The evolution of concept of matrices is the results of an attempt to obtain compact and simple method of solving system of linear equations. Matrix notation and operations are used in electronic spreadsheet, advanced statistics, etc. In this chapter, we shall study about fundamentals of matrix and matrix algebra.

MATRICES

|TOPIC 1|
Matrix and Its Types

CHAPTER CHECKLIST

- Matrix and Its Types
- Addition and Scalar Multiplication of Matrices
- Multiplication of Matrices
- Transpose of a Matrix, Symmetric, Skew-symmetric Matrices and Invertible Matrices

MATRIX

A matrix is an ordered **rectangular array** of numbers or functions. Such an array is enclosed by [ ] or ( ). The numbers (real or complex) or functions are called the **elements** or the **entries** of the matrix. A matrix is represented by a single **capital letter** like A, B, X, Y, etc., and elements of a matrix are represented by the small letters $a, b, c, \ldots$ etc. In a matrix, horizontal lines are called **rows** and vertical lines are called **columns**.

e.g. $$A = \begin{bmatrix} 2 & -5 \\ 7 & 0 \\ \sqrt{5} & 3 \end{bmatrix} \begin{array}{l} \rightarrow \text{First row } (R_1) \\ \rightarrow \text{Second row } (R_2) \\ \rightarrow \text{Third row } (R_3) \end{array} \text{ and } B = \begin{bmatrix} 2+3i & 7 & -2/3 \\ 4.2 & -1 & 0 \\ \sqrt{3} & 5 & 5/7 \end{bmatrix} \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{array}$$

$\downarrow$ First column (C_1) $\downarrow$ Second column (C_2); $\downarrow C_1$ $\downarrow C_2$ $\downarrow C_3$

Note In this chapter, we will consider only those matrices whose elements or entries are real numbers or functions taking real values.

EXAMPLE |1| A shopkeeper sells 5 rings, 8 bracelets and 10 necklaces in first week and 11 rings, 6 bracelets and 9 necklaces in next week. Represent this information in a matrix having 2 rows and 3 columns.

Sol. In table form, given information can be written as

	Rings	Bracelets	Necklaces
First week	5	8	10
Next week	11	6	9

Here, we see that table has 2 rows and 3 columns, so given information in a matrix can be written as

$$\begin{array}{c c} \begin{array}{r} \text{First week} \\ \text{Next week} \end{array} & \begin{bmatrix} 5 & 8 & 10 \\ 11 & 6 & 9 \end{bmatrix} \\ & \begin{array}{ccc} \text{Rings} & \text{Bracelets} & \text{Necklaces} \end{array} \end{array}$$

Sol. (i) Here, the matrix A has 3 rows and 4 columns, so the order of the matrix is 3×4.

(ii) Since, a_{ij} represents the element in ith row and jth column.

$\therefore \quad a_{13}$ = element in first row and third column

$\Rightarrow \quad a_{13} = 19$

Representation of Point(s) in Matrix Form

We can also represent any point (x, y) in a plane by a matrix (column or row) as $\begin{bmatrix} x \\ y \end{bmatrix}$ or $[x\ y]$. e.g. Suppose quadrilateral $ABCD$ have coordinates $A(3, 0)$, $B(-1, 2)$, $C(2, -2)$ and $D(-3, 2)$, we can represent it in matrix form as

$$X = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 3 & -1 & 2 & -3 \\ 0 & 2 & -2 & 2 \end{bmatrix} \end{array} \quad \text{or} \quad Y = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 2 & -2 \\ -3 & 2 \end{bmatrix}$$

Thus, matrices can be used as representation of vertices of geometrical figures in a plane.

Order of a Matrix

Suppose a matrix have m rows and n columns, then order of matrix is written as $m \times n$, where m represents number of rows and n represents number of columns, and read as m by n matrix. In general, $m \times n$ matrix has the following rectangular array

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

In notation form, it can be rewritten as $A = [a_{ij}]_{m \times n}$, where $1 \le i \le m$, $1 \le j \le n$ and $i, j \in N$.

Here, the ith row consists of the elements $a_{i1}, a_{i2}, a_{i3}, \ldots, a_{in}$, while jth column consists of the elements $a_{1j}, a_{2j}, a_{3j}, \ldots, a_{mj}$. In general, a_{ij} is an element lying in the ith row and jth column. We can also call it as (i, j)th element of matrix A. The number of elements in $m \times n$ matrix will be equal to mn.

e.g. $A = \begin{bmatrix} 2 & 0 \\ 4 & -3 \\ -7 & 1 \end{bmatrix}$

Here, A has 3 rows and 2 columns. So, order of matrix is 3×2 and it has $3 \times 2 = 6$ elements.

EXAMPLE |2| In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$,

(i) write the order of the matrix.

(ii) write the value of elements a_{13}, a_{21}, a_{23} and a_{33}. **[NCERT]**

On putting all above values in Eq. (i), we get

$$A = \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & \frac{2}{3} \\ \frac{5}{3} & \frac{4}{3} \end{bmatrix}$$

which is the required 3×2 order matrix.

Similarly, $a_{21} = 35$, $a_{23} = \frac{5}{2}$ and $a_{33} = -5$

METHOD TO FIND ORDER OF MATRIX WHEN NUMBER OF ELEMENTS IS GIVEN

Suppose number of elements (say P) of a matrix is given, then to find the possible orders of matrix, we use the following steps

I. As matrix of order $m \times n$ has mn elements. So, firstly put mn = number of elements, P.

II. Find all possible factors of P.

III. Now, form all possible ordered pairs with the help of these factors such that the product of elements of each ordered pair is P. These ordered pairs gives the possible orders of given matrix.

EXAMPLE |3| If a matrix has 16 elements, then what are the possible orders it can have?

Sol. Given number of elements, $mn = 16$

All factors of 16 are 1, 2, 4, 8 and 16.

Possible ordered pairs, whose element's product is 16, are (1, 16), (2, 8), (4, 4), (8, 2) and (16, 1).

Hence, the possible orders of matrix are

1×16, 2×8, 4×4, 8×2 and 16×1.

Formation of a Matrix

Sometimes, a relation between the elements and its position is given to us. Then, to form a matrix, whose elements satisfies this relation, we put different values of i and j according to given order to find all elements and then write the matrix of given order.

EXAMPLE |4| Construct a matrix of order 3×2, whose elements are determined by $a_{ij} = \frac{2i - j}{3}$.

Sol. Let A be a matrix of order 3×2.

Then, $A = [a_{ij}]_{3 \times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ ...(i)

Given, $a_{ij} = \frac{2i - j}{3}$

$\therefore \quad a_{11} = \frac{2(1) - 1}{3} = \frac{1}{3}$, $a_{12} = \frac{2(1) - 2}{3} = 0$,

$a_{21} = \frac{2(2) - 1}{3} = 1$, $a_{22} = \frac{2(2) - 2}{3} = \frac{2}{3}$,

$a_{31} = \frac{2(3) - 1}{3} = \frac{5}{3}$ and $a_{32} = \frac{2(3) - 2}{3} = \frac{4}{3}$

e.g. $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}_{3 \times 3}$ $\quad [a_{ij} = 0, \text{ for } i \ne j]$

In above, A can be written as $A = \text{diag}\,[a, b, c]$.

A diagonal matrix of order $m \times m$ having $a_{11}, a_{22}, \ldots, a_{mm}$ as diagonal elements is denoted by $\text{diag}\,[a_{11}, a_{22}, \ldots, a_{mm}]$.

TYPES OF MATRICES

There are various types of matrices, which are given below

(i) **Row Matrix** A matrix having only one row is called a row matrix.

e.g. $A = [1 \quad 2 \quad 3]_{1\times 3}$ [one row and three columns]

In general, row matrix $A = [a_{ij}]_{1\times n}$
where, n is number of columns.

(ii) **Column Matrix** A matrix having only one column is called a column matrix.

e.g. $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$ [one column and three rows]

In general, column matrix $B = [b_{ij}]_{m\times 1}$
where, m is number of rows.

(iii) **Zero or Null Matrix** If all the elements of a matrix are zero, then it is called a zero or null matrix. It is denoted by letter O.

e.g. $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$ $[a_{ij} = 0, \forall\, i \text{ and } j]$

(iv) **Square Matrix** A matrix in which number of rows and number of columns are equal, is called a square matrix.

e.g. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$ [order is 2×2]

In general, square matrix $A = [a_{ij}]_{m\times m}$ or $[a_{ij}]_{n\times n}$
where, number of rows = number of columns
$= m$ or n.

Note

(i) A matrix of order $n \times n$ is also known as a **square matrix** of order 'n'.

(ii) If $A = [a_{ij}]$ is any square matrix of order n, then the elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$ are said to constitute the diagonal of the matrix A. We call these elements as diagonal elements.

(v) **Diagonal Matrix** A square matrix $A = [a_{ij}]_{m\times m}$ is said to be a diagonal matrix, if all the elements lying outside the diagonal elements are zero.

(iii) We have, $\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$. Here, we see that matrix has two rows and two columns. So, it is a square matrix.

(iv) We have, $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Here, we see that all diagonal

(vi) **Scalar Matrix** A diagonal matrix in which all diagonal elements are equal, is called a scalar matrix. In other words, a square matrix $A = [a_{ij}]_{m\times m}$ is said to be scalar, if

$$a_{ij} = \begin{cases} 0, & \text{for } i \neq j \\ \text{constant } k, & \text{for } i = j \end{cases}.$$

e.g. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3\times 3}$

$[a_{ij} = 2, \text{ when } i = j \text{ and } a_{ij} = 0, \text{ when } i \neq j]$

(vii) **Unit or Identity Matrix** A square matrix in which all diagonals elements are 1 and rest are 0, is called an identity matrix. In other words, a square matrix $A = [a_{ij}]_{n\times n}$ is an identity matrix, if

$$a_{ij} = \begin{cases} 1, & \text{for } i = j \\ 0, & \text{for } i \neq j \end{cases}.$$

Identity matrix of order n is denoted by I_n or simply I (when order is clear from context).

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$[a_{ij} = 1, \text{ when } i = j \text{ and } a_{ij} = 0, \text{ when } i \neq j]$

Note A scalar matrix is an identity matrix, when $k = 1$ and every identity matrix is always a scalar matrix.

EXAMPLE |5| Identify the following types of matrices.

(i) $[1 \quad 4 \quad 5]$ (ii) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

(iv) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Sol. (i) We have, $[1 \; 4 \; 5]$. Here, we see that matrix has only one row and three columns, so it is a row matrix.

(ii) We have, $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$. Here, we see that matrix has three rows and only one column. So, it is a column matrix.

On solving Eqs. (i) and (iii), we get

$$x = 1 \text{ and } y = 2$$

On putting the value of x in Eq. (ii), we get

$$2 \times 1 + z = 5$$

$$\Rightarrow \quad z = 5 - 2 = 3$$

On putting the value of z in Eq. (iv), we get elements of square matrix are equal but different from one and non-diagonal elements are zero. So, it is a scalar matrix.

(v) We have, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Here, we see that diagonal elements are not equal and non-diagonal elements are zero. So, it is a diagonal matrix.

EQUALITY OF MATRICES

Two matrices, say $A=[a_{ij}]$ and $B=[b_{ij}]$, are said to be equal, if their order are same and their corresponding elements are also equal, i.e. $a_{ij}=b_{ij}, \forall\, i$ and j. Symbolically, if two matrices A and B are equal, then we write $A=B$.

e.g. $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ are equal matrices, but $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$ are not equal matrices because corresponding elements are not equal.

Note Two matrices A and B are said to be **comparable**, if they are of same order.

EXAMPLE |6| Find the values of a, b and c, when

$$\begin{bmatrix} a & 3 \\ 2 & c \end{bmatrix}=\begin{bmatrix} 1 & b \\ 2 & 2 \end{bmatrix}.$$

Sol. Given, $\begin{bmatrix} a & 3 \\ 2 & c \end{bmatrix}=\begin{bmatrix} 1 & b \\ 2 & 2 \end{bmatrix}$

Here, both matrices are equal, so we equate the corresponding elements.

Now, $a=1$, $3=b$ and $c=2$

i.e. $a=1$, $b=3$ and $c=2$

EXAMPLE |7| If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix}=\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, then find x, y, z and w. **[Delhi 2013]**

Sol. Given, $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix}=\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Here, both matrices are equal, so we equate the corresponding elements

Now, $x-y=-1$ ...(i)

$2x+z=5$ ...(ii)

$2x-y=0$ ...(iii)

and $3z+w=13$ ...(iv)

$3\times 3+w=13$

$\Rightarrow$ $w=13-9=4$

Hence, $x=1$, $y=2$, $z=3$ and $w=4$.

TOPIC PRACTICE 1

OBJECTIVE TYPE QUESTIONS

1 If A is a 3×2 matrix, B is a 3×3 matrix and C is a 2×3 matrix, then the elements in A, B and C are respectively

(a) 6, 9, 8 (b) 6, 9, 6

(c) 9, 6, 6 (d) 6, 6, 9

2 If a matrix has 8 elements, then which of the following will not be a possible order of the matrix?

(a) 1×8 (b) 2×4

(c) 4×2 (d) 4×4

3 Total number of possible matrices of order 3×3 with each entry 2 or 0 is **[NCERT Exemplar]**

(a) 9 (b) 27

(c) 81 (d) 512

4 The matrix $P=\begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is not a **[NCERT Exemplar]**

(a) square matrix (b) diagonal matrix

(c) unit matrix (d) None of these

5 Which of the given values of x and y make the following pair of matrices equal $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$? **[NCERT]**

(a) $x=\frac{-1}{3}, y=7$

(b) not possible to find

(c) $y=7, x=\frac{-2}{3}$

(d) $x=\frac{-1}{3}, y=\frac{-2}{3}$

VERY SHORT ANSWER Type Questions

6 The sale figure of three car dealers during March 2015 showed that dealer A solds 6 deluxe, 4 premium and 5 standard cars, dealer B solds

SHORT ANSWER Type I Questions

16 In the matrix, $A=\begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2-y \\ 0 & 5 & -2/5 \end{bmatrix}$. Write

8 deluxe, 3 premium and 4 standard cars and dealer C solds 4 deluxe, 2 premium and 3 standard cars. Write 3×3 matrices summarising sales data for March.

7 Write the order of each of the following matrices.

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$

(ii) $[-1]$

8 Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3. **[All India 2016]**

9 Write the element a_{23} of a 3×3 matrix $A=[a_{ij}]$, whose elements a_{ij} are given by $a_{ij}=\dfrac{|i-j|}{2}$. **[Delhi 2015]**

10 The elements a_{ij} of a 3×3 matrix are given by $a_{ij}=\dfrac{1}{2}\,|-3i+j|$. Write the value of element a_{32}. **[All India 2014C]**

11 Construct a matrix of order 2×2, whose elements are given by $a_{ij}=\dfrac{(i-2j)^2}{2}$. **[NCERT Exemplar]**

12 If matrix $A=[a_{ij}]_{2\times 2}$, where $a_{ij}=\begin{cases}2, & i\neq j\\ 0, & i=j\end{cases}$, then write the matrix A. **[Delhi 2016C]**

13 If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix}=\begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then find the value of $x\cdot y$. **[Delhi 2020]**

14 If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}=\begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$, then write the value of $a-2b$. **[Foreign 2014]**

15 If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix}=\begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, then find the value of $x+y$. **[All India 2014]**

(i) the order of the matrix A.

(ii) the number of elements.

(iii) the value of elements a_{23}, a_{31} and a_{12}. **[NCERT Exemplar]**

17 If a matrix has 24 elements, then what are the possible orders it can have? What if it has 13 elements? **[NCERT]**

18 If a matrix has 14 elements, then what are the possible orders it can have? What if it has 17 elements? **[NCERT Exemplar]**

19 Construct a 3×4 matrix, whose elements are given by $a_{ij}=\dfrac{1}{2}|-3i+j|$. **[NCERT]**

20 Construct a 3×2 matrix, whose elements are given by $a_{ij}=e^{ix}\sin jx$. **[NCERT Exemplar]**

21 Find the values of a, b, c and d from the equation $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix}=\begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$ **[NCERT]**

22 Find the values of x, y and z, if $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$. **[NCERT]**

23 Find the values of x, y and z, if $\begin{bmatrix} -2x+y \\ x+y+z \\ x+y \end{bmatrix}=\begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$.

24 Find the values of a, b, c and d, if

$$\begin{bmatrix} a+b+c+d \\ a+c-d \\ b-c+d \\ a+d \end{bmatrix}=\begin{bmatrix} 4 \\ 1 \\ 1 \\ 2 \end{bmatrix}.$$

SHORT ANSWER Type II Questions

25 For what values of x and y are the following matrices equal?

$A=\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix}$ and $B=\begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

26 If $\begin{bmatrix} x+y & z \\ 5 & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find the values of x and y.

HINTS & SOLUTIONS

1. (b) The number of elements in $m\times n$ matrix is equal to mn.
2. (d) We know that if a matrix is of order $m\times n$, then it has mn elements. Thus, to find all possible orders of a matrix with 8 elements, we will find all ordered pairs of natural numbers, whose product is 8. Thus, all possible ordered pairs are (1, 8), (8, 1), (2, 4), (4, 2).
3. (d) Number of entries in 3×3 matrix is 9. Since, each entry has 2 choices, namely 2 or 0. Therefore, number of possible matrices $=\underbrace{2\times 2\times 2\ldots\times 2}_{9\text{ times}}=2^9=512$

13. We have, $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix}=\begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$

$\Rightarrow\quad x+y=2$ and $x-y=4$

Since, $(x+y)^2-(x-y)^2=4xy$

$\therefore\quad 2^2-4^2=4xy$

$\Rightarrow\quad 4xy=4-16=-12$

$\Rightarrow\quad xy=-3$

14. Similar as Example 7. [**Ans.** 0]

15. Similar as Example 7. [**Ans.** $x+y=3$]

16. Similar as Example 2.
[**Ans.** (i) 3×3 (ii) 9 (iii) $a_{23}=x^2-y$, $a_{31}=0$, $a_{12}=1$]

4. (c) If square matrix in which all diagonals elements are 1 and rest are 0, is called unit matrix.

5. (b) Consider, $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

$\Rightarrow 3x+7=0; 5=y-2; y+1=8; 2-3x=4$

$\Rightarrow \quad x = \frac{-7}{3}; y=7; y=7; x=\frac{-2}{3}$

Hence, we have two different values of x, which is not possible.

6. Similar as Example 1.

$$\left[\textbf{Ans.} \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{ccc} \text{Deluxe} & \text{Premium} & \text{Standard} \\ \end{array}\begin{bmatrix} 6 & 4 & 5 \\ 8 & 3 & 4 \\ 4 & 2 & 3 \end{bmatrix}\right]$$

7. Similar as Example 2 (i). [**Ans.** (i) 2×3 (ii) 1×1]

8. We know that a matrix of order 2×2 has 4 entries. Since, each entry has 3 choices, namely 1, 2 or 3, therefore number of required matrices

$$= 3\times3\times3\times3 = 3^4 = 81$$

9. Given, $a_{ij} = \frac{|i-j|}{2}$

$\therefore \quad a_{23} = \frac{|2-3|}{2} = \frac{|-1|}{2} = \frac{1}{2}$ [put $i=2$ and $j=3$]

10. Solve as Question 9. $\left[\textbf{Ans.} \frac{7}{2}\right]$

11. Similar as Example 4. $\left[\textbf{Ans.} \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}\right]$

12. Clearly, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now, $a_{11}=0, a_{12}=2, a_{21}=2$ and $a_{22}=0$

$\left[\because a_{ij} = \begin{cases} 2, \text{ for } i \neq j \\ 0, \text{ for } i = j \end{cases}\right]$

$\therefore \quad A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

17. The possible orders of a matrix having 24 elements are 1×24, 24×1, 2×12, 12×2, 3×8, 8×3, 4×6 and 6×4.

The possible orders of a matrix having 13 elements are 1×13 and 13×1.

18. Solve as Question 17.

[**Ans.** 1×14, 14×1, 2×7, 7×2; 1×17, 17×1]

19. Let A be a 3×4 matrix.

Then, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}$ ...(i)

Here, $a_{ij} = \frac{1}{2}|-3i+j|$

$\therefore \quad a_{11} = \frac{1}{2}|-3+1| = 1, a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2},$

$a_{13} = \frac{1}{2}|-3+3| = 0, a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2},$

$a_{21} = \frac{1}{2}|-6+1| = \frac{5}{2}, a_{22} = \frac{1}{2}|-6+2| = 2,$

$a_{23} = \frac{1}{2}|-6+3| = \frac{3}{2}, a_{24} = \frac{1}{2}|-6+4| = 1,$

$a_{31} = \frac{1}{2}|-9+1| = 4, a_{32} = \frac{1}{2}|-9+2| = \frac{7}{2},$

$a_{33} = \frac{1}{2}|-9+3| = 3$ and $a_{34} = \frac{1}{2}|-9+4| = \frac{5}{2}$

On putting the values in Eq. (i), we get the required matrix, i.e. $A = \begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \end{bmatrix}_{3\times4}$

20. Solve as Question 19. $\left[\textbf{Ans.} \begin{bmatrix} e^x \sin x & e^x \sin 2x \\ e^{2x} \sin x & e^{2x} \sin 2x \\ e^{3x} \sin x & e^{3x} \sin 2x \end{bmatrix}\right]$

21. Similar as Example 7.

[**Ans.** $a=1, b=2, c=3$ and $d=4$]

22. Here, two matrices are equal. Therefore, equating the corresponding elements of two matrices, we get

$$x+y+z=9 \quad \text{...(i)}$$
$$x+z=5 \quad \text{...(ii)}$$
and $$y+z=7 \quad \text{...(iii)}$$

On solving Eqs. (i) and (ii), we get

$$y = 9-5 = 4$$

On putting $y=4$ in Eq. (iii), we get

$$4+z=7 \Rightarrow z=7-4=3$$

On putting $z=3$ in Eq. (ii), we get

$$x+3=5 \Rightarrow x=5-3=2$$

Hence, $x=2$, $y=4$ and $z=3$.

23. Solve as Question 22.

[**Ans.** $x=2$, $y=1$ and $z=0$]

24. Solve as Question 22.

[**Ans.** $a=1$, $b=1$, $c=1$ and $d=1$]

25. Consider, $A=B$, then we get

$$\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

$$3y = y^2+2 \quad \text{...(ii)}$$
and $$y^2-5y = -6 \quad \text{...(iii)}$$

[$\because$ corresponding elements of two equal matrices are equal]

Now, from Eq. (i), we get

$$2x+1 = x+3 \Rightarrow 2x-x = 3-1$$

$\Rightarrow \quad x = 2$

Now, from Eq. (ii), we get

$$y^2-3y+2=0$$

$\Rightarrow \quad (y-1)(y-2)=0$

$\Rightarrow \quad y=1$ or $y=2$

Now, from Eq. (iii), we get

$$y^2-5y+6=0$$

$\Rightarrow \quad (y-2)(y-3)=0$

$\Rightarrow \quad y=2$ or $y=3$

Hence, $x=2$ and $y=2$.

[$\because y=1, 2$ and $y=2, 3 \Rightarrow y=2$]

26. **Hint** Use the identity $(x-y)^2 = (x+y)^2 - 4xy$.

$\Rightarrow \quad 2x+1 = x+3 \quad \ldots(i)$ **[Ans.** $x = 4,\ y = 2$ or $x = 2,\ y = 4$**]**

|TOPIC 2|

Addition and Scalar Multiplication of Matrices

Here, we will discuss certain operations on matrices namely, addition of matrices, multiplication of a matrix by a scalar and difference of matrices.

ADDITION OF MATRICES

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be the two matrices of the same order, say $m \times n$. Then, the sum of two matrices A and B is defined as a matrix $C = [c_{ij}]_{m\times n}$, where $c_{ij} = a_{ij} + b_{ij}$ for all possible values of i and j, i.e. the sum of matrices A and B is a matrix whose elements are obtained by adding the corresponding elements of A and B.

e.g. $$\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 6 & 9 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 2+6 & 3+9 \\ 0+(-4) & -1+1 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 0 \end{bmatrix}$$

Note

(i) If A and B are not of the same order, then $A + B$ is not defined.
e.g. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$, then $A + B$ is not defined.

(ii) Addition of matrices is an example of binary operation on the set of matrices of the same order.

EXAMPLE |1| Find the sum of two matrices A and B, if
$$A = \begin{bmatrix} 1 & \sqrt{2} \\ 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -\sqrt{2} \\ 4 & 1 \end{bmatrix}.$$

Sol. Given, matrices are $A = \begin{bmatrix} 1 & \sqrt{2} \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -\sqrt{2} \\ 4 & 1 \end{bmatrix}$

Here, we see that both matrices are of same order, therefore sum of A and B is defined.

$$\text{Now, } A+B = \begin{bmatrix} 1 & \sqrt{2} \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -\sqrt{2} \\ 4 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1+3 & \sqrt{2}-\sqrt{2} \\ 3+4 & 2+1 \end{bmatrix} \quad \left[\because \text{adding corresponding elements of } A \text{ and } B\right]$$
$$= \begin{bmatrix} 4 & 0 \\ 7 & 3 \end{bmatrix}$$

MULTIPLICATION OF A MATRIX BY A SCALAR

Let $A = [a_{ij}]_{m\times n}$ be a matrix and k be a scalar, then kA is a another matrix which is obtained by multiplying each element of A by the scalar k.

Thus, $kA = k[a_{ij}]_{m\times n} = [k(a_{ij})]_{m\times n}$

i.e. (i, j)th element of kA is ka_{ij} for all possible values of i and j.

e.g. If $A = \begin{bmatrix} 7 & 18 \\ -1 & 3 \end{bmatrix}$ and $k = 3$.

$$\text{Then, } kA = 3A = 3\begin{bmatrix} 7 & 18 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 3\times 7 & 3\times 18 \\ 3\times(-1) & 3\times 3 \end{bmatrix} = \begin{bmatrix} 21 & 54 \\ -3 & 9 \end{bmatrix}$$

EXAMPLE |2| If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then find $3A + 4B$.

Sol. We have, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$$\text{Now, } 3A+4B = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + 4\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

Negative of a Matrix

If we multiply a matrix A by a scalar quantity (-1), then the negative of a matrix (i.e. $-A$) is obtained.

In negative of A, each element is multiply by (-1).

e.g. Let $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, then $-A = (-1)A = \begin{bmatrix} -1 & -3 \\ 2 & -4 \end{bmatrix}$

Difference (or Subtraction) of Matrices

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be the two matrices of the same order (say $m \times n$), then the difference of these matrices, $A - B$ is defined as a matrix $D = [d_{ij}]$, where $d_{ij} = a_{ij} - b_{ij}$ for all values of i and j.

In other words, $D = A - B = A + (-1)B$

i.e. $D =$ The sum of the matrix A and the matrix $(-B)$.

e.g. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$$\text{Then, } A-B = A+(-1)B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + (-1)\begin{bmatrix} e & f \\ g & h \end{bmatrix}$$
$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -e & -f \\ -g & -h \end{bmatrix}$$
$$= \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

$$= \begin{bmatrix} 3+8 & 0 & 0 \\ 0 & -3+12 & 0 \\ 0 & 0 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

EXAMPLE |3| If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then find the value of $(x + y)$. **[Delhi 2013C; All India 2012]**

Sol. Given, $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing the corresponding elements, we get

$$2 + y = 5 \text{ and } 2x + 2 = 8$$

$$\Rightarrow \quad y = 5 - 2 = 3 \text{ and } 2x = 8 - 2$$

$$\Rightarrow \quad y = 3 \text{ and } x = \frac{6}{2} = 3$$

$$\therefore \quad x + y = 6$$

EXAMPLE |4| If $A = \begin{bmatrix} 4 & 4 \\ 2 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 3 & -5 \end{bmatrix}$, then find $A - B - C$.

Sol. Here, A, B and C are the three matrices of same order 2×2.

Now, $A - B - C = A + (-1)B + (-1)C$

$$= \begin{bmatrix} 4 & 4 \\ 2 & 7 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 4 \\ 3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -4 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4-1-1 & 4-2-4 \\ 2+1-3 & 7-3+5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 0 & 9 \end{bmatrix}$$

EXAMPLE |5| Two farmers Ramkishan and Gurucharan Singh cultivates only three varieties of rice namely Basmati, Permal and Naura. The sales (in ₹) of these varieties of rice by both the farmers in the months of September and October are given by the matrices A and B.

September sales (in ₹)

$$\begin{array}{cc} & \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ A = & \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurucharan Singh} \end{array} \end{array}$$

October sales (in ₹)

$$\begin{array}{cc} & \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ B = & \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurucharan Singh} \end{array} \end{array}$$

(i) Find the combined sales in September and October for each farmer in each variety.

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, then compute the profit for each farmer and for each variety sold in October.

Sol. (i) Combined sales in September and October for each farmer in each variety is given by

$$A + B = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ = & \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurucharan Singh} \end{array} \end{array}$$

(ii) Changes in sales from September to October is given by

$$A - B = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} - \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$$

Basmati Permal Naura

PROPERTIES OF MATRIX ADDITION

Let $A = [a_{ij}]$, $B = [b_{ij}]$ and $C = [c_{ij}]$ be any three matrices of same order $m \times n$. Then, we have the following properties

(i) **Commutative Law** Matrix addition is commutative, i.e. $A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}] = [b_{ij} + a_{ij}]$
$= [b_{ij}] + [a_{ij}] = B + A$

(ii) **Associative Law** Matrix addition is associative, i.e. $A + (B + C) = (A + B) + C$

(iii) **Existence of Additive Identity** Let O be a zero matrix of order $m \times n$, then $A + O = O + A = A$.
So, O is the additive identity for matrix addition.

(iv) **Existence of Additive Inverse** Let A be a matrix, then $(-A)$ is another matrix of same order $m \times n$, such that $A + (-A) = O = (-A) + A$.
So, $(-A)$ or negative of A is the additive inverse of A.

EXAMPLE |6| If $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$, then

(i) verify the commutative law with respect to addition, i.e. verify $A + B = B + A$.

(ii) verify the associative law with respect to addition, i.e. verify $A + (B + C) = (A + B) + C$.

(iii) find the additive inverse of matrix A.

Sol. Given matrices are $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$

$$=\begin{array}{l}\begin{bmatrix}5000 & 10000 & 24000\\30000 & 20000 & 0\end{bmatrix}\begin{array}{l}\text{Ramkishan}\\\text{Gurucharan Singh}\end{array}\end{array}$$

(iii) 2% of $B=\frac{2}{100}\times B=0.02\times B$

$$=0.02\begin{array}{c}\begin{array}{ccc}\text{Basmati} & \text{Permal} & \text{Naura}\end{array}\\\begin{bmatrix}5000 & 10000 & 6000\\20000 & 10000 & 10000\end{bmatrix}\end{array}\begin{array}{l}\text{Ramkishan}\\\text{Gurucharan Singh}\end{array}$$

$$=\begin{array}{c}\begin{array}{ccc}\text{Basmati} & \text{Permal} & \text{Naura}\end{array}\\\begin{bmatrix}100 & 200 & 120\\400 & 200 & 200\end{bmatrix}\end{array}\begin{array}{l}\text{Ramkishan}\\\text{Gurucharan Singh}\end{array}$$

[multiply each element by 0.02]

Hence, the profits earn by Ramkishan in the sale of each variety of rice are ₹ 100, ₹ 200, ₹ 120 respectively and profits earn by Gurucharan Singh in the sale of each variety of rice are ₹ 400, ₹ 200, ₹ 200 respectively.

(ii) $A+(B+C)=\begin{bmatrix}2 & 2\\-3 & 1\\4 & 0\end{bmatrix}+\left(\begin{bmatrix}6 & 2\\1 & 3\\0 & 4\end{bmatrix}+\begin{bmatrix}-8 & -4\\2 & -4\\-4 & -4\end{bmatrix}\right)$

$$=\begin{bmatrix}2 & 2\\-3 & 1\\4 & 0\end{bmatrix}+\begin{bmatrix}6-8 & 2-4\\1+2 & 3-4\\0-4 & 4-4\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2\\-3 & 1\\4 & 0\end{bmatrix}+\begin{bmatrix}-2 & -2\\3 & -1\\-4 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2-2 & 2-2\\-3+3 & 1-1\\4-4 & 0+0\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\\0 & 0\end{bmatrix}$$

and $(A+B)+C=\begin{bmatrix}8 & 4\\-2 & 4\\4 & 4\end{bmatrix}+\begin{bmatrix}-8 & -4\\2 & -4\\-4 & -4\end{bmatrix}$ [from Eq. (i)]

$$=\begin{bmatrix}8-8 & 4-4\\-2+2 & 4-4\\4-4 & 4-4\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\\0 & 0\end{bmatrix}$$

Thus, $A+(B+C)=(A+B)+C$.

Hence, associative law verified.

(iii) We know that additive inverse of A is $(-1)A$.

$\therefore \quad (-1)A=(-1)\begin{bmatrix}2 & 2\\-3 & 1\\4 & 0\end{bmatrix}=\begin{bmatrix}-2 & -2\\3 & -1\\-4 & 0\end{bmatrix}$

[multiplying each element by (−1)]

PROPERTIES OF SCALAR MULTIPLICATION

Let A and B be the two matrices of same order. Then,

(i) $k(A+B)=kA+kB$, where k is a scalar.

(ii) $(k_1+k_2)A=k_1A+k_2A$, where k_1 and k_2 are scalars.

(iii) $(kl)A=k(lA)=l(kA)$, where l and k are scalars.

and $C=\begin{bmatrix}-8 & -4\\2 & -4\\-4 & -4\end{bmatrix}$.

Now,

(i) $A+B=\begin{bmatrix}2 & 2\\-3 & 1\\4 & 0\end{bmatrix}+\begin{bmatrix}6 & 2\\1 & 3\\0 & 4\end{bmatrix}=\begin{bmatrix}2+6 & 2+2\\-3+1 & 1+3\\4+0 & 0+4\end{bmatrix}$

$$=\begin{bmatrix}8 & 4\\-2 & 4\\4 & 4\end{bmatrix} \quad \text{...(i)}$$

and $B+A=\begin{bmatrix}6 & 2\\1 & 3\\0 & 4\end{bmatrix}+\begin{bmatrix}2 & 2\\-3 & 1\\4 & 0\end{bmatrix}=\begin{bmatrix}6+2 & 2+2\\1-3 & 3+1\\0+4 & 4+0\end{bmatrix}=\begin{bmatrix}8 & 4\\-2 & 4\\4 & 4\end{bmatrix}$

Thus, $A+B=B+A$.

Hence, commutative law verified.

$$=3\begin{bmatrix}2-1 & 1+2\\-1+0 & 0-1\\2+3 & 5+2\end{bmatrix}=3\begin{bmatrix}1 & 3\\-1 & -1\\5 & 7\end{bmatrix}=\begin{bmatrix}3 & 9\\-3 & -3\\15 & 21\end{bmatrix}$$

[multiplying each element by 3]

and RHS $=3A+3B$

$$=3\begin{bmatrix}2 & 1\\-1 & 0\\2 & 5\end{bmatrix}+3\begin{bmatrix}-1 & 2\\0 & -1\\3 & 2\end{bmatrix}=\begin{bmatrix}6 & 3\\-3 & 0\\6 & 15\end{bmatrix}+\begin{bmatrix}-3 & 6\\0 & -3\\9 & 6\end{bmatrix}$$

[multiplying each element by 3]

$$=\begin{bmatrix}6-3 & 3+6\\-3+0 & 0-3\\6+9 & 15+6\end{bmatrix}=\begin{bmatrix}3 & 9\\-3 & -3\\15 & 21\end{bmatrix}$$

Thus, LHS = RHS **Hence verified.**

(ii) LHS $=(5+7)A=12A$

$$=12\begin{bmatrix}2 & 1\\-1 & 0\\2 & 5\end{bmatrix}=\begin{bmatrix}24 & 12\\-12 & 0\\24 & 60\end{bmatrix}$$

[multiplying each element by 12]

and RHS $=5A+7A$

$$=5\begin{bmatrix}2 & 1\\-1 & 0\\2 & 5\end{bmatrix}+7\begin{bmatrix}2 & 1\\-1 & 0\\2 & 5\end{bmatrix}=\begin{bmatrix}10 & 5\\-5 & 0\\10 & 25\end{bmatrix}+\begin{bmatrix}14 & 7\\-7 & 0\\14 & 35\end{bmatrix}$$

$$=\begin{bmatrix}10+14 & 5+7\\-5-7 & 0\\10+14 & 25+35\end{bmatrix}=\begin{bmatrix}24 & 12\\-12 & 0\\24 & 60\end{bmatrix}$$

Thus, LHS = RHS **Hence verified.**

EXAMPLE |8| If $A=\begin{bmatrix}8 & 0\\4 & -2\\3 & 6\end{bmatrix}$ and $B=\begin{bmatrix}2 & -2\\4 & 2\\-5 & 1\end{bmatrix}$, then find the matrix X, such that $2A+3X=5B$.

Sol. Given, $A=\begin{bmatrix}8 & 0\\4 & -2\\3 & 6\end{bmatrix}$ and $B=\begin{bmatrix}2 & -2\\4 & 2\\-5 & 1\end{bmatrix}$

Also given, $2A+3X=5B \Rightarrow 3X=5B-2A$

EXAMPLE |7| Let A and B be two matrices such that $A=\begin{bmatrix}2 & 1\\-1 & 0\\2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2\\0 & -1\\3 & 2\end{bmatrix}$, then verify the following results.

(i) $3(A+B)=3A+3B$

(ii) $(5+7)A=5A+7A$

Sol. Given matrices are $A=\begin{bmatrix}2 & 1\\-1 & 0\\2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2\\0 & -1\\3 & 2\end{bmatrix}$

(i) LHS $=3(A+B)=3\left(\begin{bmatrix}2 & 1\\-1 & 0\\2 & 5\end{bmatrix}+\begin{bmatrix}-1 & 2\\0 & -1\\3 & 2\end{bmatrix}\right)$

$$\Rightarrow X=\frac{1}{3}[5B-2A]=\frac{1}{3}\left\{5\begin{bmatrix}2 & -2\\4 & 2\\-5 & 1\end{bmatrix}-2\begin{bmatrix}8 & 0\\4 & -2\\3 & 6\end{bmatrix}\right\}$$

$$=\frac{1}{3}\left\{\begin{bmatrix}10 & -10\\20 & 10\\-25 & 5\end{bmatrix}+\begin{bmatrix}-16 & 0\\-8 & 4\\-6 & -12\end{bmatrix}\right\}$$

$$=\frac{1}{3}\begin{bmatrix}10-16 & -10+0\\20-8 & 10+4\\-25-6 & 5-12\end{bmatrix}=\frac{1}{3}\begin{bmatrix}-6 & -10\\12 & 14\\-31 & -7\end{bmatrix}$$

$$\Rightarrow X=\begin{bmatrix}-2 & -\frac{10}{3}\\4 & \frac{14}{3}\\-\frac{31}{3} & -\frac{7}{3}\end{bmatrix}$$, which is the required matrix X.

TOPIC PRACTICE 2

OBJECTIVE TYPE QUESTIONS

1 If $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3 & 2\\4 & 3 & 1\end{bmatrix}, C=\begin{bmatrix}1\\2\end{bmatrix}$ and $D=\begin{bmatrix}4 & 6 & 8\\5 & 7 & 9\end{bmatrix}$, then which of the following is defined? **[NCERT Exemplar]**

(a) $A+B$ (b) $B+C$

(c) $C+D$ (d) $B+D$

2 If $\begin{bmatrix}1 & 2\\-2 & -b\end{bmatrix}+\begin{bmatrix}a & 4\\3 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 0\end{bmatrix}$, then a^2+b^2 is equal to

(a) 20 (b) 22

(c) 12 (d) 10

3 If $A=\begin{bmatrix}0 & 2\\3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\2b & 24\end{bmatrix}$, then the values of k, a and b are respectively

(a) $-6, -12, -18$ (b) $-6, 4, 9$

(c) $-6, -4, -9$ (d) $-6, 12, 18$

4 If A and B are two matrices of the order $3\times m$ and $3\times n$, respectively and $m=n$, then the order of the matrix $(5A-2B)$ is **[NCERT Exemplar]**

(a) $m\times 3$ (b) 3×3 (c) $m\times n$ (d) $3\times n$

VERY SHORT ANSWER Type Questions

5 Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}$ **[NCERT; Delhi 2012]**

6 If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$, then find $(x-y)$. **[Delhi 2014]**

7 If $\begin{bmatrix}1 & 0\\y & 5\end{bmatrix}+2\begin{bmatrix}x & 0\\1 & -2\end{bmatrix}=I$, where I is a 2×2 unit matrix, find $(x-y)$. **[All India 2016C]**

8 If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, then write the value of x. **[Foreign 2012]**

11 Find the values of x and y from the following matrix equation.

$$2\begin{pmatrix}x & 5\\7 & y-3\end{pmatrix}+\begin{pmatrix}3 & -4\\1 & 2\end{pmatrix}=\begin{pmatrix}7 & 6\\15 & 14\end{pmatrix}$$ **[All India 2017C]**

12 Find x, y, z and t, if $2\begin{bmatrix}x & z\\y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\4 & 6\end{bmatrix}$. **[NCERT]**

13 If $X=\begin{bmatrix}3 & 1 & -1\\5 & -2 & -3\end{bmatrix}$ and $Y=\begin{bmatrix}2 & 1 & -1\\7 & 2 & 4\end{bmatrix}$, then find matrix Z, such that $X+Y+Z$ is a zero matrix.

SHORT ANSWER Type II Questions

14 Solve the matrix equation $\begin{bmatrix}x^2\\y^2\end{bmatrix}-3\begin{bmatrix}x\\2y\end{bmatrix}=\begin{bmatrix}-2\\9\end{bmatrix}$.

15 Find X and Y, if

$2X+3Y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix}$. **[NCERT]**

16 If $A+B=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and $A-2B=\begin{bmatrix}-1 & 1\\0 & -1\end{bmatrix}$, then find the value of A. **[All India 2020]**

17 If $A=\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}, B=\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$. **[NCERT]**

HINTS & SOLUTIONS

1. (d) Only $B+D$ is defined because matrices of the same order can only be added.

2. (a) We have, $\begin{bmatrix}1 & 2\\-2 & -b\end{bmatrix}+\begin{bmatrix}a & 4\\3 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 0\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}a+1 & 6\\1 & 2-b\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 0\end{bmatrix}$$

9 If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A. **[Delhi 2012C, 2019]**

SHORT ANSWER Type I Questions

10 If $A = \text{diag}\,[2, -1, 3]$ and $B = \text{diag}\,[3, 0, -1]$, then find $4A + 2B$.

4. (d) **Hint** The order of $5A$ is $3 \times m$ and $2B$ is $3 \times n$, where $m = n$.

$\therefore$ The order of $5A - 2B$ is $3 \times m$ or $3 \times n$.

5. Consider, $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

6. Given, $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8 + y \\ 10 & 2x + 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing the corresponding elements, we get

$$8 + y = 0 \text{ and } 2x + 1 = 5$$

$$\Rightarrow y = -8 \text{ and } x = \frac{5-1}{2} = 2$$

$$\therefore x - y = 2 - (-8) = 10$$

7. Solve as Question 6. [**Ans.** 2]

8. **Hint** Here, $2x - y = 10$ ...(i)

and $3x + y = 5$ ...(ii)

On adding Eqs. (i) and (ii), we get the required value. [**Ans.** 3]

9. **Hint** $A = \frac{1}{3}\left\{\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + B\right\}$ $\left[\textbf{Ans. } A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\right]$

10. Similar as Example 2.

Hint $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

[**Ans.** $4A + 2B = \text{diag}\,[14, -4, 10]$]

11. Similar as Example 3. [**Ans.** $x = 2$ and $y = 9$]

12. Similar as Example 3. [**Ans.** $x = 3$, $y = 6$, $z = 9$ and $t = 6$]

13. **Hint** $X + Y + Z = O \Rightarrow Z = -(X + Y)$

$\left[\textbf{Ans. } \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix}\right]$

14. **Hint** $\begin{bmatrix} x^2 - 3x \\ y^2 - 6y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix} \Rightarrow x^2 - 3x = -2$ and $y^2 - 6y = 9$

[**Ans.** $x = 1, 2$ and $y = 3 \pm 3\sqrt{2}$]

$$\Rightarrow a + 1 = 5,\ 2 - b = 0$$

$$\Rightarrow a = 4,\ b = 2$$

$$\Rightarrow a^2 + b^2 = 20$$

3. (c) **Hint** $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \Rightarrow k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \Rightarrow k = -6, a = -4, b = -9$$

On multiplying Eq. (i) by 2 and Eq. (ii) by 3, we get

$$4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \quad \text{...(iii)}$$

and $$9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \quad \text{...(iv)}$$

On subtracting Eq. (iii) from Eq. (iv), we get

$$(9X + 6Y) - (4X + 6Y) = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$$

$$\Rightarrow 9X + 6Y - 4X - 6Y = \begin{bmatrix} 6-4 & -6-6 \\ -3-8 & 15-0 \end{bmatrix} = \begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{5}\begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

On substituting the value of X in Eq. (i), we get

$$2\begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{3}\begin{bmatrix} 2 - \frac{4}{5} & 3 + \frac{24}{5} \\ 4 + \frac{22}{5} & 0 - 6 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

16. We have, $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - A$

$$\because A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\therefore A - 2\left(\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - A\right) = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 3A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\therefore A = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$$

15. We have, $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ...(i)

and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ ...(ii)

17. Similar as Example 6.

$\left[\textbf{Ans. } A + B = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}, B - C = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}\right]$

|TOPIC 3|

Multiplication of Matrices

The product AB of two matrices A and B is defined, if the number of columns of A is equal to the number of rows of B. Let $A = [a_{ij}]$ be the $m \times n$ matrix and $B = [b_{jk}]$ be the $n \times p$ matrix. Then, the product AB of the matrices A and B is matrix C of order $m \times p$.

Thus, $C = [c_{ik}]_{m \times p} = AB = [a_{ij}]_{m \times n} \; [b_{jk}]_{n \times p}$

Same number

Order of resulting matrix

To get the (i, k)th element c_{ik} of the matrix C, we take the ith row of A and kth column of B, multiply them elementwise and take the sum of all these products.

e.g. $[1 \quad 4]\begin{bmatrix} 3 \\ 2 \end{bmatrix} = [(1 \times 3) + (4 \times 2)] = [3 + 8] = [11]$

[multiplying row by column]

Method of Multiplying Two Matrices

Suppose, we have two matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times p}$, then to multiply them (i.e. for finding AB), we use the following steps

I. First, write the given two matrices (A and B), then find the number of columns of first matrix A and number of rows of second matrix B. If both are same, i.e. equal, then go to next step, otherwise product is not possible.

II. Multiply first row (R_1) of A with first column (C_1) of B elementwise and take the sum of all these products.

e.g. If $[a_{11} \quad a_{12} \quad a_{13}]$ is first row of A and $\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}$ is first column of B, then their multiplication is $a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$.

Similarly, multiply first row (R_1) of A with second column (C_2) of B. Repeat this process to multiply each row of matrix A with each column of B.

III. From step II, we get the elements of product matrix C, where c_{11} = Sum of products of elements of first row of A with corresponding elements of first column of B. In general,

$$c_{ik} = a_{i1} b_{1k} + a_{i2}b_{2k} + \ldots + a_{in}b_{nk} = \sum_{j=1}^{n} a_{ij} b_{jk}.$$

IV. Now, write the matrix $C = [c_{ik}]_{m \times p}$, whose elements are obtained in step III.

Thus, we get the required product of A and B.

EXAMPLE |1| Find AB, if $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{bmatrix}$.

Sol. We have, $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}_{2 \times 2}$ and $B = \begin{bmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{bmatrix}_{2 \times 3}$

Here, number of columns of A = number of rows of B = 2.

So, product AB of matrices A and B is possible and its order will be 2×3. Now, let us multiply Ist and IInd row of A with Ist, IInd and IIIrd column of B.

Clearly, multiplication of Ist row of $A(R_1)$ with Ist column of $B(C_1) = R_1C_1 = [6 \quad 9]\begin{bmatrix} 2 \\ 7 \end{bmatrix} = 6 \times 2 + 9 \times 7$

Multiplication of Ist row of $A(R_1)$ with IInd column of $B(C_2) = R_1C_2 = [6 \quad 9]\begin{bmatrix} 6 \\ 9 \end{bmatrix} = 6 \times 6 + 9 \times 9$

Multiplication of Ist row of $A(R_1)$ with IIIrd column of $B(C_3) = R_1C_3 = [6 \quad 9]\begin{bmatrix} 0 \\ 8 \end{bmatrix} = 6 \times 0 + 9 \times 8$

Multiplication of IInd row of $A(R_2)$ with Ist column of $B(C_1) = R_2C_1 = [2 \quad 3]\begin{bmatrix} 2 \\ 7 \end{bmatrix} = 2 \times 2 + 3 \times 7$

Multiplication of IInd row of $A(R_2)$ with IInd column of $B(C_2) = R_2C_2 = [2 \quad 3]\begin{bmatrix} 6 \\ 9 \end{bmatrix} = 2 \times 6 + 3 \times 9$

Multiplication of IInd row of $A(R_2)$ with IIIrd column of $B(C_3) = R_2C_3 = [2 \quad 3]\begin{bmatrix} 0 \\ 8 \end{bmatrix} = 2 \times 0 + 3 \times 8$

Thus, we have

$c_{11} = R_1C_1 = 6 \times 2 + 9 \times 7 = 12 + 63 = 75$

$c_{12} = R_1C_2 = 6 \times 6 + 9 \times 9 = 36 + 81 = 117$

$c_{13} = R_1C_3 = 6 \times 0 + 9 \times 8 = 0 + 72 = 72$

$c_{21} = R_2C_1 = 2 \times 2 + 3 \times 7 = 4 + 21 = 25$

$c_{22} = R_2C_2 = 2 \times 6 + 3 \times 9 = 12 + 27 = 39$

and $c_{23} = R_2C_3 = 2 \times 0 + 3 \times 8 = 0 + 24 = 24$

Hence, $C = AB = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} = \begin{bmatrix} 75 & 117 & 72 \\ 25 & 39 & 24 \end{bmatrix}_{2 \times 3}$

which is the required product, i.e. AB.

Note

(i) If AB is defined, then BA need not be defined. Let $A=[a_{ij}]_{2\times 3}$ and $B=[b_{jk}]_{3\times 3}$, then $AB=[a_{ij}b_{jk}]_{2\times 3}$ is defined but BA is not defined, since B has 3 rows while A has 2 columns.

(ii) If A is a matrix of order $m\times n$ and B is a matrix of order $k\times l$, then both AB and BA will be defined, if and only if $n=k$ and $l=m$. In particular, if both A and B are square matrices of the same order, then AB and BA both are defined.

EXAMPLE |2| Compute the product of $\begin{bmatrix}2&3&4\\3&4&5\\4&5&6\end{bmatrix}$ and $\begin{bmatrix}1&-3&5\\0&2&4\\3&0&5\end{bmatrix}$. **[NCERT]**

Sol. Let $A=\begin{bmatrix}2&3&4\\3&4&5\\4&5&6\end{bmatrix}$ and $B=\begin{bmatrix}1&-3&5\\0&2&4\\3&0&5\end{bmatrix}$. Here, number of columns of A = number of rows of B = 3, so AB is defined.

Now, $$AB=\begin{bmatrix}2&3&4\\3&4&5\\4&5&6\end{bmatrix}\begin{bmatrix}1&-3&5\\0&2&4\\3&0&5\end{bmatrix}$$

Here, $$c_{11}=R_1C_1=[2\ 3\ 4]\begin{bmatrix}1\\0\\3\end{bmatrix}=2+0+12=14$$

$$c_{12}=R_1C_2=[2\ 3\ 4]\begin{bmatrix}-3\\2\\0\end{bmatrix}=-6+6+0=0$$

$$c_{13}=R_1C_3=[2\ 3\ 4]\begin{bmatrix}5\\4\\5\end{bmatrix}=10+12+20=42$$

$$c_{21}=R_2C_1=[3\ 4\ 5]\begin{bmatrix}1\\0\\3\end{bmatrix}=3+0+15=18$$

$$c_{22}=R_2C_2=[3\ 4\ 5]\begin{bmatrix}-3\\2\\0\end{bmatrix}=-9+8+0=-1$$

$$c_{23}=R_2C_3=[3\ 4\ 5]\begin{bmatrix}5\\4\\5\end{bmatrix}=15+16+25=56$$

$$c_{31}=R_3C_1=[4\ 5\ 6]\begin{bmatrix}1\\0\\3\end{bmatrix}=4+0+18=22$$

$$c_{32}=R_3C_2=[4\ 5\ 6]\begin{bmatrix}-3\\2\\0\end{bmatrix}=-12+10+0=-2$$

and $$c_{33}=R_3C_3=[4\ 5\ 6]\begin{bmatrix}5\\4\\5\end{bmatrix}=20+20+30=70$$

Hence, $$C=AB=\begin{bmatrix}c_{11}&c_{12}&c_{13}\\c_{21}&c_{22}&c_{23}\\c_{31}&c_{32}&c_{33}\end{bmatrix}=\begin{bmatrix}14&0&42\\18&-1&56\\22&-2&70\end{bmatrix}.$$

EXAMPLE |3| Use matrix multiplication to divide ₹30000 in two parts such that the total annual interest at 9% on the first part and 11% on the second part amounts ₹3060. **[NCERT Exemplar]**

Sol. Let the two parts be ₹x and ₹$(30000-x)$. Let A be the 1×2 matrix representing these parts and B be the 2×1 matrix representing the annual interest rates on two parts.

i.e. $$A=\begin{array}{cc}\text{Part I}&\text{Part II}\\ \big[x & 30000-x\big]\end{array} \text{ and } B=\begin{bmatrix}0.09\\0.11\end{bmatrix}\begin{array}{l}\text{Part I}\\\text{Part II}\end{array}$$

Clearly, the total interest is given by the matrix AB,

i.e. $$AB=[x\quad 30000-x]\begin{bmatrix}0.09\\0.11\end{bmatrix}$$
$$=[x\times 0.09+(30000-x)\times 0.11]$$
$$=[0.09x+3300-0.11x]=[3300-0.02x]$$

Thus, $3300-0.02x=3060$ [$\because$ total interest = ₹3060]

$\Rightarrow$ $0.02x=240 \Rightarrow x=12000$

Hence, two parts are ₹12000 and ₹18000, respectively.

Properties of Multiplication of Matrices

Multiplication of matrices have following properties

(i) **Associative Law** Matrix multiplication is associative, i.e. if A, B and C are three matrices, then
$A(BC)=(AB)C$, whenever both sides are defined.

(ii) **Distributive Law** Matrix multiplication is distributive over matrix addition, if A, B and C are three matrices, then
$A(B+C)=AB+AC$ or $(A+B)C=AC+BC$,
whenever both sides are defined.

(iii) **Existence of Multiplicative Identity** For every square matrix A, there exists an identity matrix I of same order such that $AI=A=IA$. So, I is the multiplicative identity for every square matrix A.

(iv) **Non-commutativity** Generally, matrix multiplication is not commutative, i.e. if A and B are two matrices and AB, BA both exists, then it is not necessary that $AB=BA$.

(v) **Zero Matrix as the Product of Two Non-zero Matrices** If the product of two matrices is a zero matrix, then it is not necessary that one of the matrices is zero matrix.

e.g. Let $A=\begin{bmatrix}0&-1\\0&2\end{bmatrix}$ and $B=\begin{bmatrix}3&5\\0&0\end{bmatrix}$.

Then, $AB=\begin{bmatrix}0&-1\\0&2\end{bmatrix}\begin{bmatrix}3&5\\0&0\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}=O.$

But, here $A\neq O$ and $B\neq O$.

Note Multiplication of diagonal matrices of same order will be commutative.

EXAMPLE |4| If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then find AB and BA. Show that A and B are not commutative, i.e. $AB \neq BA$.

Sol. Here, order of matrix A is 2×3 and order of matrix B is 3×2. So, AB will be of order 2×2 and BA will be of order 3×3.

Now, $AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-8+6 & 3-10+3 \\ -8+8+10 & -12+10+5 \end{bmatrix}$ [multiplying rows by columns]

$AB = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$

and $BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix}$ [multiplying rows by columns]

$= \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$

Clearly, $AB \neq BA$.

EXAMPLE |5| Three matrices A, B and C defined as $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, then show that matrices A, B and C satisfy

(i) the property of associativity with respect to multiplication, i.e. $A(BC) = (AB)C$.

(ii) the property of distributivity with respect to addition, i.e. $A(B+C) = AB + AC$. **[NCERT Exemplar]**

Sol. Given, $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

(i) Here, $BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8+0 & 0+0 \\ 2+5 & 0-10 \end{bmatrix}$ [multiplying rows by columns]

$= \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$ ...(i)

$\therefore$ LHS $= A(BC) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$ [using Eq. (i)]

$= \begin{bmatrix} 8+14 & 0-20 \\ -8+21 & 0-30 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$ [multiplying rows by columns]

and $AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix}$ [multiplying rows by columns]

$= \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$ ...(ii)

$\therefore$ RHS $= (AB)C = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$ [using Eq. (ii)]

$= \begin{bmatrix} 12+10 & 0-20 \\ -2+15 & 0-30 \end{bmatrix}$ [multiplying rows by columns]

$= \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$

Thus, LHS = RHS

(ii) Here, $B + C = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 4+2 & 0 \\ 1+1 & 5-2 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix}$ ...(iii)

and $AC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2+2 & 0-4 \\ -2+3 & 0-6 \end{bmatrix}$

$= \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix}$ ...(iv)

[multiplying rows by columns]

LHS $= A(B+C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix}$ [using Eq. (iii)]

$= \begin{bmatrix} 6+4 & 0+6 \\ -6+6 & 0+9 \end{bmatrix}$ [multiplying rows by columns]

$= \begin{bmatrix} 10 & 6 \\ 0 & 9 \end{bmatrix}$

and RHS $= AB + AC = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix} + \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 0 & 9 \end{bmatrix}$

[using Eqs. (ii) and (iv)]

Thus, LHS = RHS

POSITIVE INTEGRAL POWERS OF A SQUARE MATRIX

If we multiply the square matrix by itself, then it will give the positive integral powers of the square matrix.
i.e. for any square matrix, say A, we define
$A^1 = A$, $A^2 = A \cdot A$, $A^3 = A^2 \cdot A$, $A^4 = A^3 \cdot A$, ...,
$A^n = A^{n-1} \cdot A$, where n is a positive integer.

Note $I^n = I$ for any positive integer n, where I is the identity matrix.

Method to Solve Problems Based on nth Power of a Matrix

In this type of problems, a matrix (say A) is given to us and an equation involving nth power of A is given to us. We have to show/prove that the given equation is true.

For proving/showing such problems, we use the concept of mathematical induction, which is given below.
Suppose there is a statement $P(n)$ involving the natural number n such that

(i) $P(1)$ is true, i.e. statement is true for $n = 1$.

(ii) Truth of $P(k)$ implies the truth of $P(k+1)$, i.e. if the statement is true for $n = k$ (k is positive integer), then statement is also true for $n = k + 1$.

Then, by principal of mathematical induction, $P(n)$ is true for all natural numbers n.

EXAMPLE |6| If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N.$$

Sol. Let given statement be $P(n)$,

i.e. $P(n)$: If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$$

For $n = 1$, $A^1 = A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore$ $P(1)$ is true.
Let $P(n)$ be true for $n = k$, so

$$P(k): A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$$

Now, we have to prove the statement for $n = k + 1$.

Consider,

$$A^{k+1} = A^k \cdot A = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & \cos k\theta\sin\theta + \sin k\theta\cos\theta \\ -\sin k\theta\cos\theta - \cos k\theta\sin\theta & -\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k\theta+\theta) & \sin(k\theta+\theta) \\ -\sin(k\theta+\theta) & \cos(k\theta+\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Thus, the statement is true for $n = k + 1$.

Hence, by principal of mathematical induction,

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, \forall n \in N. \quad \textbf{Hence proved.}$$

MATRIX POLYNOMIAL

Let $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n$ be a polynomial and A be a square matrix.
Then, $f(A) = a_0 A^n + a_1 A^{n-1} + a_2 A^{n-2} + \ldots + a_{n-1} A + a_n I$ is called a matrix polynomial, where I is the identity matrix of order of A.

Method to Solve Problems Based on Matrix Polynomial

Sometimes, a square matrix (say A) is given to us and we have to find the value of a polynomial in which variable is a matrix. (e.g. $A^3 - 23A - 40I$). For this, firstly we find the value of each term having power of A by multiplying A by itself as many times as power and then put these values in given expression to find the required value.

EXAMPLE |7| Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ and $f(x) = x^2 + x - 1$, then find $f(A)$.

Sol. We have, $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$

$$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+2 \\ 0+2 & 1+4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

[multiplying rows by columns]

$$\text{Now, } f(A) = A^2 + A - I = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0-1 & 2+1-0 \\ 2+1-0 & 5+2-1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 6 \end{bmatrix}$$

EXAMPLE |8| If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.
[Delhi 2016]

Sol. Given, $A^2 = I$ ...(i)

$$\text{Now, } (A - I)^3 + (A + I)^3 - 7A = (A^3 - 3A^2 I + 3AI^2 - I^3) + (A^3 + 3A^2 I + 3AI^2 + I^3) - 7A$$

$$= (A^3 - 3A^2 + 3AI - I) + (A^3 + 3A^2 + 3AI + I) - 7A$$

$[\because A^2 I = A^2 \text{ and } I^3 = I^2 = I]$

$$= 2A^3 + 6AI - 7A = 2A^2 A + 6A - 7A \quad [\because AI = A]$$

$$= 2IA - A \quad [\text{from Eq. (i)}]$$

$$= 2A - A = A \quad [\because IA = A]$$

EXAMPLE |9| If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $A^2 - 5A + 4I$ and hence find a matrix X, such that $A^2 - 5A + 4I + X = 0$.
[Delhi 2015]

Sol. We have, $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

[multiplying rows by columns]

Now, consider $A^2 - 5A + 4I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

Since, $A^2 - 5A + 4I + X = O$

$\therefore\ A^2 - 5A + 4I + X - X = O - X$

[subtracting matrix X from both sides]

$\Rightarrow A^2 - 5A + 4I + O = -X$

$$\Rightarrow \quad X = -(A^2 - 5A + 4I)$$

$$\Rightarrow \quad X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$$

EXAMPLE |10| If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = 0$, find k. **[All India 2016]**

Sol. Given, $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

and $A^3 = A \cdot A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 8+0+26 \\ 0+4+8 & 0+8+0 & 0+10+13 \\ 10+0+24 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Since, $A^3 - 6A^2 + 7A + kI_3 = O$

$$\therefore \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21-30+7+k & 0-0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+k & 23-30+7+0 \\ 34-48+14+0 & 0-0+0+0 & 55-78+21+k \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2+k & 0 & 0 \\ 0 & -2+k & 0 \\ 0 & 0 & -2+k \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

On equating the corresponding elements, we get

$$-2 + k = 0$$

$$\therefore \quad k = 2$$

TOPIC PRACTICE 3

OBJECTIVE TYPE QUESTIONS

1 The product $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$ (b) $\begin{bmatrix} (a+b)^2 & 0 \\ (a+b)^2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} a^2+b^2 & 0 \\ a^2+b^2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

2 If the product of two matrices is a zero matrix, then

(a) atleast one of the matrix is a zero matrix

(b) both the matrices are zero matrices

(c) it is not necessary that one of the matrices is a zero matrix

(d) None of the above

3 If $A=\begin{bmatrix}2 & -1 & 3\\ -4 & 5 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & -2\\ 1 & 5\end{bmatrix}$, then

(a) only AB is defined

(b) only BA is defined

(c) AB and BA both are defined

(d) AB and BA both are not defined

4 If A and B are square matrices of the same order, then $(A+B)(A-B)$ is equal to [NCERT Exemplar]

(a) A^2-B^2 (b) $A^2-BA-AB-B^2$

(c) $A^2-B^2+BA-AB$ (d) A^2-BA+B^2+AB

5 The set of all 2×2 matrices which is commutative with the matrix $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ with respect to matrix multiplication is

(a) $\begin{bmatrix}p & q\\ r & r\end{bmatrix}$ (b) $\begin{bmatrix}p & q\\ q & r\end{bmatrix}$

(c) $\begin{bmatrix}p-q & p\\ q & r\end{bmatrix}$ (d) $\begin{bmatrix}p & q\\ q & p-q\end{bmatrix}$

6 Given that $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ and $A^2=3I$, then [CBSE 2021 (Term I)]

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2-\beta\gamma=0$

(c) $3-\alpha^2-\beta\gamma=0$ (d) $3+\alpha^2+\beta\gamma=0$

VERY SHORT ANSWER Type Questions

7 Show that if A and B are square matrices such that $AB=BA$, then $(A+B)^2=A^2+2AB+B^2$. [NCERT Exemplar]

8 Let A and B be two matrices of order 3×2 and 2×4, respectively. Write the order of matrix (AB). [Delhi 2017C]

9 If $[2x\ \ 3]\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 8\end{bmatrix}=O$, then find the value of x. [Delhi 2015C; NCERT Exemplar]

10 If $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, then find A^3. [Delhi 2020]

SHORT ANSWER Type I Questions

11 If $X_{m\times 3}\,Y_{p\times 4}=Z_{2\times b}$, for three matrices X, Y and Z, then find the values of m, p and b.

12 If $A=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$, then find the value of x for which $A^2=B$.

13 If $A=\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$, show that $(A-2I)(A-3I)=O$. [All India 2019]

SHORT ANSWER Type II Questions

14 Find the matrix A such that
$$A\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}.$$

15 If $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{bmatrix}$, then find A. [All India 2017]

16 Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$. Find a matrix D such that $CD-AB=O$. [NCERT; Delhi 2017]

17 If $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, then show that $f(x)\cdot f(y)=f(x+y)$. [NCERT]

18 If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then find the values of a and b. [Foreign 2015]

19 If $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$, then find the real values x and y such that $(xI+yA)^2=A$.

20 If $A=\begin{bmatrix}0 & -\tan\alpha/2\\ \tan\alpha/2 & 0\end{bmatrix}$ and I is the identity matrix of order 2, then show that
$$I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}.$$ [NCERT]

21 If A and B are square matrices of the same order such that $AB=BA$, then prove by mathematical induction that $AB^n=B^nA$ for all $n\in N$.

22 If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then prove that
$$A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}.$$ [NCERT]

23 If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, then find the value of (A^2-5A). [Delhi 2019]

LONG ANSWER Type Questions

24 If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that

$A^3 - 4A^2 - 3A + 11I = O.$ **[All India 2016C]**

25 If $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$, then show that

$A^3 - A^2 - 3A - I_3 = O.$

HINTS & SOLUTIONS

1. (a) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & -ab+ba \\ -ba+ab & b^2+a^2 \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$

2. (c) **Hint** Let $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$. Then, $AB = O$ but both A and B are non-zero.

3. (c) Let $A = [a_{ij}]_{2 \times 3}$ and $B = [b_{ij}]_{3 \times 2}$.

Since, number of columns of A = number of rows of B

$\therefore$ AB is defined

Also, as number of columns of B = number of rows of A.

$\therefore$ BA is defined.

Hence, both AB and BA are defined.

4. (c) $(A+B)(A-B) = A(A-B) + B(A-B)$

$= A^2 - AB + BA - B^2$

5. (d) Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ be a matrix which commute with matrix $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

Then, $AB = BA$

$\Rightarrow \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$\Rightarrow \begin{bmatrix} p+q & p \\ r+s & r \end{bmatrix} = \begin{bmatrix} p+r & q+s \\ p & q \end{bmatrix}$

Here, both matrices are equal, so we equate the corresponding elements.

$\therefore$ $p+q = p+r,\ p = q+s,\ r+s = p$ and $r = q$

$\Rightarrow$ $r = q$ and $s = p - q$

$\therefore$ $A = \begin{bmatrix} p & q \\ q & p-q \end{bmatrix}$

Hence, the required set is $\begin{bmatrix} p & q \\ q & p-q \end{bmatrix}$.

6. (c) Given, $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$= \begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix}$

$\because$ $A^2 = 3I \Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow$ $\alpha^2 + \beta\gamma = 3$

$\Rightarrow$ $3 - \alpha^2 - \beta\gamma = 0$

7. Given, $AB = BA$

Now, $(A+B)^2 = (A+B)\cdot(A+B)$

$= A\cdot(A+B) + B\cdot(A+B)$

$= A^2 + AB + BA + B^2$

$= A^2 + AB + AB + B^2$ [$\because BA = AB$, given]

$= A^2 + 2AB + B^2$

8. Order of AB is 3×4.

9. Given matrix equation is

$[2x\ \ 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = O$

$\Rightarrow$ $[2x\ \ 3]\left(\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix}\right) = O$

[by associative law of multiplication]

$\Rightarrow$ $[2x\ \ 3]\begin{bmatrix} x+16 \\ -3x \end{bmatrix} = O$

$\Rightarrow$ $[2x(x+16) - 9x] = [O]$

$\Rightarrow$ $2x^2 + 32x - 9x = 0$

$\Rightarrow$ $2x^2 + 23x = 0$

$\Rightarrow$ $x(2x + 23) = 0$

$\Rightarrow$ $x = 0$ and $x = -23/2$

10. We have, $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Now, $A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow$ $A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow$ $A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

11. Given, $X_{m \times 3} Y_{p \times 4} = Z_{2 \times b}$ ...(i)

XY is defined only, when $p = 3$.

Now, Eq. (i) will be of the form $Z_{m \times 4} = Z_{2 \times b}$,

which is possible only, when $m = 2$ and $b = 4$.

$\therefore$ $m = 2$, $p = 3$ and $b = 4$

12. We have, $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \quad x^2 = 1 \quad \text{...(i)}$$

$$\text{and} \quad x + 1 = 5 \quad \text{...(ii)}$$

From Eq. (ii), we get $x = 4$, which does not satisfy Eq. (i).

So, no value of x exist.

13. Given, $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

LHS $= (A - 2I)(A - 3I)$

$$= \left\{\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}\left\{\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = \text{RHS}$$

Hence proved.

14. The right side of given matrix equation is of order 2×3, so left side of the given matrix equation should be of order 2×3. It is clear that, the product of A with 2×3 matrix is a 2×3 matrix, therefore A is a 2×2 matrix.

Let $A = \begin{bmatrix} u & v \\ w & x \end{bmatrix}$.

Then,

$$\text{LHS} = \begin{bmatrix} u & v \\ w & x \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} u+4v & 2u+5v & 3u+6v \\ w+4x & 2w+5x & 3w+6x \end{bmatrix}$$

$$\therefore \begin{bmatrix} u+4v & 2u+5v & 3u+6v \\ w+4x & 2w+5x & 3w+6x \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

On comparing the elements from both sides, we get

$$u + 4v = -7,\ 2u + 5v = -8,\ 3u + 6v = -9,$$

$$w + 4x = 2,\ 2w + 5x = 4 \text{ and } 3w + 6x = 6$$

On solving, we get $u = 1$, $v = -2$, $w = 2$ and $x = 0$

$$\therefore \quad A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

15. Solve as Question 14. $\left[\text{Ans. } A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}\right]$

16. Solve as Question 14. $\left[\text{Ans. } D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}\right]$

17. Given, $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

To prove, $f(x) \cdot f(y) = f(x+y)$.

For $f(y)$, replace x with y, so that $f(x)$ becomes $f(y)$.

LHS $= f(x) \cdot f(y)$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0+0+0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

[multiplying rows by columns]

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{l} \because \cos(A+B) = \cos A \cos B - \sin A \sin B \\ \text{and } \sin(A+B) = \sin A \cos B + \cos A \sin B \end{array}\right]$$

$$= f(x+y) = \text{RHS}$$

18. Given, $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

Clearly, $A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1+a & -1+1 \\ 2+b & -1-1 \end{bmatrix} = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix}$$

$$\Rightarrow \quad (A+B)^2 = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} \cdot \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + a^2 + 2a & 0 \\ 2 + 2a + b + ab - 4 - 2b & 4 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + 2a + 1 & 0 \\ 2a - b + ab - 2 & 4 \end{bmatrix}$$

Now, $A^2 + B^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \cdot \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b - 1 & a - 1 \\ ab - b & b \end{bmatrix}$$

Since, $(A+B)^2 = A^2 + B^2$

$$\therefore \quad \begin{bmatrix} a^2 + 2a + 1 & 0 \\ 2a - b + ab - 2 & 4 \end{bmatrix} = \begin{bmatrix} a^2 + b - 1 & a - 1 \\ ab - b & b \end{bmatrix}$$

On equating the corresponding elements, we get

$$a^2 + 2a + 1 = a^2 + b - 1$$

$$\Rightarrow \quad 2a - b = -2, \quad \text{...(i)}$$

$$a - 1 = 0$$

$$\Rightarrow \quad a = 1, \quad \text{...(ii)}$$

$$\text{and} \quad 2a - b + ab - 2 = ab - b$$

$$\Rightarrow \quad 2a - 2 = 0$$

$$\Rightarrow \quad a = 1 \quad \text{...(iii)}$$

$$\text{and} \quad b = 4 \quad \text{...(iv)}$$

Since, $a = 1$, $b = 4$ satisfy all four Eqs. (i), (ii), (iii) and (iv), therefore

$$a = 1 \text{ and } b = 4$$

19. We have, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$\Rightarrow \quad xI + yA = x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + y\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} 0 & y \\ -y & 0 \end{bmatrix} = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$

Now, $(xI + yA)^2 = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}^2 = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

$$= \begin{bmatrix} x^2 - y^2 & 2xy \\ -2xy & x^2 - y^2 \end{bmatrix}$$

Since, $(xI + yA)^2 = A$, therefore

$$\begin{bmatrix} x^2 - y^2 & 2xy \\ -2xy & x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$\Rightarrow \quad x^2 - y^2 = 0,\ 2xy = 1, -2xy = -1$ and $x^2 - y^2 = 0$

$\Rightarrow \quad x^2 - y^2 = 0$ and $2xy = 1$

$\Rightarrow \quad x = \pm y$ and $2xy = 1$

***Case* I** *When $x = y$ and $2xy = 1$.*

In this case, $2x^2 = 1$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}$$

Thus, $\left(x = \frac{1}{\sqrt{2}} \text{ and } y = \frac{1}{\sqrt{2}}\right)$ or $\left(x = -\frac{1}{\sqrt{2}} \text{ and } y = -\frac{1}{\sqrt{2}}\right)$

***Case* II** *When $x = -y$ and $2xy = 1$.*

In this case, $x^2 = \frac{-1}{2} \Rightarrow x = \pm\frac{i}{\sqrt{2}}$, which is imaginary.

Hence, only real values of x and y are

$$x = \frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}} \text{ or } x = -\frac{1}{\sqrt{2}}, y = -\frac{1}{\sqrt{2}}$$

20. LHS $= I + A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

RHS $= (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}\right\}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1-0 & 0+\tan\alpha/2 \\ 0-\tan\alpha/2 & 1-0 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \tan\frac{\alpha}{2}\sin\alpha & -\sin\alpha + \tan\frac{\alpha}{2}\cos\alpha \\ -\tan\frac{\alpha}{2}\cos\alpha + \sin\alpha & \tan\frac{\alpha}{2}\sin\alpha + \cos\alpha \end{bmatrix}$$

[multiplying rows by columns]

$$= \begin{bmatrix} \dfrac{\cos\alpha\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\sin\alpha}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\alpha\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\cos\alpha}{\cos\frac{\alpha}{2}} \\ \dfrac{-\sin\frac{\alpha}{2}\cos\alpha + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\sin\frac{\alpha}{2}\sin\alpha + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\alpha/2} & \dfrac{\sin\left(\frac{\alpha}{2} - \alpha\right)}{\cos\alpha/2} \\ \dfrac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$\left[\begin{array}{l} \because \cos(A - B) = \cos A\cos B + \sin A\sin B \\ \text{and } \sin(A - B) = \sin A\cos B - \cos A\sin B \end{array}\right]$

$$= \begin{bmatrix} \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = \text{LHS}$$

Hence proved.

21. Given that $AB = BA$ ...(i)

To prove, $AB^n = B^n A$ ...(ii)

For $n = 1$, Eq. (ii) is obviously true.

Let Eq. (ii) be true for a positive integer $n = m$,

i.e. $\quad AB^m = B^m A$ ...(iii)

Then, for $n = m + 1$, $AB^{m+1} = A(B^m B) = (AB^m)B$

[using associative law of matrix multiplication]

$= (B^m A)B$ [using Eq. (iii)]

$= B^m(AB) = B^m(BA)$ [using Eq. (i)]

$= (B^m B)A = B^{m+1}A$

Hence, by mathematical induction, Eq. (ii) is true for all $n \in N$.

22. Similar as Example 6.

23. Given, $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3-0 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Now, $A^2 - 5A = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

24. Hint $A^2 = A \cdot A = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$

and $A^3 = A^2 \cdot A = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$

Now, consider LHS $= A^3 - 4A^2 - 3A + 11I$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{RHS}$$

25. Solve as Question 24.

|TOPIC 4|

Transpose of a Matrix, Symmetric, Skew-symmetric Matrices and Invertible Matrices

TRANSPOSE OF A MATRIX

The matrix obtained by interchanging the rows and columns of matrix A is called the transpose of matrix A. The transpose of matrix A is represented by A' or A^c or A^T.

In other words, if $A = [a_{ij}]$ is $m \times n$ matrix.

Then, $A^T = [a_{ji}]_{n \times m}$.

e.g. Let $A = \begin{bmatrix} 1 & 5 \\ \sqrt{2} & 1 \\ -1/5 & 0 \end{bmatrix}_{3 \times 2}$.

Then, $A' = \begin{bmatrix} 1 & \sqrt{2} & -1/5 \\ 5 & 1 & 0 \end{bmatrix}_{2 \times 3}$

EXAMPLE |1| Find the transpose of the matrix $\begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$.

Then, $A' = \begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 4 & 5 & -1 \end{bmatrix}' = \begin{bmatrix} 4 & 1 & 4 \\ 3 & -2 & 5 \\ 1 & 3 & -1 \end{bmatrix}$

[interchanging the elements of rows and columns]

Properties of Transpose of Matrices

For any matrices A and B of suitable orders, we have

(i) $(A')' = A$

(ii) $(A \pm B)' = A' \pm B'$

(iii) $(kA)' = kA'$, where k is any constant.

(iv) $(AB)' = B'A'$ [reversal law]

Note $(A^n)^T = (A^T)^n$, where n is a positive integer.

EXAMPLE |2| If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify that

(i) $(A')' = A$

(ii) $(A + B)' = A' + B'$

(iii) $(kB)' = kB'$, where k is any constant.

Sol. We have, $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}'$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then

$$A' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix} \text{ and } B' = \begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix}$$

[interchanging the elements of rows and columns]

(i) $(A')' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix}' = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$

[interchanging the elements of rows and columns]

$= A$

(ii) $A + B = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3+2 & \sqrt{3}-1 & 2+2 \\ 4+1 & 2+2 & 0+4 \end{bmatrix}$

$= \begin{bmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{bmatrix}$

$\therefore \ (A+B)' = \begin{bmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{bmatrix}' = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$ ...(i)

[interchanging the elements of rows and columns]

and $A' + B' = \begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3+2 & 4+1 \\ \sqrt{3}-1 & 2+2 \\ 2+2 & 0+4 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$ ...(ii)

Thus, $(A+B)' = A' + B'$ [from Eqs. (i) and (ii)]

(iii) $kB = k\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 2k & -k & 2k \\ k & 2k & 4k \end{bmatrix}$

and $(kB)' = \begin{bmatrix} 2k & -k & 2k \\ k & 2k & 4k \end{bmatrix}' = \begin{bmatrix} 2k & k \\ -k & 2k \\ 2k & 4k \end{bmatrix}$

$= k\begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix} = kB'$

Thus, $(kB)' = kB'$

EXAMPLE |3| For the matrices A and B, verify that $(AB)' = B'A'$, where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = [-1 \ \ 2 \ \ 1]$. **[NCERT]**

Sol. Given, $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times1}$ and $B = [-1 \ \ 2 \ \ 1]_{1\times3}$

$\therefore \quad A' = [1 \ \ -4 \ \ 3]_{1\times3}$

[interchanging the elements of rows and columns]

and $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}_{3\times1}$

[interchanging the elements of rows and columns]

Now, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times1} [-1 \ \ 2 \ \ 1]_{1\times3}$

Since, number of columns in A is equal to the number of rows in B. So, AB exists.

Thus, $AB = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3}$

[multiplying rows by columns]

$\Rightarrow (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}_{3\times3}$ ...(i)

[interchanging the elements of rows and columns]

Now, $B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}_{3\times1} [1 \ \ -4 \ \ 3]_{1\times3} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}_{3\times3}$

[multiplying rows by columns] ...(ii)

From Eqs. (i) and (ii), we get $(AB)' = B'A'$

Hence verified.

SYMMETRIC MATRIX

A square matrix A is called a symmetric matrix, if the transpose of matrix A is equal to the matrix A, i.e. $A' = A$. In other words, let $A = [a_{ij}]_{n\times n}$, then A is said to be symmetric, if $[a_{ij}] = [a_{ji}]$, for all possible values of i and j.

e.g. $A = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 7 \\ 5 & 7 & 3 \end{bmatrix}$, then $A' = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 7 \\ 5 & 7 & 3 \end{bmatrix}' = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 7 \\ 5 & 7 & 3 \end{bmatrix}$

[interchanging the elements of rows and columns]

$= A$

Hence, A is a symmetric matrix. $[\because A' = A]$

EXAMPLE |4| Show that matrix $A = \begin{bmatrix} 3 & -4 & 2 \\ -4 & 0 & 6 \\ 2 & 6 & 1 \end{bmatrix}$ is symmetric matrix.

Sol. Given, $A = \begin{bmatrix} 3 & -4 & 2 \\ -4 & 0 & 6 \\ 2 & 6 & 1 \end{bmatrix}$

Now, $A' = \begin{bmatrix} 3 & -4 & 2 \\ -4 & 0 & 6 \\ 2 & 6 & 1 \end{bmatrix}' = \begin{bmatrix} 3 & -4 & 2 \\ -4 & 0 & 6 \\ 2 & 6 & 1 \end{bmatrix}$

[interchanging the elements of rows and columns]

$= A$

$\therefore A$ is a symmetric matrix.

EXAMPLE |5| Show that all positive integral powers of a symmetric matrix are symmetric.

Sol. Let A be a symmetric matrix and $n \in N$.

Then, $(A^n)^T = [A \cdot A \cdot A \ldots n \text{ times}]^T$

$= [A^T \cdot A^T \ldots n \text{ times}]$ $\quad [\because (AB)^T = B^T A^T]$

$= [A \; A \ldots n \text{ times}] = (A)^n$

$[\because A \text{ is symmetric, therefore } A^T = A]$

SKEW-SYMMETRIC MATRIX

A square matrix A is said to be skew-symmetric matrix, if the transpose of matrix A is equal to the negative of matrix A i.e. $A' = -A$. In other words, let $A = [a_{ij}]_{n \times n}$, then A is said to be skew-symmetric matrix, if $[a_{ji}] = -[a_{ij}]$, for all values of i and j.

e.g. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 5 \\ -2 & -5 & 0 \end{bmatrix}$.

Then, $A' = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

[interchanging the elements of rows and columns]

$= -\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 5 \\ -2 & -5 & 0 \end{bmatrix}$

[taking (-1) common from the matrix]

$\Rightarrow \quad A' = -A$

Note

(i) The principal diagonal elements of a skew-symmetric matrix are always zero, because if we put $i = j$ in $a_{ji} = -a_{ij}$, then $a_{ii} = -a_{ii}$.
$\therefore\ 2a_{ii} = 0 \Rightarrow a_{ii} = 0, \forall\, i$.

(ii) The only zero matrix is both symmetric and skew-symmetric matrix.

EXAMPLE |6| Show that matrix $B = \begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & 2 \\ 4 & -2 & 0 \end{bmatrix}$ is skew-symmetric matrix.

Sol. Given, $B = \begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & 2 \\ 4 & -2 & 0 \end{bmatrix}$

Now, $B' = \begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & 2 \\ 4 & -2 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -3 & 4 \\ 3 & 0 & -2 \\ -4 & 2 & 0 \end{bmatrix}$

[interchanging the elements of rows and columns]

$= -\begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & 2 \\ 4 & -2 & 0 \end{bmatrix}$

[taking (-1) common from the matrix]

$\therefore \quad B' = -B$

Hence, B is skew-symmetric matrix.

EXAMPLE |7| For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is skew-symmetric matrix?

[All India 2013]

Sol. Given, $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$.

We know that if A is a skew-symmetric matrix, then

$$A = -A^T \quad \text{...(i)}$$

From Eq. (i), we get

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

On comparing the corresponding element, we get

$$-2 = -x \Rightarrow x = 2$$

Some Important Theorems

Theorem 1 For a square matrix A with real number entries, $A + A'$ is a symmetric and $A - A'$ is a skew-symmetric matrix.

Theorem 2 A square matrix A can be expressed as the sum of a symmetric and skew-symmetric matrices.

i.e. $\quad A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, where $\frac{1}{2}(A + A')$ is symmetric and $\frac{1}{2}(A - A')$ is skew-symmetric.

EXAMPLE |8| Prove that the square matrix $\begin{bmatrix} 5 & 2 \\ 3 & -6 \end{bmatrix}$ can be expressed as a sum of symmetric and skew-symmetric matrices.

Sol. Let $A = \begin{bmatrix} 5 & 2 \\ 3 & -6 \end{bmatrix}$. Then, $A' = \begin{bmatrix} 5 & 3 \\ 2 & -6 \end{bmatrix}$

[interchanging the elements of rows and columns]

Now, $A + A' = \begin{bmatrix} 5 & 2 \\ 3 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ 2 & -6 \end{bmatrix}$

$$\Rightarrow \quad A + A' = \begin{bmatrix} 5+5 & 2+3 \\ 3+2 & -6-6 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & -12 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 10 & 5 \\ 5 & -12 \end{bmatrix}, \text{ which is symmetric.} \quad \ldots\text{(i)}$$

Also, $A - A' = \begin{bmatrix} 5 & 2 \\ 3 & -6 \end{bmatrix} - \begin{bmatrix} 5 & 3 \\ 2 & -6 \end{bmatrix}$

$$= \begin{bmatrix} 5-5 & 2-3 \\ 3-2 & -6+6 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

which is skew-symmetric. ...(ii)

On adding Eqs. (i) and (ii), we get

$$\frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$= \frac{1}{2}\begin{bmatrix} 10 & 5 \\ 5 & -12 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 10+0 & 5-1 \\ 5+1 & -12+0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 10 & 4 \\ 6 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 3 & -6 \end{bmatrix} = A$$

Hence proved.

INVERTIBLE MATRIX

A square matrix A of order m is said to be invertible, if there exists another square matrix B of same order m, such that $AB = BA = I$, where I is a unit matrix of same order m. The matrix B, is called the inverse of matrix A and is denoted by A^{-1}.

e.g. Let $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ be two square matrices of order 2.

Then, $AB = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 21-20 & -30+30 \\ 14-14 & -20+21 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= I$, which is of order 2.

Also, $BA = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 21-20 & 70-70 \\ -6+6 & -20+21 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= I$, which is of order 2.

Thus, $AB = BA = I$.

Hence, A is invertible and $A^{-1} = B$.

Note

(i) A rectangular matrix does not possess inverse matrix. Since, for the products BA and AB to be defined and to be equal, it is necessary that matrices A and B are square matrices of the same order.

(ii) If B is the inverse of A, then A is also the inverse of B.

Some Important Theorems

Theorem 1 (Uniqueness of Inverse) Inverse of a square matrix, if it exists, is unique.

Proof Let A be a square matrix such that its inverse exists.

If possible, let B and C be two inverses of A.

Then, we need to show $B = C$.

Since, B is the inverse of A.

$\therefore \quad AB = BA = I$ ...(i)

Also, C is the inverse of A.

$\therefore \quad AC = CA = I$ ...(ii)

Now, we can write $B = BI$

$= B(AC)$ [using Eq. (ii)]

$= (BA)C$

[by associative law of matrix multiplication]

$= IC$ [using Eq. (i)]

$= C$ **Hence proved.**

Theorem 2 If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$, where A and B are matrices of the same order.

Proof By definition of inverse, we have

$(AB)(AB)^{-1} = I$

$\Rightarrow \quad A^{-1}(AB)(AB)^{-1} = A^{-1}I$

[pre-multiplying both sides by A^{-1}]

$\Rightarrow \quad (A^{-1}A)B(AB)^{-1} = A^{-1}$

[by associative law of matrix multiplication and by using $AI = A = IA$]

$\Rightarrow \quad IB(AB)^{-1} = A^{-1}$ $\quad [\because AA^{-1} = I = A^{-1}A]$

$\Rightarrow \quad B(AB)^{-1} = A^{-1}$

$\Rightarrow \quad B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$

[pre-multiplying both sides by B^{-1}]

$\Rightarrow \quad I(AB)^{-1} = B^{-1}A^{-1}$

$\Rightarrow \quad (AB)^{-1} = B^{-1}A^{-1}$ **Hence proved.**

EXAMPLE |9| If $(AB)^{-1} = A^{-1}B^{-1}$, then prove that A^{-1} and B^{-1} satisfy commutative property with respect to multiplication. **[NCERT Exemplar]**

Sol. Given, $(AB)^{-1} = A^{-1}B^{-1}$ ...(i)

Now as, $(AB)^{-1} = B^{-1}A^{-1}$ [by theorem 2]

$\therefore \quad A^{-1}B^{-1} = B^{-1}A^{-1}$ [from Eq. (i)]

Here, we see that A^{-1} and B^{-1} satisfy the property of commutative with respect to multiplication.

TOPIC PRACTICE 4

OBJECTIVE TYPE QUESTIONS

1 If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, if the value of α is **[NCERT]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$

2 If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is **[NCERT Exemplar]**

(a) $m \times m$ (b) $n \times n$ (c) $n \times m$ (d) $m \times n$

3 The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a **[NCERT Exemplar]**

(a) diagonal matrix
(b) symmetric matrix
(c) skew-symmetric matrix
(d) scalar matrix

4 If the matrix A is both symmetric and skew-symmetric, then **[NCERT]**

(a) A is a diagonal matrix (b) A is a zero matrix
(c) A is a square matrix (d) None of these

5 If A, B are symmetric matrices of same order, then $AB - BA$ is a **[NCERT]**

(a) skew-symmetric matrix
(b) symmetric matrix
(c) zero matrix
(d) identity matrix

6 Matrices A and B will be inverse of each other only if **[NCERT]**

(a) $AB = BA$ (b) $AB = BA = O$
(c) $AB = O, BA = I$ (d) $AB = BA = I$

7 If A and B are square matrices of the same order and $AB = 3I$, then A^{-1} is equal to

(a) $3B$ (b) $\frac{1}{3}B$
(c) $3B^{-1}$ (d) $\frac{1}{3}B^{-1}$

VERY SHORT ANSWER Type Questions

8 Find the transpose of matrix $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$. **[NCERT]**

9 If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of a and b. **[CBSE 2018]**

10 If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$. **[All India 2012]**

11 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then find α satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}\, I_2$, where A^T is transpose of A. **[All India 2016]**

12 Show that matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix. **[NCERT]**

13 Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, then find the values of a and b. **[Delhi 2016]**

14 Show that matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew-symmetric matrix. **[NCERT]**

15 If A is a skew-symmetric matrix, then show that A^2 is a symmetric matrix. **[NCERT Exemplar]**

16 Write a 2×2 matrix which is both symmetric and skew-symmetric. **[Delhi 2014C]**

SHORT ANSWER Type I Questions

17 Write a 3×3 skew-symmetric matrix.

18 If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)'$. **[NCERT]**

19 If $A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$, then find $(AB)'$.

20 Show that $A'A$ and AA' are both symmetric matrices for any matrix A. **[NCERT Exemplar]**

21 If A is symmetric, then show that $B'AB$ is symmetric matrix. **[NCERT]**

22 If A and B are symmetric matrices, then prove that $AB + BA$ is a symmetric matrix.

23 If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c. **[NCERT Exemplar]**

24 Show that all the diagonal elements of a skew-symmetric matrix are zero. **[Delhi 2017]**

SHORT ANSWER Type II Questions

25 If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that $(A+B)' = A' + B'$. **[NCERT]**

26 If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that $(A+B)' = A' + B'$. **[NCERT]**

27 If $A = \begin{bmatrix} 2 & 3 & -5 \\ 0 & -1 & 4 \end{bmatrix}$, then verify that $(3A)' = 3A'$.

28 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then verify that $(2A+B)' = 2A' + B'$.

29 Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$.

30 If $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$, then verify that $(AB)' = B'A'$. **[NCERT Exemplar]**

31 If $A = [a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then check whether A is symmetric or skew-symmetric matrix.

32 For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix.

(ii) $(A - A')$ is a skew-symmetric matrix. **[NCERT]**

33 Express the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ as the sum of a symmetric matrix and the skew-symmetric matrix.

34 Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices. **[All India 2015C; NCERT Exemplar]**

35 Express matrix A as the sum of a symmetric and a skew-symmetric matrices,
where $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$. **[NCERT Exemplar]**

36 Show that a matrix which is both symmetric as well as the skew-symmetric matrix is a null matrix.

HINTS & SOLUTIONS

1. (b) **Hint** $A + A' = I$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\Rightarrow 2\cos\alpha = 1$$
$$\Rightarrow \cos\alpha = \frac{1}{2}$$
$$\Rightarrow \alpha = \frac{\pi}{3}$$

2. (d) Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{p\times q}$

$\therefore \quad B' = [b_{ji}]_{q\times p}$

Now, AB' is defined, so $n = q$

and $B'A$ is also defined, so $p = m$

$\therefore$ Order of B is $m \times n$

3. (c) **Hint** Let $A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$.

Then, $A' = -A$.

4. (b) The only zero matrix is both symmetric and skew-symmetric matrix.

5. (a) **Hint** $(AB - BA)' = (AB)' - (BA)'$
$$= B'A' - A'B'$$
$$= BA - AB \quad [\because A' = A \text{ and } B' = B]$$
$$= -(AB - BA)$$

6. (d) By definition of invertible matrix.

7. (b) **Hint** $AB = BI$

$$\Rightarrow \quad \frac{1}{3}(AB) = I$$

$$\Rightarrow \quad A\left(\frac{1}{3}B\right) = I$$

$$\Rightarrow \quad A^{-1} = \frac{1}{3}B$$

8. Similar as Example 1. $\left[\textbf{Ans.} \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}\right]$

9. Given, $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

$\therefore \quad A^T = -A$

$$\Rightarrow \quad \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}^T = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$$

On equating the corresponding elements, we get

$$a = -2 \quad \text{and} \quad b = 3$$

10. $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

11. Given, $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Also, given $A + A^T = \sqrt{2}\, I_2$

$$\therefore \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}^T = \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \cos\alpha + \cos\alpha & \sin\alpha - \sin\alpha \\ -\sin\alpha + \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$\Rightarrow \quad 2\cos\alpha = \sqrt{2}$$

$$\Rightarrow \quad \cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4}$$

12. Similar as Example 4.

13. Given, $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is a symmetric matrix.

We know that a matrix A is symmetric, if $A' = A$.

$$\therefore \text{ We have, } \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}' = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

On equating the corresponding elements, we get

$$3 = 2b \text{ and } 3a = -2$$

$$\therefore \quad b = \frac{3}{2} \text{ and } a = -\frac{2}{3}$$

14. Similar as Example 6.

15. Now, $(A^2)' = (AA)'$

$$= A'A' = (-A)(-A) \qquad [\because A' = -A]$$

$$= A^2$$

16. A null or zero matrix of order 2×2 is both symmetric and skew-symmetric.

17. **Hint** Find the matrix A of order 3×3 such that $A' = -A$.

$\left[\textbf{Ans.} \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}\right]$

18. We have, $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$$\therefore \quad A = (A')' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}' = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

[interchanging the elements of rows and columns]

$$\text{Now,} \quad A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-2 & 1+0 \\ 3+2 & 2+4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\Rightarrow \quad (A + 2B)' = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

[interchanging the elements of rows and columns]

19. Now, $AB = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} -2+0 & 4+0 \\ -1+12 & 2+0 \end{bmatrix}$

[multiplying rows by columns]

$$= \begin{bmatrix} -2 & 4 \\ 11 & 2 \end{bmatrix}$$

$$\therefore \quad (AB)' = \begin{bmatrix} -2 & 4 \\ 11 & 2 \end{bmatrix}' = \begin{bmatrix} -2 & 11 \\ 4 & 2 \end{bmatrix}$$

[interchanging the elements of rows and columns]

20. Consider, $(A'A)' = (A)'(A')' \qquad [\because (AB)' = B'A']$

$$= A' \cdot A \qquad [\because (A')' = A]$$

Hence, $A'A$ is symmetric matrix for any matrix A.

Now, $(AA')' = (A')'(A)' = A\,A' \qquad [\because (AB)' = B'A']$

Hence, $A\,A'$ is also symmetric matrix for any matrix A.

21. Given, A is a symmetric matrix.

$\therefore \quad A' = A$

Now, $(B'AB)' = (AB)'(B')'$ $\quad [\because (CD)' = D'C']$

$\Rightarrow \quad (B'AB)' = B'A'B$ $\quad [\because (B')' = B]$

$\Rightarrow \quad (B'AB)' = B'AB$ $\quad [\because A' = A]$

Hence, $B'AB$ is a symmetric matrix.

22. Given, A and B are symmetric matrices.

$\therefore \; A' = A$ and $B' = B$

Now, $(AB + BA)' = (AB)' + (BA)'$ $\quad [\because (A+B)' = A' + B']$

$= B'A' + A'B'$ $\quad [\because (AB)' = B'A']$

$= BA + AB$ $\quad [\because A' = A$ and $B' = B]$

$= AB + BA$ $\quad [\because$ matrix addition is commutative$]$

Hence, $AB + BA$ is a symmetric matrix.

23. Similar as Example 7. [**Ans.** $a = -2, b = 0$ and $c = -3$]

24. Let $A = [a_{ij}]$ be a skew-symmetric matrix.

Then, $a_{ij} = -a_{ji}$ for all i, j.

Now, if we put $i = j$, we get

$a_{ii} = -a_{ii}$ for all values of i

$\Rightarrow \quad 2a_{ii} = 0$ for all values of i

$\Rightarrow \quad a_{ii} = 0$ for all values of i

$\Rightarrow \quad a_{11} = a_{22} = a_{33} = \ldots = a_{nn} = 0$

$\Rightarrow$ All the diagonal elements of a skew-symmetric matrix are zero.

25. Similar as Example 2 (ii).

26. Similar as Example 2 (ii).

27. Similar as Example 2 (iii).

28. Similar as Example 2 (ii).

29. We have, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

For finding A^{-1}, using elementary row transformation, consider

$$A = IA$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

On applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 2R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

On applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

On applying $R_1 \to R_1 - R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

30. Similar as Example 3.

31. Consider a square matrix of order 2,

i.e. $A = [a_{ij}]_{2\times 2}$

Given, $a_{ij} = i^2 - j^2$

$\therefore \quad a_{11} = 1^2 - 1^2 = 1 - 1 = 0,$

$a_{12} = 1^2 - 2^2 = 1 - 4 = -3$

$a_{21} = 2^2 - 1^2 = 4 - 1 = 3$

$a_{22} = 2^2 - 2^2 = 0$

Now, $\quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

$\Rightarrow \quad A' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

[interchanging the elements of rows and columns]

$= -\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

[taking (-1) common from the matrix]

$= -A$

Hence, A is skew-symmetric matrix.

32. (i) Here, $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}'$

$= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

and $(A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$

$\therefore (A + A')$ is a symmetric matrix.

(ii) Here, $A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}'$

$= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

and $(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$

$\therefore (A - A')$ is a skew-symmetric matrix.

33. Similar as Example 8.

$\left[\textbf{Ans.} \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\right]$

34. We have, $A=\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$.

$$\Rightarrow \quad A'=\begin{bmatrix}2 & 7 & 1\\ 4 & 3 & -2\\ -6 & 5 & 4\end{bmatrix}$$

Let $P=\frac{1}{2}(A+A')=\frac{1}{2}\left\{\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}+\begin{bmatrix}2 & 7 & 1\\ 4 & 3 & -2\\ -6 & 5 & 4\end{bmatrix}\right\}$

$$=\frac{1}{2}\begin{bmatrix}4 & 11 & -5\\ 11 & 6 & 3\\ -5 & 3 & 8\end{bmatrix}=\begin{bmatrix}2 & \frac{11}{2} & -\frac{5}{2}\\ \frac{11}{2} & 3 & \frac{3}{2}\\ -\frac{5}{2} & \frac{3}{2} & 4\end{bmatrix},$$

which is symmetric matrix.

and $Q=\frac{1}{2}(A-A')=\frac{1}{2}\left\{\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}-\begin{bmatrix}2 & 7 & 1\\ 4 & 3 & -2\\ -6 & 5 & 4\end{bmatrix}\right\}$

$$=\frac{1}{2}\begin{bmatrix}0 & -3 & -7\\ 3 & 0 & 7\\ 7 & -7 & 0\end{bmatrix}=\begin{bmatrix}0 & -\frac{3}{2} & -\frac{7}{2}\\ \frac{3}{2} & 0 & \frac{7}{2}\\ \frac{7}{2} & -\frac{7}{2} & 0\end{bmatrix},$$

which is skew-symmetric matrix.

Now, $P+Q=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$

$$=\begin{bmatrix}2 & \frac{11}{2} & -\frac{5}{2}\\ \frac{11}{2} & 3 & \frac{3}{2}\\ -\frac{5}{2} & \frac{3}{2} & 4\end{bmatrix}+\begin{bmatrix}0 & -\frac{3}{2} & -\frac{7}{2}\\ \frac{3}{2} & 0 & \frac{7}{2}\\ \frac{7}{2} & -\frac{7}{2} & 0\end{bmatrix}=\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}=A$$

Hence, A is represented as sum of symmetric and skew-symmetric matrix.

35. Solve as Question 34. $\left[\text{Ans. } \begin{bmatrix}2 & 2 & \frac{5}{2}\\ 2 & -1 & \frac{3}{2}\\ \frac{5}{2} & \frac{3}{2} & 2\end{bmatrix}+\begin{bmatrix}0 & 1 & \frac{-3}{2}\\ -1 & 0 & \frac{1}{2}\\ \frac{3}{2} & \frac{-1}{2} & 0\end{bmatrix}\right]$

36. Let A be a matrix, which is both symmetric and skew-symmetric.

Then, we have $A^T=A$ [as A is symmetric] ...(i)

and $A^T=-A$ [as A is skew-symmetric] ...(ii)

From Eqs. (i) and (ii), we get

$$A=-A$$
$$\Rightarrow \quad A+A=O$$
$$\Rightarrow \quad 2A=O$$
$$\Rightarrow \quad A=O$$

Hence, matrix A, which is both symmetric and skew-symmetric is a null matrix.

SUMMARY

- **Matrix** A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix. It is enclosed by the symbol [ ] or ( ).
- **Order of a Matrix** If a matrix have m rows and n columns, then order of matrix is written as $m \times n$ and read as m by n matrix.
- **Types of Matrices**
  (i) A matrix having only one row is called a **row matrix.**
  (ii) A matrix having only one column is called a **column matrix.**
  (iii) If all the elements of a matrix are zero, then it is called a **zero** or **null matrix.** It is denoted by letter O.
  (iv) A matrix in which number of rows and number of columns are equal, is called a **square matrix.**
  (v) A square matrix $A=[a_{ij}]_{m\times n}$ is said to be a **diagonal matrix**, if all the elements lying outside the diagonal elements are zero.
  (vi) A diagonal matrix in which all diagonal elements are equal is called a **scalar matrix.**
  (vii) A square matrix having all diagonal elements 1 and 0 elsewhere, is called an **identity matrix.**

  (b) $(k_1+k_2)A=k_1A+k_2A$, where k_1 and k_2 are scalars.
  (c) $(kl)\,A=k(lA)=l(kA)$, where l and k are scalars.
  (iii) **Difference of Matrices** Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be the two matrices of same order, say $m\times n$, then the difference of these matrices, $A-B$ is defined as a matrix $D=[d_{ij}]$, where $d_{ij}=a_{ij}-b_{ij}$ for all values of i and j.
- **Multiplication of Matrices** The product AB of two matrices A and B exists, if the number of columns of A is equal to the number of rows of B. If $A=[a_{ij}]_{m\times n}$ and $B=[b_{jk}]_{n\times p}$, then $AB=C=[c_{ik}]_{m\times p}$, where c_{ik} is the (i,k)th element of the matrix C and it is given by multiplying ith row of A and kth column of B, element-wise and take the sum of all these products.
  (i) Matrix multiplication is associative and it satisfies distributive law.
  (ii) Matrix multiplication is not commutative.
  (iii) For every square matrix A, there exists an identity matrix I of same order such that $AI=A=IA$.
  (iv) If the product of two matrices is a zero matrix, then it is not necessary that one of the matrices is zero matrix.
- **Transpose of a Matrix** The matrix obtained by interchanging the rows and columns of matrix A is called the transpose of matrix A. It is denoted by A' or A^c or A^T.

- **Equality of Matrices** Two matrices, say $A=[a_{ij}]$ and $B=[b_{ij}]$ are said to be equal, if their order are same and their corresponding elements are also equal, i.e. $a_{ij}=b_{ij}$, $\forall$ i and j.
- **Operations on Matrices**
  (i) **Addition of Matrices** Let $A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{m\times n}$ be the two matrices, then $A+B$ is defined as matrix $C=[c_{ij}]_{m\times n}$, where $c_{ij}=a_{ij}+b_{ij}$, $\forall\, i, j$.
  (a) Matrix addition is commutative and associative.
  (b) O is the additive identity for matrix addition.
  (c) $(-A)$ is the additive inverse of A.
  (ii) **Multiplication of a Matrix by a Scalar** Let $A=[a_{ij}]_{m\times n}$ be a matrix and k be a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.
  Let A and B be the two matrices of same order. Then,
  (a) $k(A+B)=kA+kB$, where k is a scalar.
- **Properties of Transpose of Matrices** For any matrices A and B of suitable orders, we have
  (i) $(A')'=A$
  (ii) $(A\pm B)'=A'\pm B'$
  (iii) $(kA)'=kA'$, where k is any constant.
  (iv) $(AB)'=B'A'$ [reversal law]
- **Symmetric and Skew-symmetric Matrices** A square matrix A is called a symmetric matrix, if $A'=A$ and a square matrix A is said to be skew-symmetric matrix, if $A'=-A$.
- **Invertible Matrix** A square matrix A of order m is said to be invertible, if there exists another square matrix B of same order m, such that $AB=BA=I$, where I is a unit matrix of same order m. The matrix B is called the inverse of matrix A and is denoted by A^{-1}.

CHAPTER PRACTICE

OBJECTIVE TYPE QUESTIONS

1 A matrix $A=[a_{ij}]_{3\times 3}$ is defined by

$$a_{ij}=\begin{cases}2i+3j, & i<j\\ 5, & i=j\\ 3i-2j, & i>j\end{cases}$$

The number of elements in A which are more than 5, is **[CBSE 2021 (Term I)]**

(a) 3 (b) 4 (c) 5 (d) 6

2 Given that matrices A and B are of order $3\times n$ and $m\times 5$, respectively, then the order of matrix $C=5A+3B$ is **[CBSE 2021 (Term I)]**

(a) 3×5 and $m=n$ (b) 3×5
(c) 3×3 (d) 5×5

3 If matrix $A=[a_{ij}]_{2\times 2}$, where $a_{ij}=\begin{cases}1, \text{ if } i\neq j\\ 0, \text{ if } i=j\end{cases}$, then A^2 is equal to **[NCERT Exemplar]**

(a) I (b) A
(c) O (d) None of these

4 The value of x such that

$$\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O, \text{ is}$$

(a) 1 (b) 0 (c) -1 (d) 3

5 If $\begin{bmatrix}2a+b & a-2b\\ 5c-d & 4c+3d\end{bmatrix}=\begin{bmatrix}4 & -3\\ 11 & 24\end{bmatrix}$, then the value of $a+b-c+2d$ is **[CBSE 2021 (Term I)]**

8 If a matrix A is both symmetric and skew- symmetric, then A is necessarily a **[CBSE 2021 (Term I)]**

(a) diagonal matrix (b) zero square matrix
(c) square matrix (d) identity matrix

VERY SHORT ANSWER Type Questions

9 If possible, then find the sum of the matrices A and B, where $A=\begin{bmatrix}\sqrt{3} & 1\\ 2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}x & y & z\\ a & b & c\end{bmatrix}$. **[NCERT Exemplar]**

10 Simplify $\begin{bmatrix}\cos^2 x & \sin^2 x\\ \sin^2 x & \cos^2 x\end{bmatrix}+\begin{bmatrix}\sin^2 x & \cos^2 x\\ \cos^2 x & \sin^2 x\end{bmatrix}$. **[NCERT]**

11 Find the value of $y-x$ from the following equation $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$ **[All India 2012]**

12 Find non-zero values of x satisfying the matrix equation $x\begin{bmatrix}2x & 2\\ 3 & x\end{bmatrix}+2\begin{bmatrix}8 & 5x\\ 4 & 4x\end{bmatrix}=2\begin{bmatrix}x^2+8 & 24\\ 10 & 6x\end{bmatrix}$. **[NCERT Exemplar]**

13 For the matrix $X=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, then find (X^2-X). **[CBSE 2021 (Term I)]**

14 If $\begin{bmatrix}9 & -1 & 4\\ -2 & 1 & 3\end{bmatrix}=A+\begin{bmatrix}1 & 2 & -1\\ 0 & 4 & 9\end{bmatrix}$, then find the

(a) 8 (b) 10 (c) 4 (d) –8

6 If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to **[NCERT]**

(a) A (b) $I - A$
(c) I (d) $3A$

7 For any two matrices A and B, we have **[NCERT Exemplar]**

(a) $AB = BA$ (b) $AB \neq BA$
(c) $AB = O$ (d) None of these

17 Find x from the matrix equation $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$.

18 If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ -1 \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, find $x + y + z$. **[Delhi 2016C]**

19 If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the value of k, a and b. **[CBSE 2021 (Term I)]**

20 If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find $A + A'$. **[Delhi 2013]**

SHORT ANSWER Type I Questions

21 If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix?

22 If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$, then find the values of a, b, c and z. **[NCERT]**

23 Assume Y, W and P are the matrices of orders $3 \times k$, $n \times 3$ and $p \times k$. Find the restrictions on n, k and p, so that $PY + WY$ will be defined. **[NCERT]**

24 Give one example of skew-symmetric matrix of order 2 and order 3.

25 Find the symmetric and skew-symmetric matrices of matrix $A = \begin{bmatrix} 0 & -2 & 4 \\ 2 & 0 & -1 \\ -4 & 1 & 0 \end{bmatrix}$.

26 If A and B are symmetric matrices, then prove that $BA - 2AB$ is neither a symmetric matrix nor skew-symmetric matrix. **[NCERT Exemplar]**

matrix A. **[Delhi 2013]**

15 Let $A = [a_{ij}]_{n \times n}$ be a diagonal matrix whose diagonal elements are different and $B = [b_{ij}]_{n \times n}$ is some another matrix. If $AB = [c_{ij}]_{n \times n}$, then find c_{ij}.

16 Suppose $A = \begin{bmatrix} 5 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 & 1 \\ 6 & 8 & 4 \end{bmatrix}$, then find AB and BA, if they exist.

28 If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify that $(BA)^2 \neq B^2A^2$. **[NCERT Exemplar]**

29 If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then show that $(A+B)(A-B) \neq A^2 - B^2$. **[NCERT Exemplar]**

30 Solve the matrix $[x \;\; -5 \;\; -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$. **[NCERT]**

31 If $[2 \;\; 1 \;\; 3]\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then find the value of A.

32 If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then prove that $A_\alpha A_\beta = A_{\alpha+\beta}$.

33 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then show that $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$.

34 Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. **[NCERT Exemplar]**

35 If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $A^2 - 5A + 6$.

36 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, then prove that $A^3 - 4A^2 + A = O$.

37 If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, then find k such that $A^2 - 8A + kI = O$.

SHORT ANSWER Type II Questions

27 Prove by mathematical induction that $(A')^n = (A^n)'$, where $n \in N$ for any square matrix A. **[NCERT Exemplar]**

40 A factory makes three products P, Q and R. The table shows the units of labour, materials and other items needed to produce one of each product.

Product	Labour	Materials	Other items
P	4	3	$2+a$
Q	5	$2+b$	3
R	2	5	4

Represent this data by a matrix A. Given that labour costs ₹ 10 per unit, materials ₹ 4 per unit and other items ₹ 6 per unit. Represent this by a column matrix B. Given that the total cost of the product P is ₹ 82 and the product Q is ₹ 88, respectively. Find the values of a and b.

41 A manufacturer sells the products x, y and z in two markets. Annual sales are indicated below

Market	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

(i) If unit sale prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively, then find the total revenue in each market with the help of matrix algebra.

(ii) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise, respectively. Find the gross profit.
[Hint Profit = Revenue − Cost**]** **[NCERT]**

42 In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways: telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

$$\text{Cost per contact, } A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \text{Telephone} \\ \text{House call} \\ \text{Letter} \end{array}$$

The number of contacts of each type made in two cities X and Y is given by

$$B = \begin{array}{c} \begin{array}{ccc} \text{Telephone} & \text{House call} & \text{Letter} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \to X \\ \to Y \end{array}$$

38 For the matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find a and b such that $A^2 + aI = bA$, where I is a 2×2 identity matrix.

39 If $A = \begin{bmatrix} 4 & 0 \\ 2k & 5k \end{bmatrix}$ and $B = \begin{bmatrix} k & 0 \\ 3 & -1 \end{bmatrix}$, such that $AB = BA$. Then, show that $2k^2 + 17k - 12 = 0$.

45 If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$ (ii) $(A-B)' = A' - B'$ **[NCERT]**

46 Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, where

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}.$$ **[NCERT]**

47 If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that $(A - A')$ is a skew-symmetric matrix, where A' is the transpose of matrix A. **[NCERT Exemplar]**

48 Show that the matrix $B^T AB$ is symmetric or skew-symmetric, according as A is symmetric or skew-symmetric. **[NCERT]**

LONG ANSWER Type Questions

49 If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then prove that A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.
[Hint Prove that $f(A) = O$**]** **[NCERT]**

50 If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that

$A^3 - 6A^2 + 7A + 2I = O$. **[Delhi 2016C]**

51 Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = O$. Use this result to find A^5.

CASE BASED Questions

|5 Marks|

52 A manufacture produces three stationery products pencil, eraser and sharpener which he sells in two markets. Annual sales are indicated below

Find the total amount spent by the group in the two cities X and Y.

43 If $AB = BA$ for any two square matrices, then prove by mathematical induction that $(AB)^n = A^n B^n$. **[NCERT Exemplar]**

44 If $A = \text{diag}\,[a, b, c]$, then show that $A^n = \text{diag}\,[a^n, b^n, c^n], \forall\, n \in N$.

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10000	2000	18000
B	6000	20000	8000

If the unit sale price of pencil, eraser and sharpener are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively and unit cost of the above three commodities are ₹ 2.00, ₹ 1.00 and ₹ 0.50 respectively. **[CBSE Question Bank]**

Based on the above information, answer the following questions.

(i) Total revenue of market A is

(a) ₹ 64000 (b) ₹ 60400
(c) ₹ 46000 (d) ₹ 40600

(ii) Total revenue of market B is

(a) ₹ 35000 (b) ₹ 53000
(c) ₹ 50300 (d) ₹ 30500

(iii) Cost incurred in market A is

(a) ₹ 13000 (b) ₹ 30100
(c) ₹ 10300 (d) ₹ 31000

(iv) Profit in market A and B respectively are

(a) (₹ 15000, ₹ 17000)
(b) (₹ 17000, ₹ 15000)
(c) (₹ 51000, ₹ 71000)
(d) (₹ 10000, ₹ 20000)

(v) Gross profit in both market is

(a) ₹ 23000 (b) ₹ 20300
(c) ₹ 32000 (d) ₹ 30200

53 Amit, Biraj and Chirag were given the task of creating a square matrix of order 2.

Below are the matrices created by them. A, B and C are the matrices created by Amit, Biraj and Chirag, respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

If $a = 4$ and $b = -2$. **[CBSE Question Bank]**

Based on the above information, answer the following questions.

(i) Sum of the matrices A, B and C, $A + (B + C)$ is

(a) $\begin{bmatrix} 1 & 6 \\ 2 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 1 \\ 7 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 7 & 6 \end{bmatrix}$

(ii) $(A^T)^T$ is equal to

(a) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

(iii) $(bA)^T$ is equal to

(a) $\begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 2 \\ -6 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 2 & 4 \end{bmatrix}$

(iv) $AC - BC$ is equal to

(a) $\begin{bmatrix} -4 & -6 \\ -4 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & -4 \\ 4 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & 4 \\ -4 & -4 \end{bmatrix}$

(v) $(a + b)B$ is equal to

(a) $\begin{bmatrix} 0 & 8 \\ 10 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 10 \\ 8 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ 8 & 10 \end{bmatrix}$

54 Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

September sales (in ₹)

$$A = \begin{array}{l} \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramakishan} \\ \text{Gurucharan} \end{array}$$

October sales (in ₹)

$$B = \begin{array}{l} \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramakishan} \\ \text{Gurucharan} \end{array}$$

[CBSE Question Bank]

Answer the following questions using the above information.

(i) The total sales in September and October for each farmer in each variety can be represented as

(a) $A + B$ (b) $A - B$

(ii) What is the value of A_{23}?

(a) 10000 (b) 20000

(c) 30000 (d) 40000

(iii) The decrease in sales from September to October is given by

(a) $A + B$ (b) $A - B$

(c) $A > B$ (d) $A < B$

(iv) If Ramakishan receives 2% profit on gross sales, compute his profit for each variety sold in October.

(a) ₹ 100, ₹ 200 and ₹ 120

(c) $A > B$ (d) $A < B$

(b) ₹ 100, ₹ 200 and ₹ 130

(c) ₹ 100, ₹ 220 and ₹ 120

(d) ₹ 110, ₹ 200 and ₹ 120

(v) If Gurucharan receives 2% profit on gross sales, compute his profit for each variety sold in September.

(a) ₹ 100, ₹ 200 and ₹ 120

(b) ₹ 1000 , ₹ 600 and ₹ 200

(c) ₹ 400, ₹ 200 and ₹ 120

(d) ₹ 1200, ₹ 200 and ₹ 120

| ANSWERS |

1. (b) **2.** (b) **3.** (a) **4.** (c) **5.** (a) **6.** (c)

7. (d) **8.** (b) **9.** Not possible **10.** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **11.** 7 **12.** $x = 4$

13. $2I$ **14.** $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$ **15.** $a_{ii}\, b_{ij}$ **16.** $AB = \begin{bmatrix} 39 & 57 & 21 \\ 24 & 34 & 14 \end{bmatrix}$; BA does not exist.

17. $x = -1$ **18.** 0 **19.** $-6, -4, -9$ **20.** $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$ **21.** $\alpha = 0°$

22. $a = -2$, $b = -7$, $c = -1$ and $z = 2$ **23.** $k = 3$, $p = n$ **24.** $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$

25. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 & -2 & 4 \\ 2 & 0 & -1 \\ -4 & 1 & 0 \end{bmatrix}$ **30.** $x = \pm 4\sqrt{3}$ **31.** $A = [-4]$ **34.** $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

35. $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ **37.** $k = 7$ **38.** $a = 8$, $b = 8$ **40.** $a = 3$, $b = 3$

41. (i) Total revenue in market I is ₹ 46000 and market II is ₹ 53000. (ii) Gross profit = ₹ 32000

42. Total amount spent by the group in two cities are ₹ 3400 and ₹ 7200, respectively.

46. $\frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ **51.** $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

52. (i) → (c), (ii) → (b), (iii) → (d), (iv) → (a), (v) → (c)

53. (i) → (c), (ii) → (a), (iii) → (b), (iv) → (c), (v) → (c)

54. (i) → (a), (ii) → (a), (iii) → (b), (iv) → (a), (v) → (b)