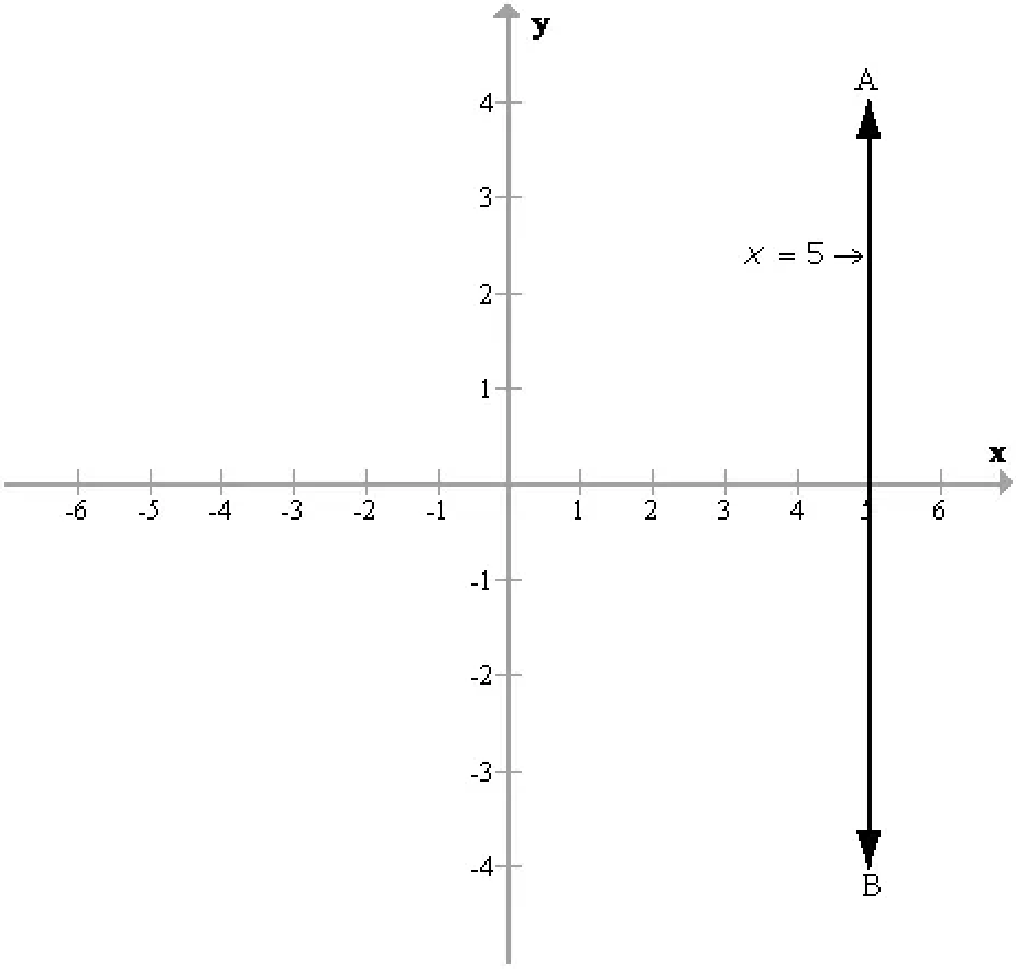
Chapter 27. Graphical Solution (Solution of Simultaneous Linear Equations, Graphically)

Exercise 27(A)

Solution 1:

(i)

The graph x= 5 in the following figure is a straight line AB which is parallel to y axis at a distance of 5 units from it.

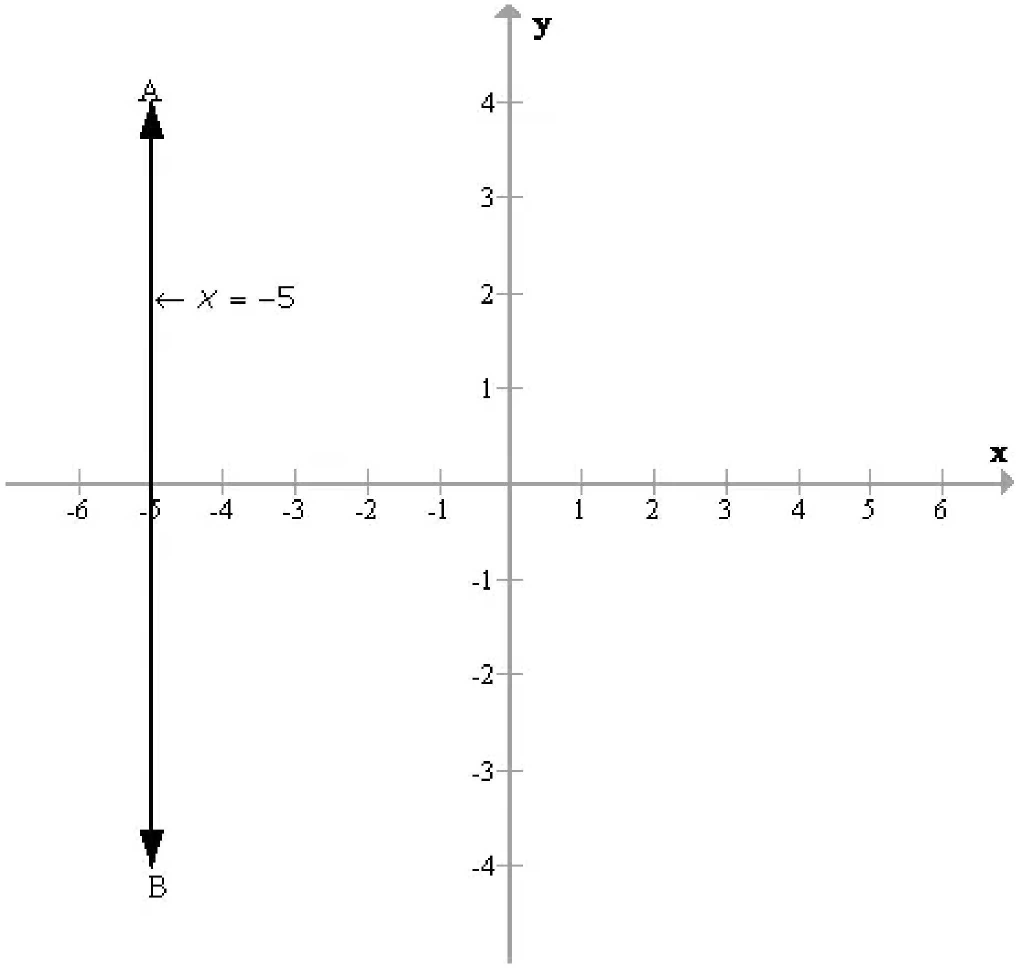


(ii)

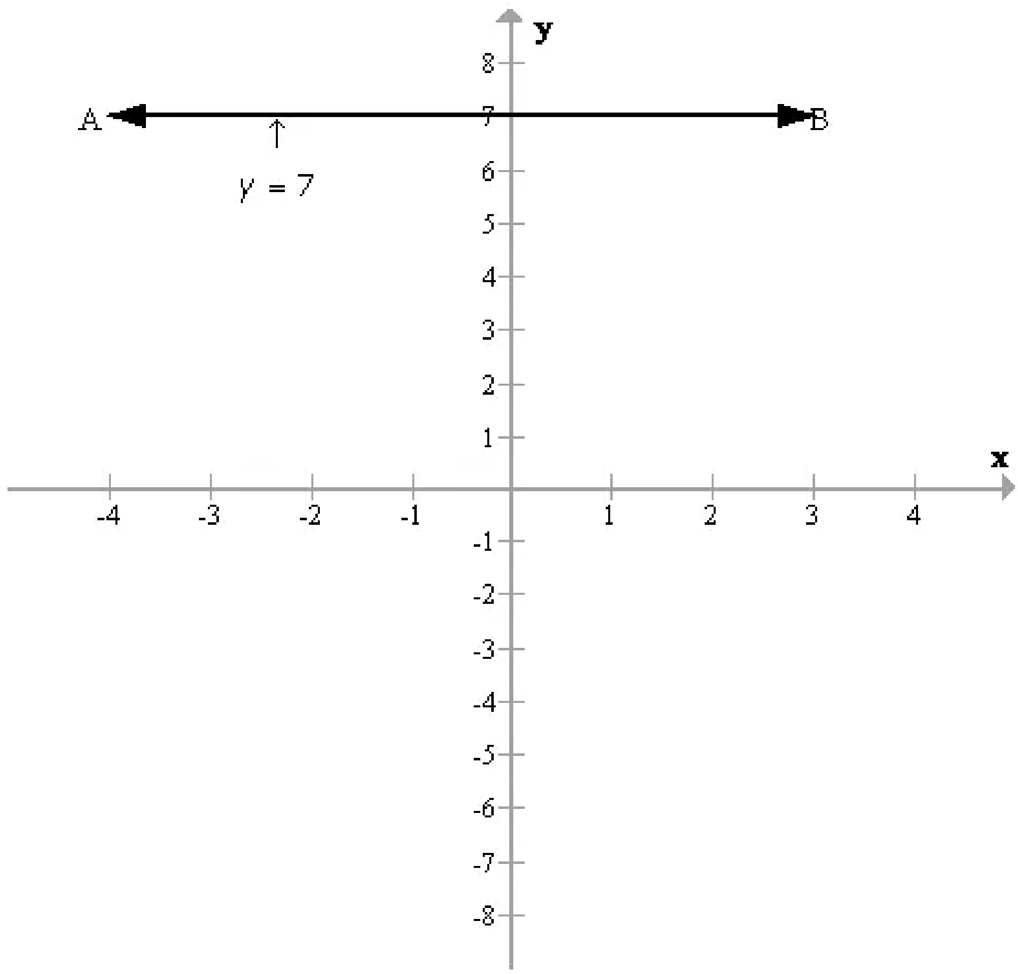
x+5=0

x = -5

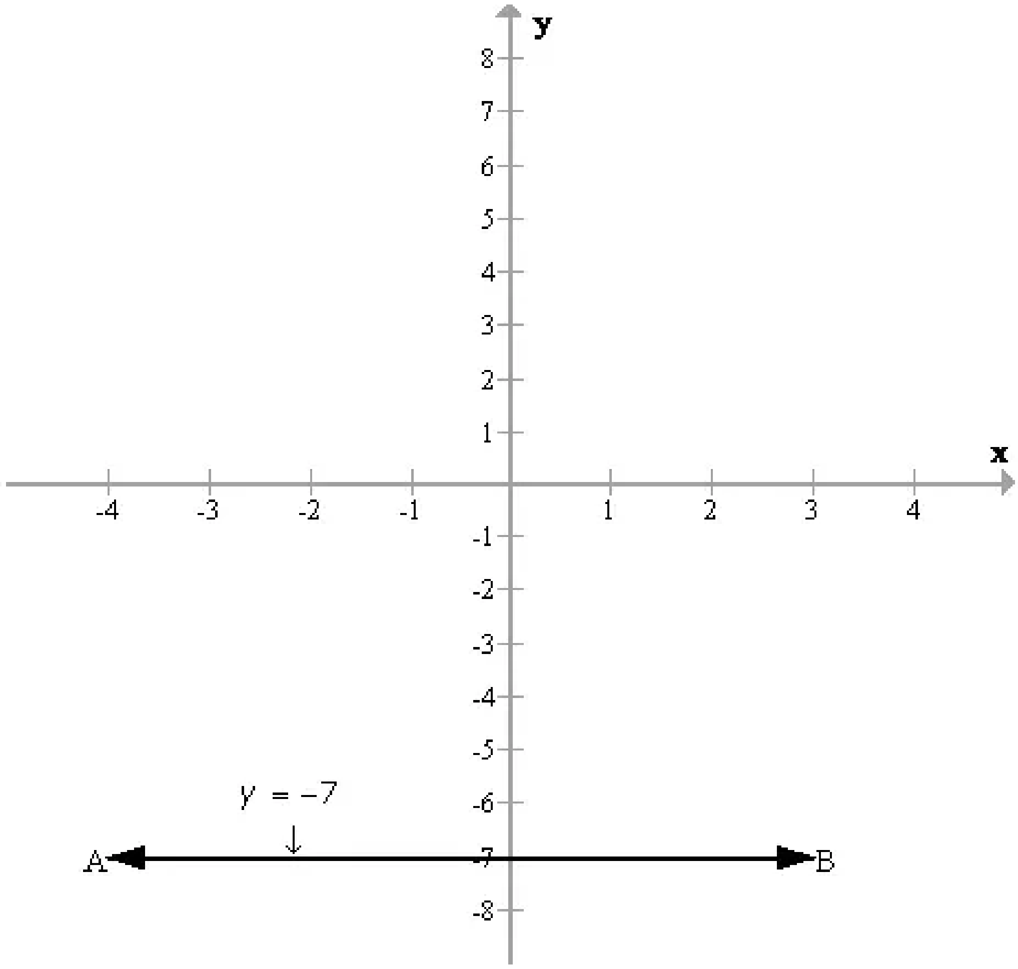
The graph x= -5 in the following figure is a straight line AB which is parallel to y axis at a distance of 5 units from it in the negative x direction.



(iii)
The graph y = 7 in the following figure is a straight line AB which is parallel to x axis at a distance of 7 units from it.



(iv)
y + 7 = 0
y = -7
The graph y = -7 in the following figure is a straight line AB which is parallel to x axis at a distance of 7 units from it in the negative y direction.



(v)

$2x + 3y = 0$

$\Rightarrow 3y = -2x$

$\therefore y = \frac{-2x}{3}$

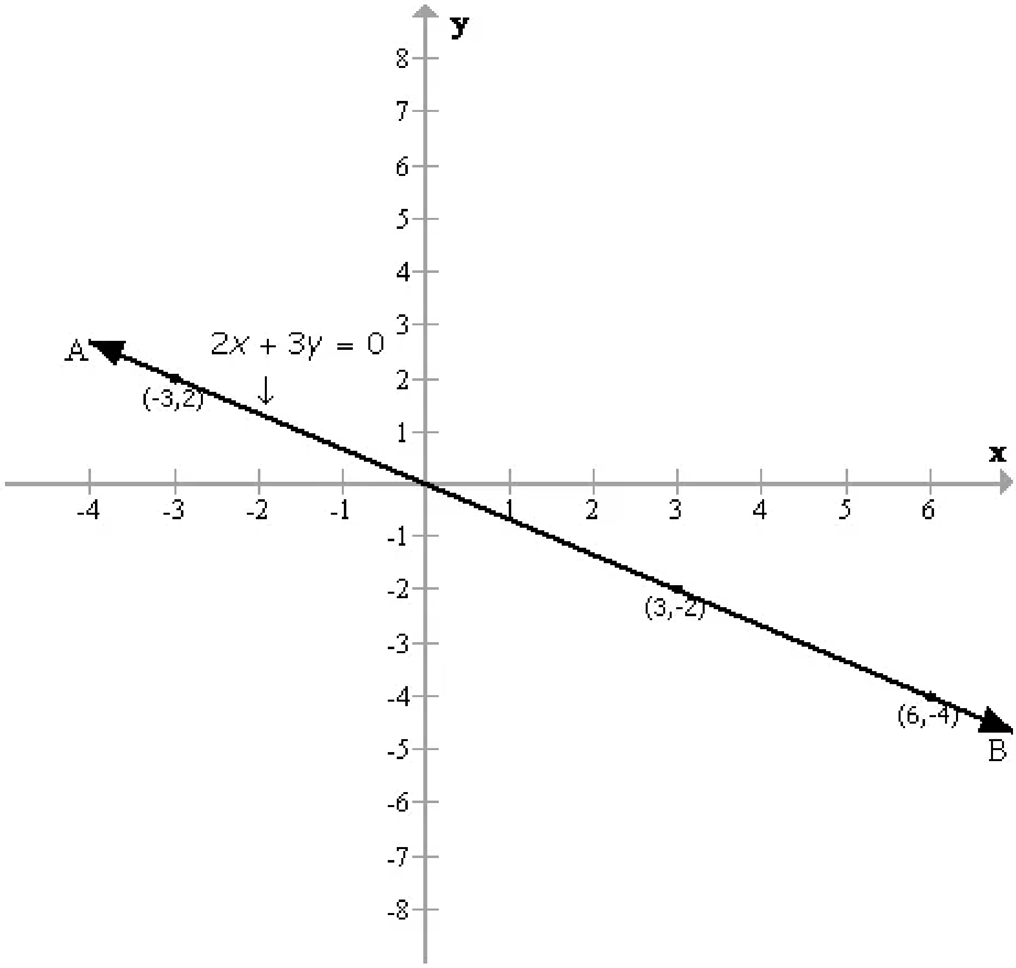
When $x = -3; y = \frac{-2(-3)}{3} = \frac{6}{3} = 2$

When $x = 3; y = \frac{-2(3)}{3} = \frac{-6}{3} = -2$

When $x = 6; y = \frac{-2(6)}{3} = \frac{-12}{3} = -4$

x	−3	3	6
y	2	−2	−4

Plotting these points we get the required graph as shown below:



(vi)

$3x + 2y = 6$

$\Rightarrow 2y = 6 - 3x$

$\therefore y = \dfrac{6-3x}{2}$

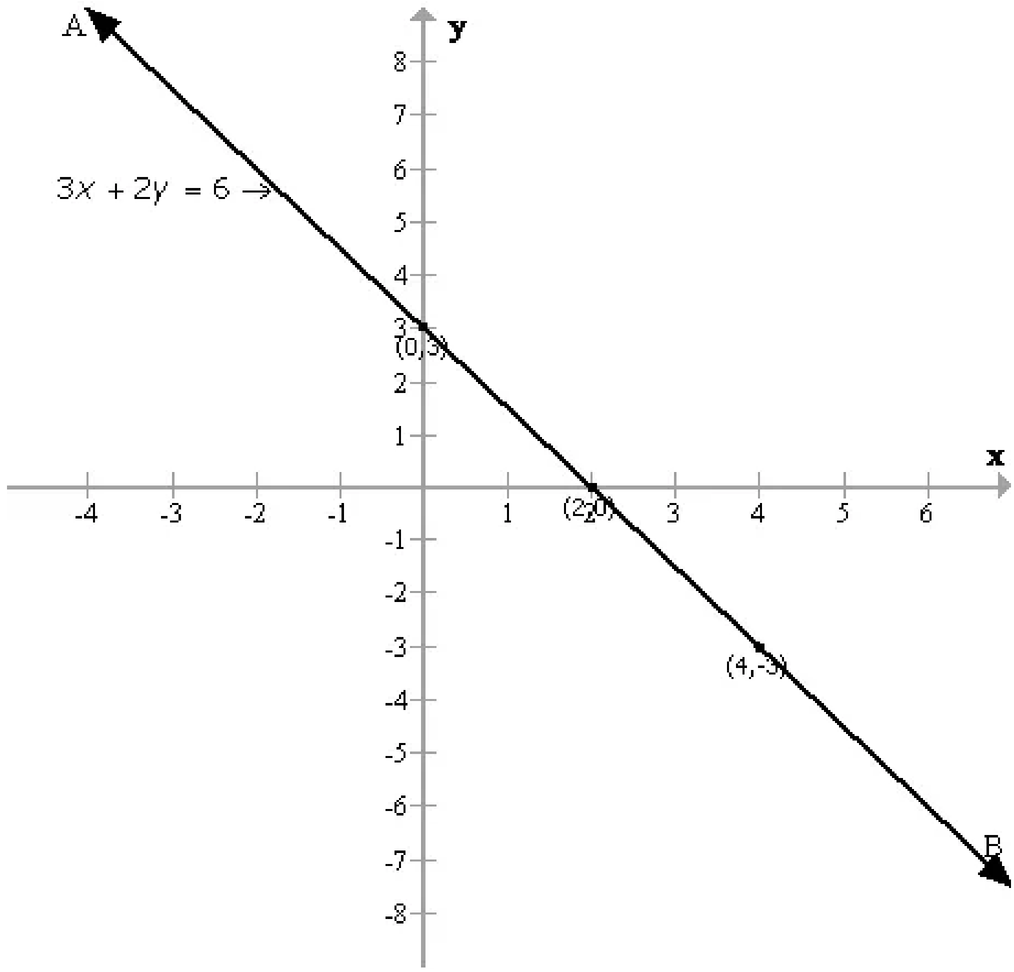
When $x=0; y = \dfrac{6-3\times0}{2} = \dfrac{6-0}{2} = 3$

When $x=2; y = \dfrac{6-3\times2}{2} = \dfrac{6-6}{2} = 0$

When $x=4; y = \dfrac{6-3\times4}{2} = \dfrac{6-12}{2} = -3$

x	0	2	4
y	3	0	−3

Plotting these points we get the required graph as shown below:



(vii)

x-5y+4=0

$\Rightarrow$ 5y=4+x

$\therefore y = \frac{x+4}{5}$

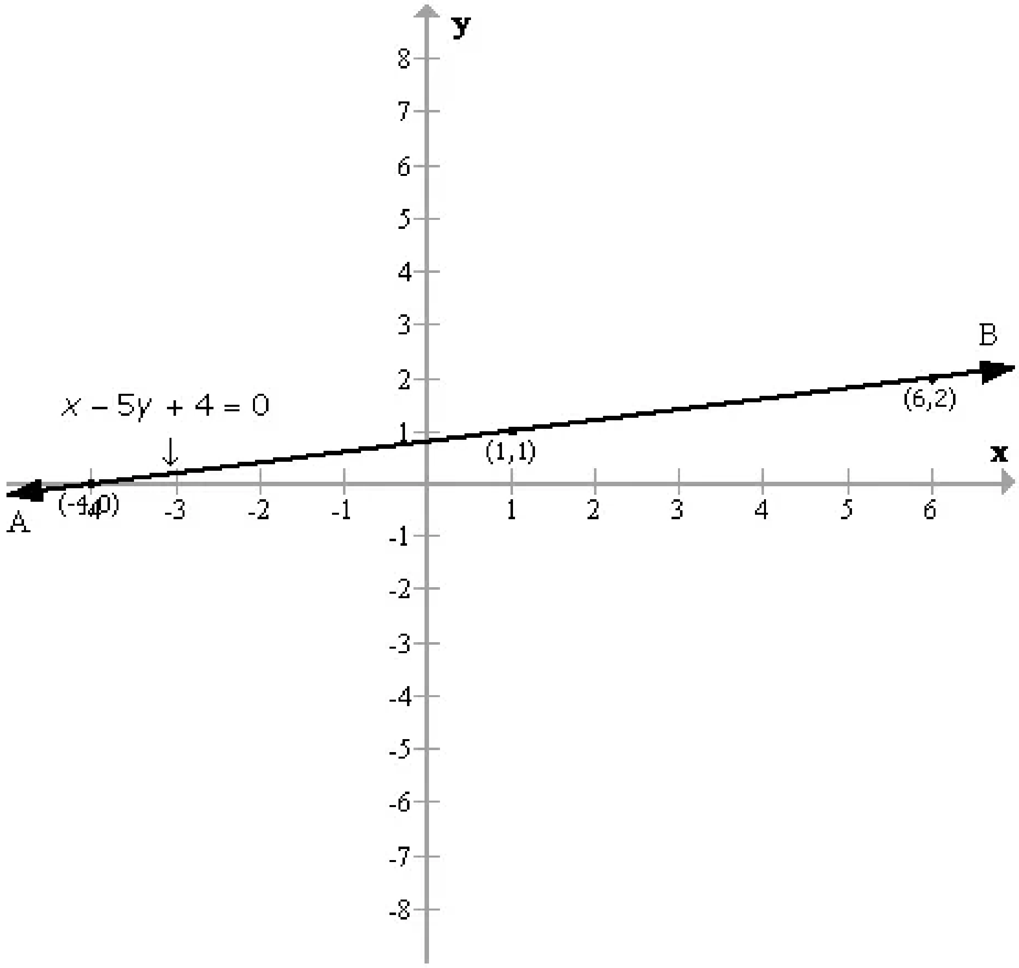
When $x=1; y = \frac{1+4}{5} = \frac{5}{5} = 1$

When $x=6; y = \frac{6+4}{5} = \frac{10}{5} = 2$

When $x=-4; y = \frac{-4+4}{5} = \frac{0}{5} = 0$

x	1	6	−4
y	1	2	0

Plotting these points we get the required graph as shown below:



(viii)

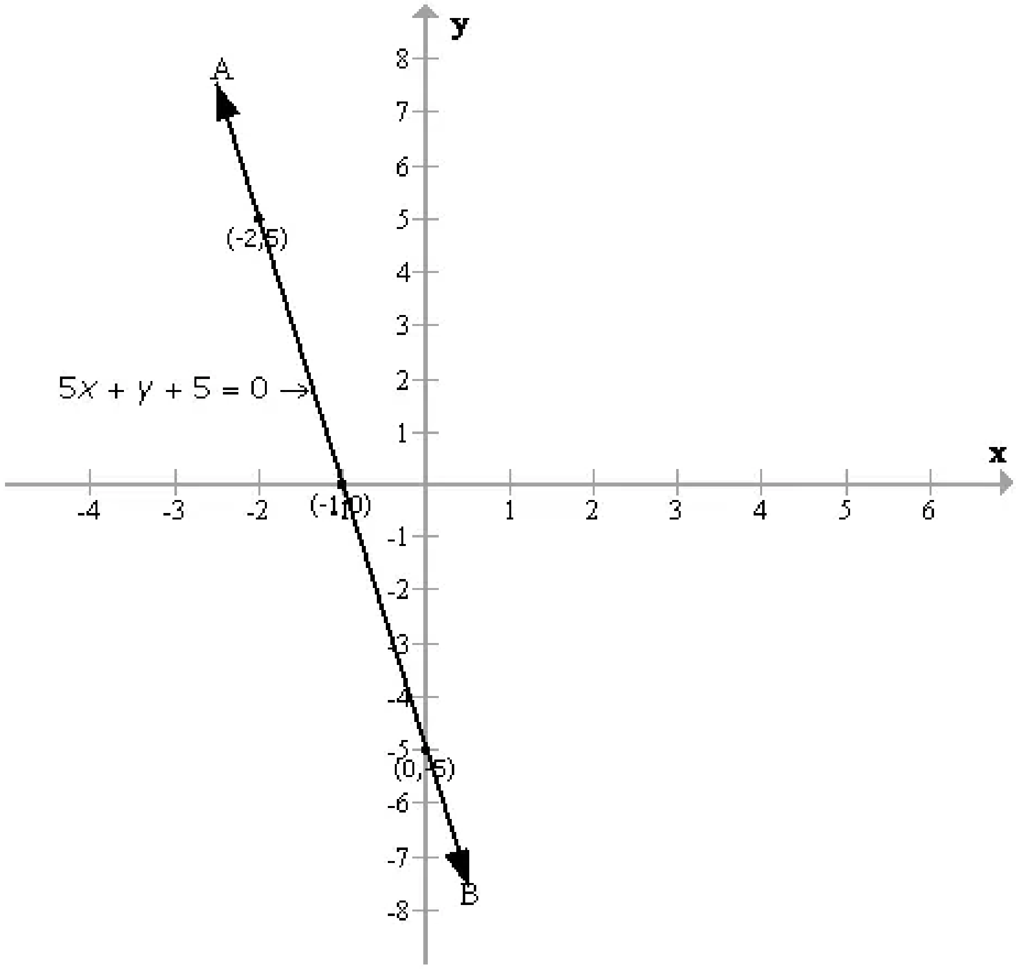
$5x + y + 5 = 0$

$\Rightarrow y = -5x - 5$

When $x=0; y = -5 \times x - 5 = -0 - 5 = -5$
When $x=-1; y = -5 \times (-1) - 5 = 5 - 5 = 0$
When $x=-2; y = -5 \times (-2) - 5 = 10 - 5 = 5$

x	0	−1	−2
y	−5	0	5

Plotting these points we get the required graph as shown below:



Solution 2:

$$(i)\ \frac{1}{3}x+\frac{1}{5}y=1$$

$$\Rightarrow \frac{5x+3y}{15}=1$$

$$\Rightarrow 5x+3y=15$$

$$\Rightarrow \quad 3y=15-5x$$

$$\Rightarrow \quad y=\frac{15-5x}{3}$$

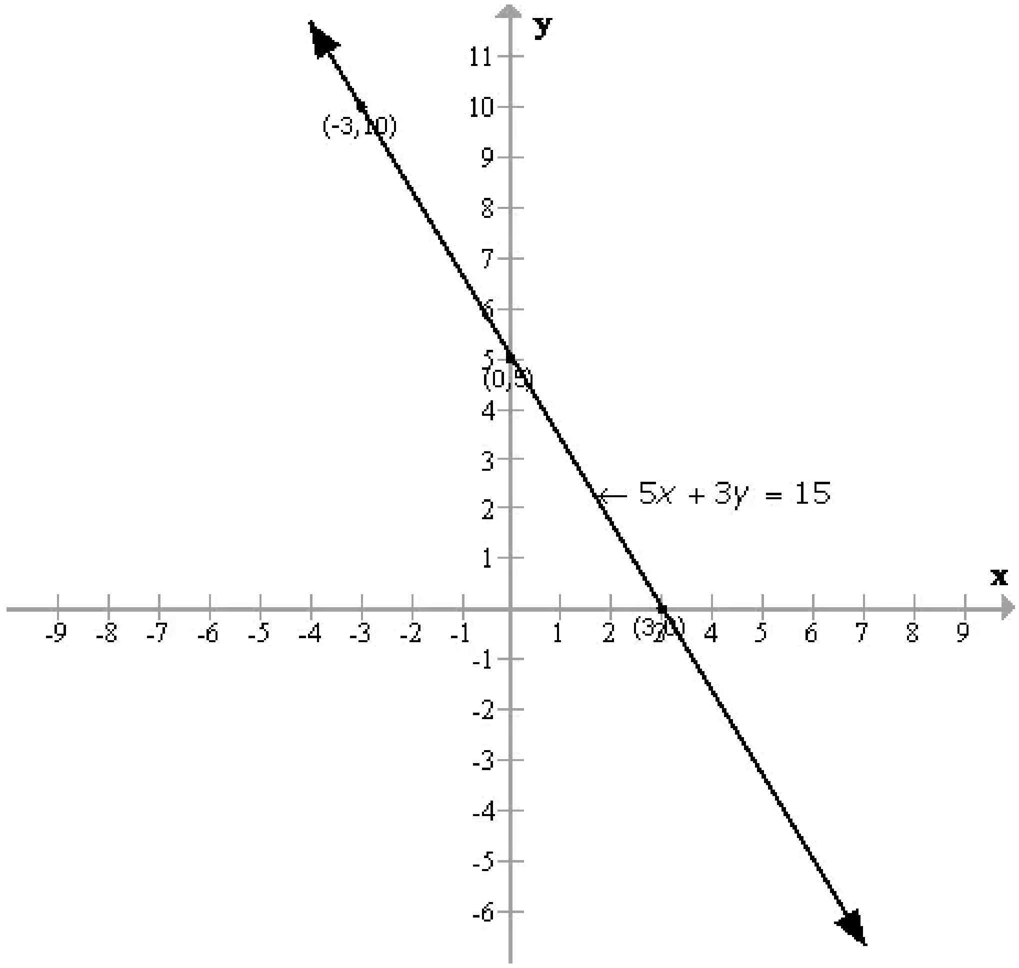
When $x=0;\ y=\frac{15-5\times 0}{3}=\frac{15-0}{3}=5$

When $x=3;\ y=\frac{15-5\times 3}{3}=\frac{15-15}{3}=0$

When $x=-3;\ y=\frac{15-5\times(-3)}{3}=\frac{15+15}{3}=10$

x	0	3	−3
y	5	0	10

Plotting these points we get the required graph as shown below:



From the figure it is clear that, the graph meets the coordinate axes at (3, 0) and (0, 5)

$$\text{(ii)} \frac{2x+15}{3} = y-1$$

$$\Rightarrow 2x+15 = 3(y-1)$$

$$\Rightarrow 2x+15 = 3y-3$$

$$\Rightarrow 2x-3y = -15-3$$

$$\Rightarrow 2x-3y = -18$$

$$\Rightarrow \quad -3y = -18-2x$$

$$\Rightarrow \qquad y = \frac{-18-2x}{-3}$$

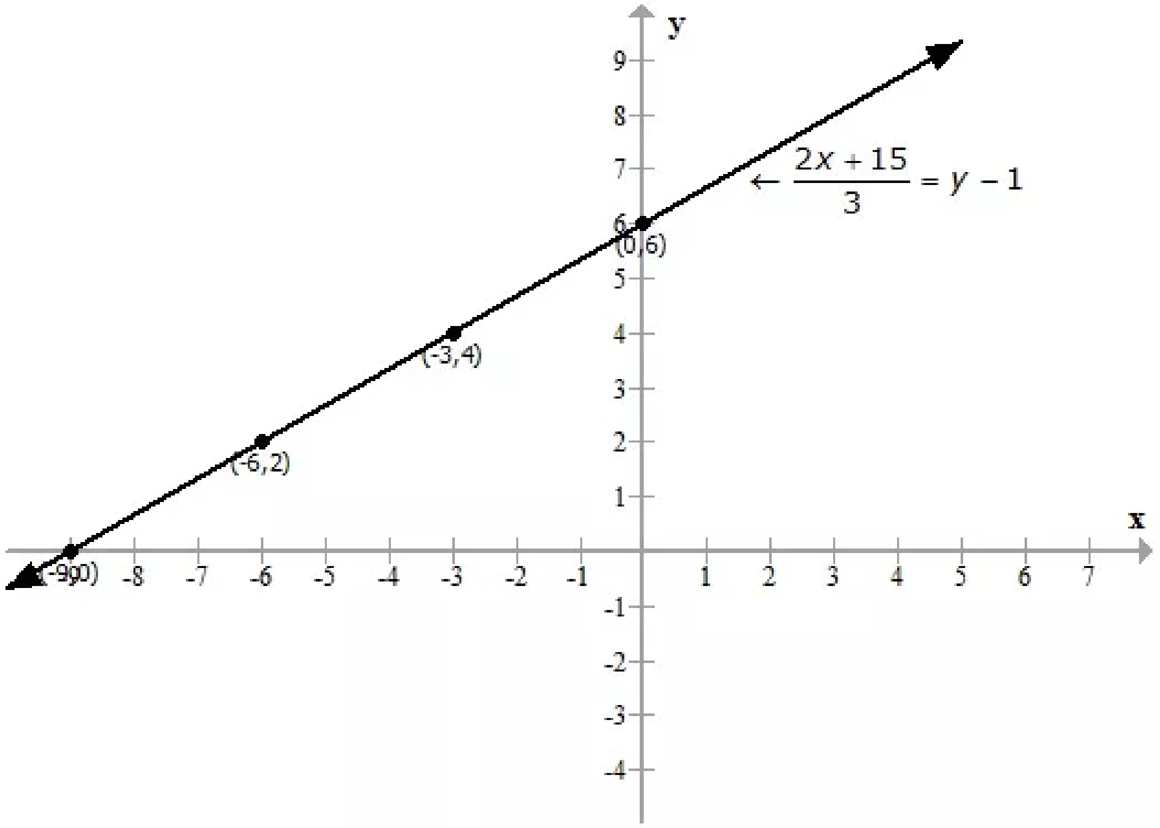
$$\text{When } x = 0,\ y = \frac{-18-[2\times 0]}{-3} = \frac{-18-0}{-3} = 6$$

$$\text{When } x = -3,\ y = \frac{-18-[2\times(-3)]}{-3} = \frac{-18+6}{-3} = 4$$

$$\text{When } x = -6,\ y = \frac{-18-[2\times(-6)]}{-3} = \frac{-18+12}{-3} = 2$$

x	0	−3	−6
y	6	4	2

Plotting these points we get the required graph as shown below:



From the figure it is clear that, the graph meets the coordinate axes at (-9, 0) and (0, 6)

Solution 3:

$4x - 3y + 36 = 0$

$\Rightarrow 4x - 3y = -36$

$\Rightarrow -3y = -36 - 4x$

$\Rightarrow 3y = 36 + 4x$

$\Rightarrow y = \frac{36 + 4x}{3}$

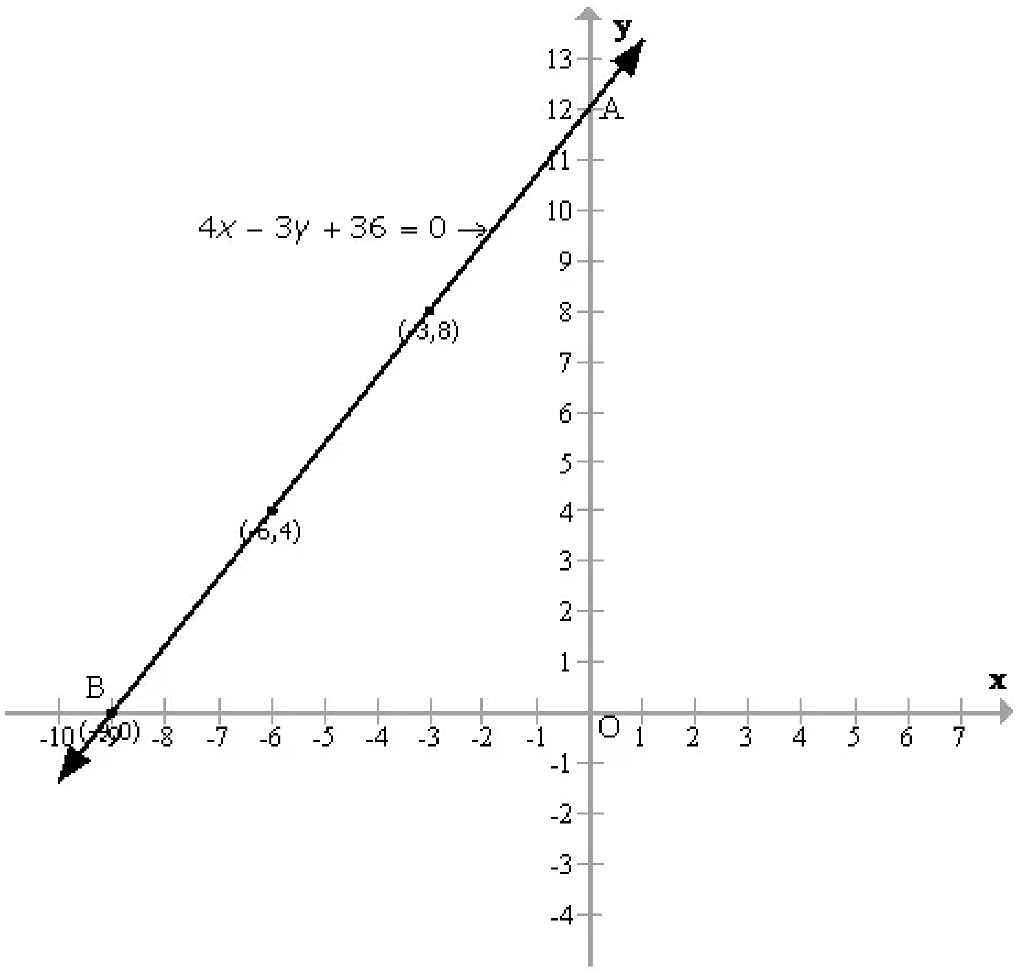
When $x = -6,\ y = \frac{36 + 4 \times (-6)}{3} = \frac{36 - 24}{3} = 4$

When $x = -3,\ y = \frac{36 + 4 \times (-3)}{3} = \frac{36 - 12}{3} = 8$

When $x = -9,\ y = \frac{36 + 4 \times (-9)}{3} = \frac{36 - 36}{3} = 0$

x	−9	−3	−6
y	0	8	4

Plotting these points we get the required graph as shown below:



The straight line cuts the co-ordinate axis at A(0, 12) and B(-9, 0).

$\therefore$ The triangle ΔAOB is formed.

Area of the triangle AOB$=\frac{1}{2} \times AO \times OB$

$$= \frac{1}{2} \times 12 \times 9$$

$$= 54 \text{ sq. units}$$

$\therefore$ Area of the triangle is 54 sq. units

Solution 4:

$2x-3y-5=0$

$\Rightarrow 2x=3y+5$

$\Rightarrow x=\dfrac{3y+5}{2}$

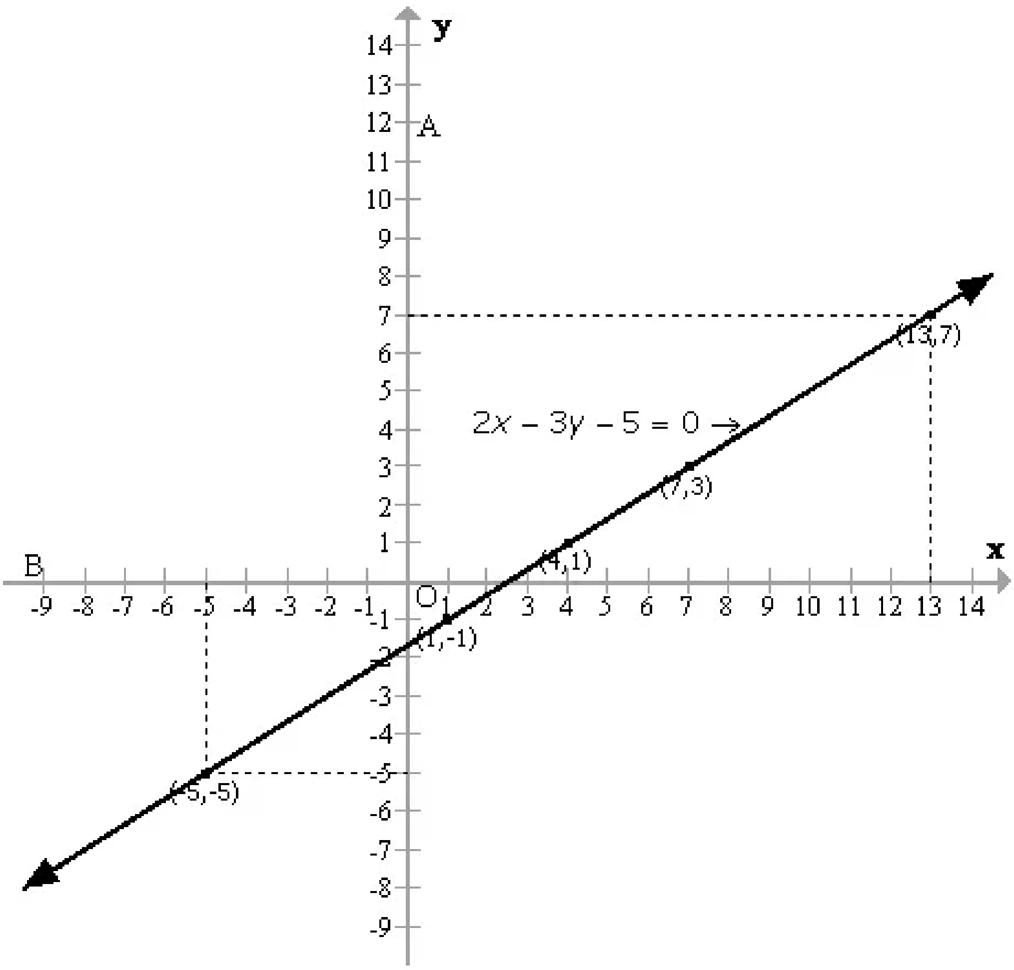
When $y=1,\ x=\dfrac{3(1)+5}{2}=\dfrac{8}{2}=4$

When $y=3,\ x=\dfrac{3(3)+5}{2}=\dfrac{9+5}{2}=7$

When $y=-1,\ x=\dfrac{3(-1)+5}{2}=\dfrac{5-3}{2}=1$

x	4	7	1
y	1	3	−1

Plotting these points we get the required graph as shown below:



The value of x, when y=7:

We have the equation of the line as

$$x = \frac{3y+5}{2}$$

Now substitute y=7 and $x=x_1$:

$$x_1 = \frac{3(7)+5}{2} = \frac{21+5}{2} = \frac{26}{2} = 13$$

The value of x, when y=−5:

Now substitute y=−5 and $x=x_2$:

$$x_2 = \frac{3(-5)+5}{2} = \frac{-15+5}{2} = \frac{-10}{2} = -5$$

Solution 5:

$$4x+3y+6=0$$

$$\Rightarrow 3y = -4x-6$$

$$\Rightarrow y = \frac{-4x-6}{3}$$

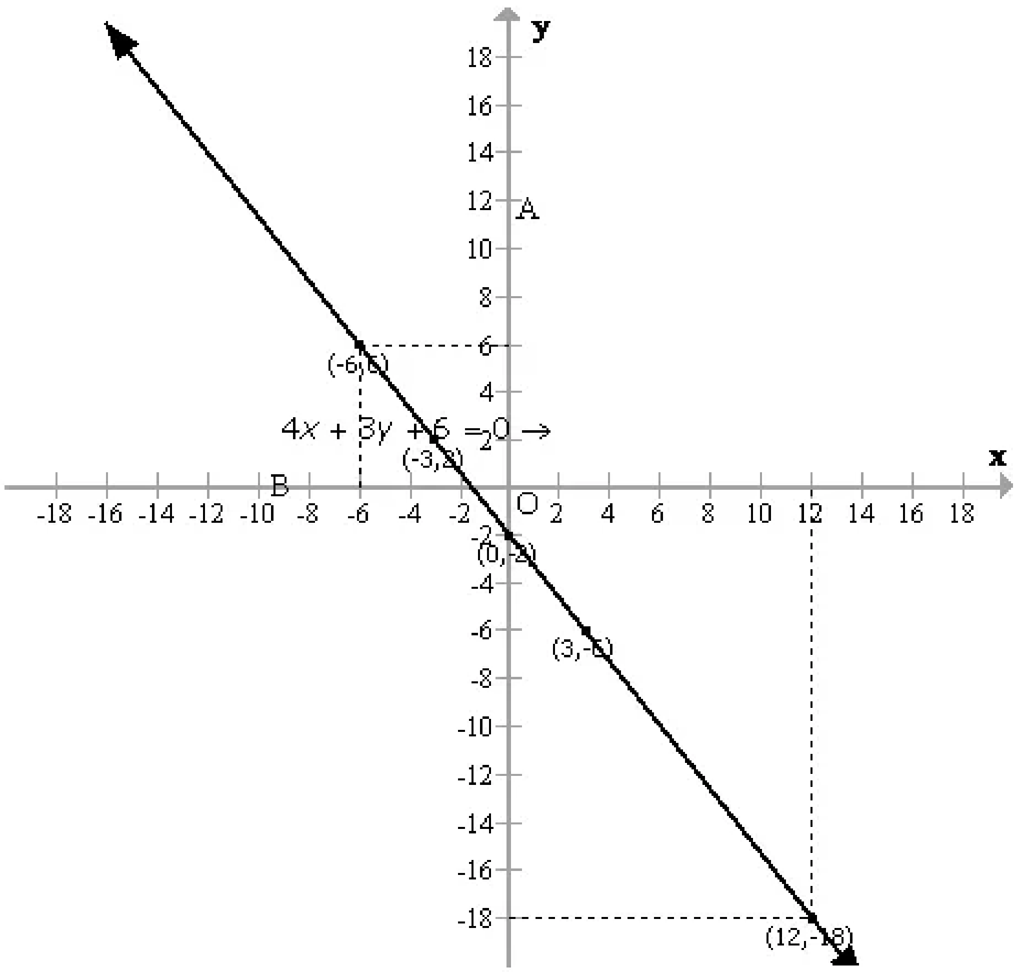
When $x=0,\ y = \frac{-4(0)-6}{3} = \frac{-6}{3} = -2$

When $x=3,\ y = \frac{-4(3)-6}{3} = \frac{-12-6}{3} = -6$

When $x=-3,\ y = \frac{-4(-3)-6}{3} = \frac{12-6}{2} = 2$

x	0	3	−3
y	−2	−6	2

Plotting these points we get the required graph as shown below:



The value of y, when x=12:

We have the equation of the line as

$$y = \frac{-4x - 6}{3}$$

Now substitute x=12 and $y=y_1$:

$$y_1 = \frac{-4(12)-6}{3} = \frac{-48-6}{3} = \frac{-54}{3} = -18$$

The value of y, when x=− 6:

Now substitute x=−6 and $y=y_2$:

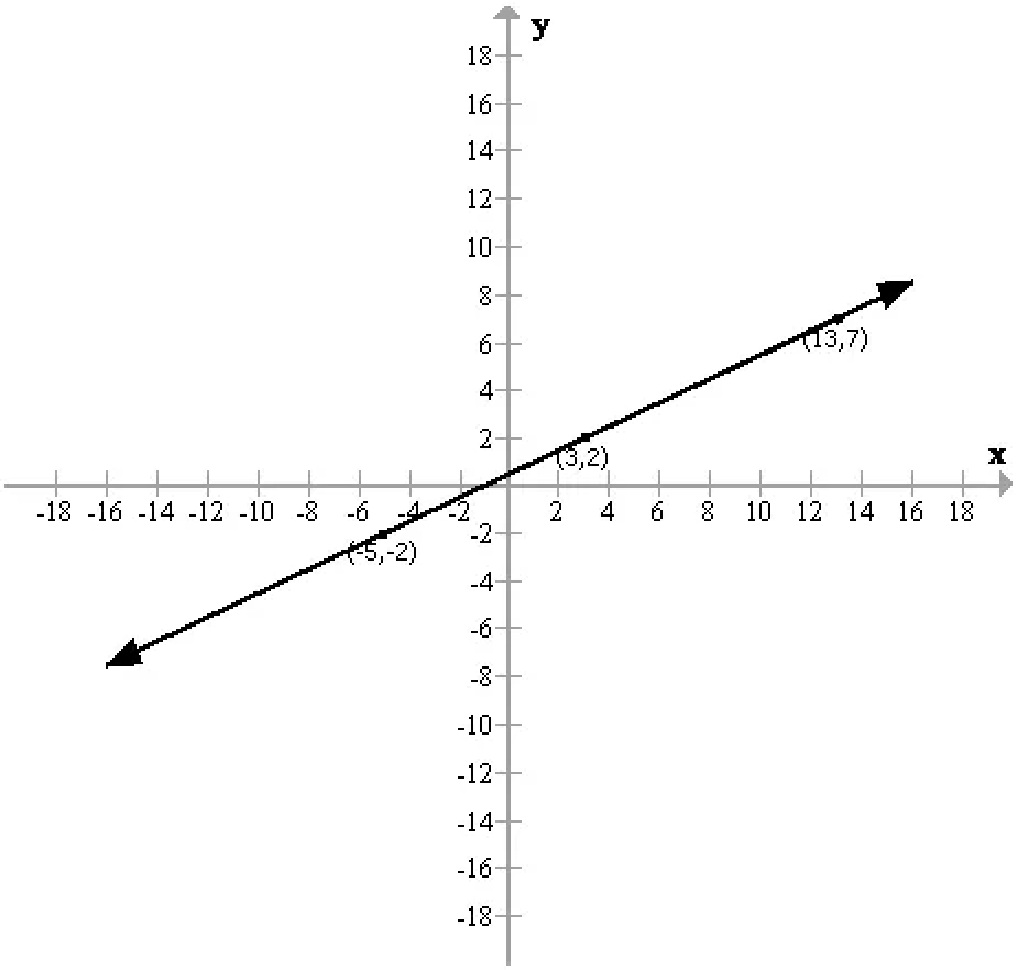
$$y_2 = \frac{-4(-6)-6}{3} = \frac{24-6}{3} = \frac{18}{3} = 6$$

Solution 6:

The table is:

x	−5	−1	3	*b*	13
y	−2	*a*	2	5	7

Plotting the points as shown in the above table,
we get the following required graph:

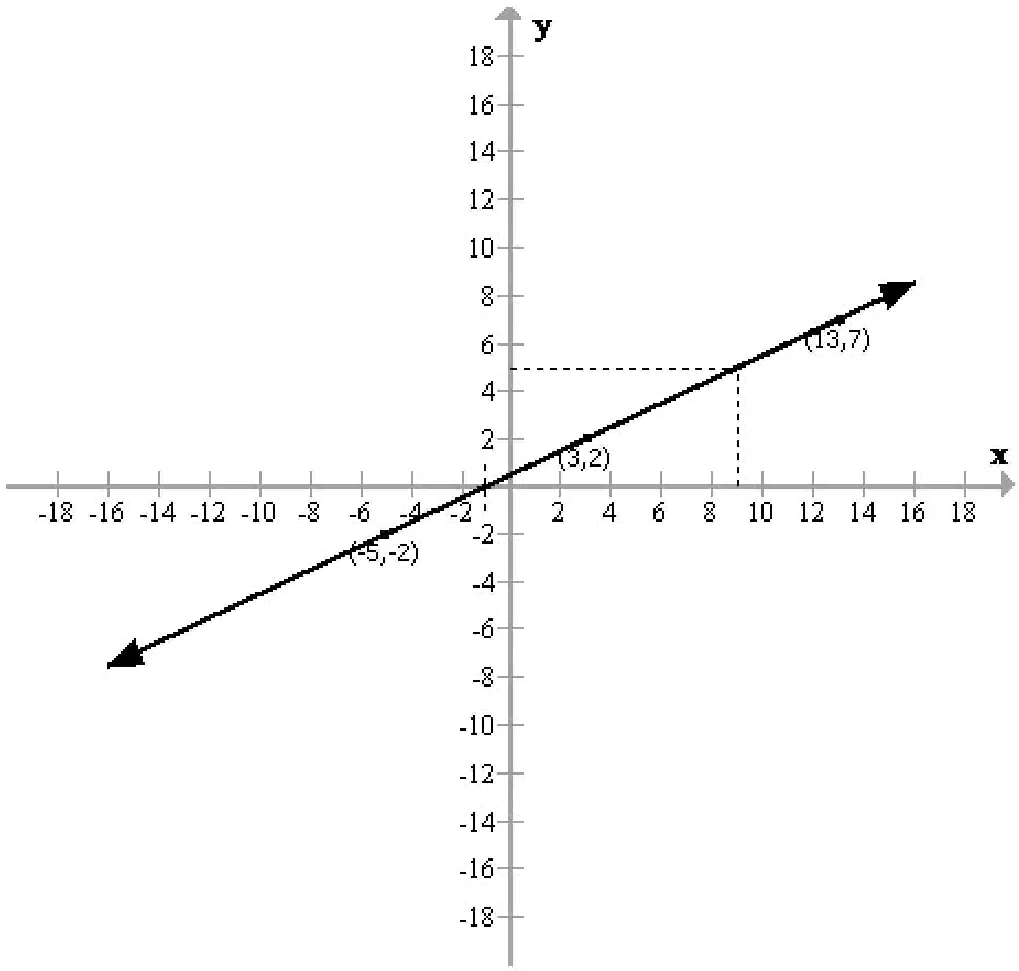


When x =− 1, then y=0

⇒ a=0

When y =5, then x=9

⇒ b=9



Let y=px+q ....(1)

be a linear relation between x and y

Substitute x=9 and y=5 in the equation (1), we have,

5=9p+q ....(2)

Substitute x=− 1 and y=0 in the equation (1), we have,

0=− p+q ....(3)

Subtracting (3) from (2), we have,

5=10p

$\Rightarrow p=\frac{5}{10}$

$\Rightarrow p=\frac{1}{2}$

From (3), we have,

p=q

$\therefore q=\frac{1}{2}$

Thus, the linear relation is

y=px+q

$\Rightarrow y=\frac{1}{2}x+\frac{1}{2}$

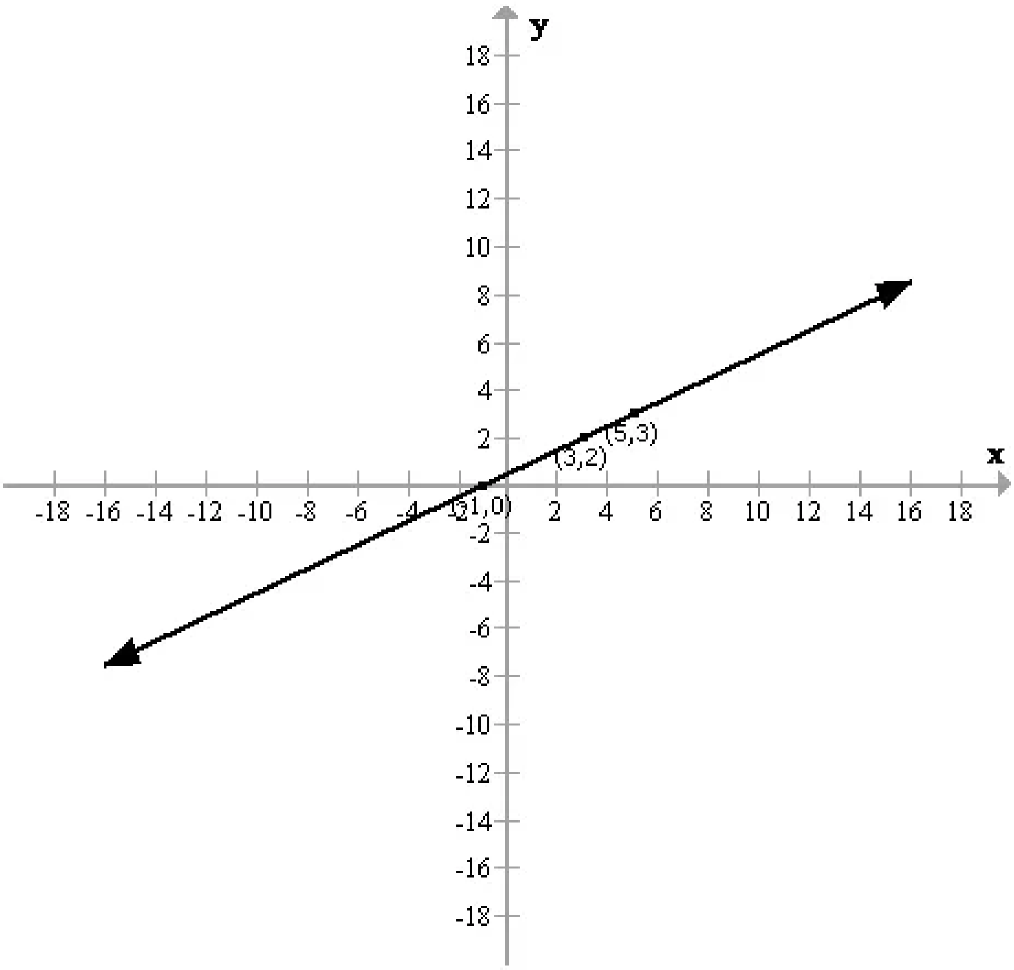
$\Rightarrow y=\frac{x+1}{2}$

Solution 7:

The table is:

x	a	3	−5	5	c	−1
y	−1	2	b	3	4	0

Plotting the points as shown in the above table,
we get the following required graph:



When $y = -1$, then $x = -3$

$\Rightarrow a = -3$

When $x = -5$, then $y = -2$

$\Rightarrow b = -2$

When $y = 4$, then $x = 7$

$\Rightarrow c = 7$

Let $y = px + q$ ....(1)

be a linear relation between x and y

Substitute $x = -3$ and $y = -1$ in the equation (1), we have,

$-1 = -3p + q$ ....(2)

Substitute $x = -5$ and $y = -2$ in the equation (1), we have,

$-2 = -5p + q$ ....(3)

Subtracting (3) from (2), we have,

$1 = 2p$

$\Rightarrow p = \frac{1}{2}$

From (3), we have,

$-2 = -5p + q$

$\Rightarrow -2 = -5\left(\frac{1}{2}\right) + q$

$\Rightarrow -4 = -5 + 2q$

$\Rightarrow 2q = 5 - 4$

$\Rightarrow 2q = 1$

$\therefore q = \frac{1}{2}$

Thus, the linear relation is

$y = px + q$

$\Rightarrow y = \frac{1}{2}x + \frac{1}{2}$

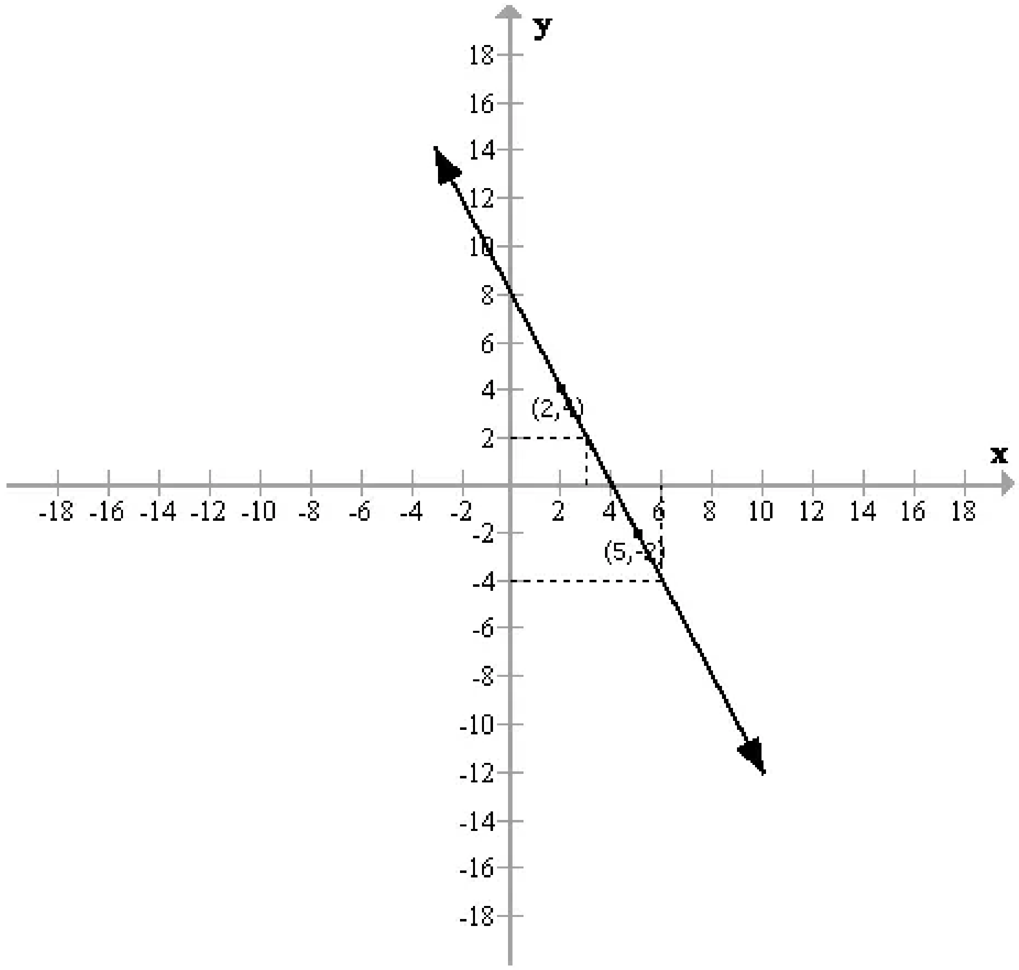
$\Rightarrow y = \frac{x+1}{2}$

Solution 8:

The table is:

x	2	3	5	m
y	4	n	−2	−4

Plotting the points as shown in the above table,
we get the following required graph:



Plotting the points in the graph we get the above required graph.

Now draw a line x=3, parallel to y-axis to meet the line

It meets the line at y=2 and therefore, n=2

Now draw a line y=− 4, parallel to x-axis to meet the line

It meets the line at x=6 and therefore, m=6

Thus the values of m and n are 6 and 2 respectively.

Solution 9:

Consider the equation

$x - 3y = 18$

$\Rightarrow -3y = 18 - x$

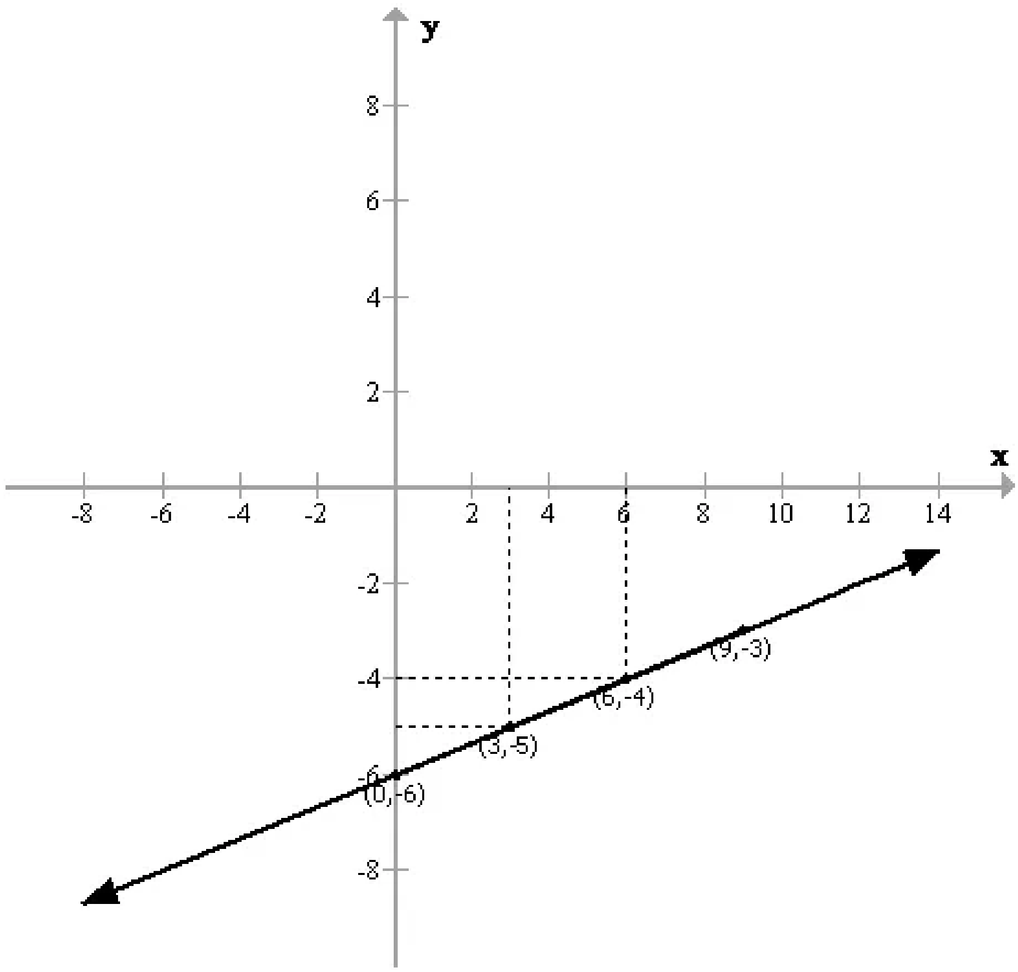
$\Rightarrow 3y = x - 18$

$\Rightarrow y = \dfrac{x - 18}{3}$

The table for $x - 3y = 18$ is

x	9	0	6	3
y	-3	-6	-4	-5

Plotting the above points, we get the following required graph:



From the above figure, we have

m=3 and n=– 4

Solution 10:

(i)

$2x + 3y = 1$

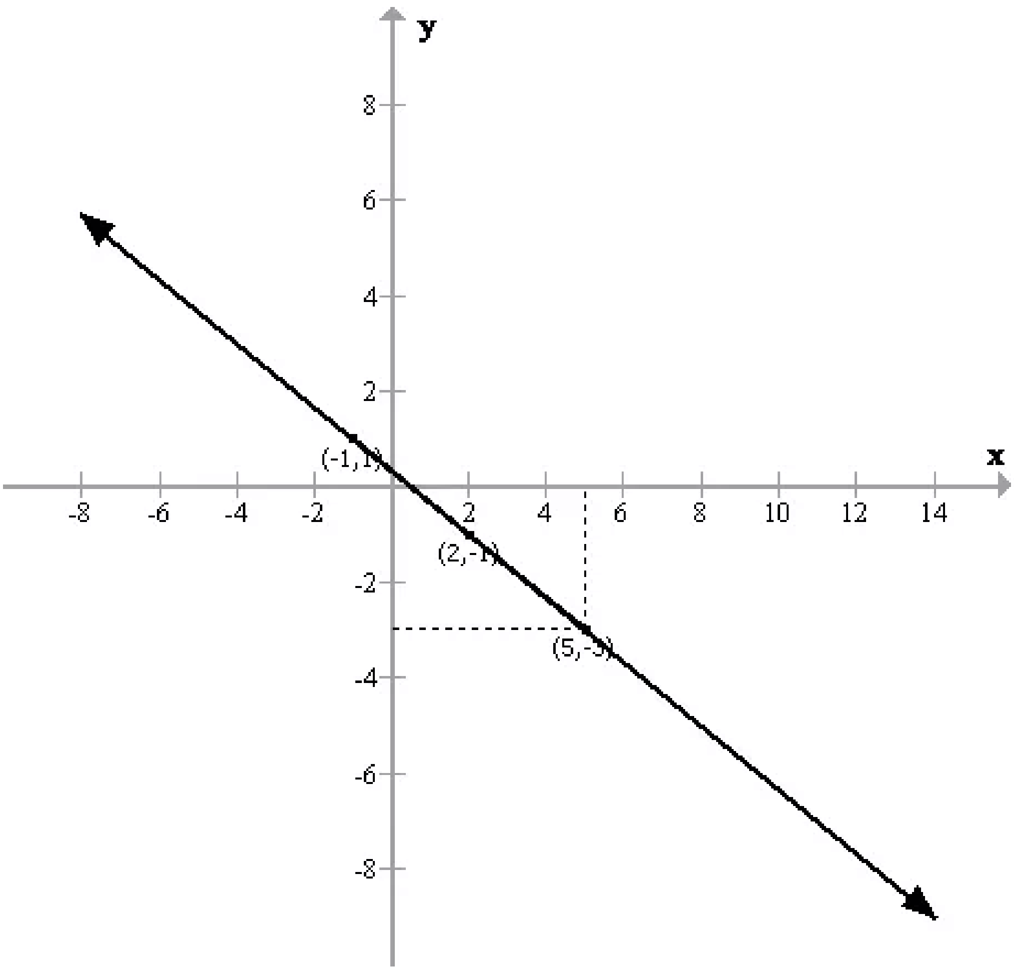
$\Rightarrow 3y = 1 - 2x$

$\Rightarrow y = \frac{1 - 2x}{3}$

The table for $2x + 3y = 1$ is

x	−1	2	5
y	1	−1	−3

Plotting the above points in a graph, we get the following graph:



From the above graph, it is clear that k=5

(ii)

$x - 2y + 1 = 0$

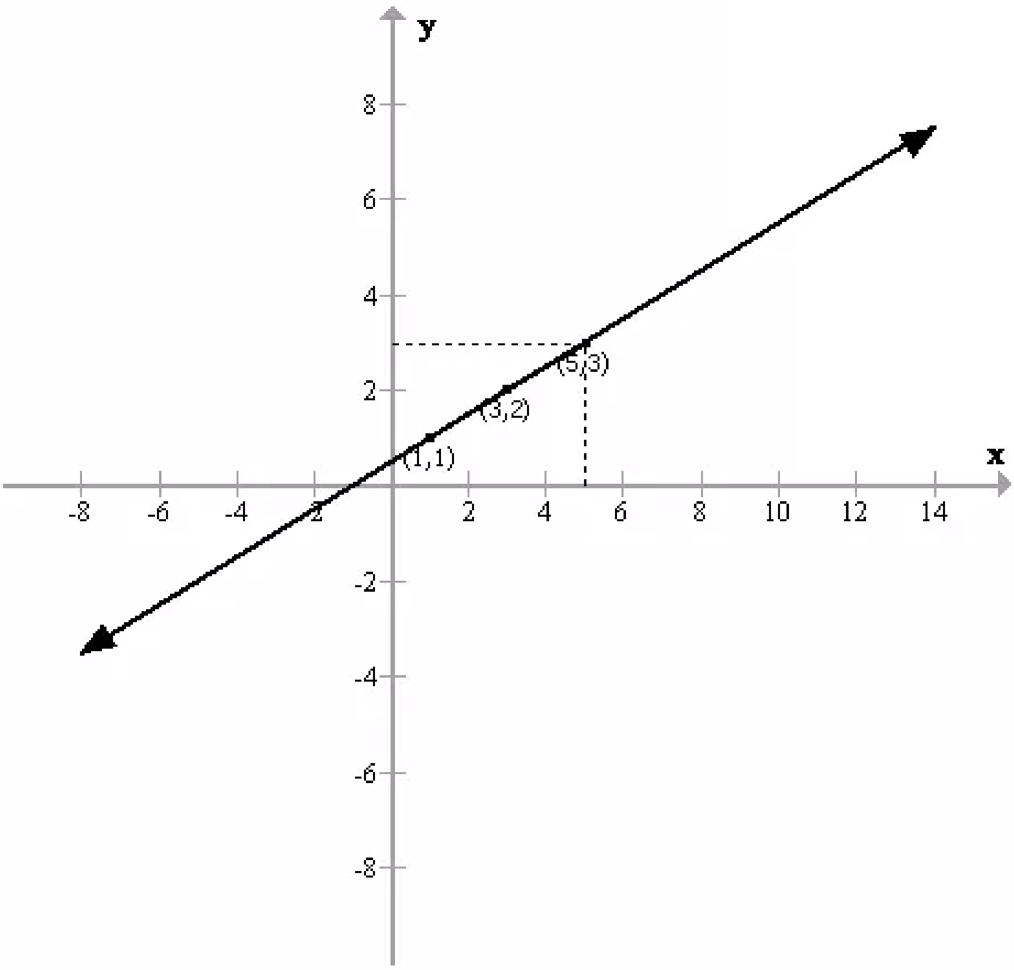
$\Rightarrow 2y = x + 1$

$\Rightarrow y = \dfrac{x+1}{2}$

The table for $x - 2y + 1 = 0$ is

x	1	3	5
y	1	2	3

Plotting the above points in a graph, we get the following graph:



From the above graph, it is clear that

k - 2=3

$\Rightarrow$ k=5

Exercise 27(B)

Solution 1:

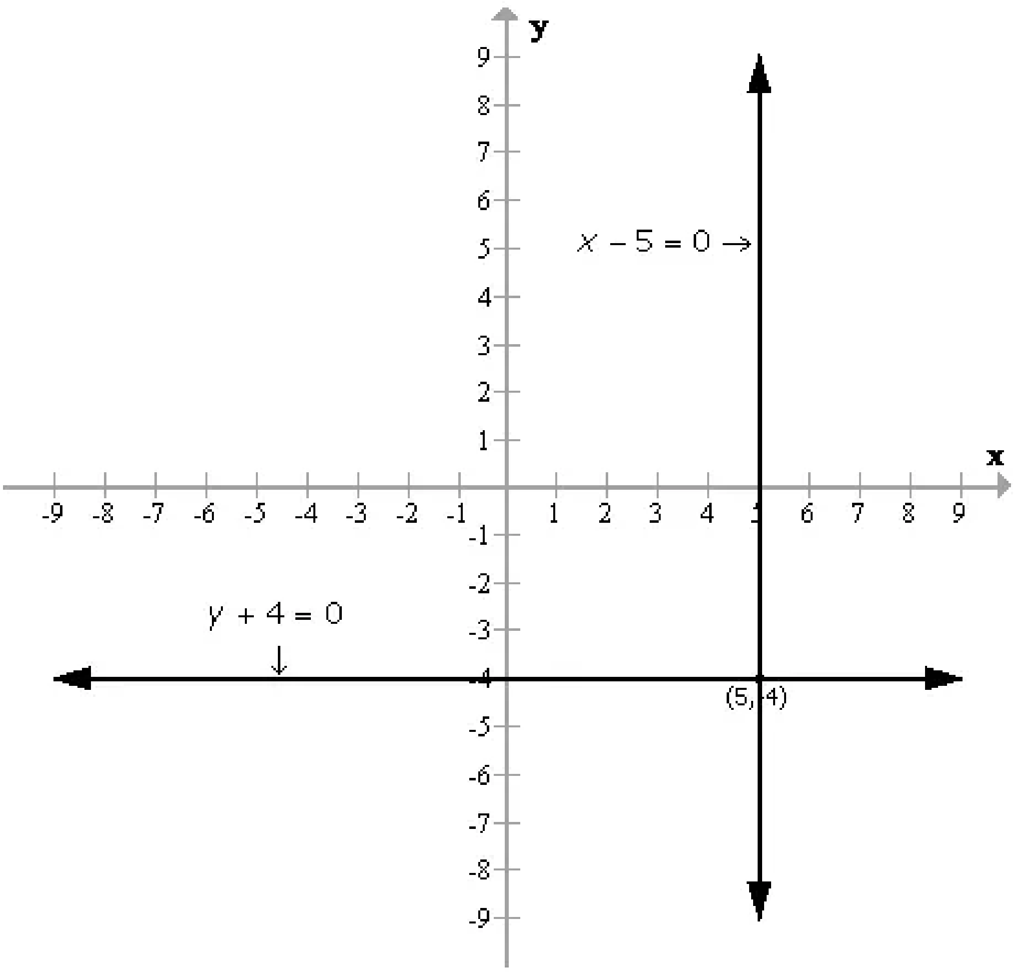
(i)

$x - 5 = 0 \Rightarrow x = 5$

$y + 4 = 0 \Rightarrow y = -4$

Following is the graph of the two equations

$x = 5$ and $y = -4$:



(ii)

$2x + y = 23 \Rightarrow y = 23 - 2x$

The table for $y = 23 - 2x$ is

x	5	10	15
y	13	3	−7

Also, we have

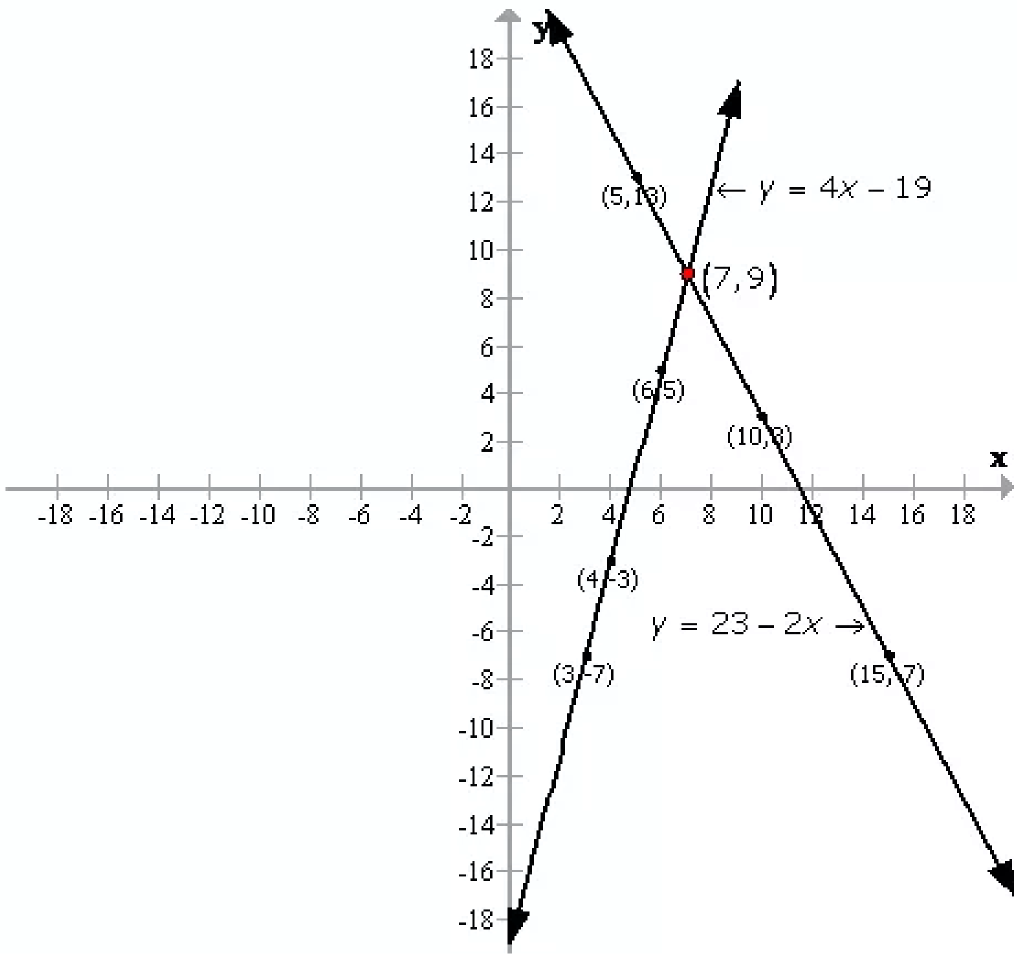
$4x - y = 19$

$\Rightarrow y = 4x - 19$

The table for $y = 4x - 19$ is

x	3	4	6
y	−7	−3	5

Plotting the points we get the following required graph:



From the above graph, it is dear
that the two lines $y=23-2x$ and $y=4x-19$
intersect at the point $(7,9)$

(iii)

$3x + 7y = 27 \Rightarrow 3x = 27 - 7y$

$\Rightarrow x = \dfrac{27-7y}{3}$

The table for $3x + 7y = 27$ is

x	9	2	−5
y	0	3	6

Also, we have

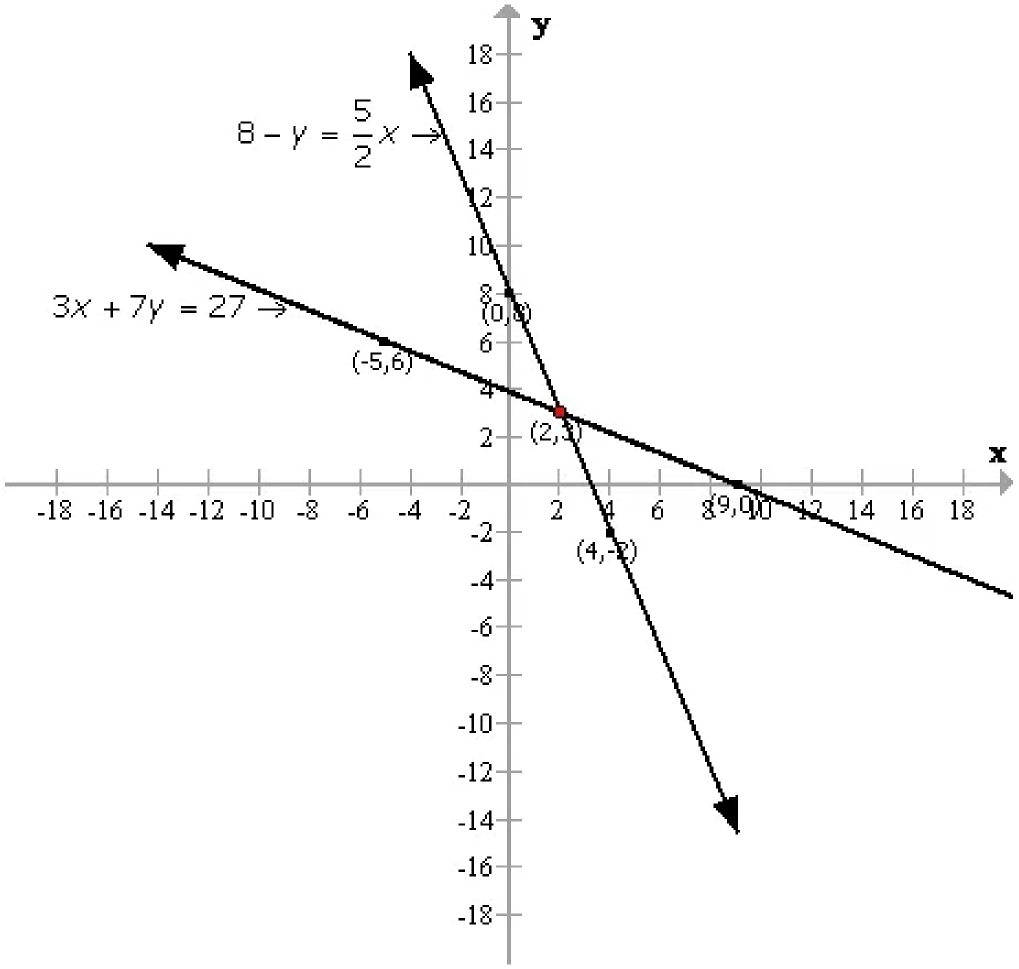
$8 - y = \dfrac{5}{2}x$

$\Rightarrow x = (8 - y) \times \dfrac{2}{5}$

The table for $5x + 2y = 16$ is

x	2	4	0
y	3	−2	8

Plotting the points we get
the following required graph:



From the above graph, it is dear

that the two lines $3x + 7y = 27$ and $8 - y = \frac{5}{2}x$

intersect at the point (2,3)

(iv)

$$\frac{x+1}{4}=\frac{2}{3}(1-2y)$$

$$\Rightarrow \quad \frac{x+1}{4}=\frac{2}{3}-\frac{4y}{3}$$

$$\Rightarrow 12\times\frac{x+1}{4}=12\times\frac{2}{3}-12\times\frac{4y}{3}$$

$$\Rightarrow \quad 3(x+1)=8-16y$$

$$\Rightarrow \quad 3x+3=8-16y$$

$$\Rightarrow 3x+3-8=-16y$$

$$\Rightarrow \quad 3x-5=-16y$$

$$\Rightarrow \quad x=\frac{5-16y}{3}$$

The table for $\frac{x+1}{4}=\frac{2}{3}(1-2y)$ is

x	7	-9	23
y	-1	2	-4

Also, we have

$$\frac{2+5y}{3}=\frac{x}{7}-2$$

$$\Rightarrow 21\times\frac{2+5y}{3}=21\times\frac{x}{7}-21\times 2$$

$$\Rightarrow \quad 7(2+5y)=3x-42$$

$$\Rightarrow \quad 14+35y=3x-42$$

$$\Rightarrow \quad 3x=14+35y+42$$

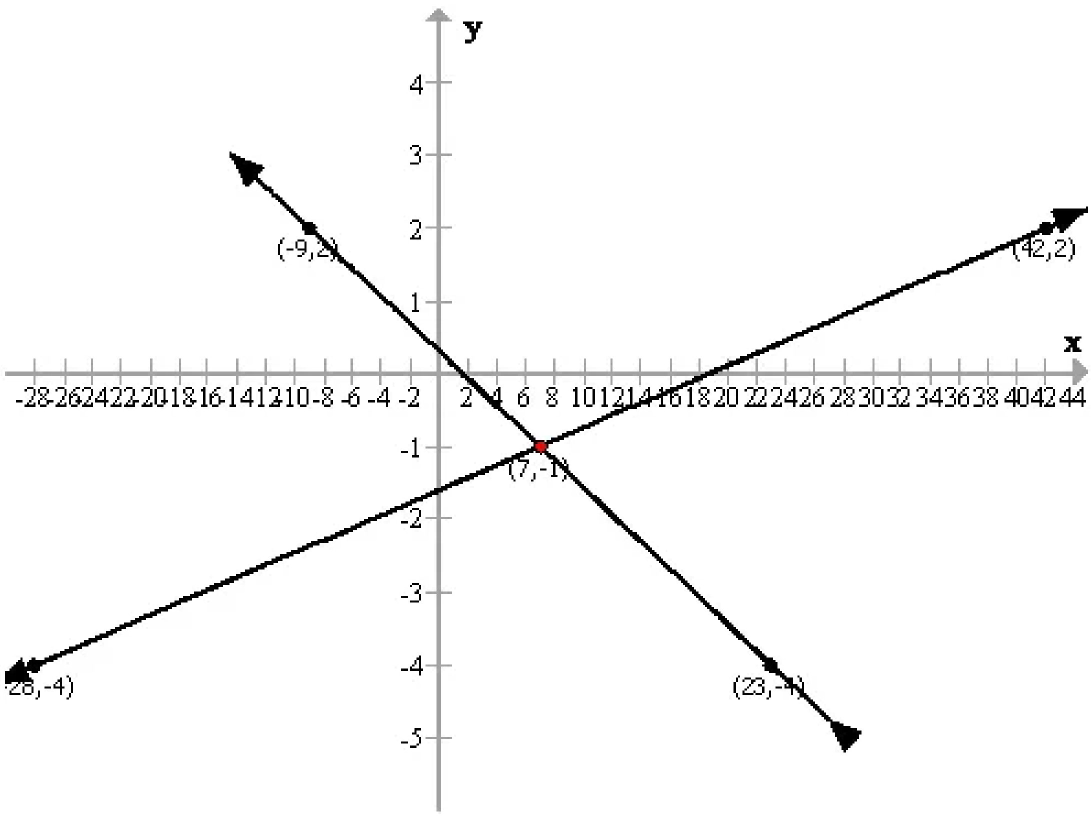
$$\Rightarrow \quad 3x=56+35y$$

$$\Rightarrow \quad x=\frac{56+35y}{3}$$

The table for $\frac{2+5y}{3}=\frac{x}{7}-2$ is

x	7	−28	42
y	−1	−4	2

Plotting the points we get
the following required graph:



From the above graph, it is dear
that the two lines $\frac{x+1}{4}=\frac{2}{3}(1-2y)$ and $\frac{2+5y}{3}=\frac{x}{7}-2$
intersect at the point $(7,-1)$

Solution 2:

$x - 2y - 4 = 0$

$\Rightarrow x = 2y + 4$

The table for $x - 2y - 4 = 0$ is

x	4	6	2
y	0	1	−1

Also we have

2x+y=3

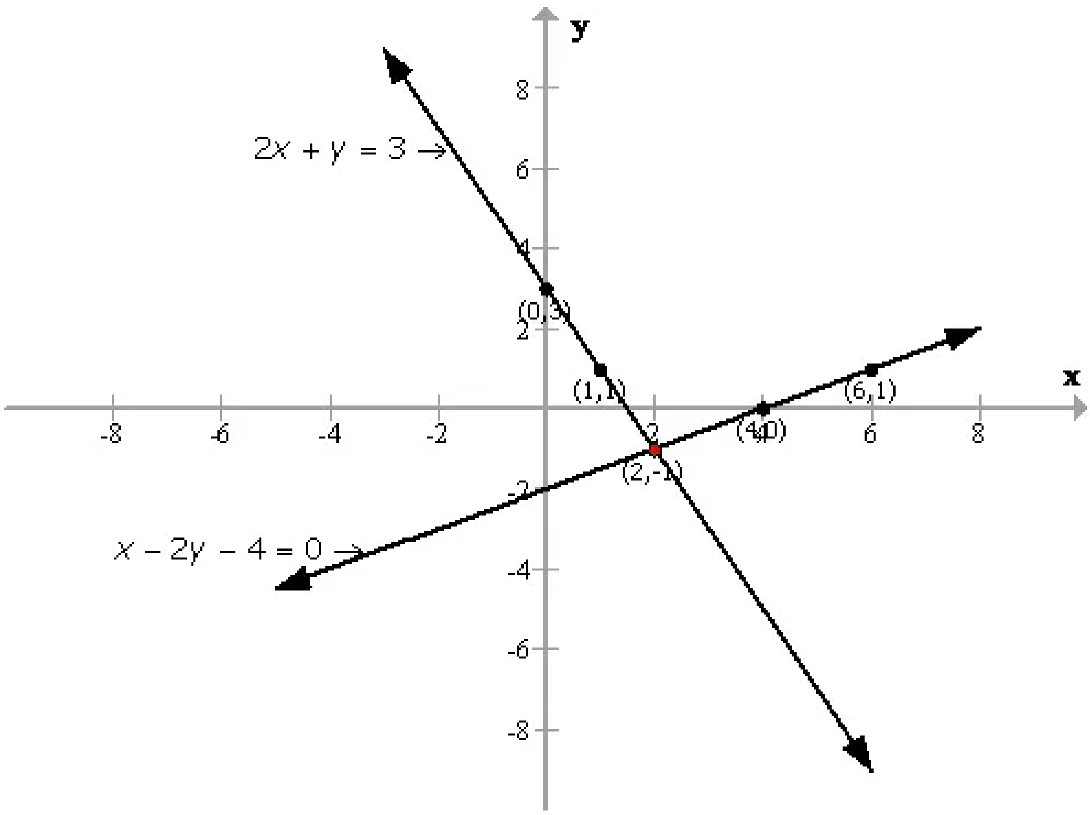
$\Rightarrow 2x = 3 - y$

$\Rightarrow x = \dfrac{3-y}{2}$

The table for 2x+y=3 is

x	1	0	2
y	1	3	−1

Plotting the above points we get the following required graph:



From the above graph, it is dear that the two lines $x - 2y - 4 = 0$ and 2x+y=3 intersect at the point $(2, -1)$

Solution 3:

$2x - y - 1 = 0$

$\Rightarrow 2x = y + 1$

$\Rightarrow x = \frac{y+1}{2}$

The table for $2x - y - 1 = 0$ is

x	2	1	0
y	3	1	−1

Also we have

$2x+y=9$

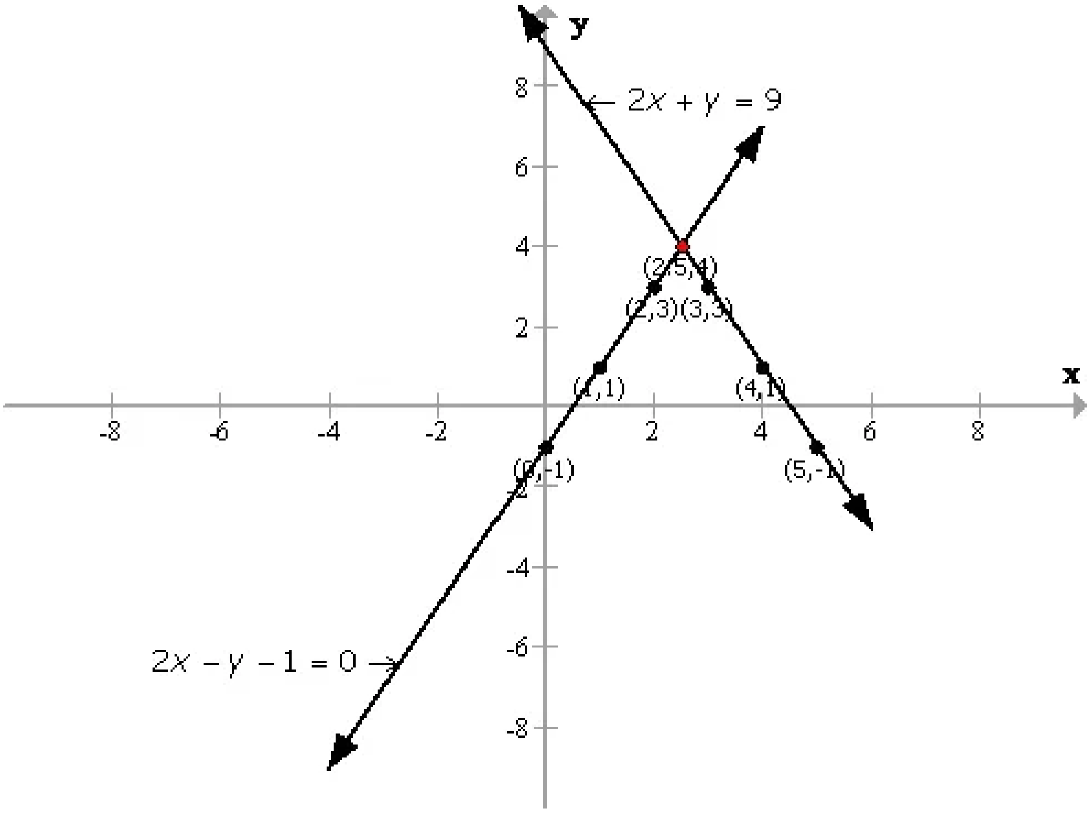
$\Rightarrow 2x=9-y$

$\Rightarrow x = \frac{9-y}{2}$

The table for $2x+y=9$ is

x	4	3	5
y	1	3	−1

Plotting the above points we get the following required graph:



From the above graph, it is dear that the two lines $2x - y - 1 = 0$ and $2x+y=9$ intersect at the point (2.5,4)

Solution 4:

$3x + 5y = 12$

$\Rightarrow 3x = 12 - 5y$

$\Rightarrow x = \dfrac{12 - 5y}{3}$

The table for $3x + 5y = 12$ is

x	4	−1	−6
y	0	3	−1

Also we have

$3x - 5y + 18 = 0$

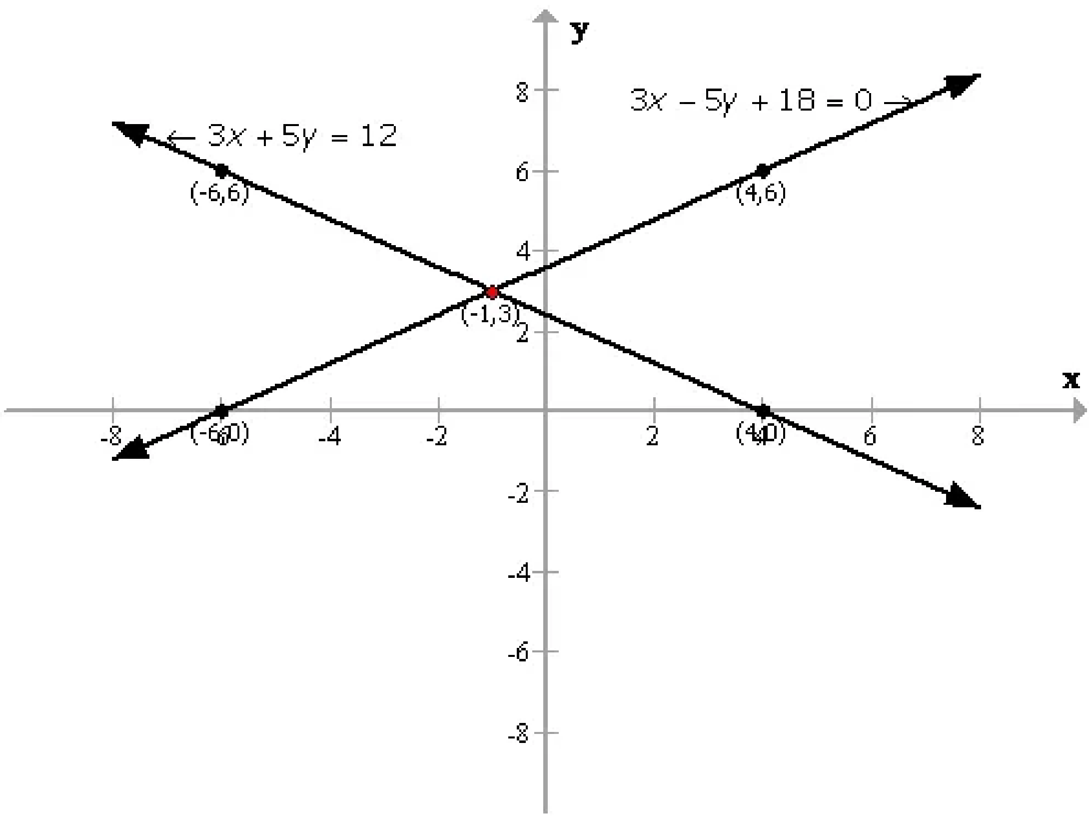
$\Rightarrow 3x = 5y - 18$

$\Rightarrow x = \dfrac{5y - 18}{3}$

The table for $3x - 5y + 18 = 0$ is

x	−6	4	−1
y	0	6	3

Plotting the above points we get the following required graph:



From the above graph, it is dear that the two lines $3x + 5y = 12$ and $3x - 5y + 18 = 0$ intersect at the point $(-1,3)$

Solution 5:

(i)

$x + y + 3 = 0$

$\Rightarrow x = -3 - y$

The table for $x + y + 3 = 0$ is

x	1	0	−2
y	−4	−3	−1

Also we have

$3x - 2y + 4 = 0$

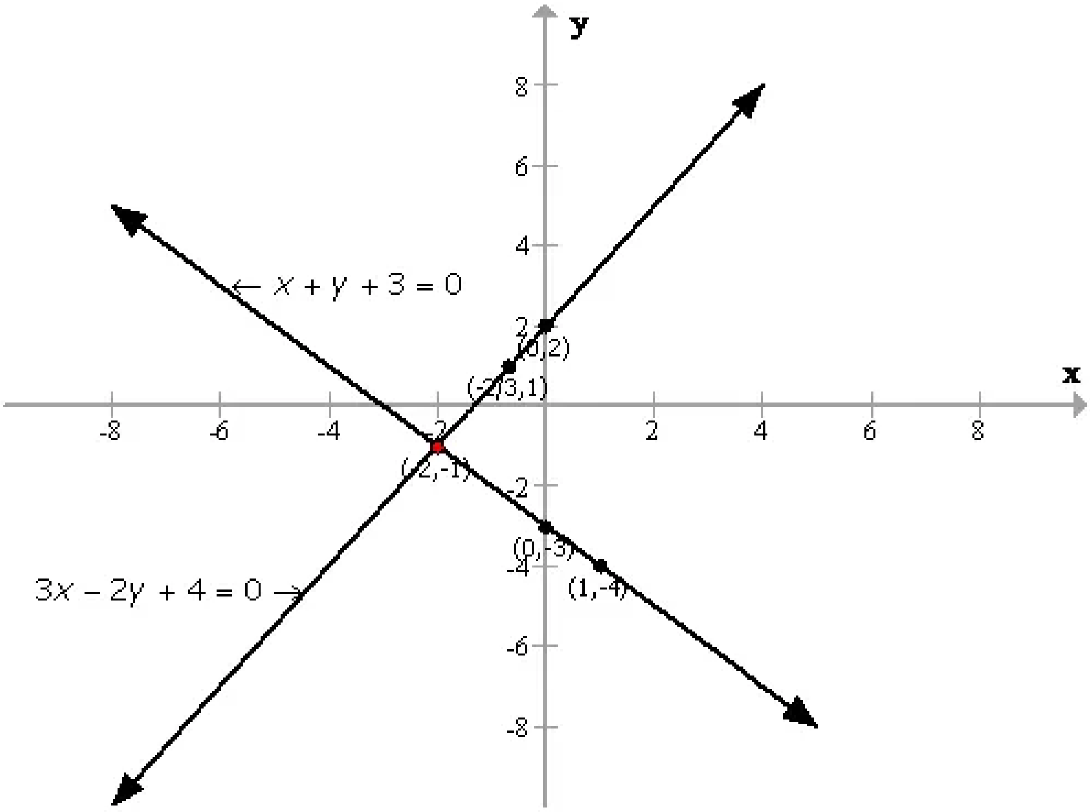
$\Rightarrow 3x = 2y - 4$

$\Rightarrow x = \frac{2y - 4}{3}$

The table for $3x - 2y + 4 = 0$ is

x	0	−2	$-\frac{2}{3}$
y	2	−1	1

Plotting the above points we get the following required graph:



(ii)

From the above graph, it is clear
that the two lines $x + y + 3 = 0$ and $3x - 2y + 4 = 0$
intersect at the point $(-2, -1)$

(iii)

Applying Pythagoras Theorem,

the distance from the origin $= \sqrt{(-2-0)^2 + (-1-0)^2}$

$= \sqrt{2^2 + 1^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$

$= 2.2\ cm\ (approx)$

Solution 6:

$y - 2 = 0$

$\Rightarrow y = 2$

$y + 1 = 3(x - 2)$

$\Rightarrow y + 1 = 3x - 6$

$\Rightarrow y = 3x - 6 - 1$

$\Rightarrow y = 3x - 7$

The table for $y + 1 = 3(x - 2)$ is

x	1	2	3
y	−4	−1	2

Also we have

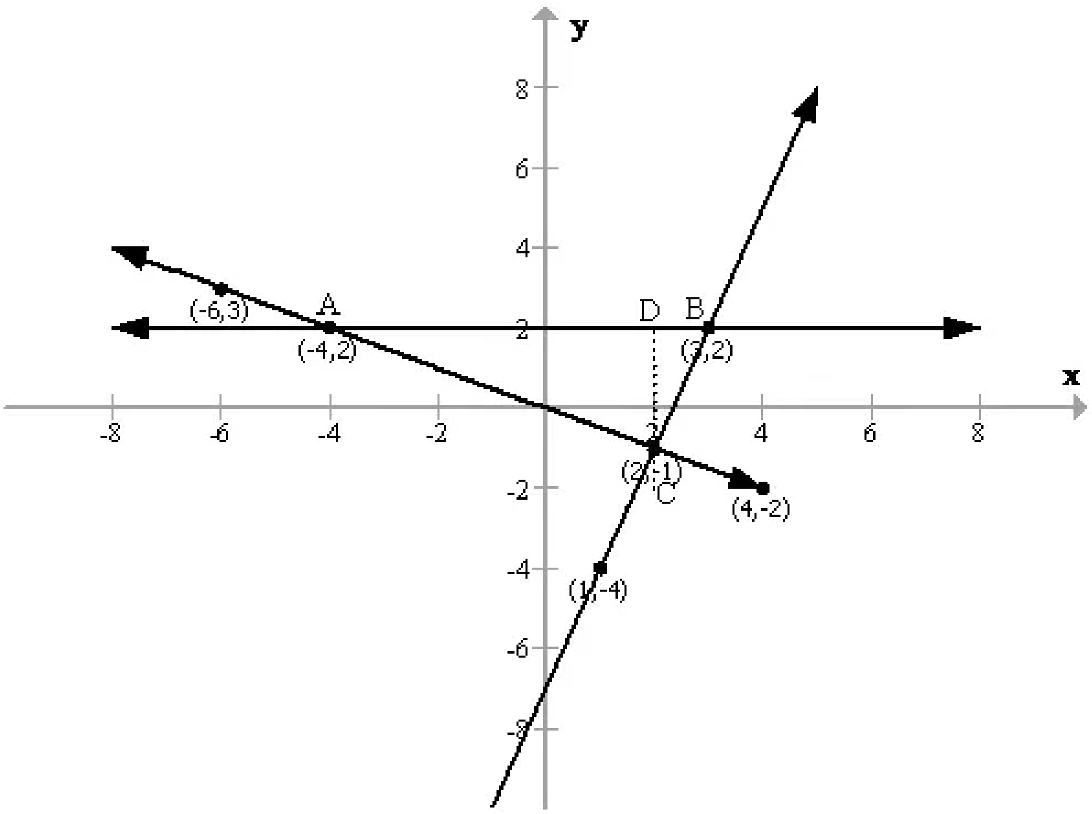
$x + 2y = 0$

$\Rightarrow x = -2y$

The table for $x + 2y = 0$ is

x	−4	4	−6
y	2	−2	3

Plotting the above points we get the following required graph:



The area of the triangle ABC $= \frac{1}{2} \times AB \times CD$

$= \frac{1}{2} \times 7 \times 3$

$= \frac{21}{2}$

$= 10.5$ sq.units

(ii)

The coordinates of the vertices of the triangle are $(-4, 2), (3, 2)$ and $(2, -1)$

Solution 7:

$3x+y+5=0 \Rightarrow y=-3x-5$

The table of $3x+y+5=0$ is

x	1	-3	-2
y	-8	4	1

$3y-x=5 \Rightarrow x=3y-5$

The table of $3y-x=5$ is

x	-2	1	7
y	1	2	4

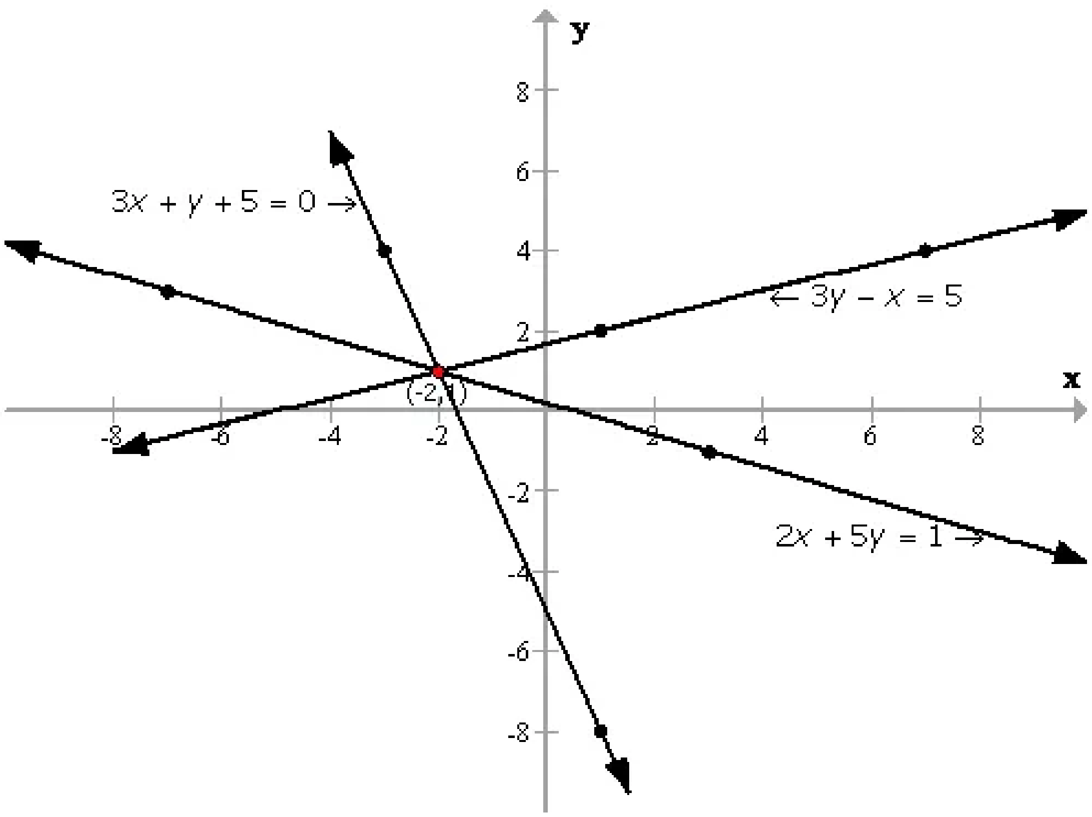
$2x+5y=1$

$\Rightarrow 2x = 1-5y \Rightarrow x = \frac{1-5y}{2}$

The table of $2x+5y=1$ is

x	3	-7	-2
y	-1	3	1

Plotting the above points, we get the following required graph:



The graph shows that the lines of these equations are concurrent.

Solution 8:

$6y = 5x + 10$

$\Rightarrow y = \dfrac{5x + 10}{6}$

The table of $6y = 5x + 10$ is

x	4	−2	−8
y	5	0	−5

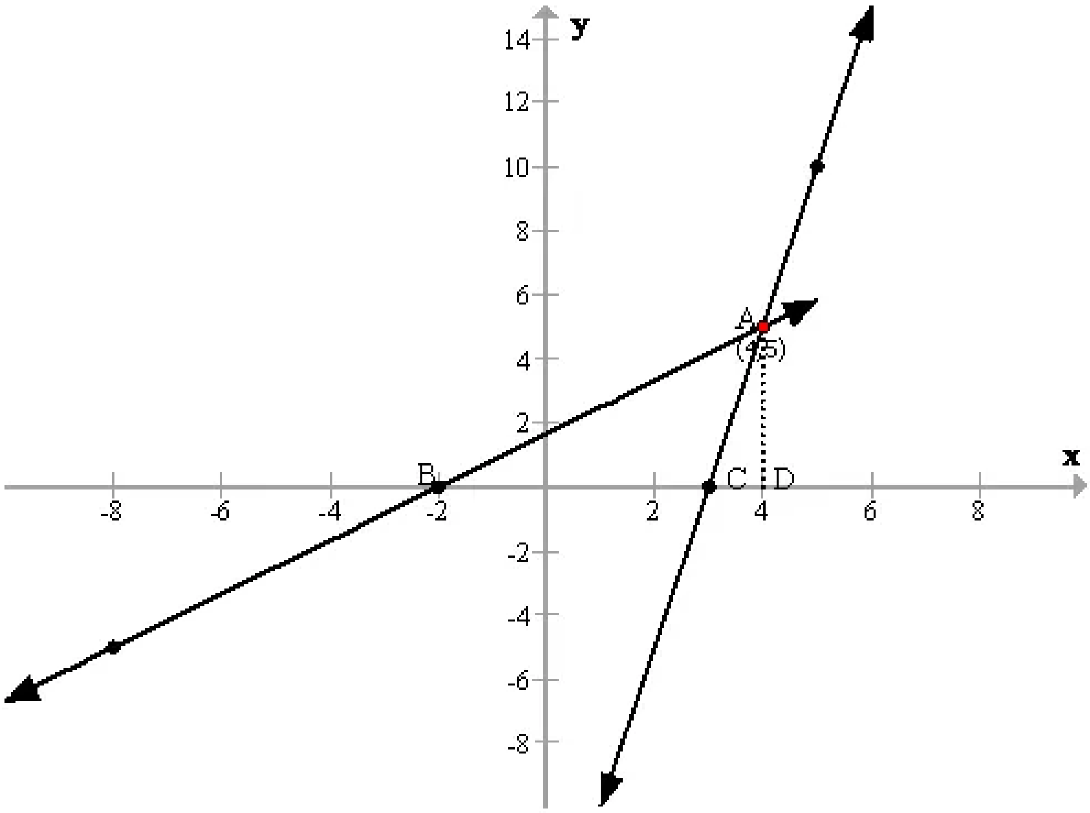
Also, we have

$y=5x - 15$

The table of $y = 5x - 15$ is

x	3	4	5
y	0	5	10

Plotting the points in a graph, we get the following graph.



(i)

The two lines intersect at (4,5)

$\therefore AD \perp BC$

AD = 5 units and BC=5 units

(ii)

The area of the triangle $= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 5 \times 5$

$= \frac{25}{2}$ sq.units

= 12.5 sq.units

Solution 9:

Given that C.P. is 50+3x

Table of C.P

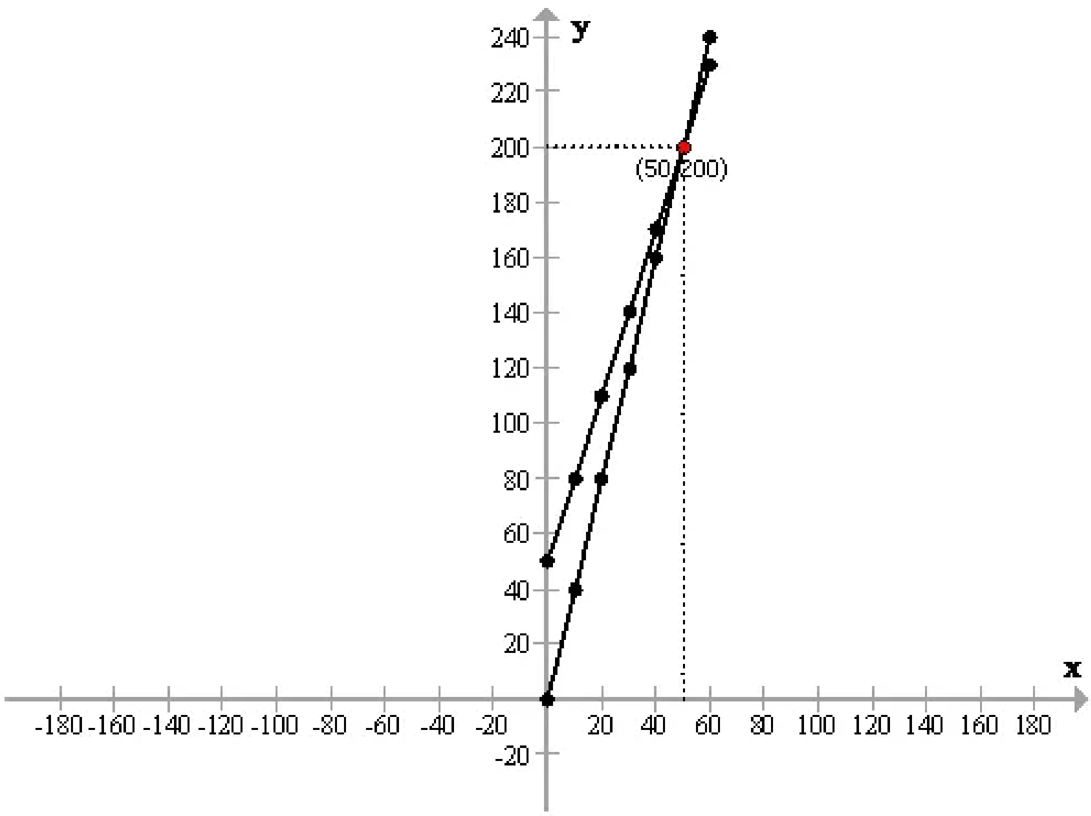
x	0	10	20	30	40	50	60
C.P	50	80	110	140	170	200	230

and S.P. =4x

$\therefore$ Table of S.P.

x	0	10	20	30	40	50	60
S.P	0	40	80	120	160	200	240

Now plotting the points on a graph and we get the following required graph:



(i)

No. of articles to be manufactured and sold are 50 when there is no loss and no profit.

C.P = S.P = Rs.200

(ii)

(a)

On article 30,

C.P = Rs.140 and S.P. = 120

Therefore Loss = 140 - 120 = Rs.20

(b)

On article 60,

C.P.=Rs.230 and S.P.= Rs.240

Therefore Profit = 240 - 230 = Rs.10

Solution 10:

$2y - x = 8;$

$y = \frac{8+x}{2};$

The table of $2y - x = 8$ is

x	−6	−2	0
y	1	3	4

$5y - x = 14 \Rightarrow x = 5y - 14$

The table of $x = 5y - 14$ is

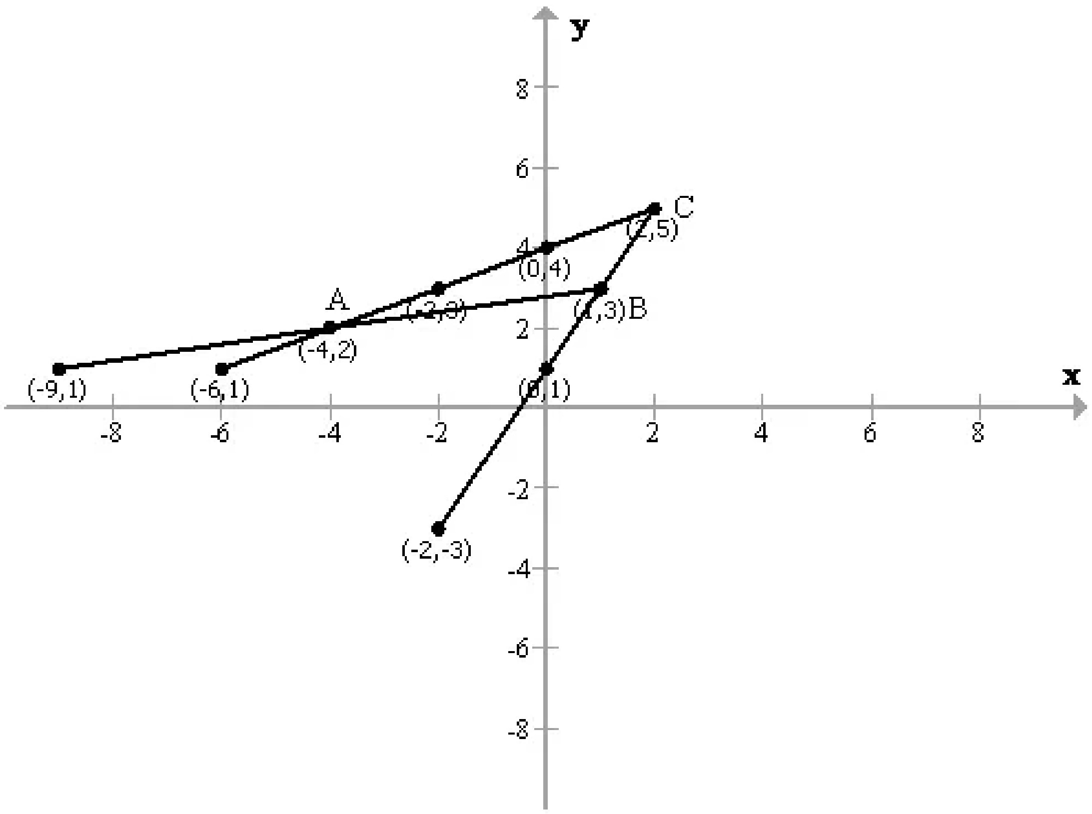
x	−9	−4	1
y	1	2	3

$y - 2x = 1 \Rightarrow y = 1 + 2x$

The table of $y - 2x = 1$ is

x	2	−2	0
y	5	−3	1

Now plotting the points on a graph and we get the following required graph:



Thus, the vertices of the triangle ΔABC are:

$A(-4,2), B(1,3)$ *and* $C(2,5)$

Solution 11:

$x + y = 0$

$y = -x;$

The table of $x + y = 0$ is

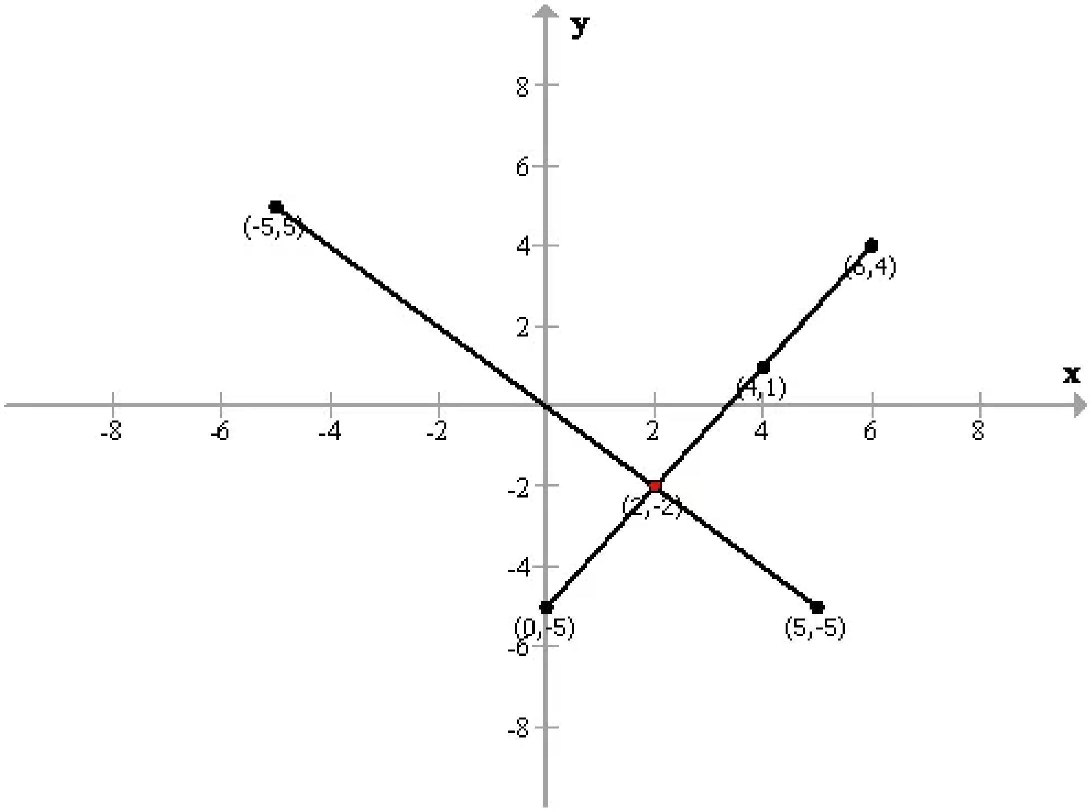
x	5	2	−5
y	−5	−2	5

$3x - 2y = 10 \Rightarrow x = \frac{10 + 2y}{3}$

The table of $3x - 2y = 10$ is

x	4	6	2
y	1	4	−2

Now plotting the points on a graph and we get the following required graph:



The two lines intersect at $(2, -2)$

$\therefore x = 2$ and $y = -2$

Solution 12:

$x + 2y = 4$

$\Rightarrow x = 4 - 2y$

The table of $x + 2y = 4$ is

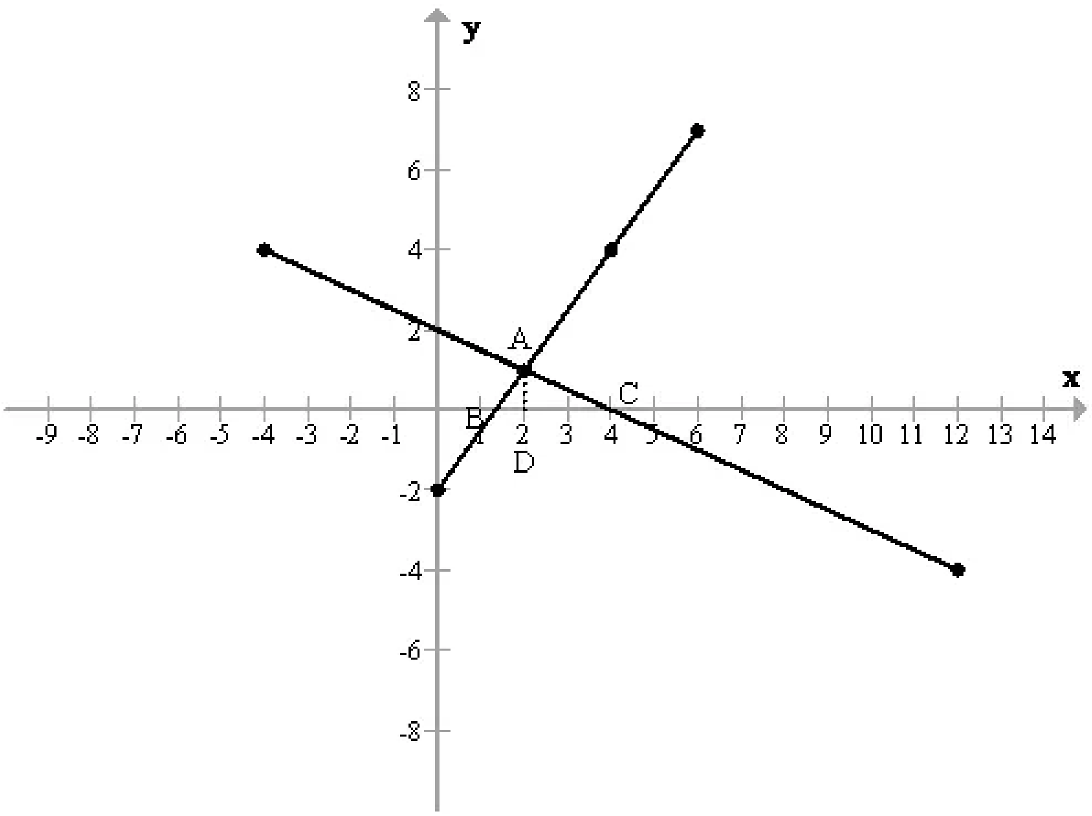
x	2	−4	12
y	1	4	−4

$3x - 2y = 4 \Rightarrow x = \dfrac{4 + 2y}{3}$

The table of $3x - 2y = 4$ is

x	2	4	6
y	1	4	7

Now plotting the points on a graph and we get the following required graph:



Therefore the solution of the given system of equations is (2,1).

Thus the vertices of the triangle are:

$A(2,1), B\left(\frac{4}{3},0\right)$ and $C(4,0)$

$AD \perp BC$ and $D \equiv (2,0)$

$\therefore AD = 1$ and $BC = 2\frac{2}{3}$ units $= \frac{8}{3}$ units

$$\text{Area of the triangle } ABC = \frac{1}{2} \times AD \times BC$$

$$= \frac{1}{2} \times 1 \times \frac{8}{3}$$

$$= \frac{4}{3} \text{ sq.units}$$

$$= 1\frac{1}{3} \text{ sq.units}$$

Solution 13:

$y = \frac{3x+2}{2}$

The table for $y = \frac{3x+2}{2}$ is

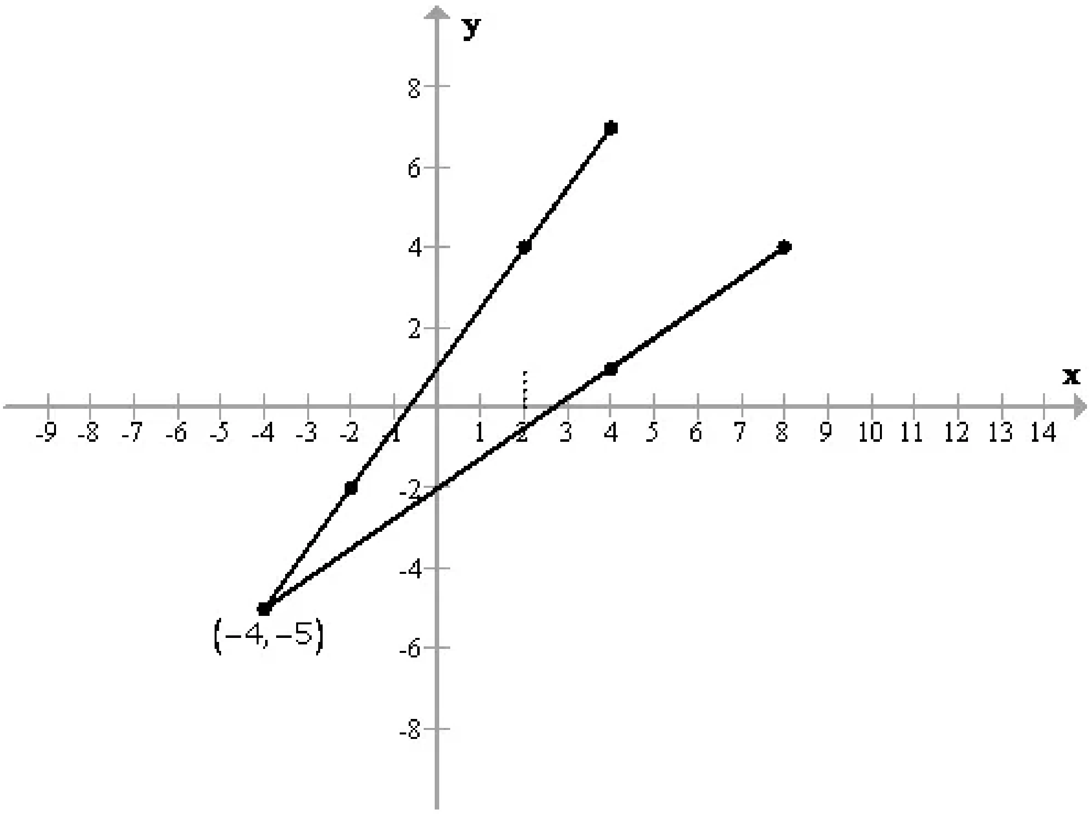
x	2	4	−2
y	4	7	−2

$y = \frac{3}{4}x - 2$

The table for $y = \frac{3}{4}x - 2$ is

x	4	−4	8
y	1	−5	4

Now plotting the points on a graph and
we get the following required graph:



Thus the value of 'x' is −4.

Solution 14:

$2x + 3y = 4$

$\Rightarrow x = \dfrac{4-3y}{2}$

The table for $2x + 3y = 4$ is

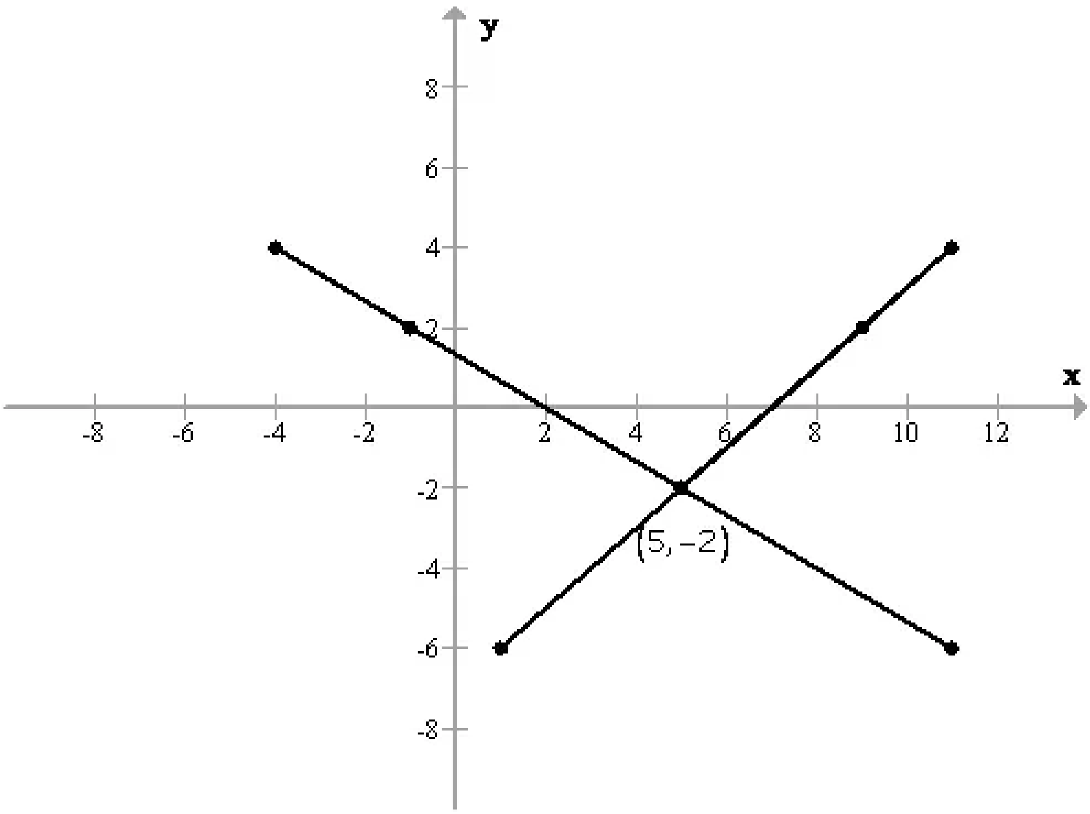
x	−1	−4	5
y	2	4	−2

$x - y = 7 \Rightarrow x = y + 7$

The table for $x - y = 7$ is

x	5	11	9
y	−2	4	2

Now plotting the points on a graph and we get the following required graph:



The point at which the paths of the submarine and the destroyer intersect are $(5, -2)$