Insulating Materials

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Complex Permittivity

When an alternating field is applied to a dielectric, the relative dielectric constant become a complex quantity whose value is given by

$$\boxed{\epsilon_r^* = \epsilon_r' - j\,\epsilon_r''}$$

Where, ϵ'_r = real part of dielectric constant

ϵ''_r = imaginary part of dielectric constant

Remember:

- When a time varying electric field is applied to a dielectric material, the response is not entirely instantaneous

$$\boxed{D = \epsilon'' E}$$

- The imaginary part is responsible for energy loss in the material.
- The real part represents the relative permittivity

Debye Equations

It gives the variation of both the inphase and out of phase component of ϵ_r as a function of angular frequency ω with relaxation time τ

$$\boxed{\epsilon_r' = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1+\omega^2\tau^2}}$$

$$\boxed{\epsilon_r'' = \frac{(\epsilon_s - \epsilon_\infty)\,\omega\tau}{1+\omega^2\tau^2}}$$

Where, ϵ_∞ = dielectric constant at infinite frequency

ϵ_s = dielectric constant under static field

τ = relaxation time

Dielectric losses

The absorption of electric energy by the dielectric material subjected to an alternating electric field is known as the dielectric losses. This results in dissipation of the electrical energy as heat in a material.

Note:

Dielectric loss occurs due to two reason

1) Oscillation of the ion
2) continuous change in orientation of dipoles

Energy absorbed per m³

$$\boxed{W = \frac{1}{2}\omega\,\epsilon_0\,\epsilon_r''E_0^2}$$

Dielectric loss tangent

It represents how lossy the material is for electrical AC signals.

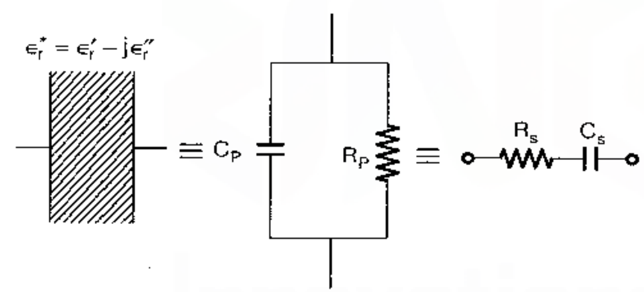
$$\boxed{\tan\delta = \frac{\epsilon_r''}{\epsilon_r'}}$$

Loss Factor

$$\boxed{\epsilon_r'\tan\delta = \epsilon_r''}$$

Lossy Capacitor

Equivalent circuits



For parallel circuit

$$\boxed{R_P = \frac{1}{\omega\,\epsilon_r''C_0}}$$

where C_0 = capacitance

$$C_0 = \frac{\epsilon_0 A}{d}$$

$$\boxed{C_P = \epsilon_r'\,C_0}$$

For series circuit

$$R_s = \frac{R_p}{1 + (\omega C_P R_P)^2}$$

$$C_s = \frac{1 + (\omega C_P R_P)^2}{\omega^2 C_P R_1^2}$$

Loss tangent

For series circuit

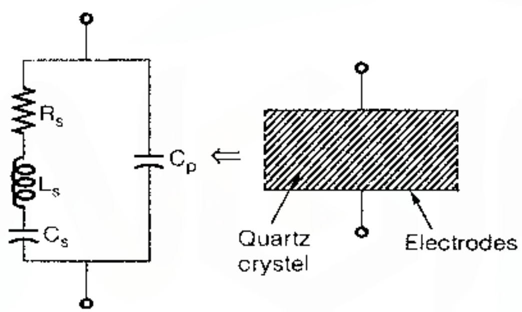
$$\tan\delta = \omega C_s R_s$$

For parallel circuit

$$\tan\delta = \frac{1}{\omega C_P R_P}$$

Quartz

Quartz is a piezoelectric material, used for high frequency oscillation



$$z = \left(\frac{1}{j\omega C_s} + j\omega L + R_1\right) \Big\| \left(\frac{1}{j\omega C_p}\right)$$

- Q-factor

$$Q = \frac{\omega_s L_s}{R_s} = \frac{1}{\omega R_s C_s}$$

- Series resonance frequency

$$\omega_s = \frac{1}{\sqrt{L_s C_s}}$$

Impedance at series resonance frequency

$$Z\big|_{\omega=\omega_s} = R_s$$

- Parallel resonance frequency

$$\omega_p = \sqrt{\frac{1}{L_s}\left(\frac{1}{C_s} + \frac{1}{C_P}\right)} = \omega_s\sqrt{1 + \frac{C_s}{C_p}}$$

- Impedance at parallel resonance frequency

$$Z\big|_{\omega=\omega_p} = \frac{R_s^2 + \left(\omega_p L_s - \frac{1}{\omega_p C_s}\right)}{R_s}$$

$$Z\big|_{\omega=\omega_p} > Z\big|_{\omega=\omega_s}$$

Note:

Quartz crystal can be modelled as an electrical network with a low impedance (series) and a high impedance (parallel).