CBSE Test Paper 03
Chapter 3 Pair of Linear Equation

1. The pair of equations x = 2 and y = – 3 has **(1)**
    a. no solution
    b. one solution
    c. infinitely many solutions
    d. two solutions
2. Solution of $\frac{a^2}{x} - \frac{b^2}{y} = 0$ and $\frac{a^2b}{x} + \frac{b^2a}{y} = a + b$ where $x, y \neq 0$ is **(1)**
    a. $x = -a^2 \; and \; y = -b^2$
    b. $x = a^2 \; and \; y = -b^2$
    c. $x = a^2$ and $y = b^2$
    d. $x = -a^2 \; and \; y = b^2$
3. A system of two linear equations in two variables has a unique solution, if their graphs **(1)**
    a. do not intersect at any point
    b. coincide
    c. cut the x – axis
    d. intersect only at a point
4. A system of two linear equations in two variables is inconsistent, if their graphs **(1)**
    a. intersect only at a point
    b. coincide
    c. do not intersect at any point
    d. cut the x – axis
5. Every linear equation in two variables has **(1)**
    a. two solutions
    b. no solution

c. an infinite number of solutions
d. one solution

6. Find whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident: **(1)**
$2x - 3y + 6 = 0; 4x - 5y + 2 = 0$

7. The larger of two supplementary angles exceeds the smaller by 20 degrees. Find the angles. **(1)**

8. Find the value of k for which the pair of linear equations 4x + 6y - 1 = 0 and 2x + ky - 7 - 0 represents parallel lines. **(1)**

9. Which type of solution will equations x + 2y = 4 and 2x + y = 5 have? **(1)**

10. Without drawing the graph, find out wheather the lines representing the following pair of linear equations intersect at a point, are parallel or coincident.
$18x - 7y = 24; \frac{9}{5}x - \frac{7}{10} = \frac{9}{10}$ **(1)**

11. Two lines are given to be parallel. The equation of one of the lines is 4x + 3y = 14, then find the equation of the second line. **(2)**

12. The difference between two numbers is 26 and one number is three times the other. Find the numbers. **(2)**

13. Draw the graph of the equation 3x + 2y = 12. Also, find the co-ordinates of the points where the line meets the x-axis and the y-axis. **(2)**

14. A man invested an amount at 12% per annum simple interest and another amount at 10% per annum simple interest. He received an annual interest of Rs 2600. But, if he had interchanged the amounts invested, he would have received Rs 140 less. What amounts did he invest at the different rates? **(3)**

15. Find the values of a and b for which 2x + 3y = 7, 2ax + (a + b)y = 28 has an infinite number of solutions. **(3)**

16. Solve: $\frac{3}{x+y} + \frac{2}{x-y} = 2$ and $\frac{9}{x+y} - \frac{4}{x-y} = 1$. **(3)**

17. Solve the following system of equation by elimination method:
$\frac{x}{2} - \frac{y}{5} = 4$ and $\frac{x}{7} + \frac{y}{15} = 3$ **(3)**

18. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream. **(4)**

19. A man travels 370 km, partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car. **(4)**

20. Solve for x and y : $\frac{8}{2x-3y} + \frac{21}{2x+3y} = 11$; $\frac{5}{2x-3y} + \frac{7}{2x+3y} = 6$, ($2x - 3y \neq 0$, $2x + 3y \neq 0$) **(4)**

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Solution

1. b. one solution

   Explanation: Here, a unique solution of each variable of a pair of linear equations is given, therefore, it has one solution to a system of linear equations.

2. c. $x = a^2$ and $y = b^2$

   Explanation: First equation:
   $\frac{a^2}{x} - \frac{b^2}{y} = 0$ or $\frac{a^2}{x} = \frac{b^2}{y}$
   Second Equation:
   $\frac{a^2b}{x} + \frac{b^2a}{y} = a + b$
   $\Rightarrow \left(\frac{b^2}{y}\right) \times b + \frac{b^2a}{y} = a + b$
   $\Rightarrow \left(\frac{b^2}{y}\right) \times (b + a) = a + b$
   $\Rightarrow \frac{b^2}{y} = \frac{a+b}{a+b} = 1$
   $\Rightarrow y = b^2$
   $\frac{a^2}{x} = \frac{b^2}{y}$
   $\Rightarrow \frac{a^2}{x} = \frac{b^2}{b^2} = 1$
   $\Rightarrow x = a^2$
   Hence **$x=a^2$ and $y=b^2$**

3. d. intersect only at a point

   Explanation: Number of solutions of a system of two linear equations in two variables are equal to number of common points between the graphs of given linear equations.
   If a system has unique solution then their graphs must intersect in only one point.

4. c. do not intersect at any point

   Explanation: A system of two linear equations in two variables is inconsistent, if their graphs do not intersect at any point.

In this case, a pair of lines represented by the system are parallel to each other. so they do not intersect each other at any point.
the system is an inconsistent system of linear equations and the equations are independent.

5. c. an infinite number of solutions

Explanation: A linear equation in two variables is of the form, ax + by + c = 0, where geometrically it does represent a straight line and every point on this graph is a solution for a given linear equation.
As a line consists of an infinite number of points,
A linear equation has an infinite number of solutions.

6. If two lines are parallel then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ and if intersect then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Now given equations are 2x - 3y + 6 = 0 ...(i)
$\Rightarrow$ $a_1 = 2$, $b_1 = -3$, $c_1 = 6$
And, 4x - 5y + 2 = 0 ...(ii)
$\Rightarrow$ $a_2 = 4$, $b_2 = -5$, $c_2 = 2$
Now, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$
$\frac{c_1}{c_2} = \frac{6}{2} = 3$
$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Therefore lines representing the given pair of linear equations intersect each orher at a point.

7. Suppose larger angle $= x^o$ and smaller angle $= y^o$
Now $x - y = 20$ ...(i)
$x + y = 180$ ...(ii)
Adding equation (i) and (ii)
$\Rightarrow 2x = 200$
$\Rightarrow x = 100$
$\Rightarrow y = 80$

8. Given pair of linear equations are 2x - ky + 3 = 0 ; 4x + 6y - 5 = 0 Since the given equations is consistent, we have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Here $a_1 = 2$, $a_2 = 4$, $b_1 = -k$, $b_2 = 6$

$\Rightarrow \frac{2}{4} \neq \frac{-k}{6}$
$\Rightarrow -k \neq \frac{2}{4} \times 6 = 3$
Hence k is a real number except 3.

9. Since, $a_1 = 1,\ b_1 = 2,\ c_1 = -4$
$a_2 = 2,\ b_2 = 1,\ c_2 = -5$
$\therefore \frac{a_1}{a_2} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{2}{1}$
$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\therefore$ System of equations has unique solution.

10. The condition for two lines to be parallel is that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Given lines are 18x - 7y = 24.Here a_1=18,b_1=-7,c_1=-24 and
$\frac{9}{5}$x - $\frac{7}{10}$y = $\frac{9}{10}$ or $\frac{18x-7y}{10}$ = $\frac{9}{10}$
$\Rightarrow$18x - 7y = 9.Here, a_2=18,b_2=-7,c_2=-9
Therefore, $\frac{a_1}{a_2}$ = $\frac{18}{18}$ = 1 and $\frac{b_1}{b_2}$ = $\frac{-7}{-7}$ = 1
and $\frac{c_1}{c_2}$ = $\frac{-24}{-9}$ = $\frac{8}{3}$
Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Hence the two lines are parallel.

11. Equation of a line ax + by + c = 0 parallel to given line is ax + by + k = 0, here k is any real number.
The equation of one line is 4x + 3y = 14.
a_1 = 4 , b_1 = 3 and c_1= - 14
Two lines are given to be parallel. So, No solutions and the pair of linear equations is inconsistent.
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
or $\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2}$
$\Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$
Hence, one of the possible, second parallel line is 12x + 9y = 5.

12. Let the two numbers be x and y (x > y)
We are given that,
The difference between two numbers is 26.

x - y = 26 ...(i)

And one number is three times the other.

x = 3y ...(ii)

On substituting the value x from eqn. (ii) in eqn. (i), we get

3y - y = 26

2y =26

∴ y = 13

Put y = 13 in (ii) we get

x = 3(13) = 39

Hence, the two numbers are 39 and 13.

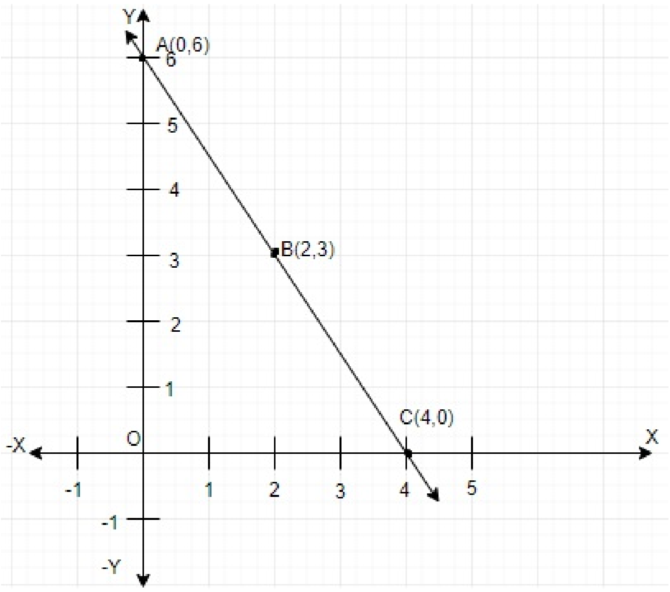
13. 3x + 2y = 12 $\Rightarrow y = \frac{12-3x}{2}$

x	0	2	4
y	6	3	0

Steps

i. Given equation.
ii. Write y in terms of x.
iii. Complete the table.
iv. Plot the points A(0, 6), B(2, 3) and C(4, 0) on the graph paper.
v. Join the points.

The Line meets the x-axis at (4, 0) and the y-axis at (0, 6).



14. Let the amount invested at 12% be Rs x and that invested at 10% be Rs y.

Then, total annual interest

$= \left(\frac{x \times 12 \times 1}{100} + \frac{y \times 10 \times 1}{100}\right) = \left(\frac{6x+5y}{50}\right)$

$\therefore \quad \frac{6x+5y}{50} = 2600 \Rightarrow 6x + 5y = 130000$.......(i)

Again, the amount invested at 12% is Rs y and that invested at 10% is Rs x.

Total annual interest at the new rates

$= \left(\frac{y \times 12 \times 1}{100} + \frac{x \times 10 \times 1}{100}\right) = \left(\frac{6y+5x}{50}\right)$

But, interest received at the new rates = $Rs. (2600 - 140) = Rs. 2460.$

$\therefore \quad \frac{6y+5x}{50} = 2460 \Rightarrow 5x + 6y = 123000$........(ii)

Adding (i) and (ii), we get

$11x + 11y = 253000$

$\Rightarrow 11(x + y) = 253000 \Rightarrow x + y = 23000.$ ... (iii)

Subtracting (ii) from (i), we get

$x - y = 7000.$ ...(iv)

Adding (iii) and (iv), we get 2x = 30000 ⇒ x = 15000.

Putting $x = 15000$ in (i), we get

$15000 + y = 23000 \Rightarrow y = 23000 - 15000 = 8000$

$\therefore x = 15000 \; and \; y = 8000.$

Hence, the amount at 12% is Rs 15000 and that at 10% is Rs 8000.

15. We know that,

if a system of linear equations

$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$

has infinite number of solutions, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Given that, $2x + 3y = 7, 2ax + (a + b)y = 28$

have an infinite number of solutions.

$\Rightarrow 2x + 3y - 7 = 0, 2ax + (a + b)y - 28 = 0$

Since, the pair of lines have an infinite number of solutions,

so, $\frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$

$\Rightarrow \frac{1}{a} = \frac{3}{a+b} = \frac{7}{28}$

a = 4

and a + b = 3a $\Rightarrow 4 + b = 12 \Rightarrow b = 8$

Hence, a = 4 and b = 8.

16. According to question the given system of equations are as

$\frac{3}{x+y}+\frac{2}{x-y}=2$

and $\frac{9}{x+y}-\frac{4}{x-y}=1$

Put $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v

So, given system of equations become

3u + 2v = 2................(i)

and 9u - 4v = 1..........(ii)

Multiply equation (i) by 2

$\Rightarrow$ 6u + 4v = 4 ....(iii)

Adding equation (ii) and equation (iii) we get

9u - 4v + 6u + 4v = 1 + 4

$\Rightarrow$ 15u = 5

$\Rightarrow$ u = $\frac{1}{3}$

Substitute the value of u = $\frac{1}{3}$ in equation (i), we get $v=\frac{1}{2}$

$\Rightarrow \frac{1}{x+y}=\frac{1}{3}$

and $\frac{1}{x-y}=\frac{1}{2}$

$\Rightarrow$ x+ y = 3...........(iv)

and x - y = 2........(v)

Adding (iv) and (v), we get

$2x=5 \Rightarrow x=\frac{5}{2}$

Substituting x = $\frac{5}{2}$ in (iv), we get y = $\frac{1}{2}$

Hence, $x=\frac{5}{2}$ and $y=\frac{1}{2}$

17. The given system of equation is

$\frac{x}{2}-\frac{y}{5}=4$ ...(1)

$\frac{x}{7}+\frac{y}{15}=3$ ...(2)

Multiplying equation (2) by 3, we get

$\frac{3x}{7}+\frac{y}{5}=9$ ...(3)

Adding equation (1) and equation (3), we get

$\frac{x}{2}+\frac{3x}{7}=13$

$\Rightarrow \frac{13}{14}x=13 \Rightarrow x=\frac{13\times14}{13}=14$

Substituting this value of x in equation (2), we get

$\frac{14}{7}+\frac{y}{15}=3$

$\Rightarrow 2 + \frac{y}{15} = 3 \Rightarrow \frac{y}{15} = 3 - 2$
$\Rightarrow \frac{y}{15} = 1 \Rightarrow y = 15$

So, the solution of the given system of equations is x = 14, y = 15

Verification; Substituting x = 14, y = 15.

We find that both the equations (1) and (2) are satisfied as shown below;

$\frac{x}{2} - \frac{y}{5} = \frac{14}{2} - \frac{15}{5} = 7 - 3 = 4$
$\frac{x}{7} + \frac{y}{15} = \frac{14}{7} + \frac{15}{15} = 2 + 1 = 3$

Hence, the solution is correct.

18. Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Then, Speed of the boat while going upstream = (x - y) km/hr

Speed of the boat while going downstream = (x + y) km/hr

Also we know that, time taken to cover 'd' Km with speed 's' Km/hr is $\frac{d}{s}$

Hence, Time taken by the boat to cover 12 km upstream = $\frac{12}{x-y}$ hrs

And, Time taken by the boat to cover 40 km downstream = $\frac{40}{x+y}$ hrs

According to the question, Total time taken = 8 hrs

$\therefore \frac{12}{x-y} + \frac{40}{x+y} = 8$.........(1)

Again, time taken by the boat to cover 16 km upstream = $\frac{16}{x-y}$

And,Time taken by the boat to cover 32 km downstream = $\frac{32}{x+y}$

According to the question,Total time taken = 8 hrs

$\therefore \frac{16}{(x-y)} + \frac{32}{(x+y)} = 8$.........(2)

Putting $\frac{1}{(x-y)} = u$ and $\frac{1}{(x+y)} = v$ in equation (1) & equation (2), so that we may get linear equations in the variables u & v as following :-

12u + 40v = 8

$\Rightarrow$ 3u + 10v = 2........(3)

and 16u + 32v = 8

$\Rightarrow$2u + 4v = 1.........(4)

Multiplying equation (3) by 4 and equation (4) by 10, we get ;

12u + 40v = 8..........(5)

20u + 40v = 10........(6)

Subtracting equation (5) from equation (6), we get

$8u = 2 \Rightarrow u = \frac{1}{4}$

Putting u = $\frac{1}{4}$ in equation (3), we get

$3 \times \frac{1}{4} + 10v = 2 \Rightarrow 10v = \frac{5}{4} \Rightarrow v = \frac{1}{8}$

$u = \frac{1}{4} \Rightarrow \frac{1}{x-y} = \frac{1}{4} \Rightarrow x - y = 4$.....(7)

$v = \frac{1}{8} \Rightarrow \frac{1}{x+y} = \frac{1}{8} \Rightarrow x + y = 8$......(8)

On adding (7) and (8), we get

2x = 12 ⇒ x = 6

Putting x = 6 in (8), we get

6 + y = 8 ⇒ y = 8 - 6 = 2

∴ x = 6, y = 2

Hence, the speed of the boat in still water = 6 km/hr and speed of the stream = 2 km/hr

19. Let the speed of the train be x km/hr and that of the car be y km/hr.

Case I Distance covered by train = 250 km.

Distance covered by car = (370 - 250) km = 120 km.

Time taken to cover 250 km by train = $\frac{250}{x}$ hours

Time taken to cover 120 km by car = $\frac{120}{y}$ hours

Total time taken =4 hours

∴ $\frac{250}{x} + \frac{120}{y} = 4 \Rightarrow \frac{125}{x} + \frac{60}{y} = 2$.......(i)

Case II Distance covered by train = 130 km.

Distance covered by car = (370 -130) km = 240 km.

Time taken to cover 130 km by train = $\frac{130}{x}$ hours

Time taken to cover 240 km by car = $\frac{240}{y}$ hours

Total time taken = $4\frac{18}{60}$ hours $= 4\frac{3}{10}$ hours $= \frac{43}{10}$ hours

∴ $\frac{130}{x} + \frac{240}{y} = \frac{43}{10} \Rightarrow \frac{1300}{x} + \frac{2400}{y} = 43$......(ii)

Putting $\frac{1}{x}$ = u and $\frac{1}{y}$ =v, equations (i) and (ii) become

$125u + 60v = 2$ ... (iii) and $1300u + 2400v = 43$. ... (iv)

On multiplying (iii) by 40 and subtracting (iv) from the result, we get

5000u - 1300v = 80 - 43 ⇒ 3700u = 37

$\Rightarrow u = \frac{37}{3700} = \frac{1}{100} \Rightarrow \frac{1}{x} = \frac{1}{100} \Rightarrow x = 100$

Putting u = $\frac{1}{100}$ in (iv), we get

$\left(1300 \times \frac{1}{100}\right) + 2400v = 43 \Rightarrow 2400v = 43 - 13 = 30$

$\Rightarrow v = \frac{30}{2400} = \frac{1}{80} \Rightarrow \frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$

∴ x = 100 and y = 80.

Hence, the speed of the train is 100 km/hr and that of the car is 80 km/hr

20. Given equations are

$\frac{8}{2x-3y} + \frac{21}{2x+3y}$ = 11...(i)

$\frac{5}{2x-3y} + \frac{7}{2x+3y}$ = 6 ....(ii)

Putting $\frac{1}{2x-3y}$ = A and $\frac{1}{2x+3y}$ = B in equation (i) & (ii) so that we may get the pair of linear equations in variables A & B as following :-

8A + 21B = 11 ...(iii).   and 5A + 7B = 6...(iv)

Multiplying eq. (iv) by 3 & then subtracting eq. (iii) from it , we get ;

$$\begin{array}{r} 15A + 21B = 18 \\ 8A + 21B = 11 \\ - \quad - \quad - \\ \hline 7A = 7 \end{array}$$

$\Rightarrow$ A = 1

Substituting A = 1 in eq. (iii) ,

8 × 1 + 21B = 11

$\Rightarrow$ 21B = 3

$\Rightarrow$ B = $\frac{1}{7}$

Since, A = 1

$\Rightarrow \frac{1}{2x-3y}$ = 1

$\Rightarrow$ 2x - 3y = 1...(vi)

Where B = $\frac{1}{7}$

$\Rightarrow \frac{1}{2x+3y} = \frac{1}{7}$

$\Rightarrow$2x + 3y = 7...(vii)

Adding (vi) and (vii), we get

$$\begin{array}{r} 2x - 3y = 1 \\ 2x + 3y = 7 \\ \hline 4x = 8 \end{array}$$

$\Rightarrow$x = 2

Substituting x = 2 in eq.(vi),

2 × 2 - 3y = 1

$\Rightarrow$ -3y = -3 $\Rightarrow$ y = 1

$\therefore$ x = 2, y = 1.