12 Linear Programming

Fastrack Revision

- A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum) of a linear function of several variables subject to conditions that the variables are non-negative and satisfy a set of linear inequalities (called linear constraints). Variables are sometimes called decision variables.
- The common region determined by all the constraints including the non-negative constraints $x \geq 0, y \geq 0$ of a linear programming problem is called the feasible solution (or solution region) for the problem.
- Points within and on the boundary of the feasible region represent feasible solutions of the constraints. Any point outside the feasible region is an infeasible solution.
- Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
- If the feasible region is unbounded, then a maximum or minimum may or may not exist. However, if it exists, it must occur at a corner point of *R*.

  Suppose $Z = ax + by$

  (i) For maximum, $Z > M$ (maxima) and the resulting half plane has point in common with the feasible region, then we say that no maximum value exist.

  (ii) For minimum, $Z < m$ (minima) and the resulting half plane has point in common with feasible region, then we say that no minimum value exist.
- If two corner points of the feasible region are both optimal solutions of the same type *i.e.*, both produce the same maximum or minimum, then any point on line segment joining these two points is also an optimal solution of the same type.
- **Fundamental Extreme Point Theorem:** An optimum solution of a LPP, if it exist, occurs at one of the extreme points (*i.e.*, corner points) of the convex polygon of the set of all feasible solution.
- **Corner Point Method:** This method of solving a LPP graphically is based on the principle of extreme point theorem.

Procedure:

(i) Consider each constraint as an equation.

(ii) Plot each equation on graph, as each one will geometrically represent a straight line.

(iii) The common region, thus obtained satisfying all the constraints and the non-negative restrictions is called the feasible region. It is a convex polygon.

(iv) Determine the vertices (corner points) of the convex polygon. These vertices are known as the extreme points of corners of the feasible region.

(v) Find the values of the objective function at each of the extreme points. The point at which the value of the objective function is optimum (maximum/ minimum) is the optimal solution of the given LPP.

Practice Exercise

Multiple Choice Questions

Q 1. The solution set of the inequation $3x + 5y < 7$ is: *(CBSE 2023)*

a. whole *XY*-plane except the points lying on the line $3x + 5y = 7$

b. whole *XY*-plane along with the points lying on the line $3x + 5y = 7$

c. open half plane containing the origin except the points of line $3x + 5y = 7$

d. open half plane not containing the origin

Q 2. The solution set of the inequality $3x + 5y < 4$ is: *(CBSE SQP 2022-23)*

a. an open half plane not containing the origin

b. an open half plane containing the origin

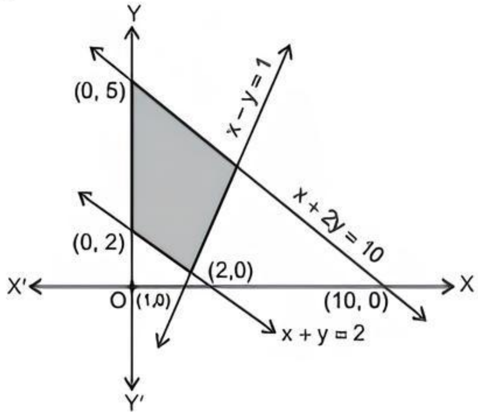
c. the whole *XY*-plane not containing the line $3x + 5y = 4$

d. a closed half plane containing the origin

Q 3. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$? *(CBSE 2023)*

a. (−2, 4) b. (3, 2) c. (−5, 6) d. (4, 2)

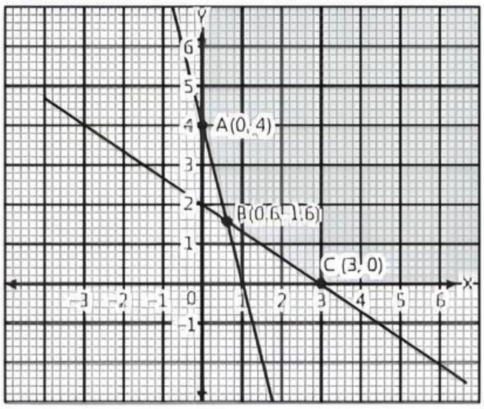
Q 4. The feasible region corresponding to the linear constraints of a linear programming problem is given below:



Which of the following is not a constraint to the given linear programming problem. *(CBSE SQP 2023-24)*

a. $x+y\geq 2$ b. $x+2y\leq 10$
c. $x-y\geq 1$ d. $x-y\leq 1$

Q 5. **The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function $Z=4x+6y$ occurs at:** *(CBSE SQP 2022-23)*



a. (0.6, 1.6) only
b. (3, 0) only
c. (0.6, 1.6) and (3, 0)
d. at every point of the line segment joining the points (0.6, 1.6) and (3, 0)

Q 6. **For an objective function $Z=ax+by$, where $a, b>0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0, 20), (10, 10), (30, 30) and (0, 40). The condition on a and b such that the maximum Z occurs at both the points (30, 30) and (0, 40) is:** *(CBSE SQP 2021 Term-1)*

a. $b-3a=0$ b. $a=3b$
c. $a+2b=0$ d. $2a-b=0$

Q 7. **In a linear programming problem, the constraints on the decision variables x and y are $x-3y\geq 0$, $y\geq 0$, $0\leq x\leq 3$. The feasible region:** *(CBSE SQP 2021 Term-1)*

a. is not in the first quadrant
b. is bounded in the first quadrant
c. is unbounded in the first quadrant
d. does not exist

Q 8. **A linear programming problem is as follows:**
Minimise $Z=30x+50y$
Subject to the constraints,

$$3x+5y\geq 15$$
$$2x+3y\leq 18$$
$$x\geq 0,\quad y\geq 0$$

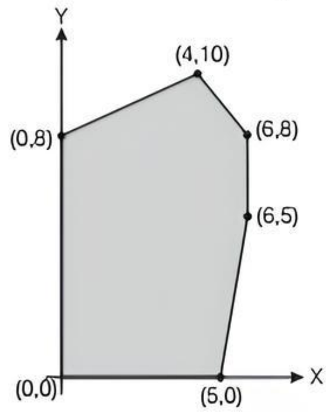
In the feasible region, the minimum value of Z occurs at: *(CBSE SQP 2021 Term-1)*

a. a unique point b. no point
c. infinitely many points d. two points only

Q 9. **A linear programming problem is as follows:**
Minimise $Z=2x+y$
Subject to the constraints $x\geq 3, x\leq 9, y\geq 0$
$$x-y\geq 0, x+y\leq 14$$
The feasible region has: *(CBSE 2021 Term-1)*

a. 5 corner points including (0, 0) and (9, 5)
b. 5 corner points including (7, 7) and (3, 3)
c. 5 corner points including (14, 0) and (9, 0)
d. 5 corner points including (3, 6) and (9, 5)

Q 10. **In the given graph, the feasible region for a LPP is shaded.**
The objective function $Z=2x-3y$, will be minimum at: *(CBSE SQP 2021 Term-1)*



a. (4, 10) b. (6, 8)
c. (0, 8) d. (6, 5)

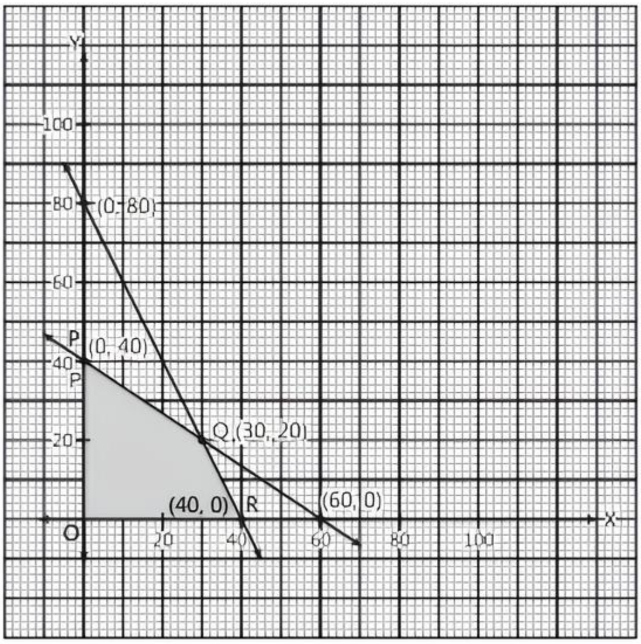
Q 11. **The corner points of the feasible region for a linear programming problem are P (0, 5), Q (1, 5), R (4, 2) and S (12, 0). The minimum value of the objective function $Z=2x+5y$ is at the point:** *(CBSE 2021 Term-1)*

a. P b. Q
c. R d. S

Q 12. **A linear programming problem is as follows:**
Maximise/Minimise objective function
$$Z=2x-y+5$$
Subject to the constraints
$$3x+4y\leq 60$$
$$x+3y\leq 30$$
$$x\geq 0, y\geq 0$$
If the corner points of the feasible region are A (0, 10), B (12, 6), C (20, 0) and O (0, 0), then which of the following is true? *(CBSE 2021 Term-1)*

a. Maximum value of Z is 40
b. Minimum value of Z is -5
c. Difference of maximum and minimum values of Z is 35
d. At two corner points, value of Z are equal

Q 13. **For an LPP the objective function is $Z=4x+3y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.** *(CBSE 2021 Term-1)*



Which one of the following statements is true?

a. Maximum value of Z is at R
b. Maximum value of Z is at Q
c. Value of Z at R is less than the value at P
d. Value of Z at Q is less than the value at R

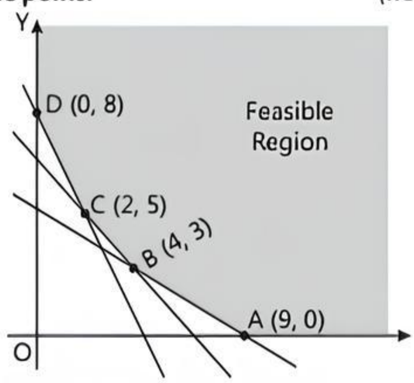
Q 14. The graph of the following linear programming problem when the conditions are $50x + 25y \leq 500$, $x + y \leq 12$ and $x \geq 0, y \geq 0$ is:

a. feasible b. unbounded
c. bounded d. None of these

Q 15. The region represented by the inequalities $x \geq 6$, $y \geq 2, 2x + y \leq 10, x \geq 0, y \geq 0$ is:

a. unbounded b. a polygon
c. a triangle d. None of these

Q 16. Feasible region for an LPP is shown shaded in the following figure. Minimum of $Z = 4x + 3y$ occurs at the point: *(NCERT EXEMPLAR)*



a. (0, 8) b. (2, 5) c. (4, 3) d. (9, 0)

Q 17. The objective function $Z = 4x + 3y$ can be minimum subject to the constraints $3x + 4y \leq 24$, $8x + 6y \leq 48$, $x \leq 5$, $y \leq 6$; $x, y \geq 0$:

a. at only one point
b. at two points only
c. at an infinite number of points
d. None of the above

Q 18. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is $Z = 4x + 3y$. *(NCERT EXEMPLAR)*
Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

a. The quantity in column A is greater
b. The quantity in column B is greater
c. The two quantities are equal
d. The relationship cannot be determined on the basis of the information supplied

Q 19. The corner points of the bounded feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. The condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is:
(NCERT EXEMPLAR; CBSE SQP 2023-24)

a. $p = 2q$ b. $p = \frac{q}{2}$ c. $p = 3q$ d. $p = q$

Q 20. A vertex of a feasible region by the linear constraints $3x + 4y \leq 18$, $2x + 3y \geq 3$ and $x, y \geq 0$ is:

a. (0, 2) b. (4.8, 0)
c. (0, 3) d. None of these

Q 21. Maximise $Z = 4x + 6y$, subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$.

a. 16 at (4, 0) b. 24 at (0, 4)
c. 24 at (6, 0) d. 36 at (0, 6)

Q 22. $Z = 7x + y$, subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at:

a. (3, 0) b. $\left(\frac{1}{2}, \frac{5}{2}\right)$ c. (7, 0) d. (0, 5)

Q 23. The maximum value of $Z = 3x + 2y$, subject to $x + 2y \leq 2$, $x + 2y \geq 8$; $x, y \geq 0$ is:

a. 32 b. 24
c. 40 d. None of these

Q 24. The maximum value of Z, where $Z = 4x + 2y$ subject to constraints $4x + 2y \geq 46$, $x + 3y \leq 24$ and $x, y \geq 0$, is:

a. 46 b. 96
c. 52 d. None of these

Q 25. Solve the linear programming problem, Maximise $Z = x + 2y$ Subject to the constraints $x - y \leq 10$, $2x + 3y \leq 20$ and $x \geq 0, y \geq 0$:

a. $Z = 10$ b. $Z = 30$
c. $Z = 40$ d. None of these

Q 26. Consider, the linear programming problem.
Maximise $Z = 4x + y$
subject to the constraints $x + y \leq 50$, $x + y \geq 100$ and $x, y \geq 0$. Then, maximum value of Z is:

a. 0 b. 50
c. 100 d. does not exist

Q 27. The point which provides the solution of the linear programming problem, maximise $Z = 45x + 55y$ subject to the constraints $x, y \geq 0$, $6x + 4y \leq 120$ and $3x + 10y \leq 180$ is:

a. (15, 10) b. (10, 15) c. (0, 18) d. (20, 0)

Q 28. The maximum value of $P = x + 3y$ such that $2x + y \leq 20$, $x + 2y \leq 20$, $x \geq 0$, $y \geq 0$, is:

a. 10 b. 60

c. 30 d. None of these

Q 29. Maximise $Z = 6x + 4y$, subject to $x \leq 2$, $x + y \leq 3$, $-2x + y \leq 1$, $x \geq 0$, $y \geq 0$:

a. 12 at (2, 0) b. $\frac{140}{3}$ at $\left(\frac{2}{3}, \frac{1}{3}\right)$

c. 16 at (2, 1) d. 4 at (0, 1)

Q 30. $Z = 6x + 21y$, subject to $x + 2y \geq 3$, $x + 4y \geq 4$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at:

a. (4, 0) b. (28, 8) c. $\left(2, \frac{1}{2}\right)$ d. (0, 3)

Q 31. Maximise $Z = 11x + 8y$, subject to $x \leq 4$, $y \leq 6$, $x + y \leq 6$, $x \geq 0$, $y \geq 0$.

a. 44 at (4, 2) b. 60 at (4, 2)

c. 62 at (4, 0) d. 48 at (4, 2)

Q 32. Maximum value of $Z = 3x + 4y$ subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$ is given by:

a. 1 b. 4

c. 6 d. no feasible solution

Q 33. The graph of the inequality $2x + 3y > 6$ is: *(CBSE 2020)*

a. half plane that contains the origin

b. half plane that neither contains the origin nor the points of the line $2x + 3y = 6$

c. whole *XOY*-plane excluding the points on the line $2x + 3y = 6$

d. entire *XOY*-plane

Assertion & Reason Type Questions

Directions (Q. Nos. 34-40): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false and Reason (R) is true

Q 34. **Assertion (A):** Maximum value of $Z = 3x + 2y$, subject to the constraints $x + 2y \leq 2$; $x \geq 0$; $y \geq 0$ will be obtained at point (2, 0).

Reason (R): In a bounded feasible region, it always exist a maximum and minimum value.

Q 35. **Assertion (A):** The linear programming problem, maximise $Z = x + 2y$ subject to the constraints $x - y \leq 10$, $2x + 3y \leq 20$ and $x \geq 0$, $y \geq 0$.

It gives the maximum value of Z as $\frac{40}{3}$.

Reason (R): To obtain the optimal value of Z, we need to compare value of Z at all the corner points of the shaded region.

Q 36. **Assertion (A):** Consider the linear programming problem.

Maximise $Z = 4x + y$

Subject to constraints

$$x + y \leq 50, \quad x + y \geq 100 \quad \text{and} \quad x, y \geq 0$$

Then, maximum value of Z is 50.

Reason (R): If the shaded region is not bounded then maximum value cannot be determined.

Q 37. **Assertion (A):** The constraints $-x_1 + x_2 \leq 1$, $-x_1 + 3x_2 \geq 9$ and x_1, $x_2 \geq 0$ defines an unbounded feasible space.

Reason (R): The maximum value of $Z = 4x + 2y$ subject to the constraints $2x + 3y \leq 18$, $x + y \geq 10$ and $x, y \geq 0$ is 5.

Q 38. **Assertion (A):** For an objective function $Z = 15x + 20y$, corner points are (0, 0), (10, 0), (0, 15) and (5, 5). Then optimal values are 300 and 0 respectively.

Reason (R): The maximum or minimum value of an objective function is known as optimal value of LPP. These values are obtained at corner points.

Q 39. Let the feasible region of the linear programming problem with the objective function $Z = ax + by$ is unbounded and let M and m be the maximum and minimum value of Z, respectively.

Now, consider the following statements.

Assertion (A): M is the maximum value of Z, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.

Reason (R): m is the minimum value of Z, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

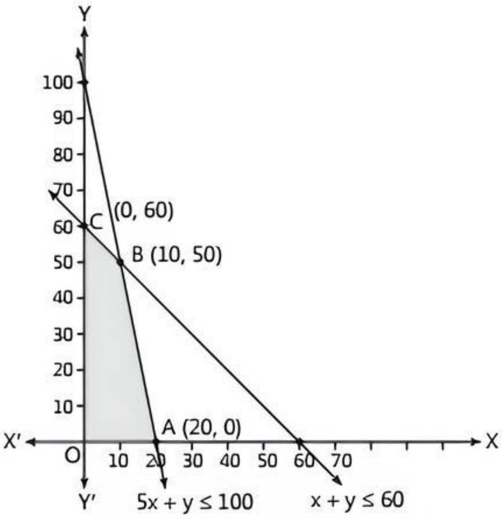
Q 40. The linear inequalities are:

$$5x + y \leq 100 \quad \text{...(1)}$$

$$x + y \leq 60 \quad \text{...(2)}$$

$$x \geq 0 \quad \text{...(3)}$$

$$y \geq 0 \quad \text{...(4)}$$



Where x and y are numbers of tables and chairs on which a furniture dealer wants to make his profit.

Assertion (A): The region $OABCO$ is the feasible region for the problem.

Reason (R): The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of a linear programming problem.

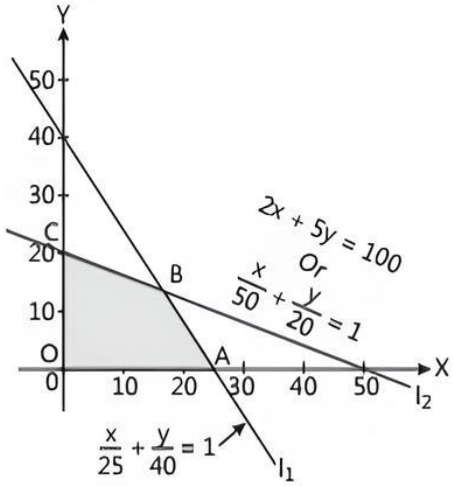
Answers

1. (c)	2. (b)	3. (d)	4. (c)	5. (d)	6. (a)	7. (b)	8. (d)	9. (b)	10. (c)
11. (c)	12. (b)	13. (b)	14. (a)	15. (d)	16. (b)	17. (a)	18. (b)	19. (b)	20. (d)
21. (d)	22. (d)	23. (d)	24. (b)	25. (d)	26. (d)	27. (b)	28. (c)	29. (c)	30. (c)
31. (b)	32. (d)	33. (b)	34. (a)	35. (a)	36. (d)	37. (c)	38. (a)	39. (b)	40. (a)

Case Study Based Questions

Case Study 1

Deepa rides her car at 25 km/hr. She has to spend ₹ 2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹ 5 per km. She has ₹ 100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. The feasible region for the LPP is shown in the adjacent figure:



Based on the above information, solve the following questions:

Q 1. What is the point of intersection of line l_1 and l_2?

a. $\left(\frac{40}{3}, \frac{50}{3}\right)$ b. $\left(\frac{50}{3}, \frac{40}{3}\right)$

c. $\left(\frac{-50}{3}, \frac{40}{3}\right)$ d. $\left(\frac{-50}{3}, \frac{-40}{3}\right)$

Q 2. The corner points of the feasible region shown in above graph are:

a. (0, 25), (20, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$

b. (0, 0), (25, 0), (0, 20)

c. (0, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$, (0, 20)

d. (0, 0), (25, 0), $\left(\frac{50}{3}, \frac{40}{3}\right)$, (0, 20)

Q 3. If $Z = x + y$ be the objective function and max. $Z = 30$. The maximum value occurs at point:

a. $\left(\frac{50}{3}, \frac{40}{3}\right)$ b. (0, 0)

c. (25, 0) d. (0, 20)

Q 4. If $Z = 6x - 9y$ be the objective function, then maximum value of Z is:

a. −20 b. 150 c. 180 d. 20

Q 5. If $Z = 6x + 3y$ be the objective function, then what is the minimum value of Z?

a. 120 b. 130

c. 0 d. 150

Solutions

1. Let $B(x, y)$ be the point of intersection of the given lines

$$2x + 5y = 100 \quad \ldots(1)$$

and $$\frac{x}{25} + \frac{y}{40} = 1$$

$\Rightarrow$ $$8x + 5y = 200 \quad \ldots(2)$$

Solving eqs. (1) and (2), we get

$$x = \frac{50}{3}, y = \frac{40}{3}$$

$\therefore$ The point of intersection $B(x, y) = \left(\frac{50}{3}, \frac{40}{3}\right)$.

So, option (b) is correct.

2. The corner points of the feasible region shown in the given graph are

$$O(0, 0), A(25, 0), B\left(\frac{50}{3}, \frac{40}{3}\right), C(0, 20).$$

So, option (d) is correct.

3. Here $Z = x + y$

Corner Points	Value of $Z = x + y$
(0, 0)	$Z = 0 + 0 = 0$
(25, 0)	$Z = 25 + 0 = 25$
$\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = \frac{50}{3} + \frac{40}{3} = \frac{90}{3} = 30$ (maximum)
(0, 20)	$Z = 0 + 20 = 20$

Thus, max $Z = 30$ occurs at point $\left(\frac{50}{3}, \frac{40}{3}\right)$.

So, option (a) is correct.

4.

Corner Points	Value of $Z = 6x - 9y$
(0, 0)	$Z = 6 \times 0 - 9 \times 0 = 0$
(25, 0)	$Z = 6 \times 25 - 9 \times 0 = 150$ (maximum)
(0, 20)	$Z = 6 \times 0 - 9 \times 20 = -180$
$\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = 6 \times \frac{50}{3} - 9 \times \frac{40}{3} = -20$

Thus maximum value of Z is 150.

So, option (b) is correct.

5.

Corner Points	Value of $Z = 6x + 3y$
(0, 0)	$Z = 6 \times 0 + 3 \times 0 = 0$ (minimum)
(25, 0)	$Z = 6 \times 25 + 3 \times 0 = 150$
(0, 20)	$Z = 6 \times 0 + 3 \times 20 = 60$
$\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = 6 \times \frac{50}{3} + 3 \times \frac{40}{3} = 140$

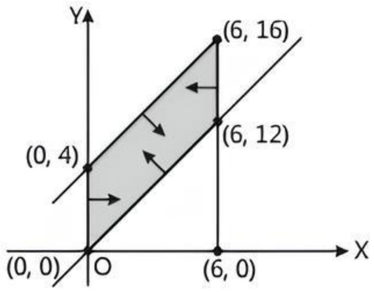
Thus, minimum value of Z is 0.

So, option (c) is correct.

Case Study 2

The feasible region for an LPP is shown shaded in the figure. Let $F = 3x - 4y$ be objective function.

(NCERT EXEMPLAR)



Based on the above information, solve the following questions:

Q 1. Find the maximum value of F.

Q 2. Find (Maximum of F + Minimum of F).

Solutions

1. Construct the following table of values of the objective function F:

Corner Points	Value of $F = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$ (maximum)
(6, 12)	$3 \times 6 - 4 \times 12 = -30$
(6,16)	$3 \times 6 - 4 \times 16 = -46$ (minimum)
(0, 4)	$3 \times 0 - 4 \times 4 = -16$

Hence, maximum value of F is 0

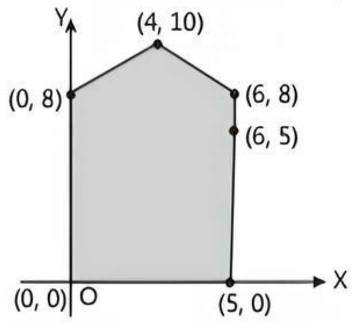
2. Minimum of $F = -46$.

$\therefore$ Maximum of F + Minimum of $F = 0 + (-46) = -46$.

Case Study 3

The feasible region for a LPP is shown shaded in the figure. Let $Z = 3x - 4y$ be the objective function.

(NCERT EXEMPLAR)



Based on the above information, solve the following questions:

Q 1. Find the points at which maximum and minimum of Z occurs.

Q 2. Find (Maximum Value of Z – Minimum Value of Z).

Solutions

1. Construct the following table of values of the objective function:

Corner Points	Value of $Z = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$
(5, 0)	$3 \times 5 - 4 \times 0 = 15$ (Maximum)
(6, 5)	$3 \times 6 - 4 \times 5 = -2$
(6, 8)	$3 \times 6 - 4 \times 8 = -14$
(4, 10)	$3 \times 4 - 4 \times 10 = -28$
(0, 8)	$3 \times 0 - 4 \times 8 = -32$ (Minimum)

∴ Minimum of $Z = -32$ at (0, 8)
and maximum of $Z = 15$ at (5, 0).

2. Here, Max. $Z = 15$ and Min. $Z = -32$

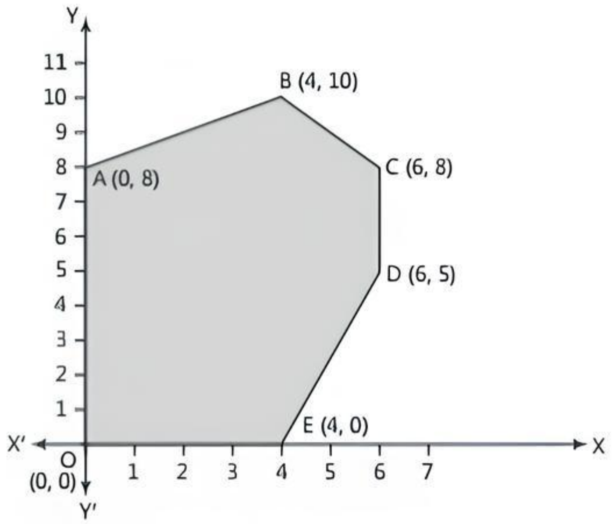
∴ Maximum value of Z – Minimum value of Z

$= 15 - (-32)$

$= 15 + 32 = 47.$

Case Study 4

The corner points of the feasible region determined by the system of linear constraints are as shown below:



Based on the above information, solve the following questions: (NCERT EXEMPLAR)

Q 1. Let $Z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

Q 2. Let $Z = px + qy$, where $p, q > 0$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B (4, 10) and C (6, 8). Also mention the number of optimal solutions in this case.

Solutions

1. The values of Z at corner points are as follows:

Corner Points	$Z = 3x - 4y$
O (0, 0)	$3 \times 0 - 4 \times 0 = 0$
A (0, 8)	$3 \times 0 - 4 \times 8 = -32$ (minimum)
B (4, 10)	$3 \times 4 - 4 \times 10 = -28$
C (6, 8)	$3 \times 6 - 4 \times 8 = -14$
D (6, 5)	$3 \times 6 - 4 \times 5 = -2$
E (4, 0)	$3 \times 4 - 4 \times 0 = 12$ (maximum)

So, the maximum value of Z is 12 at (4, 0) and the minimum value of Z is -32 at (0, 8).

2. Since the maximum value of $Z = px + qy$ occurs at B (4, 10) and C (6, 8).

∴ $4p + 10q = 6p + 8q$

⇒ $6p - 4p = 10q - 8q$

⇒ $2p = 2q$

⇒ $p = q$

which is the required condition.

The number of optimal solution are infinite.

Short Answer Type Questions

Q 1. Under the following conditions (constraints), Maximise $Z = x^2 + y^2$

$x + y \le 5, \quad x \ge 0, y \ge 0$

Q 2. Solve the following linear programming problem graphically:

Maximise $Z = 5x + 3y$

Subject to the constraints

$3x + 5y \le 15$

$5x + 2y \le 10$

$x, y \ge 0$ (CBSE 2023)

Q 3. Solve the following linear programming problem graphically:

Minimise $Z = -3x + 4y$

Subject to the constraints

$x + 2y \le 8$

$3x + 2y \le 12$

$x, y \ge 0$ (CBSE 2023)

Q 4. Solve the following linear programming problem graphically:

Maximise $Z = -3x - 5y$

Subject to the constraints

$-2x + y \le 4$

$x + y \ge 3$

$x - 2y \le 2$

$x \ge 0, y \ge 0$ (CBSE 2023)

Q 5. Solve the following linear programming problem graphically:

Minimise $Z = x + 2y$,

Subject to the constraints

$x + 2y \ge 100$

$2x - y \le 0$

$2x + y \le 200$

$x, y \ge 0$ (CBSE SQP 2023-24)

Q 6. Solve the following linear programming problem graphically:

Maximise $Z = 400x + 300y$

Subject to $x + y \le 200$, $x \le 40$, $x \ge 20$, $y \ge 0$

(CBSE SQP 2022-23)

Q 7. Solve the following linear programming problem graphically:

Maximise $Z = -x + 2y$

Subject to the constraints

$$x \ge 3$$
$$x + y \ge 5$$
$$x + 2y \ge 6$$
$$y \ge 0$$

(CBSE SQP 2023-24)

Q 8. Solve the following LPP graphically:

Minimise $Z = 5x + 10y$

Subject to constraints

$$x + 2y \le 120$$
$$x + y \ge 60$$
$$x - 2y \ge 0$$

and $x, y \ge 0$ *(CBSE 2017)*

Long Answer Type Questions

Q 1. Solve the following linear programming problem graphically:

Minimise $Z = 60x + 80y$

Subject to constraints:

$$3x + 4y \ge 8$$
$$5x + 2y \ge 11$$
$$x,\ y \ge 0$$

(CBSE 2023)

Q 2. Solve the following linear programming problem by Graphical method:

Under the following constraints:

$$x - y \ge 0$$
$$-x + 2y \ge 2$$
$$x \ge 3$$
$$y \le 4,\ y \ge 0$$

Find the minimum value of $Z = 2x + 3y - 1$.

Q 3. Solve the following linear programming problem graphically:

Maximise $P = 70x + 40y$

Subject to $3x + 2y \le 9$

$$3x + y \le 9$$
$$x \ge 0,\ y \ge 0$$

(CBSE 2023)

Q 4. Solve the following linear programming problem by Graphical Method:

Under the following constraints:

$$x + y \le 50000$$
$$x \ge 20000$$
$$y \ge 10000$$
$$x \ge y,\ x \ge 0,\ y \ge 0$$

Find the maximum value of $Z = \frac{x}{10} + \frac{9}{100}y$.

Q 5. Solve the following linear programming problem graphically:

Maximise $Z = 100x + 120y$

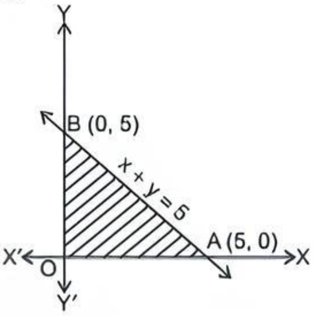
Under the following constraints:

$$5x + 8y \le 200$$
$$10x + 8y \le 240$$
$$x \ge 0,\ y \ge 0$$

Solutions

Short Answer Type Questions

1. Equation of line corresponding to inequality $x + y \le 5$ is $x + y = 5$.



Put $x = 0$ and $y = 0$ in the above equation, we get $y = 5$ and $x = 5$.

It is clear that, the above line cuts the axes at point A (5, 0) and B (0, 5) respectively.

In XY-plane, plot the points A and B and join them to get line AB.

Feasible solution region of the problem lies on the line $x + y = 5$ and below it in the first quadrant, which shows by lining.

The corner points of solution region are O (0, 0), A (5, 0) and B (0, 5).

Corner Points	$Z = x^2 + y^2$
O (0, 0)	$Z = 0^2 + 0^2 = 0$
A (5, 0)	$Z = 5^2 + 0^2 = 25$ (maximum)
B (0, 5)	$Z = 0^2 + 5^2 = 25$ (maximum)

Hence, the maximum value of Z is 25, which lies on each point of AB.

2. Our problem is to maximise $Z = 5x + 3y$

Subject to constraints

$3x + 5y \le 15$, $5x + 2y \le 10$, $x, y \ge 0$

Table for line $3x + 5y = 15$ is:

x	0	5
y	3	0

Put (0, 0) in the inequality $3x + 5y \le 15$, we get

$0 + 0 \le 15 \Rightarrow 0 \le 15$ (which is true)

So, the half plane is towards the origin.

Table for line $5x + 2y = 10$ is:

x	0	2
y	5	0

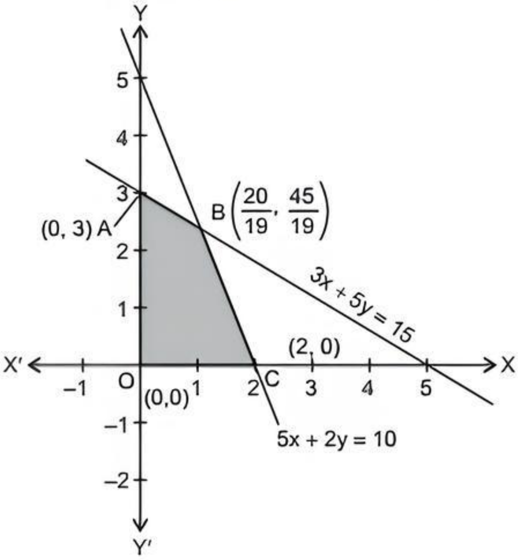
Put (0, 0) in the inequality $5x + 2y \le 10$, we get

$0 + 0 \le 10$ (which is true)

So, the half plane is towards the origin.

Since, $x, y \ge 0$

$\therefore$ The feasible region lies in the first quadrant.



Clearly, feasible region is *OABCO*. On solving equations,

$3x + 5y = 15$ and $5x + 2y = 10$, we get

$$B\left(\frac{20}{19}, \frac{45}{19}\right)$$

The corner points of the feasible region are $O(0, 0)$, $A(0, 3)$, $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(2, 0)$

The value of Z at these points are as follows:

Corner Points	$Z = 5x + 3y$
$O(0, 0)$	$5(0) + 3(0) = 0$
$A(0, 3)$	$5(0) + 3(3) = 9$
$B\left(\frac{20}{19}, \frac{45}{19}\right)$	$5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = 12.37$ (maximum)
$C(2, 0)$	$5(2) + 3(0) = 10$

Therefore, the maximum value of Z is 12.37 at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$.

3. Our problem is to minimise $Z = -3x + 4y$

Subject to constraints

$x + 2y \le 8$, $3x + 2y \le 12$, $x, y \ge 0$

Table for line $x + 2y = 8$ is:

x	0	8
y	4	0

Put (0, 0) in the inequality $x + 2y \le 8$, we get

$0 + 0 \le 8$ (which is true)

So, the half plane is towards the origin

Table for line $3x + 2y = 12$ is:

x	4	0
y	0	6

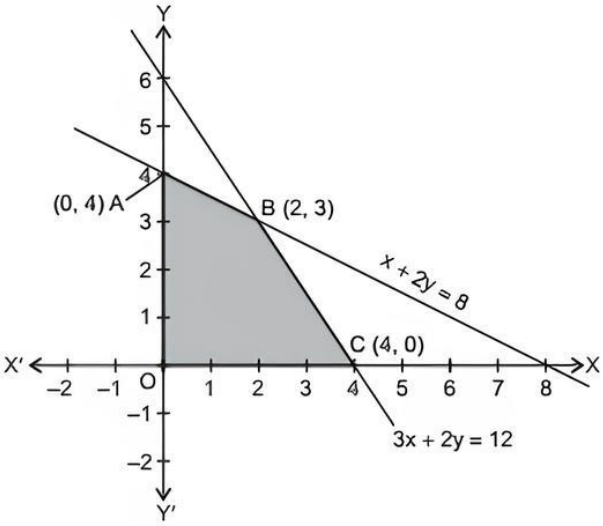
Put (0, 0) in the inequality $3x + 2y \le 12$, we get

$0 + 0 \le 12$ (which is true)

So, the half plane is towards the origin

Since, $x, y \ge 0$

$\therefore$ The feasible region lies in first quadrant.



Clearly, feasible region is *OABCO*.

On solving equations

$x + 2y = 8$ and $3x + 2y = 12$, we get $B(2, 3)$

The corner points of the feasible region are $O(0, 0)$, $A(0, 4)$, $B(2, 3)$ and $C(4, 0)$

The value of Z at these points are as follows:

Corner Points	$Z = -3x + 4y$
$O(0, 0)$	$-3(0) + 4(0) = 0$
$A(0, 4)$	$-3(0) + 4(4) = 16$
$B(2, 3)$	$-3(2) + 4(3) = 6$
$C(4, 0)$	$-3(4) + 4(0) = -12$ (minimum)

Therefore, the minimum value of Z is -12 at the point (4, 0).

4. Our problem is to maximise $Z = -3x - 5y$

Subject to constraints

$-2x + y \le 4$, $x + y \ge 3$, $x - 2y \le 2$, $x \ge 0, y \ge 0$

Table for line $-2x + y = 4$ is:

x	0	−2
y	4	0

Put (0, 0) in the inequality $-2x + y \le 4$, we get

$0 + 0 \le 4$ (which is true)

So, the half plane is towards the origin.

Table for line $x + y = 3$ is:

x	0	3
y	3	0

Put (0, 0) in the equality $x + y \ge 3$, we get

$0 + 0 \ge 3$ (which is false)

So, the half plane is aways from the origin.

Table for line $x - 2y = 2$ is:

x	0	2
y	−1	0

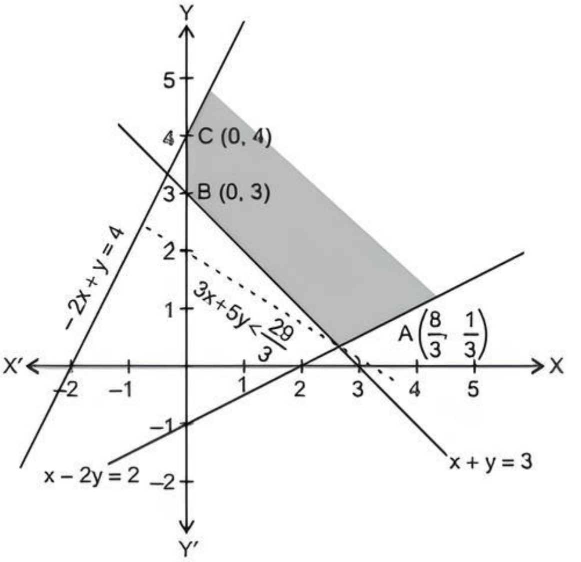
Put (0, 0) in the inequality $x - 2y \le 2$, we get

$0 - 0 \le 3$ (which is true)

So, the half plane is towards the origin.

Since, $x, y \ge 0$

$\therefore$ The feasible region lies in the first quadrant.



Clearly, feasible region is *ABC*. On solving equations $x - 2y = 2$ and $x + y = 3$, we get $x = \frac{8}{3}, y = \frac{1}{3}$

The corner points of the feasible region are

$A\left(\frac{8}{3}, \frac{1}{3}\right), B(0, 3)$ and $C(0, 4)$

The value of *Z* at these points are as follows:

Corner Points	$Z = -3x - 5y$
$A\left(\frac{8}{3}, \frac{1}{3}\right)$	$-3 \times \frac{8}{3} - 5 \times \frac{1}{3} = -\frac{29}{3}$ (maximum)
$B(0, 3)$	$-3(0) - 5(3) = -15$
$C(0, 4)$	$-3(0) - 5(4) = -20$

As the feasible region is unbounded, therefore $Z = -\frac{29}{3}$ may or may not be the maximum value.

For this, we draw the graph of the inequality $-3x - 5y > -\frac{29}{3}$ or $3x + 5y < \frac{29}{3}$ and check whether the resulting half plane has point is common with the feasible region or not.

At point (0, 0), $0 + 0 < \frac{29}{3}$ (true), therefore half plane is towards the origin.

Hence, there is no point common with the feasible region.

Hence, *Z* has maximum value $-\frac{29}{3}$ at the point $\left(\frac{8}{3}, \frac{1}{3}\right)$.

5. Our problem is to minimise $Z = x + 2y$

Subject to constraints,

$x + 2y \ge 100, \quad 2x - y \le 0, \quad 2x + y \le 200, \quad x, y \ge 0$

Table for line $x + 2y = 100$ is:

x	0	100
y	50	0

Put (0, 0) in the inequality $x + 2y \ge 100$, we get

$0 + 0 \ge 100$ (which is false)

So, the half plane is away from the origin.

Table for line $2x - y = 0$ is:

x	0	50
y	0	100

Put (2, 0) in the inequality $2x - y \le 0$, we get

$4 - 0 \le 0$ (which is false)

So, the half plane is towards the *Y*-axis.

Table for line $2x + y = 200$ is:

x	0	100
y	200	0

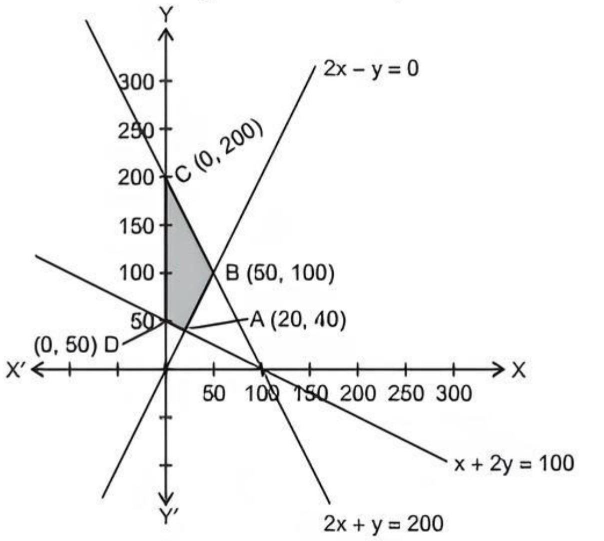
Put (0, 0) in the inequality $2x + y \le 200$, we get

$0 + 0 \le 200$ (which is true)

So, the half plane is towards the origin.

Since, $x, y \ge 0$

$\therefore$ The feasible region lies in first quadrant.



Clearly, feasible region is *ABCDA*.

On solving equations $2x + y = 200$ and $2x - y = 0$ is $B(50, 100)$ and solving equations $x + 2y = 100$ and $2x - y = 0$ is $A(20, 40)$.

The corner points of the feasible region are $A(20, 40), B(50, 100), C(0, 200)$ and $D(0, 50)$.

The value of *Z* at these points are as follows:

Corner Points	$Z = x + 2y$
$A(20, 40)$	$20 + 2 \times 40 = 100$ (minimum)
$B(50, 100)$	$50 + 2 \times 100 = 250$
$C(0, 200)$	$0 + 2 \times 2 \times 100 = 400$
$D(0, 50)$	$0 + 2 \times 50 = 100$ (minimum)

Thus, *Z* is minimum at two points *A* and *D*.

Hence, minimum value of *Z* is 100 at all points of line segment *AD*.

6. Our problem is to maximise $Z = 400x + 300y$

Subject to constraints

$x + y \le 200$, $x \le 40$, $x \ge 20$ and $y \ge 0$.

Table for the line $x + y = 200$ is:

x	0	200
y	200	0

Put (0, 0) in the inequality $x + y \le 200$, we get

$0 + 0 \le 200 \Rightarrow 0 \le 200$ (true)

So, the half plane is towards the origin.

Secondly, draw the graph of the line $x = 40$.

Put (0, 0) in the inequality $x \le 40$, we get

$0 \le 40$ (true)

So, the half plane is towards the origin.

Thirdly, draw the graph of the line $x = 20$.

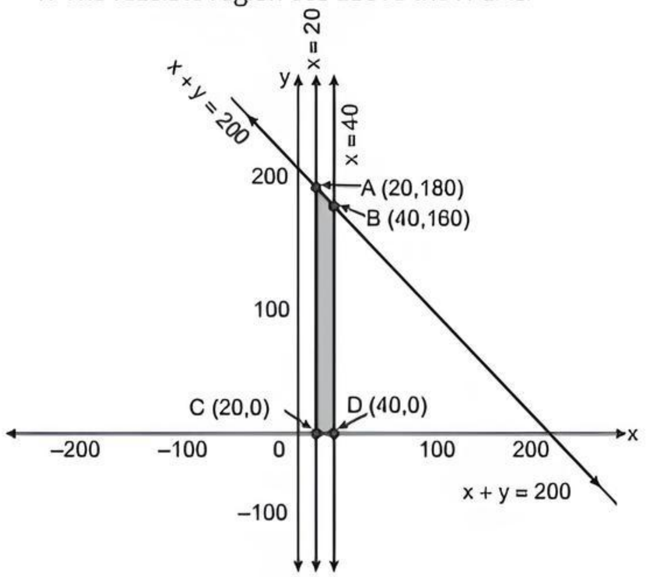
Put (0, 0) in the inequality $x \ge 20$, we get

$0 \ge 20$ (false)

So, the half plane is away from the origin.

Since, $y \ge 0$

$\therefore$ The feasible region lies above the X-axis.



Clearly, feasible region is $ABDCA$. On solving equations $x + y = 200$ and $x = 40$, we get B (40, 160). On solving equations $x = 20$ and $x + y = 200$, we get A (20, 180) and $x = 20$, $x = 40$ cuts X-axis at points C (20, 0), D (40, 0) respectively. The values of Z at these points are as follows:

Corner points	$Z = 400x + 300y$
A (20, 180)	$Z = 400 \times 20 + 300 \times 180 = 62{,}000$
B (40, 160)	$Z = 400 \times 40 + 300 \times 160 = 64{,}000$ (maximum)
C (20, 0)	$Z = 400 \times 20 + 300 \times 0 = 8{,}000$
D (40, 0)	$Z = 400 \times 40 + 300 \times 0 = 16{,}000$

Therefore, the maximum value of Z is 64,000 at the point (40, 160).

7. Our problem is to maximise $Z = -x + 2y$

Subject to constraints

$x \ge 3$, $x + y \ge 5$, $x + 2y \ge 6$, $y \ge 0$

Firstly draw the graph of the line $x = 3$, which is perpendicular to the X-axis.

Put (0, 0) in the inequality $x \ge 3$, we get

$0 \ge 3$ (which is false)

So, the half plane is right side of the line.

Table for line $x + y = 5$ is:

x	0	5
y	5	0

Put (0, 0) in the inequality $x + y \ge 5$, we get

$0 + 0 \ge 5$ (which is false)

So, the half plane is away from the origin.

Table for line $x + 2y = 6$ is:

x	0	6
y	3	0

Put (0, 0) in the inequality $x + 2y \ge 6$, we get

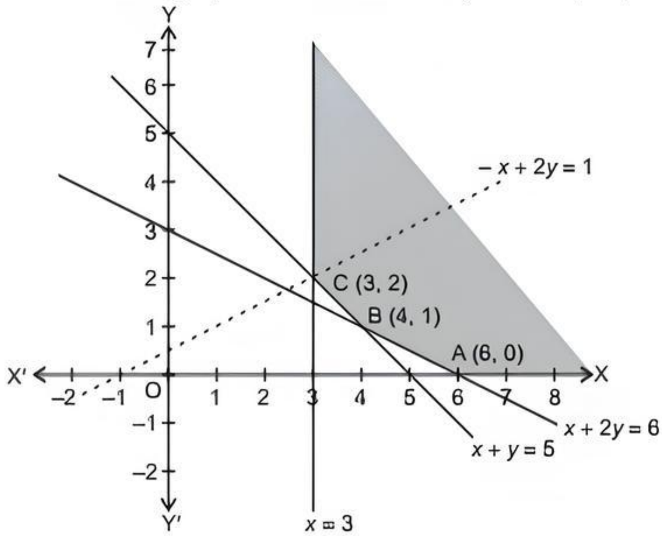
$0 + 0 \ge 6$ (which is false)

So, the half plane is away from the origin.

Also, $y \ge 0$ which lies in first quadrant.

Clearly feasible region is ABC.

On solving equations $x + y = 5$ and $x + 2y = 6$ is $B(4, 1)$ and solving equations $x = 3$ and $x + y = 5$ is C (3, 2)



The corner points of the feasible region are A(6, 0), B(4, 1) and C (3, 2)

The value of Z at these points are as follows:

Corner Points	$Z = -x + 2y$
A (6, 0)	$-6 + 0 = -6$
B (4, 1)	$-4 + 2 = -2$
C (3, 2)	$-3 + 2 \times 2 = 1$ (maximum)

As the feasible region is unbounded, therefore $Z = 1$ may or may not be the maximum value. For this, we draw the graph of the inequality $-x + 2y > 1$ and check whether the resulting half plane has points in common with the feasible region or not.

Here we see that resulting feasible region has points in common with the feasible region.

Hence, Z has no maximum value.

8. Our problem is to minimise

$$Z = 5x + 10y$$

Subject to constraints

$$x + 2y \le 120$$
$$x + y \ge 60$$
$$x - 2y \ge 0$$

and $\quad x, y \ge 0$

Table for line $x + 2y = 120$ is:

x	0	120
y	60	0

Put (0, 0) in the inequality $x + 2y \le 120$, we get

$$0 + 2 \times 0 \le 120$$

$\Rightarrow \quad 0 \le 120 \quad$ (which is true)

So, the half plane is towards the origin.

Table for line $x + y = 60$ is:

x	0	60
y	60	0

On putting (0, 0) in the inequality $x + y \ge 60$, we get

$$0 + 0 \ge 60$$

$\Rightarrow \quad 0 \ge 60 \quad$ (which is false)

So, the half plane is away from the origin.

Now, draw the graph of the line $x - 2y = 0$.

On putting (60, 20) in the inequality $x - 2y \ge 0$, we get

$$60 - 2 \times 20 \ge 0$$

$\Rightarrow \quad 60 - 40 \ge 0$

$\Rightarrow \quad 20 \ge 0 \quad$ (which is true)

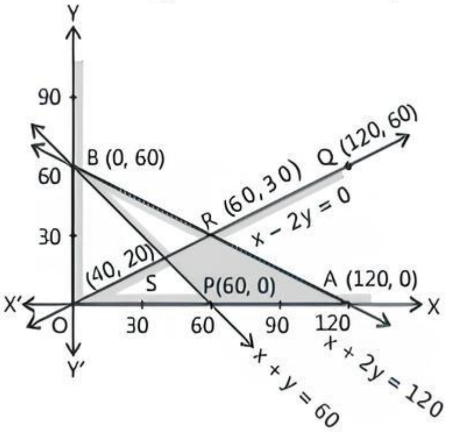
TiP

The common region determined by all the constraints including non-negative constraints $x, y \ge 0$ of a LPP is the feasible region for the problem.

So, the half plane is towards the X-axis.

Since, $\quad x, y \ge 0$

$\therefore$ The feasible region lies in the first quadrant.



Clearly, feasible region is $PARSP$. On solving equations $x - 2y = 0$ and $x + y = 60$, we get S (40, 20) and on solving equations $x - 2y = 0$ and $x + 2y = 120$, we get R (60, 30). The corner points of the feasible region are P (60, 0), A (120, 0), R (60, 30) and S(40, 20). The values of Z at these points are as follows:

Corner Points	$\boldsymbol{Z = 5x + 10y}$
P (60, 0)	$5 \times 60 + 10 \times 0 = 300$ (Minimum)
A (120, 0)	$5 \times 120 + 10 \times 0 = 600$
R (60, 30)	$5 \times 60 + 10 \times 30 = 600$
S (40, 20)	$5 \times 40 + 10 \times 20 = 400$

Therefore, the minimum value of Z is 300 at the point (60, 0).

Long Answer Type Questions

1. Our problem is to minimise $Z = 60x + 80y$

Subject to constraints,

$3x + 4y \ge 8, \quad 5x + 2y \ge 11$ and $x, y \ge 0$

Table for line $3x + 4y = 8$ is

x	0	$\frac{8}{3}$
y	2	0

Put (0, 0) in the inequality $3x + 4y \ge 8$, we get

$0 + 0 \ge 8 \quad$ (which is false)

So, the half plane is away from the origin.

Table for line $5x + 2y = 11$ is:

x	0	$\frac{11}{5}$
y	$\frac{11}{2}$	0

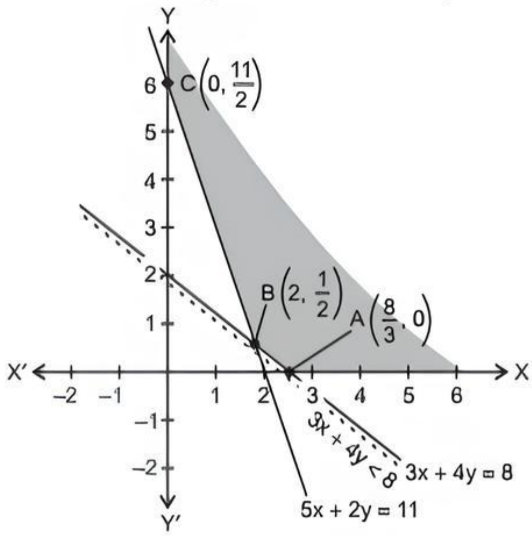
Put (0, 0) in the inequality $5x + 2y \ge 11$, we get

$0 + 0 \ge 11 \quad$ (which is false)

So, the half plane is away from the origin.

Since, $\quad x, y \ge 0$

$\therefore$ The feasible region lies in the first quadrant.



Clearly feasible region is ABC.

On solving equations

$3x + 4y = 8$ and $5x + 2y = 11$ we get $\left(2, \frac{1}{2}\right)$

The corner points of the feasible region are $A\left(\frac{8}{3}, 0\right)$,

$$B\left(2, \frac{1}{2}\right) \text{ and } C\left(0, \frac{11}{2}\right)$$

The value of Z at these points are as follows:

Corner Points	$Z = 60x + 80y$
$A\left(\frac{8}{3}, 0\right)$	$60 \times \frac{8}{3} + 80 \times 0 = 160$ (minimum)
$B\left(2, \frac{1}{2}\right)$	$60 \times 2 + 80 \times \frac{1}{2} = 160$ (minimum)
$C\left(0, \frac{11}{2}\right)$	$60 \times 0 + 80 \times \frac{11}{2} = 440$

As the feasible region is unbounded, therefore $Z = 160$ may or may not be the minimum value. For this, we draw the graph of the inequality $60x + 80y < 160$ or $3x + 4y < 8$ and check whether the resulting half plane has points in common with the feasible region or not.

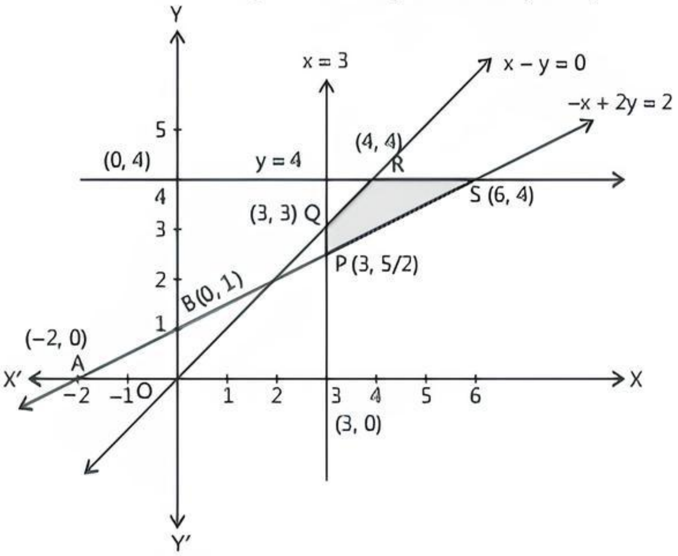
At point $(0, 0)$, $0 + 0 < 8$ (true), therefore half plane is towards the origin.

Here we see that resulting feasible region has no points in common with the feasible region.

Hence, Z has minimum value 160, which occurs at every point of the line segment AB.

2. Given, objective function: $Z = 2x + 3y - 1$

Constraints: $x - y \geq 0, -x + 2y \geq 2, x \geq 3, y \leq 4, y \geq 0$



(i) Graph of $x - y \geq 0$:

The line $x - y = 0$ or $y = x$ passing through the origin.

$\therefore$ Put $x = 1, y = 0$ in $x - y \geq 0$

$1 \geq 0$, which is true.

$\therefore$ Points of $x - y \geq 0$ lies on the line and below it.

(ii) Graph of $-x + 2y \geq 2$:

The line $-x + 2y = 2$, passing through the points $A(-2, 0)$ and $B(0, 1)$.

$\therefore$ Put $x = 0, y = 0$ in $-x + 2y \geq 2$.

$0 \geq 2$, which is not true.

$\therefore$ Points of $-x + 2y \geq 2$ lies on the line AB and above it.

(iii) Points of the region $x \geq 3$, lies on $x = 3$ and its right direction.

(iv) Points of the region $y \leq 4$, lies on $y = 4$ and below it.

(v) Points of the region $y \geq 0$, lies on X-axis and above it.

$\therefore$ Feasible region of the problem is $PSRQP$ which is bounded.

Coordinates of the corner point P, Q, R and S are $\left(3, \frac{5}{2}\right)$, $(3, 3)$, $(4, 4)$ and $(6, 4)$ respectively from the graph.

Corner Points	Objective function $Z = 2x + 3y - 1$
$P\left(3, \frac{5}{2}\right)$	$Z = 2 \times 3 + 3 \times \frac{5}{2} - 1 = \frac{25}{2}$ (minimum)
$Q(3, 3)$	$Z = 2 \times 3 + 3 \times 3 - 1 = 14$
$R(4, 4)$	$Z = 2 \times 4 + 3 \times 4 - 1 = 19$
$S(6, 4)$	$Z = 2 \times 6 + 3 \times 4 - 1 = 23$

Hence, the minimum value of Z is $\frac{25}{2}$ at point $\left(3, \frac{5}{2}\right)$.

3. Our problem is to maximise $P = 70x + 40y$

Subject to constraints

$3x + 2y \leq 9, \quad 3x + y \leq 9, \quad x, y \geq 0$

Table for line $3x + 2y = 9$ is:

x	0	3
y	$\frac{9}{2}$	0

Put $(0, 0)$ in the inequality $3x + 2y \leq 9$, we get

$0 + 0 \leq 9$ (which is true)

So, the half plane is towards the origin.

Table for $3x + y = 9$ is:

x	0	3
y	9	0

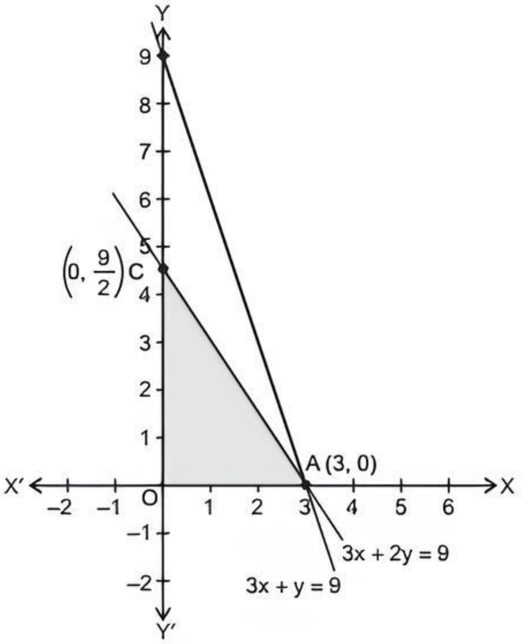
Put $(0, 0)$ in the inequality $3x + y \leq 9$, we get

$0 + 0 \leq 9$ (which is true)

So, the half plane is towards the origin.

Since, $x, y \geq 0$

$\therefore$ The feasible region lies in the first quadrant.



Clearly feasible region is $OACO$.

The corner points of the feasible region are $O(0, 0)$ $A(3, 0)$, $C\left(0, \frac{9}{2}\right)$.

The value of P at these points are as follows:

Corner Points	$P = 70x + 40y$
A (3, 0)	$70 \times 3 + 40 \times 0 = 210$ (maximum)
O (0, 0)	$70 \times 0 + 40 \times 0 = 0$
$C\left(0, \frac{9}{2}\right)$	$70 \times 0 + 40 \times \frac{9}{2} = 180$

Clearly, the maximum value of P is 210 at point (3, 0).

4. Given objective function is

$$Z = \frac{x}{10} + \frac{9}{100}y$$

Subject to constraints

$$x + y \le 50{,}000;$$
$$x \ge 20{,}000;$$
$$y \ge 10{,}000,\ x \ge y$$
$$x \ge 0,\ y \ge 0$$

Table for line $x + y = 50{,}000$ is:

x	0	50000
y	50000	0

Put (0, 0) in the inequality $x + y \le 50000$, we get

$$0 + 0 \le 50000$$

$\Rightarrow$ $0 \le 50000$ (which is true)

So, the half plane is towards the origin.

Table for line $x = y$ is:

x	0	25000
y	0	25000

Put (15000, 0) in the inequality $x \ge y$, we get

$15000 \ge 0$ (which is true)

So, the half plane lies towards the X-axis.

Now, put (25000, 0) in the inequality $x \ge 20000$, we get

$25000 \ge 20000$ (which is true)

So, the half plane lies away from the Y-axis

Again, put (0, 20000) in the inequality $y \ge 10{,}000$, we get

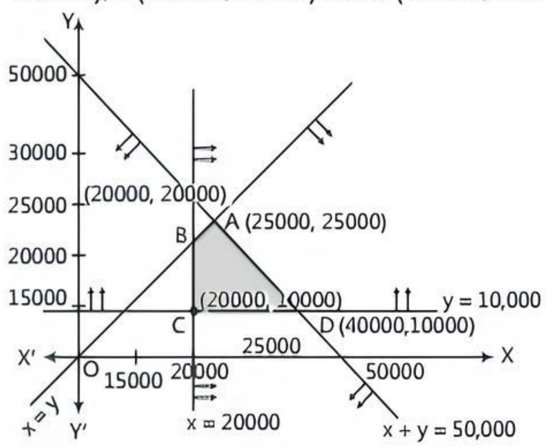
$20000 \ge 10000$ (which is true)

So, the half plane lies away from the X-axis.

Since $x, y \ge 0$

$\therefore$ The region lies in the first quadrant.

Clearly, the feasible region is $ABCDA$. On solving $x = y$ and $x + y = 50{,}000$, we get the intersection point A (25000, 25000). On solving $x = 20000$ and $x = y$, we get the intersection point B (20000, 20000). On solving $x = 20000$ and $y = 10000$, we get the intersection point C (20000, 10000). On solving $x + y = 50000$ and $y = 10000$, we get the intersection point D (40000, 10000). The corner points of the feasible region are A (25000, 25000), B (20000, 20000), C (20000, 10000) and D (40000, 10000).



The values of Z at these points are as follows:

Corner Points	$Z = \frac{x}{10} + \frac{9}{100}y$
A (25,000, 25,000)	$2{,}500 + 2{,}250 = 4{,}750$
B (20,000, 20,000)	$2{,}000 + 18{,}00 = 3{,}800$
C (20,000, 10,000)	$2{,}000 + 900 = 2{,}900$
D (40,000, 10,000)	$4{,}000 + 900 = 4{,}900$ (maximum)

Clearly, the maximum value of Z is 4900 at the point (40,000, 10,000).

5. The given problem is, Maximize $Z = 100x + 120y$

Subject to the constraints,

$$5x + 8y \le 200$$
$$10x + 8y \le 240$$
$$x \ge 0 \text{ and } y \ge 0$$

Table for line $5x + 8y = 200$ is:

x	0	40
y	25	0

Put (0, 0) in the inequality $5x + 8y \leq 200$, we get

$$5 \times 0 + 8 \times 0 \leq 200$$

$\Rightarrow \quad 0 + 0 \leq 200$

$\Rightarrow \quad 0 \leq 200$ (which is true)

So, the half plane is towards the origin.

Table for line $10x + 8y = 240$ is

x	0	24
y	30	0

On putting (0, 0) in the inequality $10x + 8y \leq 240$, we get

$$10 \times 0 + 8 \times 0 \leq 240$$

$\Rightarrow \quad 0 + 0 \leq 240$

$\Rightarrow \quad 0 \leq 240$ (which is true)

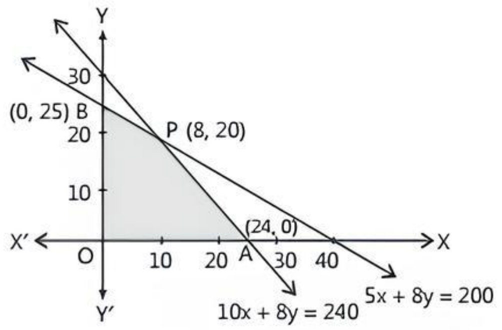
So, the half plane is towards the origin.

Since, $\quad x, y \geq 0$

$\therefore$ The feasible region lies in the first quadrant.

TiP

The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of a LPP is the feasible region for the problem.



Clearly, feasible region is *OAPBO*. On solving the equations $10x + 8y = 240$ and $5x + 8y = 200$, we get the intersection point P (8, 20). The corner points of the feasible region are O (0, 0), A (24, 0), P (8, 20) and B (0, 25). The values of Z at these points are as follows:

Corner Points	$Z = 100x + 120y$
O (0, 0)	$100 \times 0 + 120 \times 0 = 0$
A (24, 0)	$100 \times 24 + 120 \times 0 = 2400$
P (8, 20)	$100 \times 8 + 120 \times 20 = 3200$ (maximum)
B (0, 25)	$100 \times 0 + 120 \times 25 = 3000$

The maximum value of Z is 3200 at the point (8, 20).

Chapter Test

Multiple Choice Questions

Q 1. The graph of the inequality $3x + 4y < 12$ is:

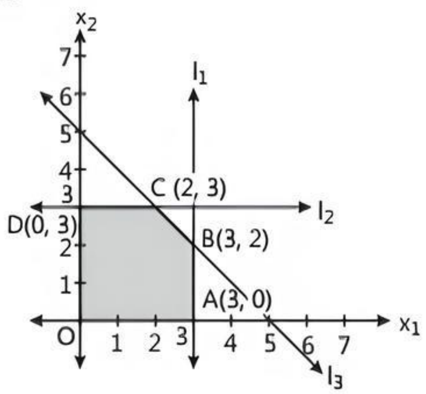
a. half plane that contains the origin

b. half plane that neither contains the origin nor the points of the line $3x + 4y = 12$

c. whole XOY-plane excluding the points on line $3x + 4y = 12$

d. None of the above

Q 2. The corner points of the feasible region determined by the system of linear inequalities are:



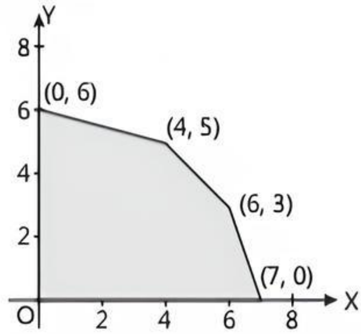
a. (0, 0), (–3, 0), (3, 2), (2, 3)

b. (3, 0), (3, 2), (2, 3), (0, –3)

c. (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)

d. None of the above

Q 3. The feasible region for an LPP is shown in the figure. Let $Z = 2x + 5y$ be the objective function. Maximum of Z occurs at:



a. (7, 0) b. (6, 3) c. (0, 6) d. (4, 5)

Assertion and Reason Type Questions

Directions (Q. Nos. 4-5): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

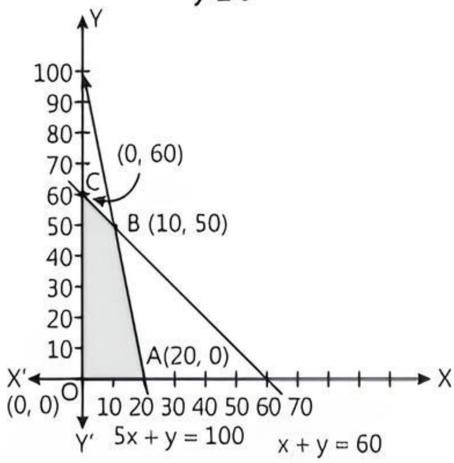
Consider, the graph of constraints stated as linear inequalities as below:

$$5x + y \le 100 \qquad ...(1)$$

$$x + y \le 60 \qquad ...(2)$$

$$x \ge 0 \qquad ...(3)$$

$$y \ge 0 \qquad ...(4)$$



Q 4. **Assertion (A): The points (10, 50), (0, 60) and (20, 0) are feasible solutions.**

Reason (R): Points within and on the boundary of the feasible region represent feasible solutions of the constraints.

Q 5. **Assertion (A): (25, 40) is an infeasible solution of the problem.**

Reason (R): Any point inside the feasible region is called an infeasible solution.

Case Study Based Questions

Q 6. Case Study 1

Corner points of the feasible region for an LPP are (0, 0), (7, 0), (6, 2) and (0, 5). Let $Z = 3x + 4y$ be the objective function.

Based on the above information, solve the following questions:

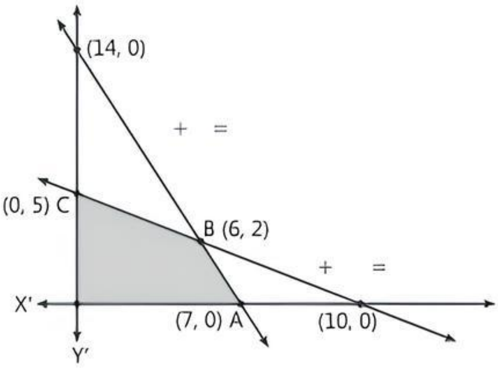
(i) Find the point at which the minimum value of Z occurs.

(ii) Find the point at which the maximum value of Z occurs.

(iii) Find the value of Maximum of Z – Minimum of Z.

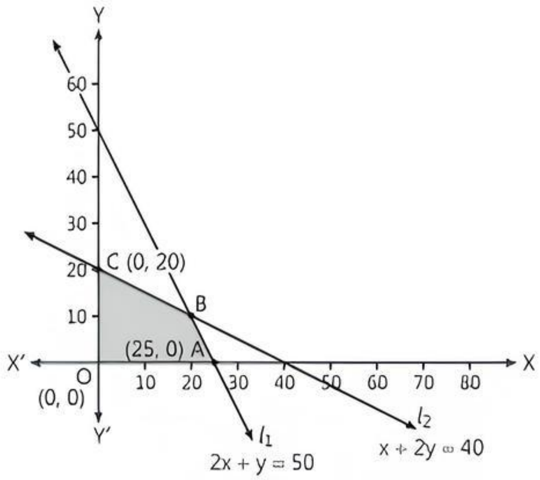
Or

If the objective function be $Z = x + y$, then find the maximum value of the following LPP.



Q 7. Case Study 2

Let subject to the constraints of LPP are $x + 2y \le 40$; $2x + y \le 50$ and $x \ge 0$, $y \ge 0$, whose graph is shown below:



Based on the above information, solve the following questions:

(i) What is the point of intersection of lines l_1 and l_2?

(ii) Find the corner points of the feasible region shown in the above graph.

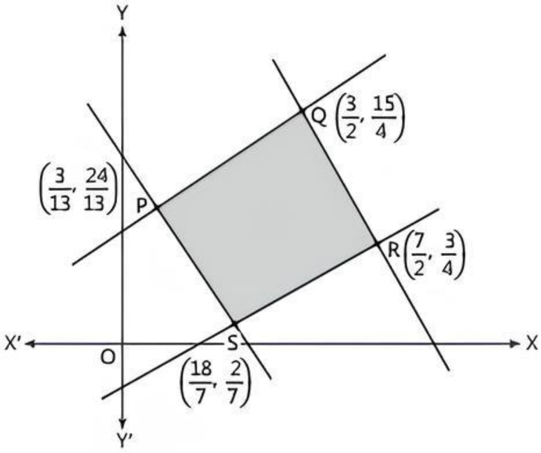
(iii) If $Z = x + y$ be the objective function and maximum $Z = 30$. Write the point at which maximum value occurs.

Or

If $Z = 2x + y$ be the objective function, then find the minimum value of Z.

Short Answer Type Questions

Q 8. In the given figure, the feasible region for a LPP is shown. Find the maximum and minimum value of $Z = x + 2y$.



Q 9. Draw the graph of the following linear programming problem when the condition are $50x + 25y \leq 500$, $x + y \leq 12$ and $x \geq 0$, $y \geq 0$.

Q 10. Find the maximum value of $Z = 3x + 4y$ by linear programming method under the following constraints:

$x - 2y \geq 2$, $x + y \geq 3$, $-2x + y \leq 4$, $x \geq 0$ and $y \geq 0$.

Long Answer Type Questions

Q 11. Under the following constraints:

$$x + y \leq 8$$
$$3x + 5y \geq 15$$
$$x \geq 0, y \geq 0$$

Find the minimum value of $Z = x + 3y$.

Q 12. Solve the following linear programming problem graphically:

Minimise $Z = 200x + 500y$

$$x + 2y \geq 10$$
$$3x + 4y \leq 24$$
$$x \geq 0, y \geq 0$$

(NCERT EXERCISE)

Q 13. Solve the following LPP graphically:

Maximise $Z = 20x + 10y$

Subject to the following constraints

$$x + 2y \leq 28$$
$$3x + y \leq 24$$
$$x \geq 2$$
$$x, y \geq 0.$$