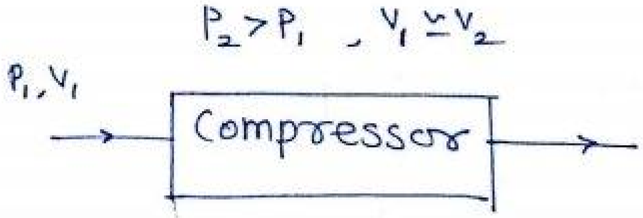
Air Compressor



Motor cap. More
less power use

- Incompressible
- Density & temp Constant – Pump.
- Density & temp. Change – Comp.
- Compressible

*The function of a compressor is to compress the gases or vapour from a lower pressure to higher pressure and for that work is done from outside.

* Classification

① Based on working principle:-

Positive displacement Compressor:- In these compressor the pressure of gas increased by decreasing it's volume in a confined cylinder i.e ~~and~~ rise in pressure is obtained by positively displacing fluid to a lower volume.

Eg reciprocating compressor

Dynamic Compressor :- In these compressure high K.E. is given to fluid by a rotaing element called rotor and then this high K.E. is transformed into pressure energy in diffusor i.e. rise in pressure is obtained due to dynamic action of gas.

eg. Centrifugal and axial flow compressor.

2) Based upon Pressure developed

→ low pressure – upto 10 bar.

→ Medium Pressure – ~~upto~~ 10 to 80 bar.

→ High pressure – above 80 bar

3) Pressure Ratio $\left(P_r = P_2/P_1\right)$

→ Fan $1.0 < P_r < 1.1$

→ Blower $1.1 < P_r < 2.3$

→ Compressor $P_r > 2.3$

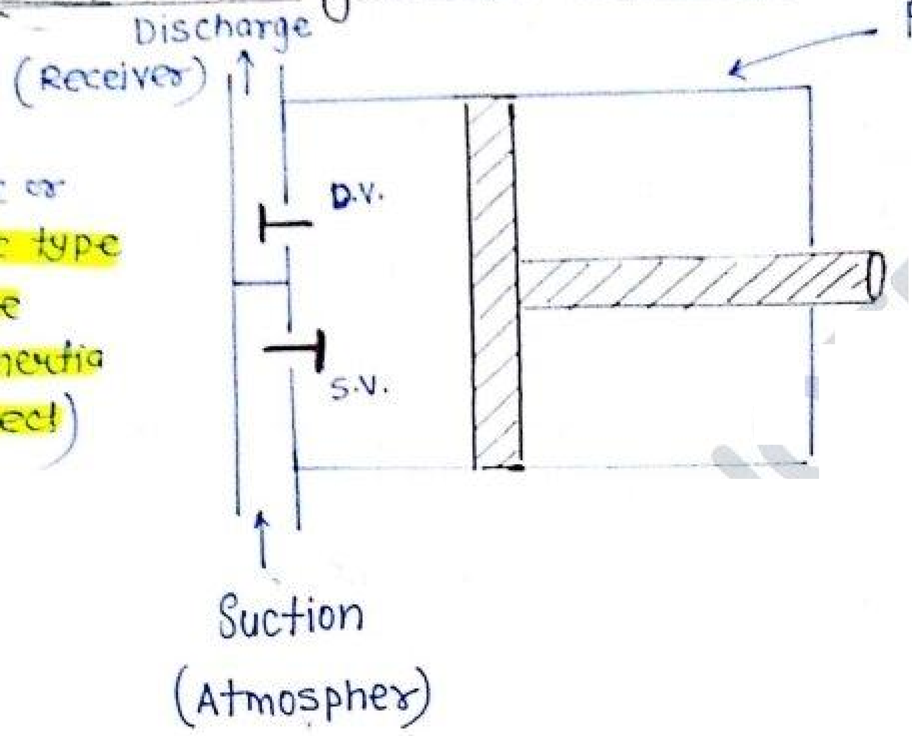
4) Volume handing capacity :-

→ small capacity — upto 9 m^3/min

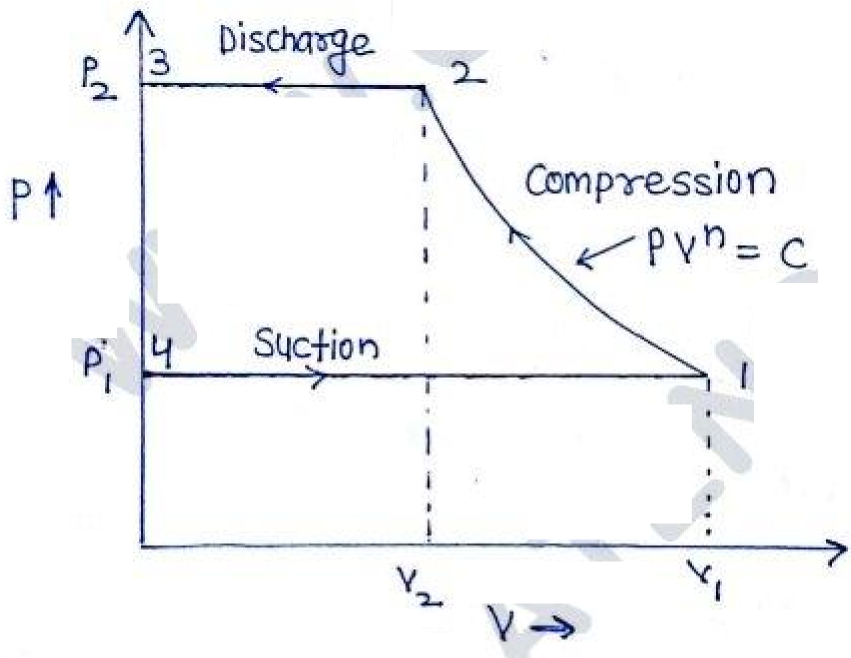
→ Medium capacity — 9 to 3000 m^3/min

→ large capacity — above 3000 m^3/min

Reciprocating Compressor :-



Disc or plate type Valve (less Inertia effect)



This is not complately open flow or not complately close flow.

* In construction it consist of a piston cy. arrangment with two valves one on the suction side and otherone on the discharge side These are simple automatic Valve which works due to pressure diff. across the Valve.

Disadvantages:

(i) Intermitent supply
(ii) Volume handling capacity is very less (8-12 m^3/min.)
(iii) Poor efficiency.
(iv) low rpm.

Advantages:-

(i) Able to develope very high pressure (upto 1000 bar)
(ii) low investment
(iii) low space requirement.

* These compressor are used in Refrigrator, A/c, air filling station, drilling, air breaks, mining work, chemical industries, fertilizer factory etc.

Work input require without clearance Volume:-

Work done per cycle is given by area 1-2-3-4-1

$$W_{cycle} = \text{Area } 1-2-3-4-1$$

work done by air during suction (4-1)

$$W_s = P_1.(V_1 - 0) = P_1 V_1$$

Work done on air during ~~discharge~~ Compression (1-2)

$$W_c = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{-(P_2 V_2 - P_1 V_1)}{n-1}$$

work done on air during discharge (2-3)

$$W_d = P_2(0 - V_2) = -P_2V_2$$

$$W_{cycle} = -\left[P_2 \cdot V_2 - P_1V_1 + \frac{P_2V_2 - P_1V_1}{n-1}\right]$$

$$W_{cycle} = -\left[P_2V_2 - P_1V_1\right]\left[1 + \frac{n}{n-1}\right]$$

$$* \boxed{W_{cycle} = \left(\frac{-n}{n-1}\right)\left(P_2V_2 - P_1V_1\right)}$$

Now as compressor work is always negative taking only the magnitude.

$$* \boxed{W_{cyce} = \left(\frac{n}{n-1}\right)\left(P_2V_2 - P_1V_1\right)}$$

$$W_{cycle} = \left(\frac{n}{n-1}\right)P_1 \cdot V_1 \left(\frac{P_2 \cdot V_2}{P_1 \cdot V_1} - 1\right)$$

For process 1-2 $\quad P_1V_1^n = P_2V_2^n$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{-\frac{1}{n}}$$

$$W_{cycle} = \left(\frac{n}{n-1}\right)P_1V_1\left[\frac{P_2}{P_1}\left(\frac{P_2}{P_1}\right)^{-1/n} - 1\right]$$

* $$\boxed{W/_{cycle} = \left(\frac{n}{n-1}\right) \cdot P_1 \cdot V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]}$$ KJ/cycle.

P_1 - KPa, V_1 - m^3/cycle

if N - rpm

* $$\boxed{Power = W/_{cycle} \times \frac{N}{60}}$$ KW

Q27
WB
Pg-21

$V = 10\ m^3/min$ $P_1 = 1$ bar. $T_1 = 300\ K$

$P_2 = 8$ bar.

i) $n = 1.3$

a) $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 300(8)^{\frac{1.3-1}{1.3}}$

$T_2 = 484.75\ K.$

b) $W_s = P_1(0 - V_1) = -1 \times 10^5\ Pa \times 10\ m^3/min$

$W_s = +1666.67\ KW$

c) $W_c = -\left(\frac{P_2V_2 - P_1V_1}{n-1}\right)$

$P_1V_1^n = P_2V_2^n$

$(1)(10)^{1.3} = 8(V_2)^{1.3}$

$V_2 = 2.019\ m^3/min$

$W_c = \frac{-(8 \times 2.019 - 1 \times 10) \times 10^2}{1.3 - 1}$

$W_c = -\frac{2052.88}{60}\ KW = -34.21\ KW$

d) $W_d = P_2 (V_2 - V_1)$

$= 8 \times 10^2 (2.019)\ \frac{m^3}{min} \times kPa$

$W_d = -\underline{26.92}\ kW$

e) $W_{net} = +16.66.67 - 34.21 + 26.92$

$W_{net} = \cancel{23.957}\ kW$

$\cancel{W_{net} =} \ -\cancel{+673.96\ kW}$ per cycle

$W_{net} = \underline{\underline{-44.42\ kW}}$

f) $Q = -\left(\frac{P_2 V_2 - P_1 V_1}{n-1}\right) + m\, c_v (T_2 - T_1)$

$$\cancel{Q = \left(\frac{8 \times 2.01 - 1 \times 10}{0.3}\right) \times \frac{10^2}{60}\ kW}$$

$$\cancel{Q = 33.77\ kW}$$

$$\dot{m} = \frac{P_1 V_1}{R T_1} = \frac{10^5\ Pa \times 10}{287 \times 300} = 0.1935\ kg/s$$

$$Q_{1-2} = \left(\frac{\gamma - n}{\gamma - 1}\right) \times W_c$$

$$Q_{1-2} = \frac{1.4 - 1.3}{1.4 - 1} \times (-34.2) = \underline{\underline{-8.55\ kW}}$$

g) $\Delta U = m c_v (T_2 - T_1) = \frac{c_v}{R}(P_2 V_2 - P_1 V_1) = 25.66\ kW$

$dQ = dU + dW$

$dU = -8.55 + 34.2 = 25.66\ kW$

ii) $n = 1.4$

$T_2 = 543.4 K$

$W_s = 16.67 KW$

$W_c = 33.81 KW$

$W_d = 30.48 KW$

$W_{net} = 47.3 KW$

$Q_{1-2} = 0$

$\Delta U = 33.81 KW$

(iii) $n = 1$

i) $T_2 = T_1 = 300 K$

ii) $W_s = 16.67 W$

$W_c = P_1 V_1 \ln\left(\frac{P_2}{P_1}\right) = 34.65 KW$

$W_d = P_2 V_2 = P_1 V_1 = 16.67 KW$

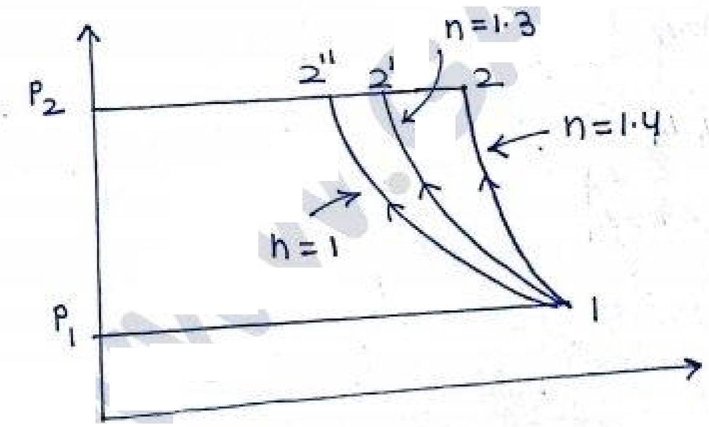
$W_{net} = W_c = 34.65 KW$

Q_{net} $W_2 = W_c = -34.65 KW$

$\Delta U = \underline{0}$

---

*



$n \simeq 1.25$ to 1.35

Isothermal efficiency (η_{iso}) it is measure of reciprocating Compressor

$$\eta_{iso} = \frac{W_{iso}}{W_{act.}}$$

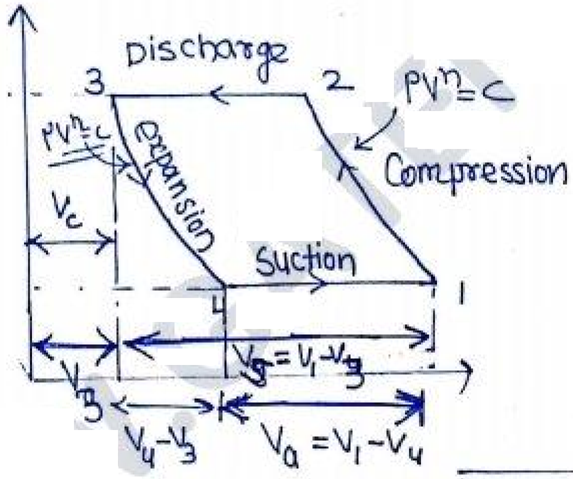
$$* \; W_{iso} = P_1 V_1 \ln\left(\frac{P_2}{P_1}\right)$$

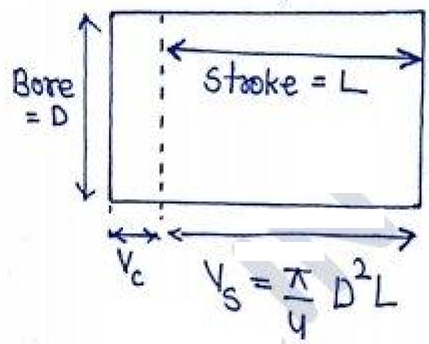
$$* \; W_{act} = \left(\frac{n}{n-1}\right) P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\eta_{iso} \simeq 80 \text{ to } 90\%$$

* Work input require with clearance Volume:-

In actual working there is always left a min. volume between piston & Cylinder head known as clearance Volume. It is provided for thermal expansion, valve tolerance and to avoid piston striking the Cylinder head.





Clearance ratio, $\boxed{C = \frac{V_c}{V_s}}$

$C\% \simeq 5\%$

Work input per cycle is given by Area 1-2-3-4-1

$$W/_{cycle} = \left(W_{1-2} - W_{4-3}\right)_{\text{without clearance.}}$$

$$W/_{cycle} = \left(\frac{n}{n-1}\right) P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] - \left(\frac{n}{n-1}\right) P_4 V_4 \left[\left(\frac{P_3}{P_4}\right)^{\frac{n-1}{n}} - 1\right]$$

as $P_2 = P_3$
$P_4 = P_1$

$$* \boxed{W/_{cycle} = \left(\frac{n}{n-1}\right) P_1 (V_1 - V_4) \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]}$$

Note:- Due to clearance Volume there is no change in work input require by Compression the only diff. the actual Volume of air deliver per cycle is decreas. thus for the same volume to be delivered the number of cycle require increases.

* Volumetric efficiency :- It indicates the volume of air that is delivered during the total displacement of piston.

It is the ratio of actual Volume of air delivered to the total piston displacement Vol.

$$\eta_v = \frac{V_a}{V_s} = \frac{\text{Actual Volume}}{\text{Swept Volume}}$$

$$\eta_v = \frac{V_s - (V_4 - V_3)}{V_s}$$

$$\eta_v = 1 - \frac{V_3}{V_s}\left(\frac{V_4}{V_3} - 1\right)$$

$V_3 = V_c$

$\frac{V_c}{V_s} = c$

for expansion process (3-4)

$P_3 V_3^n = P_4 V_4^n \qquad P_1 = P_4$

$P_2 = P_3$

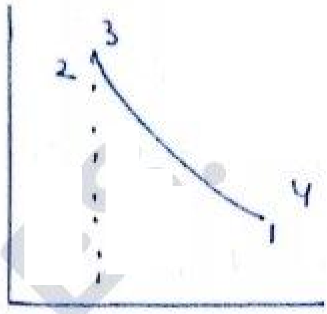
$\frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{1/n}$

$$\boxed{\eta_v = 1 - c\left[\left(\frac{P_2}{P_1}\right)^{1/n} - 1\right]}$$

if $\eta_v = 0$

$$0 = 1 - c\left[\left(\frac{P_2}{P_1}\right)^{1/n} - 1\right]$$

$$\boxed{\frac{P_2}{P_1} = \left[\frac{1}{c} + 1\right]^{n}}$$



Imp

Conclusion :-

→ when $P_2 = P_1$ there is no compression but volumentric efficiency becomes 100%.

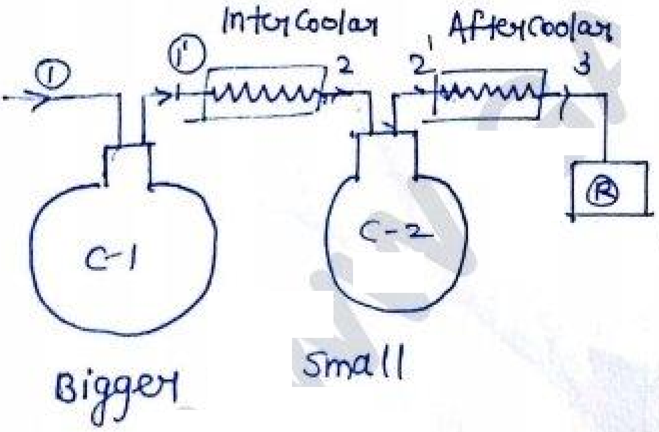
→ With increase in P_2 for a fix value of P_1 volumetric efficiency decrease.

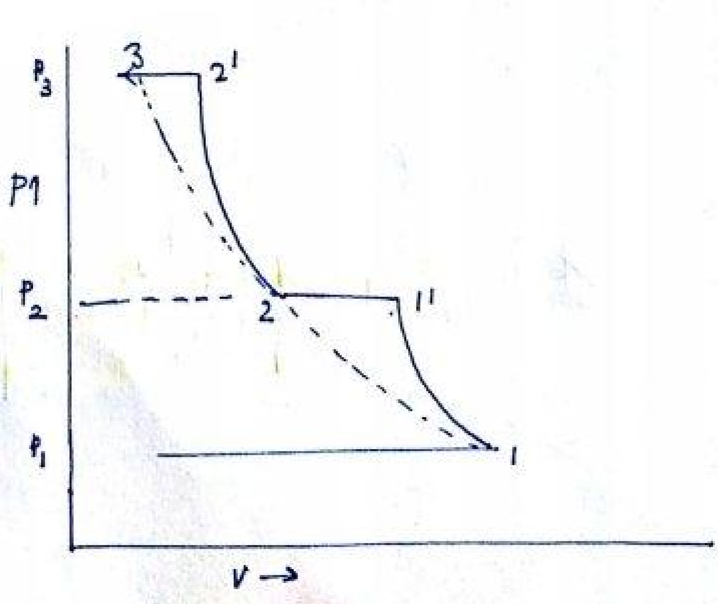
→ Volumetric efficiency decrease with increase in clearance ratio

→ η_{vol} increase with increase in expansion coeff.

22/11/2016

Multistage Compressor :-





if $P_1 = 1$ bar

Single stage - 5.6 bar

double stage - 5.6 to 35 bar

triple stage - 35 to 180 bar

In single stage as pressure ratio increase, Volumetric eff. decrease & therefore multistageng become compulsary beyond certain pressure ratio It also save the construction cost of material and leakage across the piston. If intercooling is used in b/w stages work input required by compressor can also be minimised

Intermediate pressure for minimum work input.

$$W_c = W_{c_1} + W_{c_2}$$

$$W_c = \left(\frac{n}{n-1}\right) mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] + \left(\frac{n}{n-1}\right) mRT_2 \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\text{let } x = \frac{n-1}{n}$$

$$W_c = \frac{mRT_1}{x}\left[\left(\frac{P_2}{P_1}\right)^{x} - 1\right] + \frac{mRT_2}{x}\left[\left(\frac{P_3}{P_2}\right)^{x} - 1\right]$$

for W_c to be minimum

$$\frac{dW_c}{dP_2} = 0 \qquad P_1 \,\&\, P_3 = \text{Const.}$$

$$T_1 \frac{x P_2^{x-1}}{P_1^{x}} - T_2 \frac{x P_3^{x}}{P_2^{x+1}} = 0$$

$$T_1 P_2^{2x} = T_2 P_1^{x} P_3^{x}$$

$$P_2 = \sqrt{\left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} P_1 P_2}$$

$$\boxed{P_2 = \sqrt{P_1 P_3}\left(\frac{T_2}{T_1}\right)^{\frac{n}{2(n-1)}}}$$

For perfect Inter Cooling, $T_2 = T_1$

$$\boxed{P_2 = \sqrt{P_1 P_3}}$$

$$\boxed{\frac{P_2}{P_1} = \frac{P_3}{P_2}}$$

* $\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \frac{P_5}{P_4} \; --- = \frac{P_{N+1}}{P_N} = K$

* If there are N stages in multistage compressor with perfect intercooling the pressure ratio across each stage must be equal

$$P_2 = KP_1$$

$$P_3 = KP_2 = K^2 P_1$$

$$\vdots$$

$$P_{N+1} = K^N P_1$$

$\frac{P_{N+1}}{P_1} = P_{r,0}$ = overall pressure ratio

$k = P_{r,s}$ = Pressure ratio for each stage

$\boxed{P_{r,0} = (P_{r,s})^N}$ N – No. of stages.

Now $W_{c_1} = \left(\frac{n}{n-1}\right) mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$

$$W_{c_2} = \left(\frac{n}{n-1}\right) mRT_2 \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$T_1 = T_2 \ \& \ P_2/P_1 = P_3/P_2$$

$$* \ \boxed{W_{c_1} = W_{c_2}}$$

Note:- for perfect intercooling with min. work input pressure ratio accros each stage will be equal & each stage compressor require equal amount of work.
same

Que Find the min. stage required to when air enters at 1 bar & 15°C and delivers at 350 bar. the max. discharge temp. is limited to 143°C take perfect intercooling in b/w Stages & $n = 1.3$

Sol^n

$$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2'}{T_1}\right)^{\frac{n}{n-1}} = \left(\frac{416}{288}\right)^{\frac{1.3}{0.3}} = 4.92$$

$$350 = (4.92)^N$$

$$N = 3.6765 \simeq 4$$

↳ in Gate remain same

Free Air Delieverad (FAD)

inlet

C_1 — C_2

$P_2 = 35$ bar $P_2 = 80$ bar

$T_2 = 470$ K $T_2 = 485$ K

$V_2 = 1.3\ m^3/min$ $V_2 = 0.8\ m^3/min$

$P_t = 1$ bar

$T_t = 288$ bar $m_t = m_2$

$V_t = FAD$

$$\frac{P_t V_t}{T_t} = \frac{P_2 V_2}{T_2} \qquad \frac{P_t V_t}{T_t} = \frac{P_2 V_2}{T_2}$$

$$(V_t)_{c_1} = 27.8\ m^3/min \qquad (V_t)_2 = 38.02\ m^3/min$$

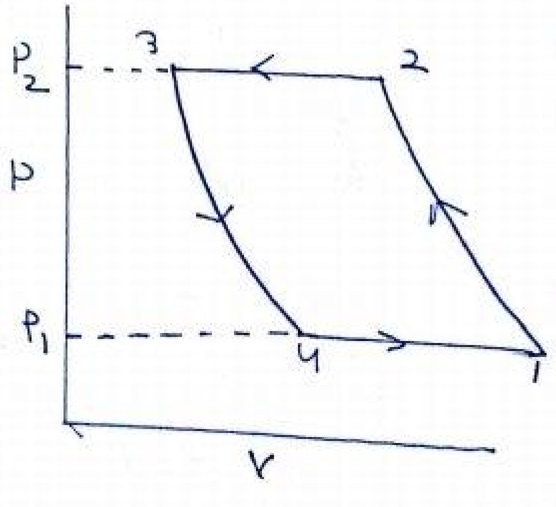
It works as a reference point to Comparing Volume handling capacity of different Compressor. The volume of air when converted to 1 bar pressure & 15°C temp. is known as FAD.

mass of air during suction

$$\dot{m}_s = \frac{P_1(V_1 - V_4)}{RT_1}$$

Mass of air during discharging

$$\dot{m}_d = \frac{P_2(V_2 - V_3)}{RT_2}$$



Mass of air at reference

$$\dot{m}_t = \frac{P_t V_t}{R T_t}$$

$P_t = 1$ bar, $T_t = 288K$, $V_t = FAD$

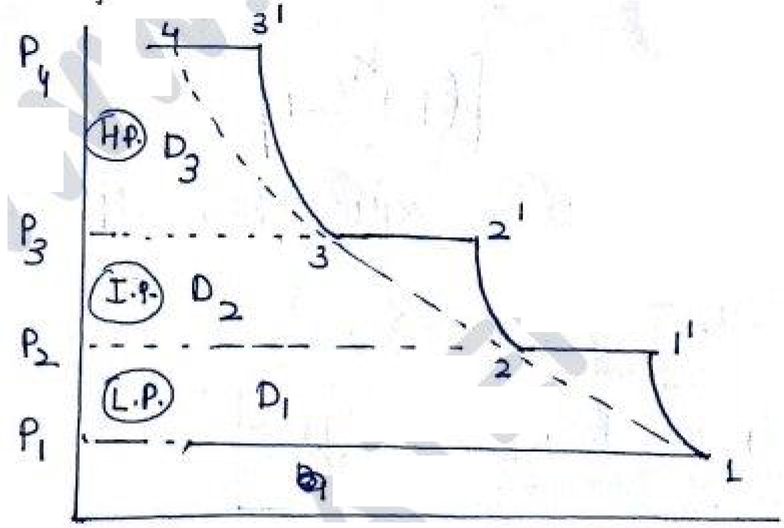
$$\dot{m}_s = \dot{m}_d = \dot{m}_t$$

FAA

$$* \quad \boxed{\frac{P_1(V_1 - V_4)}{T_1} = \frac{P_2(V_2 - V_3)}{T_2} = \frac{P_t V_t}{T_t}}$$

Cylinder dimension in Multistages Compression:

Under ideal Condition, mass of air passing through each cylinder must be equal in multi stage compression.



$$m_{a_1} = m_{a_2} = m_{a_3}$$

$$\frac{P_1 V_{a_1}}{R \cdot T_1} = \frac{P_2 V_{a_2}}{R \cdot T_2} = \frac{P_3 \cdot V_{a_3}}{R \cdot T_3}$$

For perfect intercooling

$$\boxed{T_1 = T_2 = T_3}$$

$$P_1 V_{a_1} = P_2 V_{a_2} = P_3 V_{a_3}$$

$$\Rightarrow \eta_v = \frac{V_a}{V_s}$$

$$\Rightarrow V_a = \eta_v \cdot \frac{\pi}{4} D^2 L$$

$$\boxed{P_1 \eta_{v_1} D_1^2 L_1 = P_2 \eta_{v_2} D_2^2 L_2 = P_3 \eta_{v_3} \cdot D_3^2 L_3}$$

* In multi cylinder normally stroke length is kept equal to get uniform torque, minimum constructio cost, & there will be no need of balancing.

$$L_1 = L_2 = L_3$$

$$P_1 \eta_{v_1} D_1^2 = P_2 \eta_{v_2} D_2^2 = P_3 \eta_{v_3} D_3^2$$

$$\eta_v = 1 - c\left[(k)^{1/n} - 1\right]$$

if clearance ratio is equal for all stages then.

$$\eta_{v_1} = \eta_{v_2} = \eta_{v_3}$$

$$\boxed{P_1 D_1^2 = P_2 D_2^2 = P_3 D_3^2}$$

Question
Q. 30
Pg. 21
CUB

$D = 300$ mm

$L = 400$ mm

$N = 400$ rpm

$C = \frac{V_c}{V_s} = 0.05$

$P_1 = 97$ kPa

$T_1 = 293$K

$P_2 = 650$ kPa.

$P_t = 105$ kPa

$T_t = 288$ K.

$n = 1.3$

$$\eta_v = 1 - c\left[\left(\frac{P_2}{P_1}\right)^{1/n} - 1\right]$$

$$\eta_v = 1 - 0.05\left[\left(\frac{650}{97}\right)^{1/1.3} - 1\right]$$

$$\eta_v = 83.39\%$$

$$V_s = \frac{\pi}{4} D^2 L = \frac{\pi}{4}(0.300)^2(0.400)$$

$$V_s = 0.02826 \ m^3/cycle$$

$$\frac{V_{a1}}{V_s} = \eta_v \Rightarrow V_{a_1} = 0.02826 \times 0.8339$$

$$V_{a_1} = 0.02357 \ m^3/cy$$

$$V_{a_1} = 0.02357 \times 400 \ m^3/min$$

$$V_{a_1} = 9.431 \ m^3/min$$

2.38

$$\frac{P_1 V_1}{T_1} = \frac{P_t V_t}{T_t} \qquad P_1 V_1 =$$

$$W/cycle = \left(\frac{n}{n-1}\right) \frac{P_1 V_1}{60}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W/cycle = \frac{1.3}{0.3}\left(\frac{97 \times 9.431}{60}\right)\left[\left(\frac{680}{97}\right)^{\frac{0.3}{1.3}} - 1\right]$$

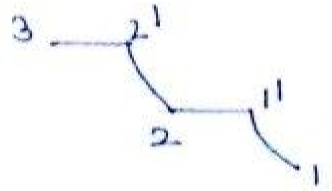
$$W/cycle = 36.4 \ kW$$

$$\frac{P_1 V_1}{T_1} = \frac{P_t V_t}{T_t}$$

$$V_t = \frac{97 \times 9.431 \times 288}{293 \times 105} \qquad V_t = 8.56 \ m^3/min$$

Ques
Q. 29
P.g 21
W.B

$\dot{m}_a = 5\ kg/min$



$T_1 = T_2$

$T_1 = 300K$

$P_1 = 1 bar$, $P_3 = 16 bar$

$n = 1.3$

$$P_2 = \sqrt{P_1 P_3} = \sqrt{1 \times 16} = 4 \text{ bar.}$$

(1) $$W/cycle = \left(\frac{n}{n-1}\right) mRT_1 \left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right)$$

$$= \frac{1.3}{0.3} \times \frac{5}{60} \times 0.287 \times 300 \left(\left(\frac{4}{1}\right)^{\frac{0.3}{1.3}} - 1\right)$$

$$W/cycle = 23.4\ kW$$

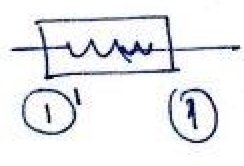
(2) $$\eta_{iso} = \frac{W_{iso}}{W_{act}} \qquad W_{iso} = mRT_1 \ln\left(\frac{P_3}{P_1}\right)$$

$$W_{iso} = \frac{5}{60} \times 0.287 \times 300 \ln(16)$$

$$W_{iso} = 19.89\ kW$$

$$\eta_{iso} = \frac{19.89}{23.4} \times 100 = 84.89\%$$

(3) $$\frac{T'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$



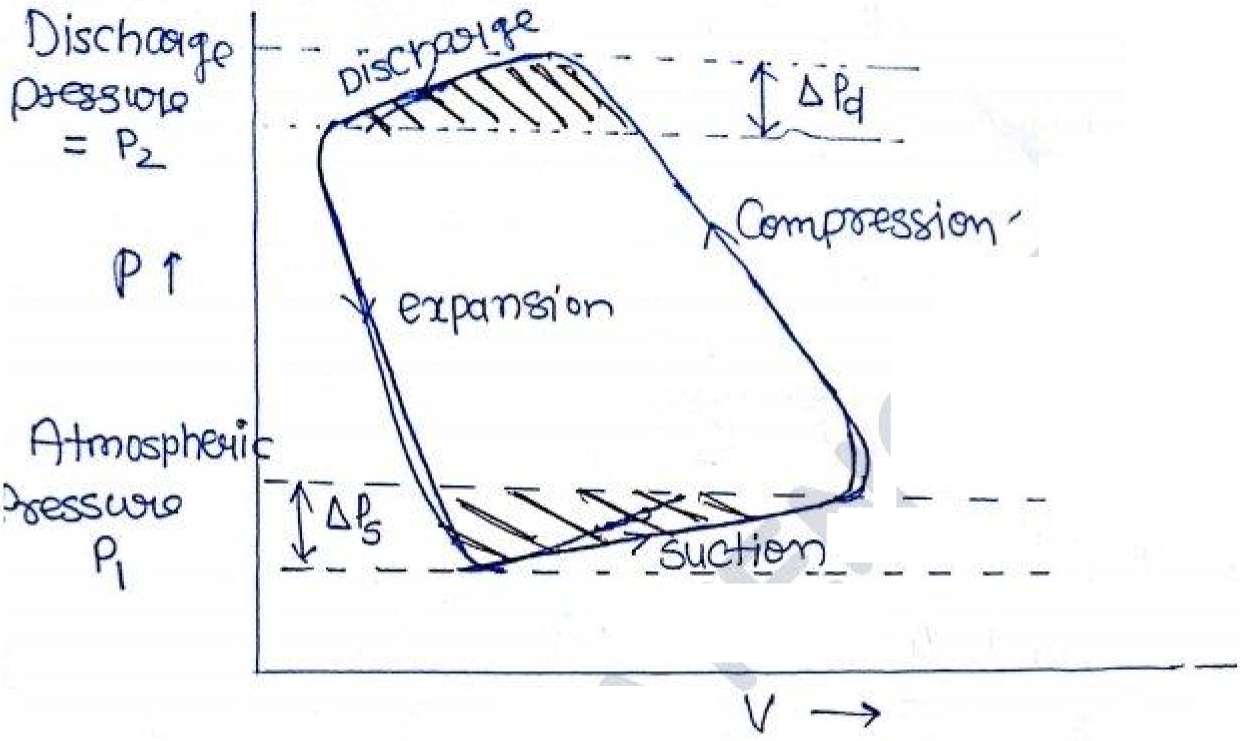
$$T_1 = 300(4)^{0.3/1.3}$$

$$T_1 = 413.1\ K$$

$$Q_R = \dot{m}C_p(T_2 - T_1')$$

$$= \frac{5}{60} \times 1\,(300 - 413.1) = -9.47\ kW$$

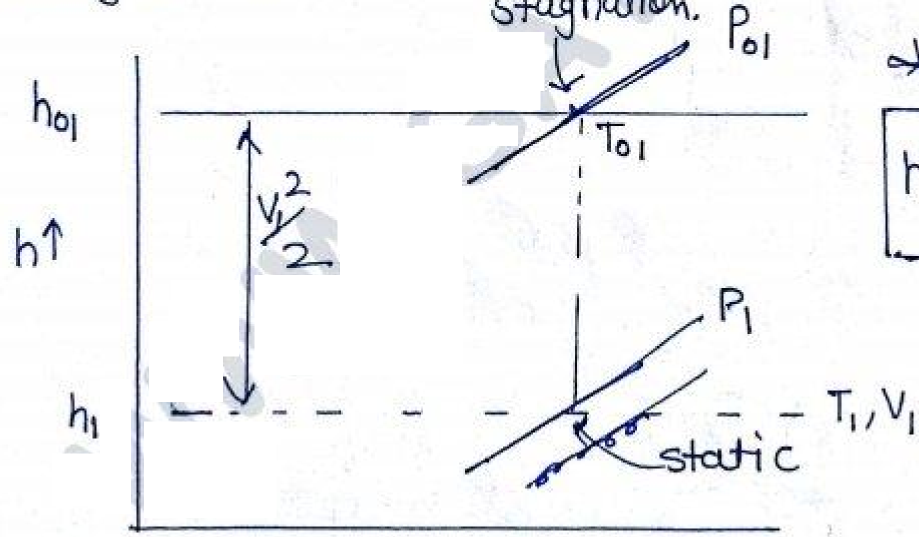
Actual Indicator diagram:-



In actual working there is resistance offered by suction & discharge value due to which actual work input required increas by shaded area.

Dynamic Compressor:-

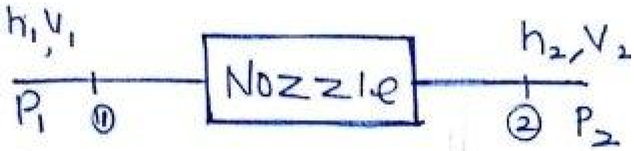
Stagnation stage.



$$\boxed{h_1 + \frac{V_1^2}{2} = h_{01}}$$

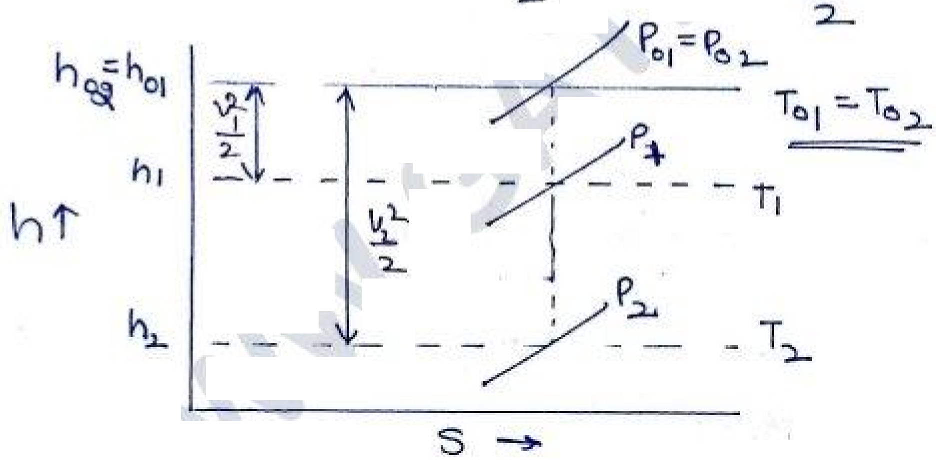
The state of fluid obtained when it is isentropically deaccelerated to '0' Velocity state is referred as stagnation state.

energy transformation:-



by SFEE ① & ②

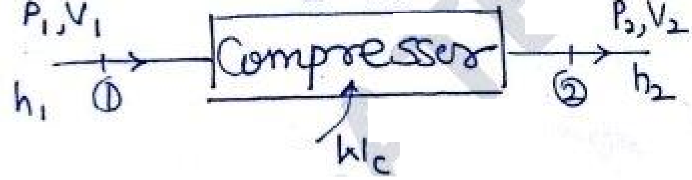
$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$



$$\boxed{h_{01} = h_{02}}$$

Energy transfer

$P_2 > P_1$, $V_2 \approx V_1$

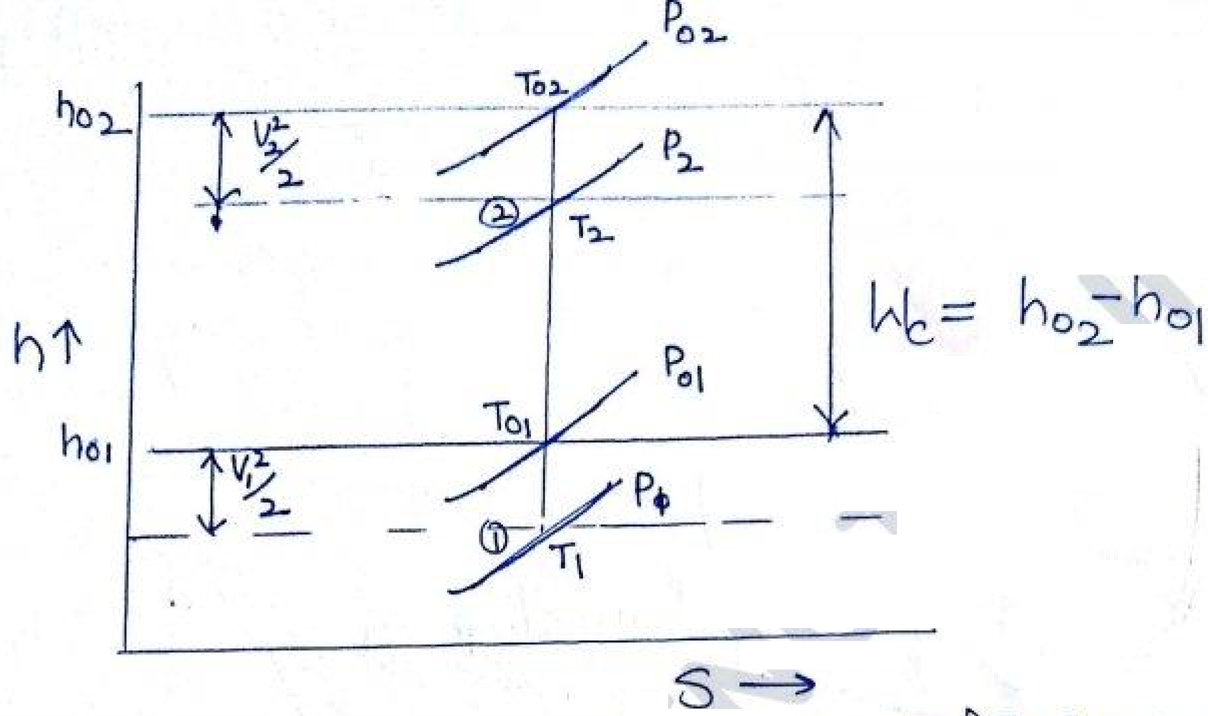


by SFEE ① & ②

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} - W_c$$

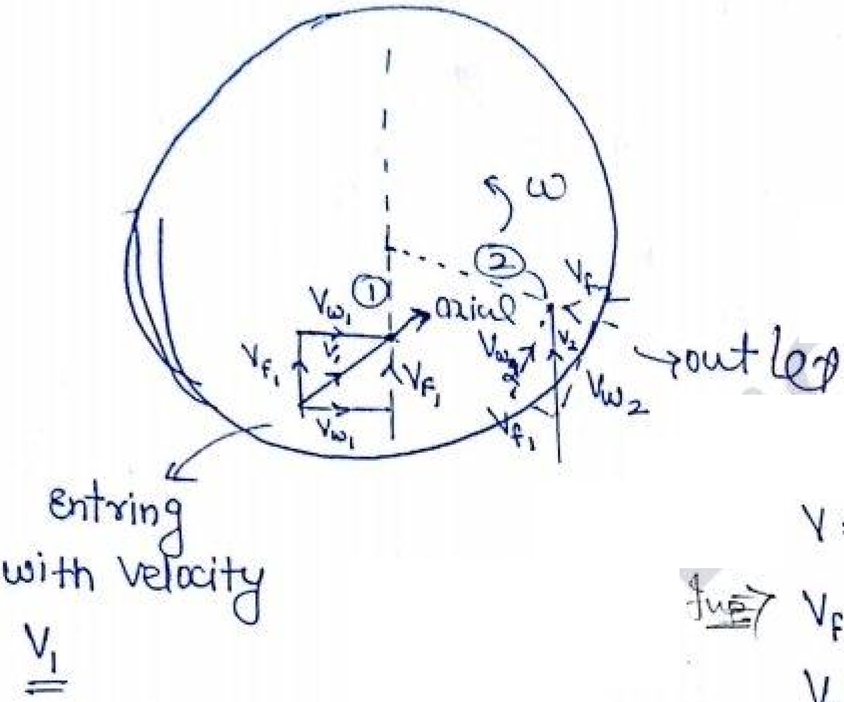
$$\left(h_1 + \frac{V_1^2}{2}\right) + W_c = h_2 + \frac{V_2^2}{2}$$

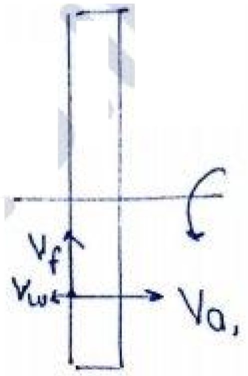
$$\Rightarrow \boxed{W_c = h_{02} - h_{01}}$$



Stagnation state works as a reference point to denote total energy of fluid. Stagnation enthapy always remains Constant for energy transformation process & for energy transfer it may increase for compressor & decrease for turbine.

Principle of Rotating Compressor:-





V = absolute Velocity

V_{f_1} = Flow velocity

V_w = whirl (tengential) Velocity

V_a = axial velocity (generally Neglected)

let Fluid enter and exit rotar at section ① & ② at radius r_1 and r_2 respectvely under equilibrium Condition when rotar is rotating at a Constant angular velocity ω and the absolute velocity at inlet and outlet are resolved into 2 Component → V_w = whirl or tengential Component

→ V_F = Flow or radial component.

Rotor speed at Inlet, $U_1 = \omega \cdot r_1$

Rotor speed at oulet $U_2 = \omega \cdot r_2$

Torque 'T' = Rate of change of angular momentum

Angular momentum at Inlet $= \dot{m} V_{w_1} \cdot r_1 - \cancel{\dot{m} V_{f_1} \cdot 0}$

$= \dot{m} V_{w_1} \cdot r_1$

Angular Momentum at outlet $= \dot{m} V_{w_2} \cdot r_2$

$$T = \dot{m} V_{w_2} \cdot r_2 - \dot{m} V_{w_1} \cdot r_1$$

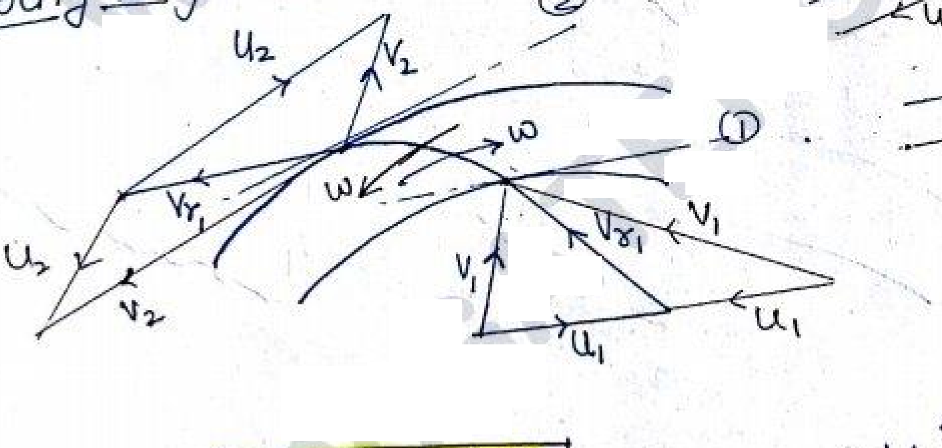
$$\text{Power} = T.\omega = \dot{m} \left(V_{w_2} \cdot \omega \cdot r_2 - V_{w_1} \cdot \omega \cdot r_1 \right)$$

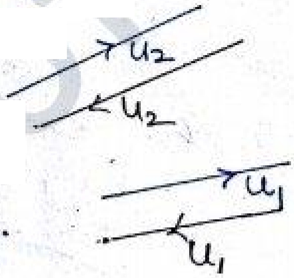
$$\boxed{P = \dot{m} \left(V_{w_2} \cdot u_2 - V_{w_1} \cdot u_1 \right)}$$ ..euler's energy eq^n

1) $V_{w_2} \cdot u_2 > V_{w_1} \cdot u_1$ Compressor

2) $V_{w_1} \cdot u_1 > V_{w_2} \cdot u_2$ Turbine

<u>Velocity Diagram</u>





$$\boxed{\vec{u} + \vec{V_r} = \vec{V}}$$

$\vec{u_1} + \vec{V_{r_1}} = \vec{V_1}$

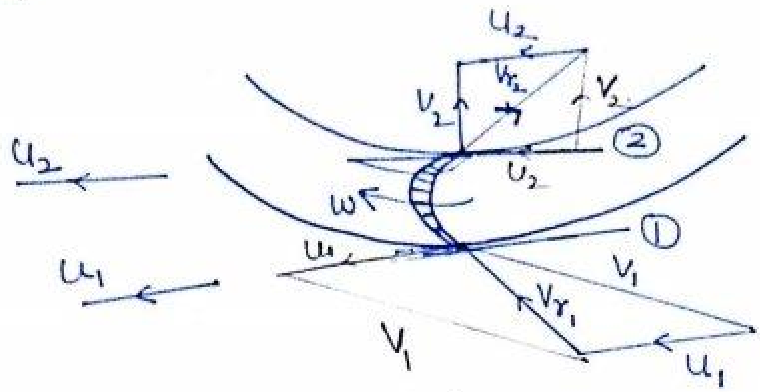
$\vec{u_2} + \vec{V_{r_2}} = \vec{V_2}$

u = blade velocity → Rotor speed

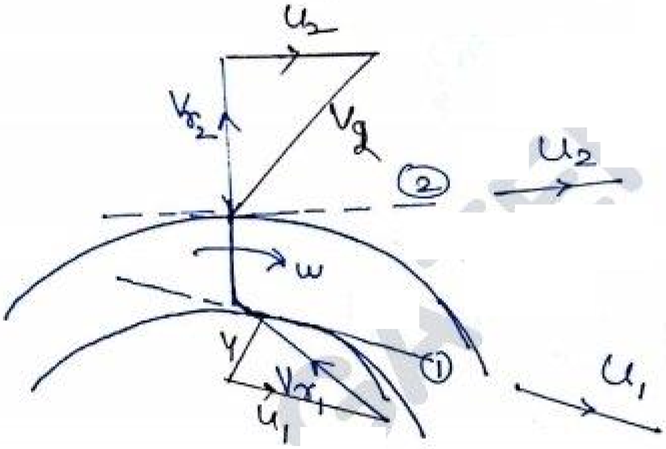
V_r = Relative Velocity

V = Absolute Velocity

*



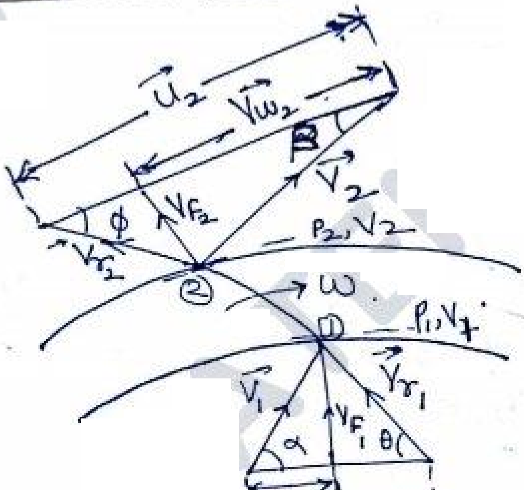
*



$u_2 + V_{r_2} = V_2$

$u_1 + V_{r_1} = V_1$

Velocity diagram for compressor:-



Compressor outlet more energy

$P_2 > P_1$

$V_2 > V_1$

$\vec{V} = \vec{u} + \vec{V}_r$

$\vec{V} = \vec{V}_w + \vec{V}_f$

$V_w \perp V_f$

after this flow comes in impeller

→ α = Nozzle angle or Fixed blade outlet angle or Guide blade outlet angle or absolute velocity angle at inlet.

→ β = ~~fixed~~ Diffuser blade inlet angle / Diffuser inlet angle or absolute vel. angle at outlet

* θ and ϕ are blade or Vane angle at Inlet and outlet.

$$P = V_{w_2} \cdot u_2 - V_{w_1} \cdot u_1 \quad \text{per unit mass}$$

From outlet Δ

$$V_{f_2}^2 = V_2^2 - V_{w_2}^2$$

$$V_{f_2}^2 = V_{r_2}^2 - (u_2 - V_{w_2})^2$$

on equating

$$V_2^2 - V_{w_2}^2 = V_{r_2}^2 - u_2^2 - V_{w_2}^2 + 2V_{w_2} \cdot u_2$$

$$V_{w_2} \cdot u_2 = \frac{V_2^2 + u_2^2 - V_{r_2}^2}{2}$$

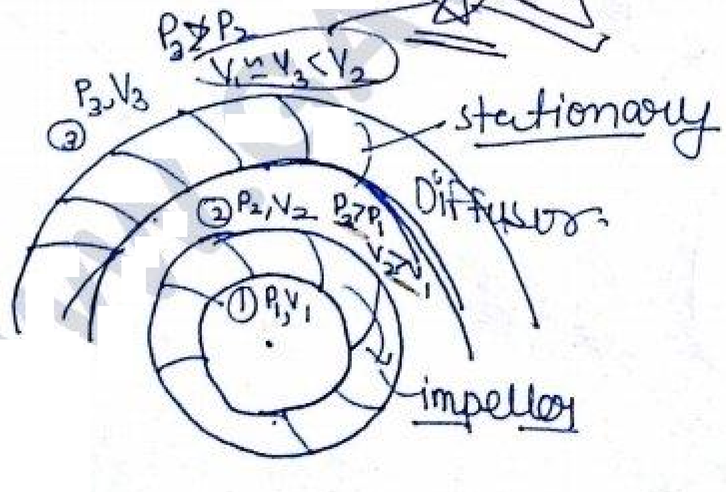
From inlet Δ

$$V_{w_1} \cdot u_1 = \frac{V_1^2 + u_1^2 - V_{r_1}^2}{2}$$

Now

$$\boxed{P = V_{w_2} \cdot u_2 - V_{w_1} \cdot u_1 = \left(\frac{V_2^2 - V_1^2}{2}\right) + \left(\frac{u_2^2 - u_1^2}{2}\right) + \left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)}$$

$u_2 > u_1$



$\left(\frac{V_2^2 - V_1^2}{2}\right)$ = impulse effect it denotes the increase in kinetic energy K.E. of fluid in the impeller which we need to convert into pressure in diffusor.

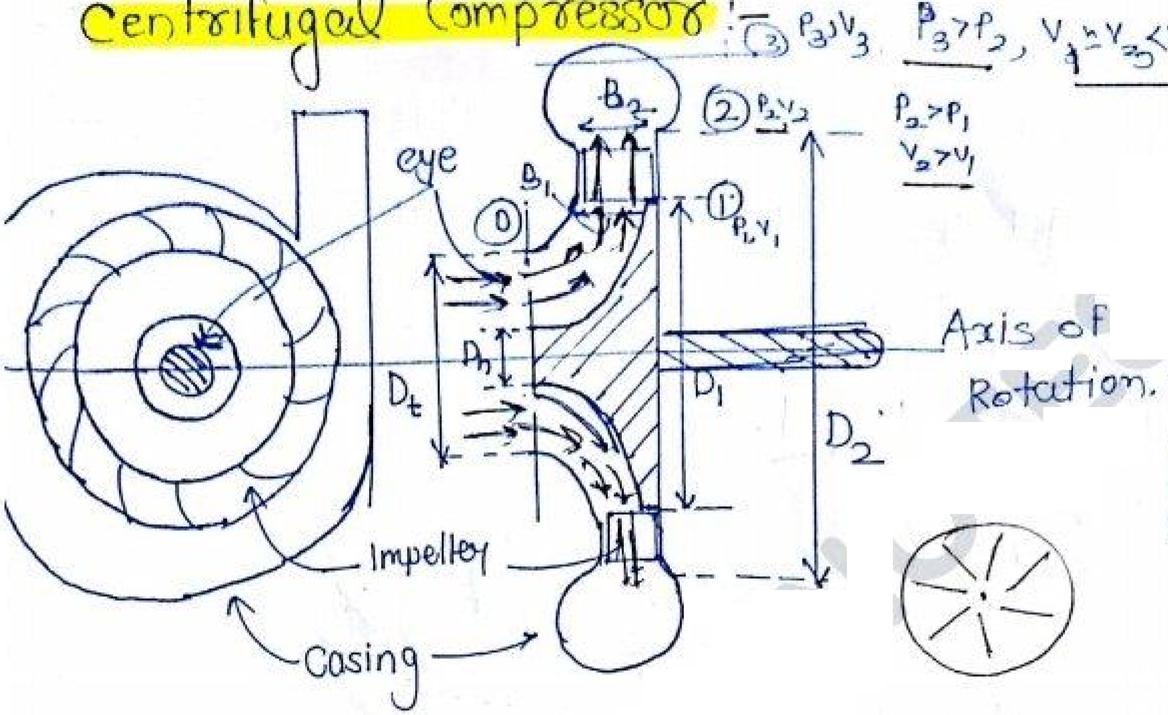
$\left(\frac{u_2^2 - u_1^2}{2}\right) \rightarrow$ (centrifugal effect)

it denotes the rise in pressure ~~if~~ in the impeller because fluid enterse at a lower dia. and comes out a higher diameter.

$\left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)$ - Diffusor/ Reaction effect

it ~~deto~~ denote the ~~in~~ increase pressure in the impeller because flow taken place in diverging passage due to which relative velocity ~~passage~~ decrease ~~wth~~ with corresponding increase in static pressure.

Centrifugal Compressor :-



D_t = tip dia
D_h = hub dia
D_1 & D_2 are impeller dia at inlet & outlet
B_1 & B_2 width.

Radial Flow :- while passing through the moving ~~part~~ part if flow takes place in a direction ⊥ to the axis of rotation it is termed as radial flow machine.

Axial Flow :- while passing through the moving part if the flow takes place along or parallel to the axis of rotation it is termed as axial flow machine :-

Centrifugal Compressor :- it is a radial flow machine in which air enter in a axial direction and enters in impeller in radially outward dirn.

In inlet casing, there is little drop in pressure with increase in absolute velocity. while flowing through the impeller energy is transferred due to which static pressure increase alonge with increase in K.E.. KE is obtained in the impeller is transforred into pressure energy in the diffusser.

→ $A_0 = \frac{\pi}{4}(D_t^2 - D_h^2)$

$A_1 = \pi D_1 \cdot B_1$, $A_2 = \pi D_2 \cdot B_2$

→ if the blade thickness is considered and let 'n' is the number of blade and 't' is thickness of each blade.

$A_1 = (\pi D_1 - nt) B_1$

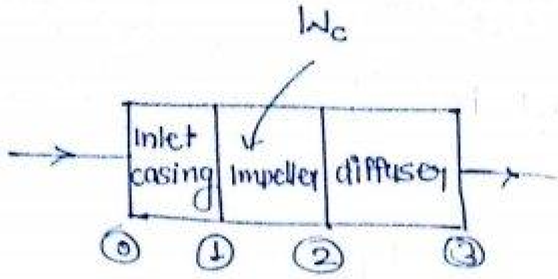
$A_2 = (\pi D_2 - nt) \cdot B_2$

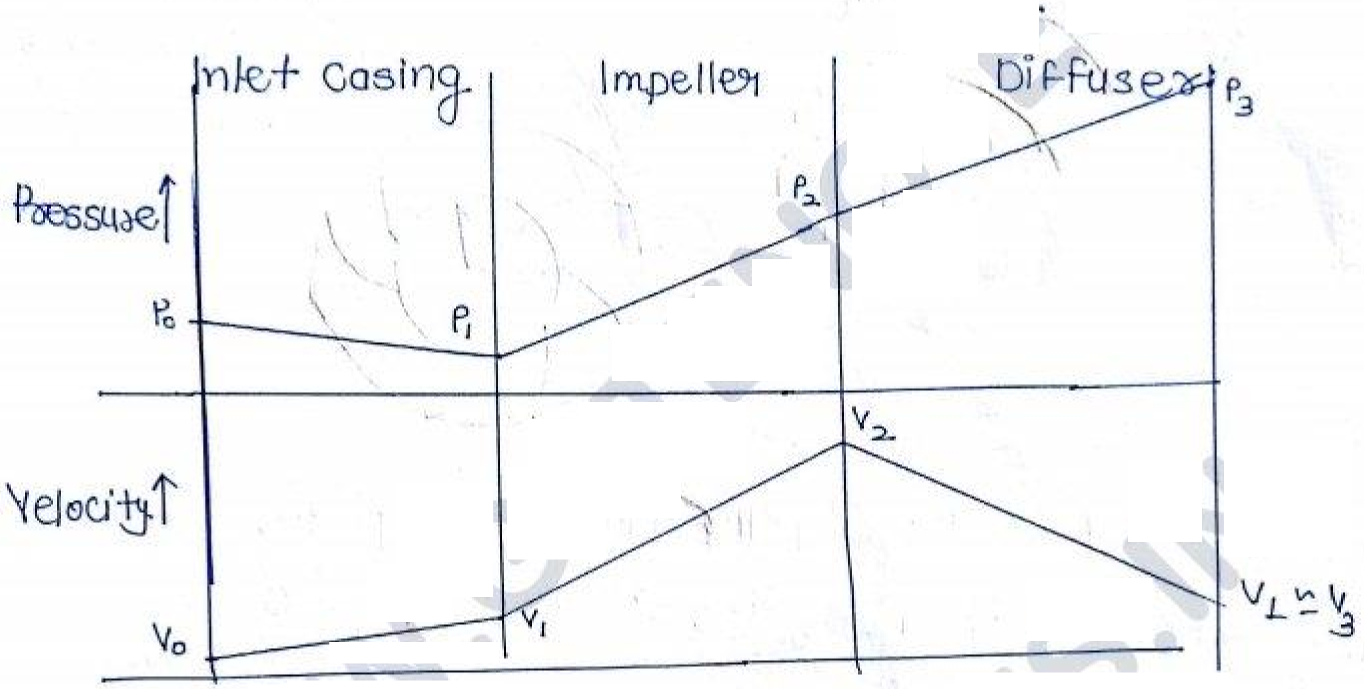
→ if blade are coefficient is given to consider blade thickness then area at inlet & outlet

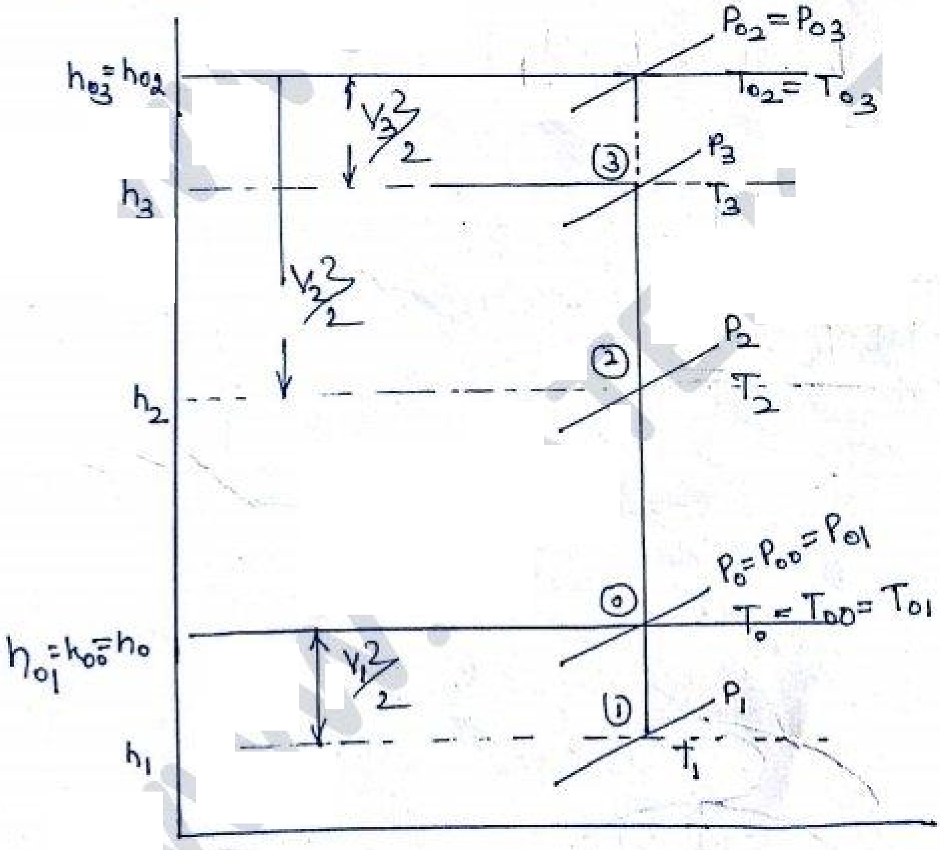
$A_1 = \pi D_1 \cdot B_1 \cdot k_1$

$A_2 = \pi D_2 \cdot B_2 \cdot k_2$

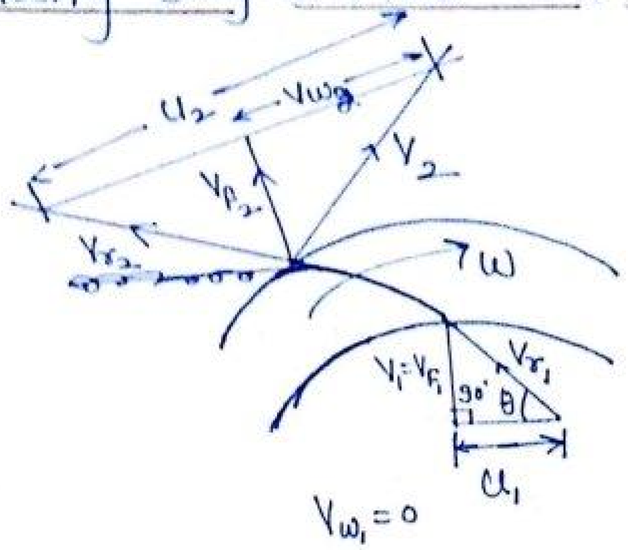
$k \simeq 0.9$


W_c
Inlet casing
Impeller
diffuser
0
1
2
3


Inlet casing
Impeller
Diffuser
Pressure
Velocity
P_0
P_1
P_2
P_3
V_0
V_1
V_2
$V_1 \approx V_3$

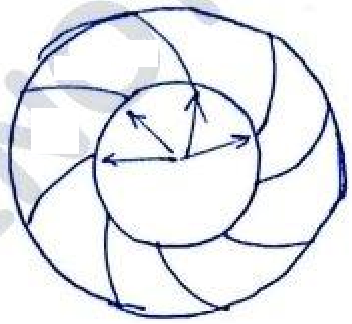

$h_{03} = h_{02}$
h_3
h_2
$h_{01} = h_{00} = h_0$
h_1
$\frac{V_3^2}{2}$
$\frac{V_2^2}{2}$
$\frac{V_1^2}{2}$
$P_{02} = P_{03}$
$T_{02} = T_{03}$
P_3
T_3
P_2
T_2
$P_0 = P_{00} = P_{01}$
$T_0 = T_{00} = T_{01}$
P_1
T_1
0
1
2
3

Velocity Diagram for centrifugal Compressor:



$V_{w_1} = 0$

Fluid हमेशा Min. dist. travel करता है।
Min. dist. के लिए
$\alpha = 90$



At the inlet to the impeller fluid always enters in the radial direct i.e. $\alpha = 90°$ and $V_{w_1} = 0$

$$P = \dot{m}\left(V_{w_2} \cdot U_2 - V_{w_1} \cdot U_1\right)$$

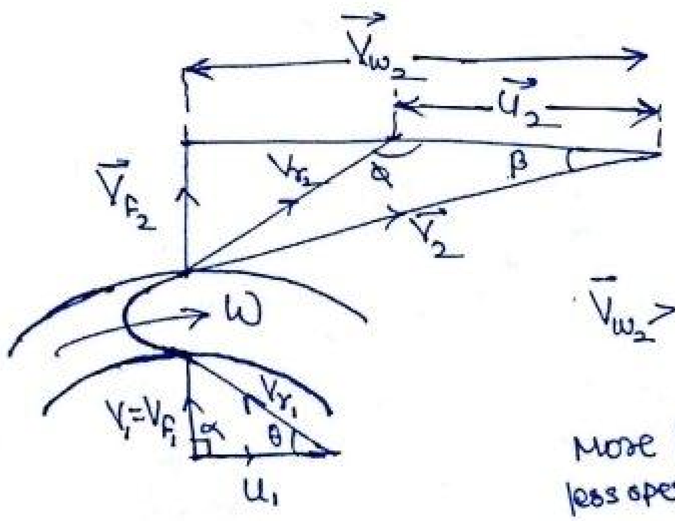
as $V_{w_1} = 0$

$$\boxed{P = \dot{m}\, V_{w_2} \cdot U_2}$$

Types of Blade:–

Forward Curved Blade :→

$\phi > 90°$

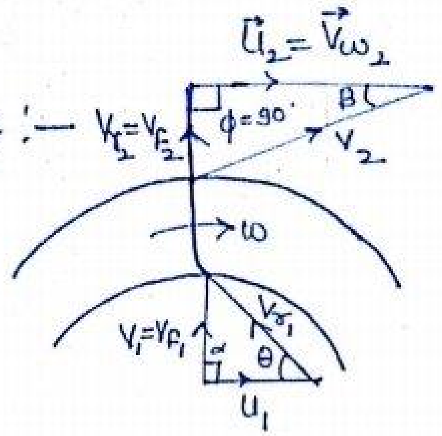


$\vec{V}_{w_2} > \vec{U}_2$

More loss
less operating range

Redial Blade :—

$\phi = 90^\circ$

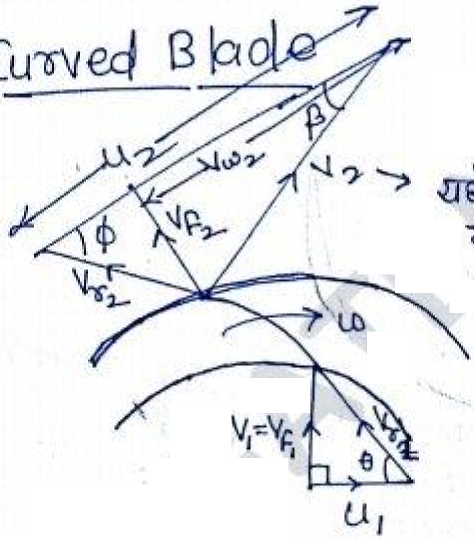


$\vec{u}_2 = \vec{V}_{w_2}$

less stress → (सबसे कम)
run at high rpm (use in aircraft.

Backward Curved Blade

$\phi < 90^\circ$



$\vec{u}_2 > \vec{V}_{w_2}$

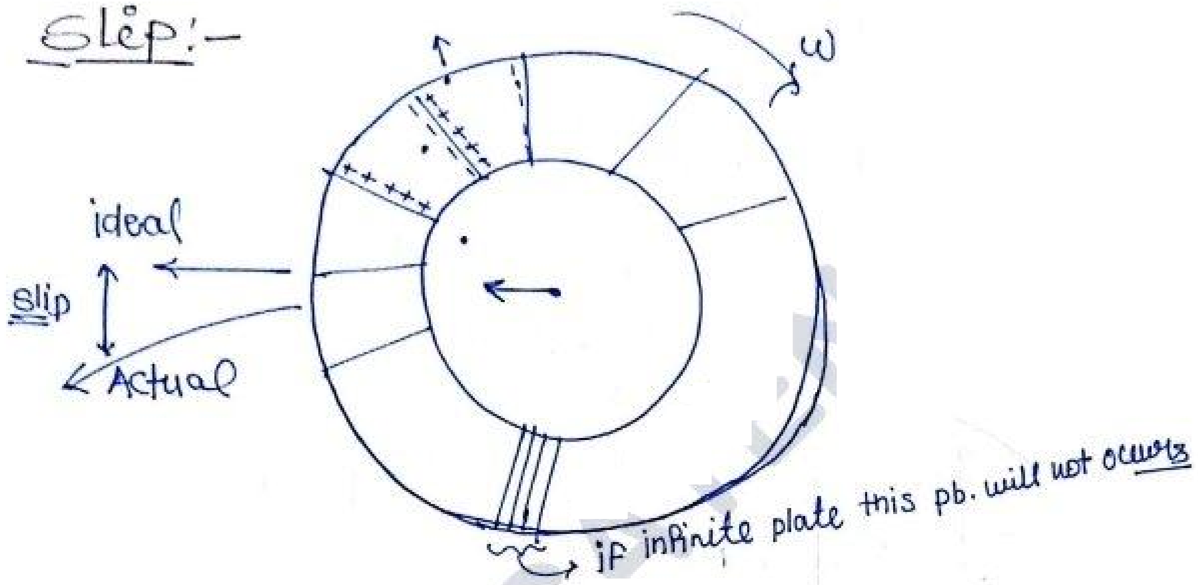
V_2 → यहाँ सबसे कम Vel. आयेगी (V_2)
→ loss less
→ low rpm
→ preferred for pump.

→ Centrifugal effect of curved plate create bending moment and develope high stress which may change the blade profile and therefor for the impellers runing at higher rpm Radial blades are preferred. → Forward curved blade consume maximum power, develope highest pressure ratio but has poor efficiency and narrow operating range.

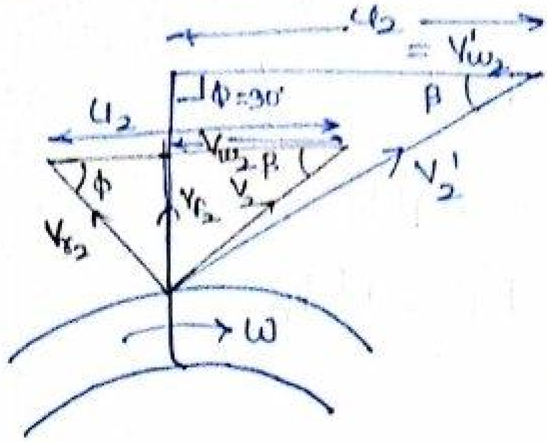
• Backward curved blade give best efficiency and has wide operating range.

Note:- if nothing is mention always assume radial blade at outlet.

Slip:-



Under ideal condition we assume that the fluid particle exactly follow the blade profile but in actual working where the number of plates are finite air is trapped between the blade passage due to its inertial effect which creat pressure diff. across the blade due to this pressure diff fluid deviate from the ideal path and this deviation is callad as slip. Due to slip actual blade angle at outlet always decrease which ~~the~~ decreases the whirl component at outlet.



$V'_{w_2} > V_{w_2}$

$\downarrow$ ideal $\quad \downarrow$ Actual

ϕ = decrease

$$\boxed{\text{Slip} = V'_{w_2} - V_{w_2}}$$

Slip factor :- (ϕ_s)

$$\phi_s = \frac{\text{Theoretical power (finite blade)}}{\text{Ideal power } (\infty \text{ blade})}$$

$$\phi_s = \frac{\dot{m}\, V_{w_2} \cdot u_2}{\dot{m}\, V'_{w_2} \cdot u_2}$$

$$\phi_s = \frac{V_{w_2}}{V'_{w_2}}$$

$$V'_{w_2} = u_2 \Rightarrow \phi_s = \frac{V_{w_2}}{u_2}$$

$$P_{theo.} = \dot{m}\, V_{w_2} \cdot u_2$$

$$\boxed{P_{theo.} = \dot{m}\, \phi_s \cdot u_2^2}$$

Power or work input factor (ϕ_w)

$$\phi_w = \frac{\text{Actual power}(P_{act})}{\text{Theoretical power}(P_{the.})}$$

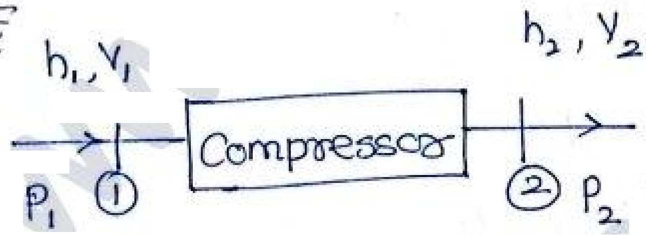
$$P_{act.} > P_{theo.}$$

$$\phi_w \geq 1$$

$$\phi_w \simeq 1.04 \text{ to } 1.05$$

$$\boxed{P_{act.} = \phi_w \cdot P_{theo.} = \dot{m} \cdot \phi_s \cdot \phi_w \cdot u_2^2}$$

By SFEE



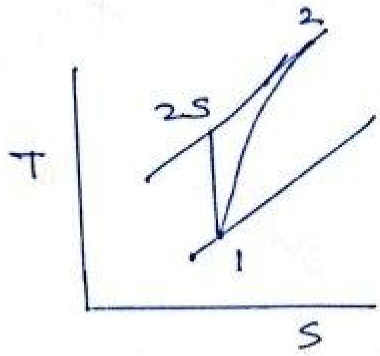
$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) - P_{act.}$$

$$P_{act.} = \dot{m}(h_{02} - h_{01}) = \dot{m}\, C_p (T_{02} - T_{01})$$

if $V_1 \simeq V_2$

$$P_{act} = \dot{m} C_p (T_2 - T_1)$$

$$\boxed{\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}}$$



$$\boxed{P_{act} = \phi_s \cdot \phi_w \cdot U_2^2 = C_p(T_2 - T_1)}$$

$$\boxed{U_2 = \frac{\pi D_2 N}{60}}$$

Pressure coeff. (ϕ_p)

$$\phi_p = \frac{\text{Isentropic work}}{\text{Euler's work}}$$

$$\phi_p = \frac{C_p(T_{2s} - T_1)}{V_{w_2} \cdot U_2}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$(T_{2s} - T_1) = \eta_c (T_2 - T_1)$$

$$\phi_p = \frac{\eta_c \cdot C_p (T_2 - T_1)}{V_{w_2} \cdot U_2}$$

$$C_p (T_2 - T_1) = \phi_s \cdot \phi_w \cdot U_2^2$$

$$\phi_p = \frac{\eta_c \cdot \phi_s \cdot \phi_w \cdot U_2^2}{V_{w_2} \cdot U_2}$$

For radial blade $V_{w_2} = U_2$

$$\boxed{\phi_p = \eta_c \phi_s \cdot \phi_w}$$

Q.32
P.g.21
W.B

$T_1 = 320k$ $N = 15000 rpm$

$\phi_s = 0.9$ $U_2 = \frac{\pi D_2 N}{60}$

$\phi_w = 1.05$ $\eta_c = 85\%$ $U_2 = 628.3 m/s$

$d = 80$ cm $r = 40cm$

$U_2 = \omega_2 r_2$

$$P = \phi_s . \phi_w . U_2^2$$

$$P = 0.9 \times 1.05 \times \left\{\frac{2\pi \times 15000}{60}\right\}^2 \times (0.40)^2 = 593.76$$

$$P = C_P(T_2 - T_1) = 1.005(T_2 - 320)$$

$$T_2 = 691.2K$$

$$\eta_C = 0.85 = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{T_{2s} - 321}{691.2 - 321}$$

$$T_{2s} = 635.5K.$$

As process 1–2s is isentropic

$$\frac{T_{2s}}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \gamma = 1.4$$

$$r_p = 11.03$$

Q. 33
Pg 22

$N = 9000\ rpm$

$v_1\ \cancel{Q} = 600\ m^3/min$

$P_1 = 1 bar,\ 20°C$

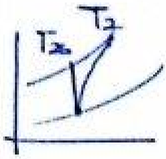
$\frac{P_2}{P_1} = 4,\ \eta_c = 0.82$

$V = 62\ m/s$

$D_2 = 2D_1$

$\phi_s = 0.9$

$\phi_w = 1.00$



$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}$$

$$T_{2s} = 293\,(4)^{2/7}$$

$$T_{2s} = 435.39\ K$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = 0.82$$

$$\frac{435.39 - 293}{T_2 - 293} = 0.82$$

$$T_2 = 466.65\ K$$

2) $P = \dot{m} C_p (T_2 - T_1)$

$$\dot{m} = \frac{P_i v_1}{R \cdot T_1} = \frac{1 \times 10^5 \times 600}{60 \times 287 \times 293} = 11.89\ kg/s.$$

$$P = 11.89 \times 1.005\,(466.65 - \cancel{435.39}\ 293)$$

$$P = 2075.3\ \underline{K}$$

3) $\phi_s \cdot \phi_w \cdot U_2^2 = C_p(T_2 - T_1) \Rightarrow 0.9 \times 1 \cdot U_2^2 = 1.005(466.65 - 293)$

$U_2 = 440.35 \text{ m/s}$

$U_2 = \frac{\pi D_2 N}{60} = 440.35$

$D_2 = 0.934 \text{ m}$

$D_1 = 0.467 \text{ m}$

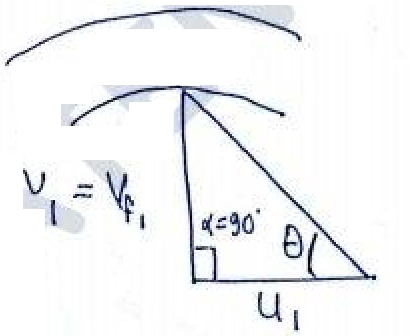
4) $\dot{m} = \rho_1 A_1 V_{f_1}$

$\frac{P_1 V_1}{RT_1} = \frac{P_1}{RT_1}(\pi D_1 B_1 k_1) V_{f_1}$

$\frac{600}{60} = \pi \times 0.467 \times B_1 \times 0.9 \times 62$

$B_1 = 0.122 \text{ m} = 12.2 \text{ cm}$

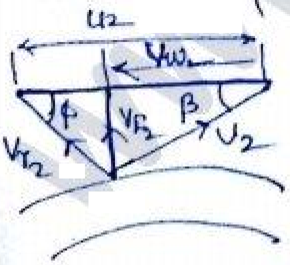
5)



$\tan\theta = \frac{V_{f_1}}{U_1}$

$\tan\theta = \left(\frac{60 \times 62}{\pi \times 0.462 \times 9000}\right) \Rightarrow \theta = 15.73^\circ$

6)

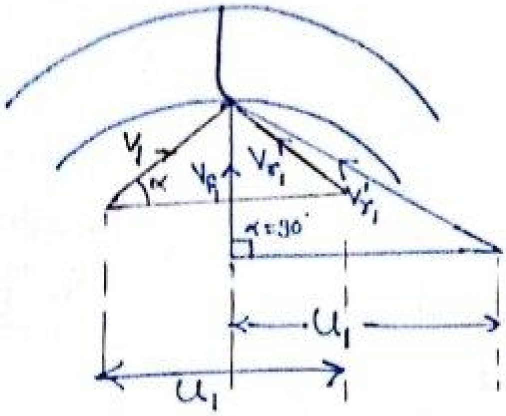


$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{62}{\phi_s U_2} = \frac{62}{0.9 \times 440.35} \Rightarrow \beta = 8.89^\circ$

$\tan\phi = \frac{U_2 - V_{w_2}}{V_{f_2}} = \cancel{\frac{440.35 - 0.9 \times 440.35}{60}} = \frac{62}{440.35 - 0.9 \times 440.35} = \phi = 54$

Pre-Whirl :-

$$M = \frac{V_{r_1}}{\sqrt{\gamma R T_1}}$$



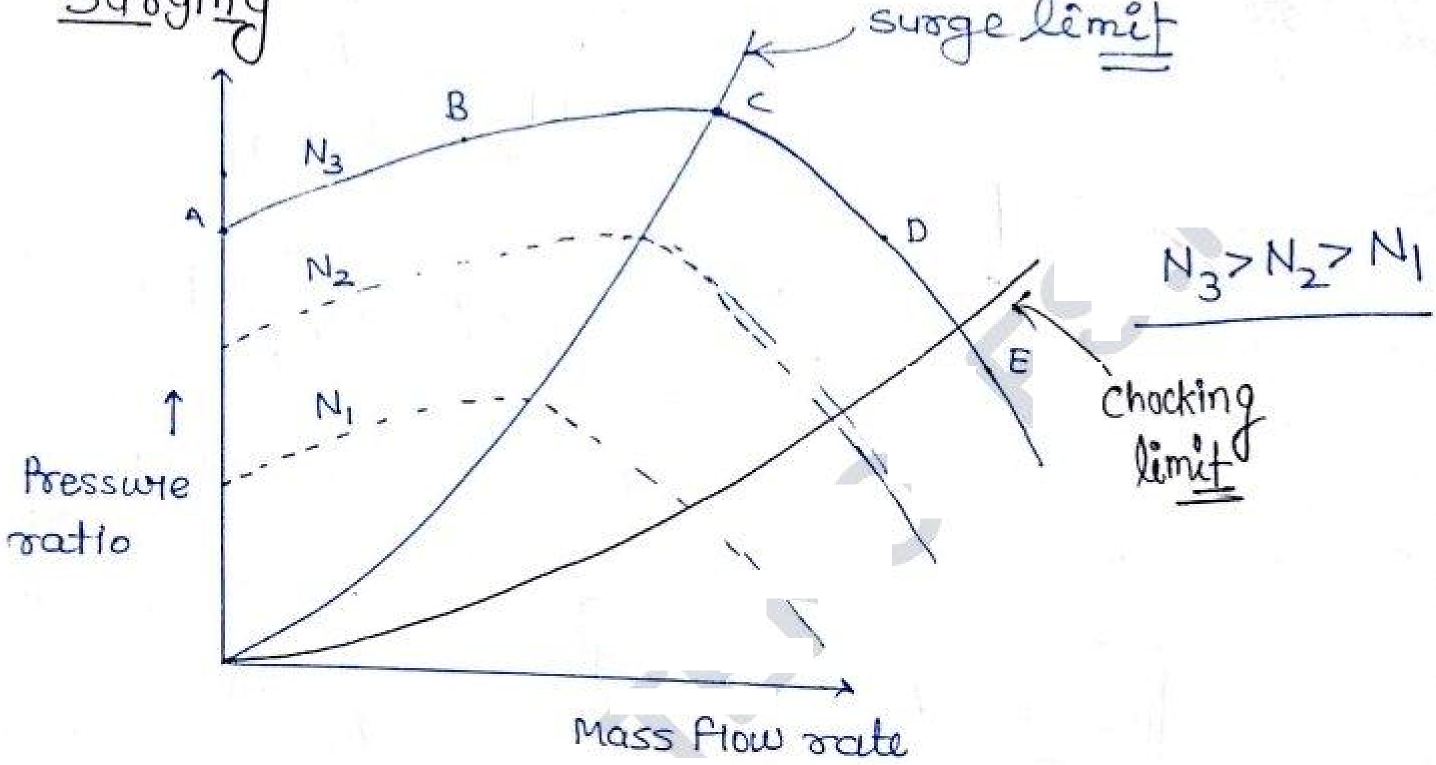
$\boxed{V_{r_1}' < V_{r_1}}$ relative Vel.

when $M > 1$
pre whirl needed
it ↑ Construction Cost
→ Aircraft it is necessary

$$\boxed{P = \dot{m}\left(V_{w_2} \cdot u_2 - V_{w_1} \cdot u_1\right)}$$

when compressor running at very high speed there is always possibility of shock wave at the inlet if the $M > 1$ (Mech number). In that case with the help of inlet guide vane fluid is given a ~~pree~~ Pre rotation to reduce relative velocity at inlet due to this construction cost increases with decrease in pressure developed.

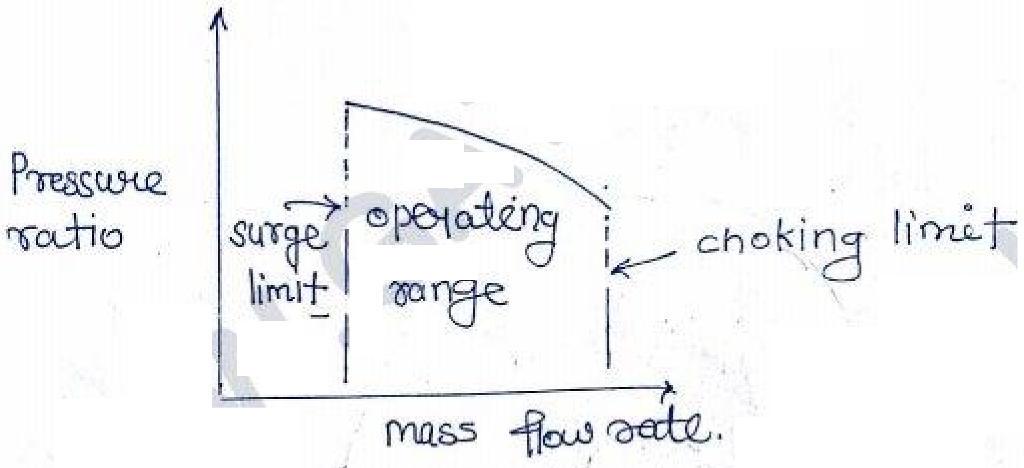
Surging



Surging is a complate breakdown of steady flow through the compressor due to periodic flow reversal This reversal of flow occurs during closing of valve in the operating range having mass flow rate less than corresponding to max pressure ratio This reversal of flow causes decrease in efficiency abnormal sound, vibration and the intensity is more it may lead to mechanical damage.

Choking limit :-

The maximum mass flow rate possible through a compressor is termed as choking limit. It occurs when mech Number corrospoing to relative velocity at inlet become sonic. Choking means fixed mass flow rate irrespective to pressure ration.

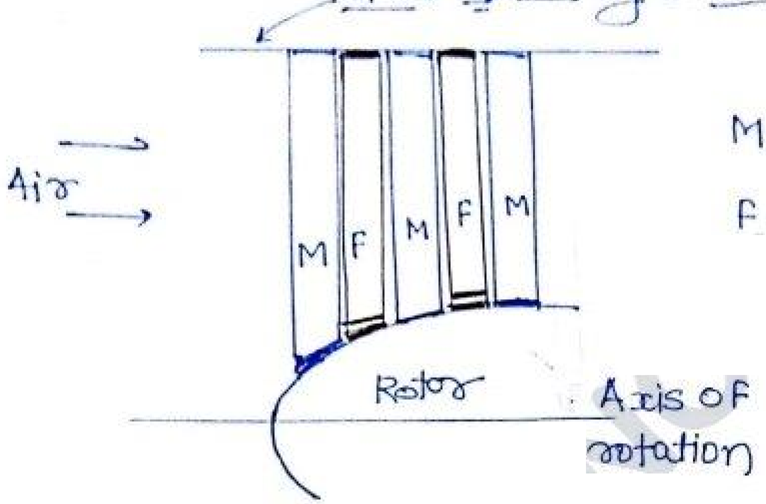


Stalling :- it is aerodynamic flow sepration from blade surface due to improper design of Blade.

It is a local phenomena and the chanses of flow sepration are more at low mass flow rate, no uniform surface of blade and defect in design.

$\left\{ \frac{dP}{dx} > 0 \right\}$ flow sepration

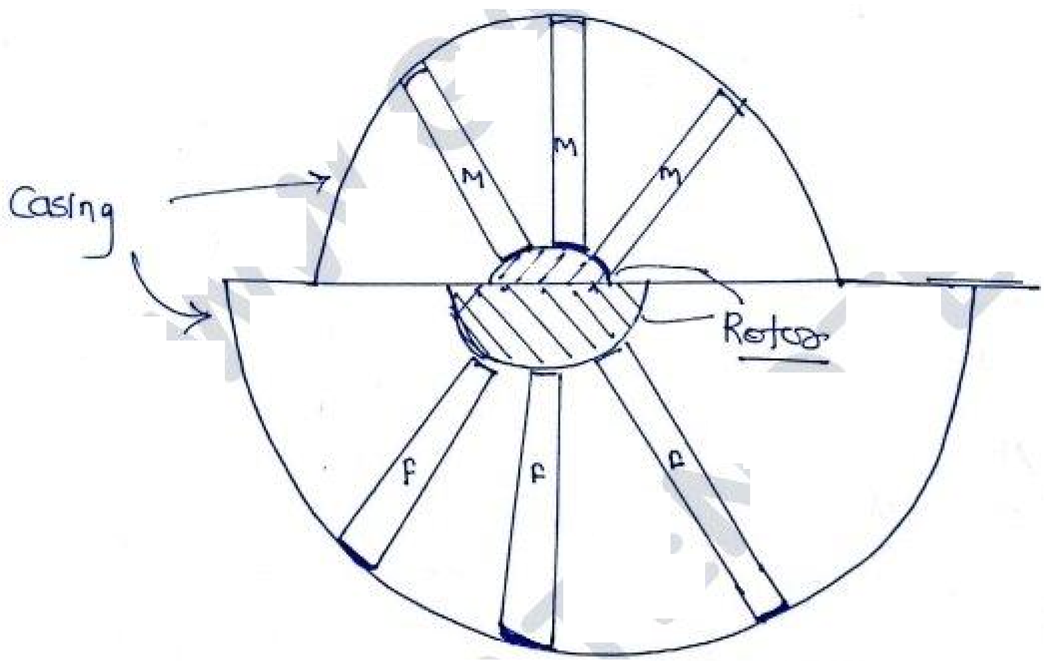
Axial flow Compressor :-

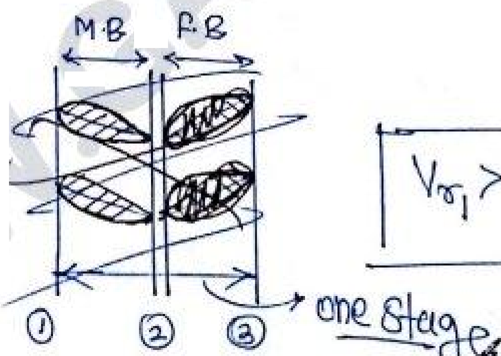


M -- Moving blades

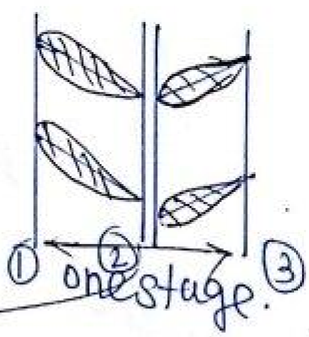
F - Fixed blades

Moving blades on rotor





$\boxed{V_{r_1} > V_{r_2}}$



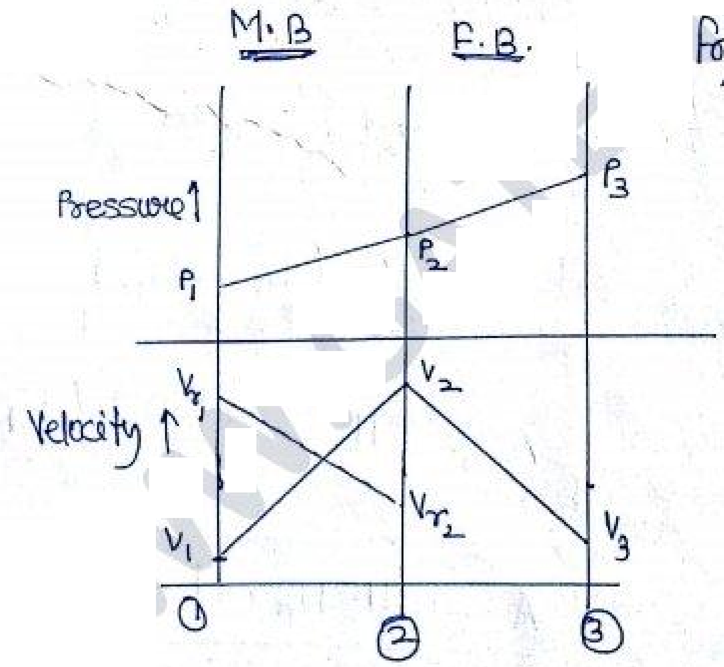
$P_1 < P_2 < P_3$

$V_1 < V_2 > V_3 \simeq V_1$

$\boxed{V_{r_1} > V_{r_2}}$

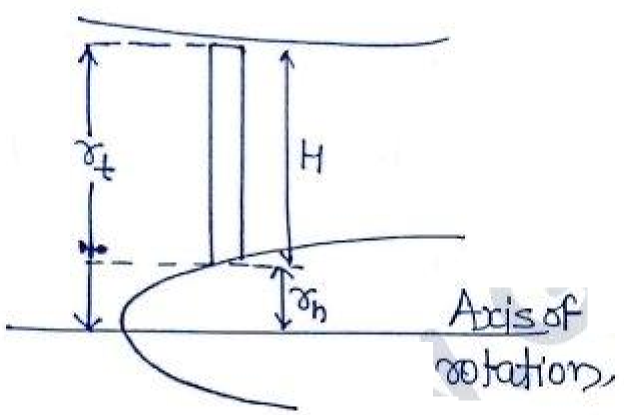
In Axial flow compressor air flow along the axis of rotation and it consist of no of stages. each stage consist of a set of moving and fixed blade and the cross section between the blade passege is made diverging type both for moving & fixed blade.

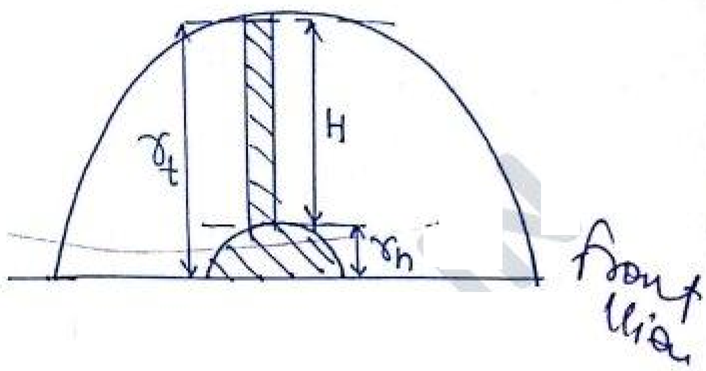
There is increase in static pressure in the moving blade due to reaction $\left\{\left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)\right\}$ effect along with increase in kinetic head. kinetic head obtain in moving blade is transform into pressure in the fixed blade the pressure ratio per stage is very less (1.2 − 1.4). Compare to centrifugal compressor (4−6).



Note: Height of blade is decrease from inlet to outlet as pressure increases to get a constant axial flow velocity inorder to neutralise axial thrust on the rotor

$$\boxed{\dot{m} = \underset{\uparrow}{\rho}\ \underset{\downarrow}{A}\ \underset{Const}{V_f}}$$





r_t = Tip radius

r_h = hub radius

$H = r_t - r_h$ blade height.

Cross sectional Area $A = \pi(r_t^2 - r_h^2)$

$$\boxed{A = \frac{\pi}{4}(D_t^2 - D_h^2)}$$

mass flow rate

$$\boxed{\dot{m} = \rho A V_f}$$

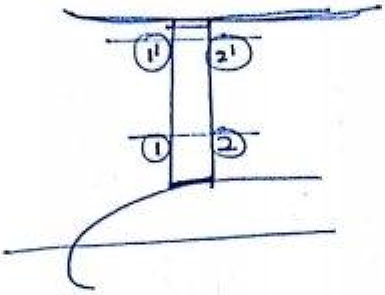
$$A = \pi \cdot \left(\frac{D_t + D_h}{2}\right)\left(\frac{D_t - D_h}{2}\right)$$

$$\boxed{A = \pi D_m \cdot H}$$

Mean dia $D_m = \dfrac{D_t + D_h}{2}$

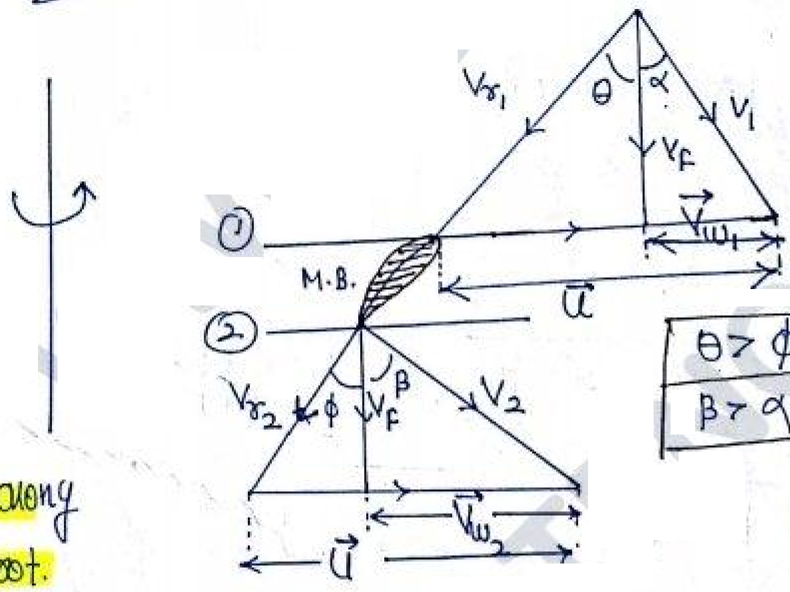
Velocity Diagram:-

As Fluid enter and exit at same radius therefore blade speed is taken as constant corresponding to mean diameter



$$u_1 = u_2 = u = \frac{\pi D_m N}{60}$$

Practice



Flow along Axis of rot.

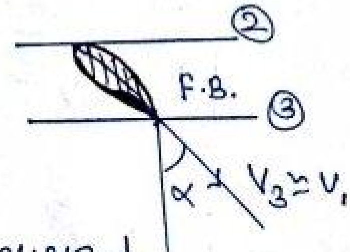
$$\boxed{V_{f_1} = V_{f_2} = V}$$

$u_1 = u_2 = u$

Area of both Δ Same

$\theta > \phi$

$\beta > \alpha$



* α & β are the flow angle measured from the ~~axial~~ axial dir^n

* θ & φ are blade angle at inlet & outlet measured from axial direction.

From Inlet Δ

$$\tan\alpha = \frac{V_{w1}}{V_f}$$

$$V_{w_1} = V_f \cdot \tan\alpha \quad -(a)$$

$$\tan\theta = \frac{u - V_{w_1}}{V_f}$$

$$u - V_{w_1} = V_f \cdot \tan\theta \quad -(b)$$

on adding (a) & (b)

$$u = V_f \cdot (\tan\alpha + \tan\theta) \quad (1)$$

from outlet Δ

$$\tan\beta = \frac{V_{w2}}{V_F}$$

$$V_{w_2} = V_f \cdot \tan\beta \quad -(c)$$

$$\tan\phi = \frac{u - V_{w_2}}{V_f}$$

$$u - V_{w_2} = V_f \cdot \tan\phi \quad -(d)$$

on adding (c) & (d)

$$u = V_f \cdot (\tan\beta + \tan\phi) \quad (2)$$

From (1) & (2)

$$\boxed{\tan\alpha + \tan\theta = \tan\beta + \tan\phi}$$

$$\boxed{\tan\theta - \tan\phi = \tan\beta - \tan\alpha}$$

Work per stage: $\boxed{W/_{st} = \dot{m}(V_{w_2} - V_{w_1}) \cdot u}$

as from eq. (a) & (c) $V_{w_2} = V_f \tan\beta$ & $V_{w_1} = V_f \tan\alpha$

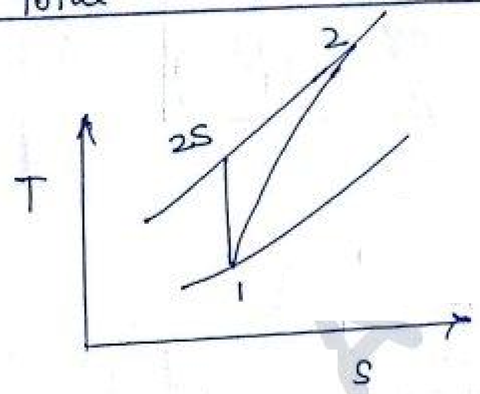
$$\boxed{W/_{st} = \dot{m} \cdot u \cdot V_f (\tan\beta - \tan\alpha)}$$

$$\boxed{W/_{st} = \dot{m} \cdot u \cdot V_f \cdot (\tan\theta - \tan\phi)}$$

if n- ~~stap~~ stages of equal work input

$$\boxed{W_{total} = n \times W/_{stage}}$$

$$\boxed{W_{total} = \dot{m} C_p (T_2 - T_1)}$$



$$\eta_c = \frac{T_{2S} - T_1}{T_2 - T_1}$$

Degree of Reaction :- It is term use to indicate the contribution of Moving part Compare to Complete stage. In case of Compressor it may define as the enthalpy or ~~pe~~ pressure rise in the moving blade Compare to enthapy & pressure rise in a stage.

$$\boxed{R = \frac{(\Delta h_m)}{(\Delta h_m) + (\Delta h_f)}}$$

m - moving part

F - fixed part

For centrifugal Compressor (c.c.)

$$\left(V_{w_2}u_2 - V_{w_1}u_1\right) = \left(\frac{V_2^2 - V_1^2}{2}\right) + \left(\frac{u_2^2 - u_1^2}{2}\right) + \left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)$$

$$\boxed{R = \frac{\left(u_2^2 - u_1^2\right) + \left(V_{r_1}^2 - V_{r_2}^2\right)}{2\left(V_{w_2} \cdot u_2 - V_{w_1}u_2\right)}}$$

$$R = 1 - \frac{\left(V_2^2 - V_1^2\right)}{2\left(V_{w_2} \cdot u_2 - V_{w_1}u_1\right)}$$

if $\alpha = 90^\circ$, $V_{w_1} = 0$, $V_1 = V_{f_1}$

$$R = 1 - \frac{\left(V_2^2 - V_{f_1}^2\right)}{2V_{w_2} \cdot u_2}$$

$$V_2^2 = V_{w_2}^2 + V_{f_2}^2$$

$$R = 1 - \frac{\left(V_{w_2}^2 + V_{f_2}^2 - V_{f_1}^2\right)}{2V_{w_2} \cdot u_2}$$

if $V_{f_1} = V_{f_2}$

$$\boxed{R = 1 - \frac{V_{w_2}}{2 \cdot u_2}}$$

degree of reaction for Centrifugal Compressor.

for Radial blade

$$\phi = 90^\circ$$

$$V_{w_2} = u_2 \qquad \boxed{R = 50\%}$$

* for Radial blade centrifugal compressor under ideal condition degree of reaction is 50%.

For axial Flow compressor

$$u_1 = u_2 = u$$

$$\left(V_{w_2} \cdot u_2 - V_{w_1} \cdot u_1\right) = \left(\frac{V_2^2 - V_1^2}{2}\right) + \cancel{\left(\frac{u_2^2 - u_1^2}{2}\right)}^{0} + \left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)$$

$$u\left(V_{w_2} - V_{w_1}\right) = \left(\frac{V_2^2 - V_1^2}{2}\right) + \left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)$$

$$R = \frac{V_{r_1}^2 - V_{r_2}^2}{2\left(V_{w_2} - V_{w_1}\right) \cdot u}$$

$$\cos\theta = \frac{V_f}{V_{r_1}} \Rightarrow V_{r_1} = V_f \sec\theta$$

$$\cos\phi = \frac{V_f}{V_{r_2}} \Rightarrow V_{r_2} = V_f \sec\phi$$

$$(V_{w_2} - V_{w_1})\cdot u = u\cdot V_f(\tan\theta - \tan\phi)$$

$$R = \frac{V_f^2(\sec^2\theta - \sec^2\phi)}{2\cdot u\cdot V_f(\tan\theta - \tan\phi)}$$

$$R = \frac{V_f}{2u}\cdot\frac{(1+\tan^2\theta - 1 - \tan^2\phi)}{(\tan\theta - \tan\phi)}$$

$$\boxed{R = \frac{V_f}{2u}(\tan\theta + \tan\phi)}$$

Degree of Reaction for axial flow Compressor

→ <u>50% stage</u> :- In a 50% stage pressure rise is equally devided b/w the fixed & moving blade.

$$R = \frac{1}{2} = \frac{\left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)}{\left(\frac{V_2^2 - V_1^2}{2}\right) + \left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)}$$

$$(V_2^2 - V_1^2) + (V_{r_1}^2 - V_{r_2}^2) = 2(V_{r_1}^2 - V_{r_2}^2)$$

$$\boxed{(V_2^2 - V_1^2) = (V_{r_1}^2 - V_{r_2}^2)}$$

$$R = \frac{1}{2}\left(\frac{V_f}{u}\right)(\tan\theta + \tan\phi) = \frac{1}{2}$$

$$u = V_f(\tan\theta + \tan\phi) \quad \text{— for 50\% Stage}$$

$$u = V_f(\tan\theta + \tan\alpha) \rightarrow \text{General eq}^n$$

eq (1)

$$\tan\theta + \tan\phi = \tan\theta + \tan\alpha$$

$$\boxed{\alpha = \phi}$$

$$\Rightarrow \tan\alpha + \tan\theta = \tan\beta + \tan\phi$$

General eqn

$$\boxed{\beta = \theta}$$

Now $\cos\beta = \cos\theta$

$$\frac{V_f}{V_2} = \frac{V_f}{V_{r_1}} \Rightarrow \boxed{V_2 = V_{r_1}}$$

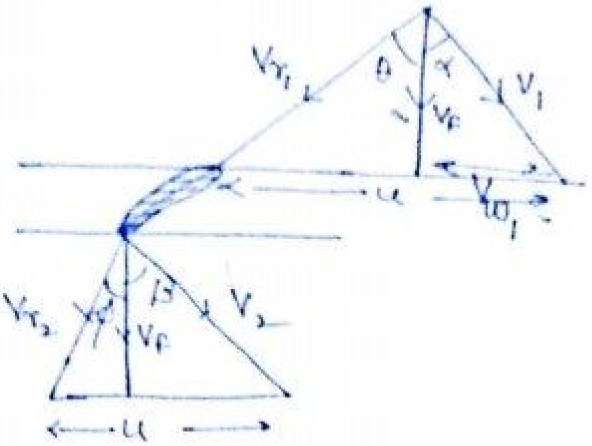
for 50% stage

$$\alpha = \phi \Rightarrow \cos\alpha = \cos\phi$$

$$\frac{V_f}{V_1} = \frac{V_f}{V_{r_2}} \Rightarrow \boxed{V_1 = V_{r_2}}$$

for 50% stage

Q.35
P.g.22
WB



$N = 15000\ rpm$

$D_m = 0.6\ m$

$V_f = 225\ m/s$

$V_{w_1} = 85\ m/s$

$W = 45\ kJ/kg.$

$P_1 = 1\ bar$

$T_1 = 300K \quad \eta_c = 89\%$

$$u = \frac{\pi D_m N}{60} = \frac{\pi \times 0.6 \times 15000}{60}$$

$$u = 471.23\ m/s$$

$$W_{/st} = (V_{w_2} - V_{w_1})u = 45 \times 10^3$$

$$(V_{w_2} - 85) \times 471.23 = 45 \times 10^3$$

$$V_{w_2} = 180.42\ m/s.$$

$$\tan\theta = \frac{u - V_{w_1}}{V_f} = \frac{471.23 - 85}{225}$$

$$\theta = 59.77^\circ$$

$$\tan\phi = \frac{u - V_{w_2}}{V_f} = \frac{471.23 - 180.42}{225}$$

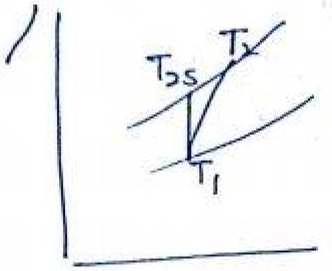
$$\phi = 52.26^\circ$$

$$\theta - \phi = 7.5^\circ$$

(ii) $W/st = C_p(T_2 - T_1)$

$45 = 1.005(T_2 - 300)$

$T_2 = 344.7 \underline{K}$



$$\frac{T_{2s} - T_1}{T_2 - T_1} = 0.89$$

$T_{2s} - 300 = 0.89(44.7)$

$T_{2s} = 339.8 \underline{K}$

$$\frac{T_{2s}}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}} \qquad r_p = \left(\frac{339.8}{300}\right)^{7/2}$$

$$\boxed{r_p = 1.547}$$

(3) $$R = \frac{V_{r_1}^2 - V_{r_2}^2}{2(V_{w_2} - V_{w_1})\cdot u} = \frac{V_f}{2u}(\tan\theta + \tan\phi)$$

$$V_{r_1} = \frac{V_f}{\cos\theta} = \frac{225}{\cos 59.71} = 446.89 \text{ m/s}$$

$$V_{r_2} = \frac{V_f}{\cos\phi} = \frac{225}{\cos 52.26} = 367.59 \text{ m/s}$$

$$R = \frac{225}{2 \times 471.23}(\tan 59.71 + \tan 52.26)$$

$R = 0.718$

Q.36
pg22

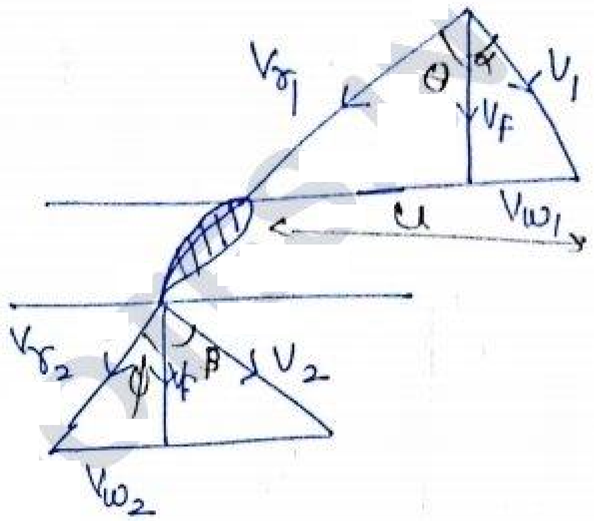
$r_p = 5$

$\eta_c = 87\%$

$T_1 = 288\,K$

$R = 50\%$ $u = 210\,m/s$.

$V_f = 170\,m/s$.



$$\frac{T_{2s}}{T_1} = (r_p)^{\frac{r-1}{r}} \qquad r = 1.4$$

$$T_{2s} = 288(5)^{2/7}$$

$$T_{2s} = 456.14\ K$$

$$\frac{T_{2s} - T_1}{T_2 - T_1} = 0.87$$

$$\frac{456.14 - 288}{T_2 - 288} = 0.87$$

$$T_2 = 481.26\ \underline{\underline{K}}$$

$$\cancel{W_{stay}} = W_{total} = C_p(T_2 - T_1)$$

$$W/_{stag} = \frac{C_p(T_2 - T_1)}{10} = \frac{1.005(481.26 - 288)}{10}$$

$$W_{stage} = 19.42\ KJ$$

$$W_{stag} = 19.42 \cancel{= (V_{w_2} - V_{w_1})\,u}$$

$$W/\text{stg.} = u \cdot V_f(\tan\theta - \tan\phi)$$

$$19.423 \times 10^3 = 210 \times 170(\tan\theta - \tan\phi)$$

$$\tan\theta - \tan\phi = 0.544 \quad \text{—(1)}$$

as 50%. $(\alpha = \phi)$

$$\tan\theta + \tan\phi = \frac{u}{V_f} = \frac{210}{170} = 1.235 \quad \text{—(2)}$$

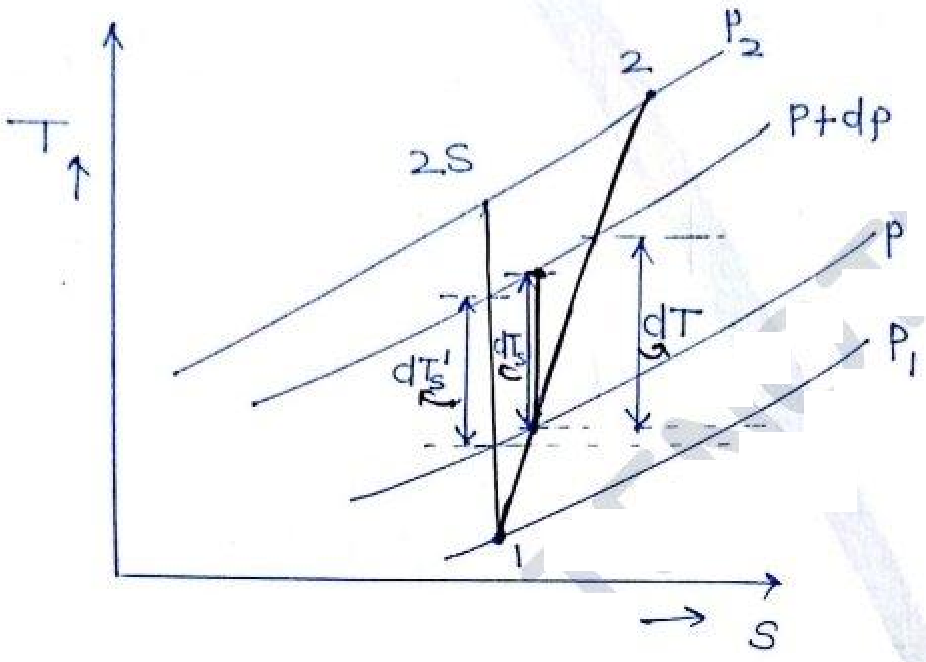
(1) & (2)

$$2\tan\theta = 1.779$$

$$\theta = 41.65^\circ$$

$$\phi = 19.06^\circ \quad \underline{\text{Ans}}$$

Polytropic efficiency / Small stage efficiency



$$\eta_{p,c} = \frac{dT_s}{dT}$$

$dT_s > dT_s'$ because P lines are diverging.

$$\frac{dT_s}{dT} > \frac{dT_s'}{dT}$$

☆ $\boxed{\eta_{p,c} > \eta_{o,c}}$

* Polytropic efficiency for a compressor always greater than overall efficiency of compressor

<u>Polytropic efficiency</u>:- it is a isentropic eff. for a small stage which remain constant throughout. It is the ratio of isentropic temp. rise across the small stage to the actual temp. rise across the small stage.

for polytropic Compression

$$PV^n = C$$

$$PV = RT$$

$$V = \frac{RT}{P}$$

$$\Rightarrow P\left(\frac{RT}{P}\right)^n = C$$

$$P^{-(n-1)} \cdot T^n = C$$

taking log both side.

$$(1-n)\ln P + n \ln T = \ln C_1$$

differenciate

$$(1-n)\frac{dP}{P} + n \cdot \frac{dT}{T} = 0$$

$$\frac{dT}{T} = -\frac{(1-n)}{n}\frac{dP}{P}$$

$$\boxed{dT = \left(\frac{n-1}{n}\right)\frac{dP}{P} \cdot T}$$

for isentropic Compression

Similarly

$$\boxed{dT_s = \left(\frac{\gamma-1}{\gamma}\right)\frac{dP}{P} \cdot T}$$

$$\eta_{p,c} = \frac{dT_s}{dT} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)$$

$$\boxed{\eta_{p,c} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)}$$

$$\boxed{\left(\frac{n-1}{n}\right) = \frac{1}{\eta_{p,c}}\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}}$$

$$\eta_{p,c} = \frac{dT_s}{dT} = \left(\frac{\gamma-1}{\gamma}\right) \cdot \frac{dP}{P} \cdot \frac{T}{dT}$$

$$\eta_{p,c} \cdot \frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right) \frac{dP}{P}$$

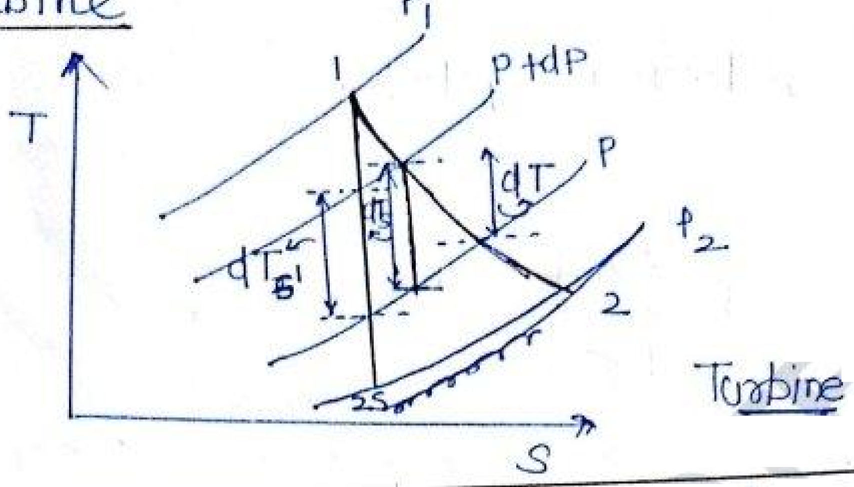
$\eta_{p,c}$ remain constant throught.
So on integrating from $1 \to 2$

$$\eta_{p,c} \ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\eta_{p,c} = \frac{\ln\left(\frac{T_{2s}}{T_1}\right)}{\ln\left(\frac{T_2}{T_1}\right)}}$$

→ polytropic efficiency for Compressor

for turbine



$$\boxed{\eta_{p,T} = \frac{dT}{dT_s} = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{n-1}{n}\right)}$$

$$dT_s > dT_s' \Rightarrow \frac{dT}{dT_s'} > \frac{dT}{dT_s}$$

$$\boxed{\eta_{o,T} > \eta_{p,T}}$$