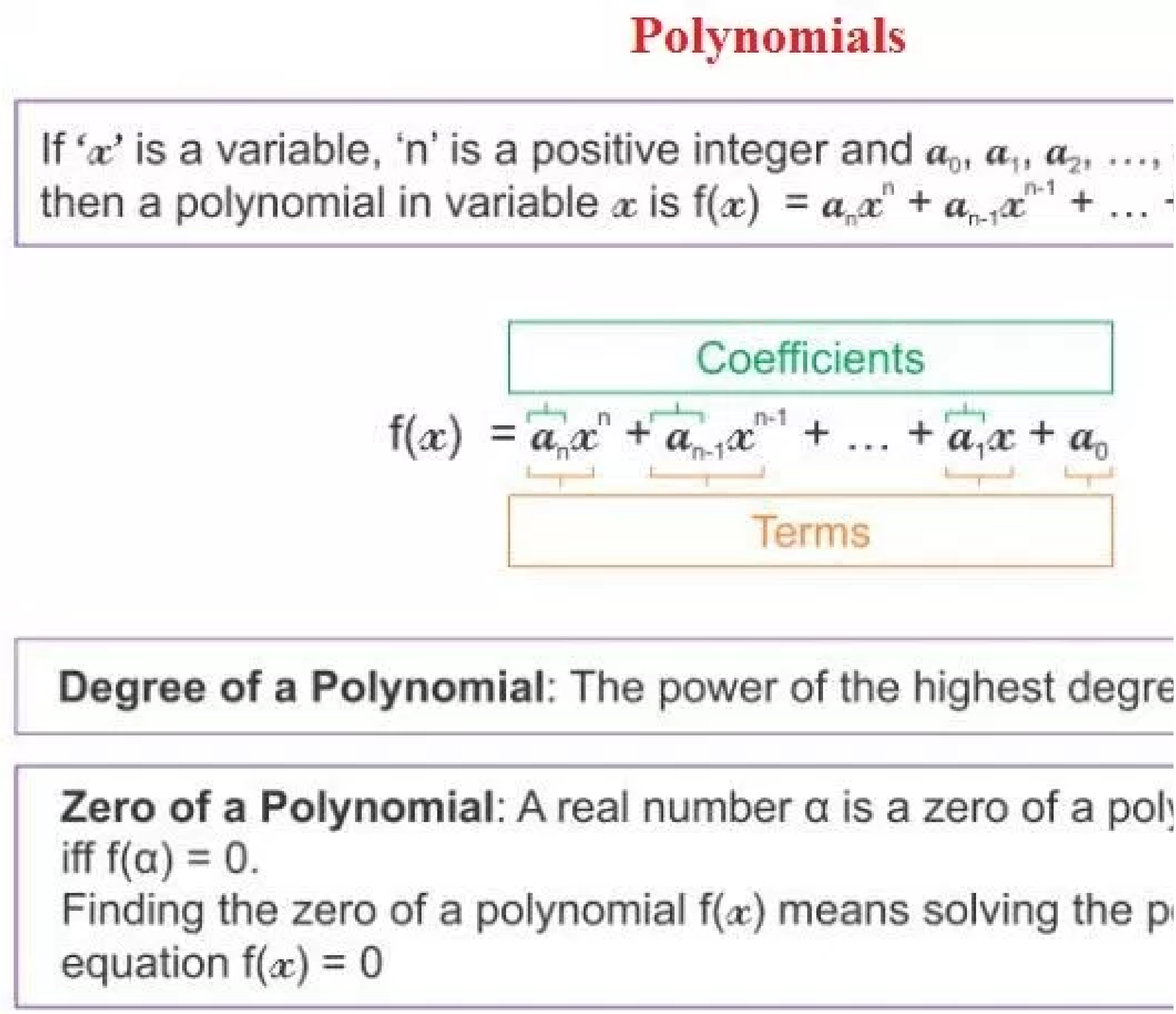
6. Polynomials

Polynomials

Algebraic expression containing many terms of the form ax^n, n being a non-negative integer is called a polynomial. i.e., $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + …….. + a_{n-1}x^{n-1} + a_nx_n$, where x is a variable, $a_0, a_1, a_2, ……$ a_n are constants and $a_n \neq 0$.

Example: $4x^4 + 3x^3 – 7x^2 + 5x + 3$, $3x^3 + x^2 – 3x + 5$.



Polynomials

If 'x' is a variable, 'n' is a positive integer and $a_0, a_1, a_2, \ldots,$
then a polynomial in variable x is $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots$

Degree of a Polynomial: The power of the highest degre

Zero of a Polynomial: A real number α is a zero of a pol
iff f(α) = 0.
Finding the zero of a polynomial $f(x)$ means solving the p
equation $f(x) = 0$

(1) Real polynomial:

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots. + a_nx_n$ is called real polynomial of real variable x with real coefficients.

Example: $3x^3 – 4x^2 + 5x – 4$, $x^2 – 2x + 1$ etc. are real polynomials.

(2) Complex polynomial:

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots. + a_nx_n$ is called complex polynomial of complex variable x with complex coefficients.

Example: $3x^2 – (2+4i)x + (5i-4)$, $x^3 – 5ix^2 + (1+2i)x + 4$ etc. are complex polynomials.

(3) Degree of polynomial:

Highest power of variable x in a polynomial is called degree of polynomial.

Example: $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots. + a_nx_n$ is a n degree polynomial.

$f(x) = 4x^3 + 3x^2 – 7x + 5$ is a 3 degree polynomial.

A polynomial of second degree is generally called a quadratic polynomial. Polynomials of degree 3 and 4 are known as cubic and biquadratic polynomials respectively.

(4) Polynomial equation:

If f(x) is a polynomial, real or complex, then f(x) = 0 is called a polynomial equation.

Types Of Polynomials

(i) Based on degree :

If degree of polynomial is

			Examples
1.	One	Linear	$x + 3$, $y – x + 2$, $\sqrt{3}x – 3$
2.	Two	Quadratic	$2x^2 – 7$, $\frac{1}{3}x^2 + y^2 - 2xy$, $x^2 + 1 + 3y$
3.	Three	Cubic	$x^3 + 3x^2 – 7x+8$, $2x^2+5x^3+7$,

4.	Four	bi-quadratic	$x^4 + y^4 + 2x^2y^2$, $x^4 + 3$,…

(ii) Based on Terms :

If number of terms in polynomial is

			Examples
1.	One	Monomial	$7x$, $5x^9$, $3x^{16}$, xy, ……
2.	Two	Binomial	$2 + 7y^6$, $y^3 + x^{14}$, $7 + 5x^9$, …
3.	Three	Trinomial	$x^3 - 2x + y$, $x^{31}+y^{32}+ z^{33}$, …..

Note:

(1) Degree of constant polynomials (Ex.5, 7, –3, 8/5, …) is zero.

(2) Degree of zero polynomial (zero = 0 = zero polynomial) is not defined.

Monomials, Binomials, and Polynomials

1. A **monomial** is the product of non-negative integer powers of variables. Consequently, a monomial has NO variable in its denominator. It has one term. (mono implies one)
   $13,\quad 3x,\quad -57,\quad x^2,\quad 4y^2,\quad -2xy,\ \text{or}\ 520x^2y^2$
   (notice: no negative exponents, no fractional exponents)
2. A **binomial** is the sum of two monomials. It has two unlike terms. (bi implies two)
   $3x + 1,\quad x^2 - 4x,\quad 2x + y,\quad \text{or}\quad y - y^2$
3. A **trinomial** is the sum of three monomials. It has three unlike terms. (tri implies three)
   $x^2 + 2x + 1,\qquad 3x^2 - 4x + 10,\qquad 2x + 3y + 2$
4. A **polynomial** is the sum of one or more terms. (poly implies many)

$$x^2 + 2x, \qquad 3x^3 + x^2 + 5x + 6, \qquad 4x - 6y + 8$$

Polynomials are in simplest form when they contain no like terms.

$x^2 + 2x + 1 + 3x^2 - 4x$ when simplified becomes $4x^2 - 2x + 1$

Polynomials are generally written in descending order.

Descending: $4x^2 - 2x + 1$ *exponents of variables decrease from left*

How Do You Determine The Degree Of A Polynomial

Degree Of A Polynomial

The greatest power (exponent) of the terms of a polynomial is called degree of the polynomial.
For example :
In polynomial $5x^2 – 8x^7 + 3x$:
(i) The power of term $5x^2 = 2$
(ii) The power of term $–8x^7 = 7$
(iii) The power of $3x = 1$
Since, the greatest power is 7, therefore degree of the polynomial $5x^2 – 8x^7 + 3x$ is 7
The degree of polynomial :
(i) $4y^3 – 3y + 8$ is 3
(ii) $7p + 2$ is $1(p = p^1)$
(iii) $2m – 7m^8 + m^{13}$ is 13 and so on.

Degree Of A Polynomial With Example Problems With Solutions

Example 1: Find which of the following algebraic expression is a polynomial.

(i) $3x^2 – 5x$ (ii) $x + \frac{1}{x}$ (iii) $\sqrt{y} – 8$ (iv) $z^5 – \sqrt[3]{z} + 8$

Sol.

(i) $3x^2 - 5x = 3x^2 - 5x^1$
It is a polynomial.
(ii) $x + \frac{1}{x} = x^1 + x^{-1}$
It is not a polynomial.
(iii) $\sqrt{y} - 8 = y^{1/2} - 8$
Since, the power of the first term ($\sqrt{y}$) is $\frac{1}{2}$, which is not a whole number.
(iv) $z^5 - \sqrt[3]{z} + 8 = z^5 - z^{1/3} + 8$
Since, the exponent of the second term is 1/3, which in not a whole number. Therefore, the given expression is not a polynomial.

Example 2: Find the degree of the polynomial :

(i) $5x - 6x^3 + 8x^7 + 6x^2$ (ii) $2y^{12} + 3y^{10} - y^{15} + y + 3$ (iii) x (iv) 8

Sol.

(i) Since the term with highest exponent (power) is $8x^7$ and its power is 7.
∴ The degree of given polynomial is 7.
(ii) The highest power of the variable is 15
∴ degree = 15
(iii) $x = x^1 \Rightarrow$ degree is 1.
(iv) $8 = 8x^0 \Rightarrow$ degree = 0

Zeros Of A Polynomial Function

If for x = a, the value of the polynomial p(x) is 0 i.e., p(a) = 0; then x = a is a zero of the polynomial p(x).

For Example:

(i) For polynomial $p(x) = x - 2$; $p(2) = 2 - 2 = 0$
∴ x = 2 or simply 2 is a zero of the polynomial
$p(x) = x - 2$.
(ii) For the polynomial $g(u) = u^2 - 5u + 6$;
$g(3) = (3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$
∴ 3 is a zero of the polynomial g(u)
= u2 – 5u + 6.
Also, $g(2) = (2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$
∴ 2 is also a zero of the polynomial

$g(u) = u^2 - 5u + 6$
(a) Every linear polynomial has one and only one zero.
(b) A given polynomial may have more than one zeroes.
(c) If the degree of a polynomial is n; the largest number of zeroes it can have is also n.

For Example:

If the degree of a polynomial is 5, the polynomial can have at the most 5 zeroes; if the degree of a
polynomial is 8; largest number of zeroes it can have is 8.
(d) A zero of a polynomial need not be 0.

For Example: If $f(x) = x^2 - 4$,

then f(2) = (2)2 – 4 = 4 – 4 = 0
Here, zero of the polynomial $f(x) = x^2 - 4$ is 2 which itself is not 0.
(e) 0 may be a zero of a polynomial.

For Example: If $f(x) = x^2 - x$,

then $f(0) = 0^2 - 0 = 0$
Here 0 is the zero of polynomial
$f(x) = x^2 - x$.

Zeros Of A Polynomial Function With Examples

Example 1: Verify whether the indicated numbers are zeroes of the polynomial corresponding to them in the following cases :
(i) p(x) = 3x + 1, x = $-\frac{1}{3}$
(ii) p(x) = (x + 1) (x – 2), x = – 1, 2
(iii) $p(x) = x^2$, x = 0
(iv) p(x) = lx + m, x = $-\frac{m}{l}$
(v) p(x) = 2x + 1, x = $\frac{1}{2}$

Sol.
(i) p(x) = 3x + 1
$\Rightarrow p\left(-\frac{1}{3}\right) = 3 \times -\frac{1}{3} + 1 = -1 + 1 = 0$
∴ x = $-\frac{1}{3}$ is a zero of p(x) = 3x + 1.
(ii) p(x) = (x + 1) (x – 2)

⇒ $p(-1) = (-1 + 1)(-1 - 2) = 0 \times -3 = 0$
and, $p(2) = (2 + 1)(2 - 2) = 3 \times 0 = 0$
∴ $x = -1$ and $x = 2$ are zeroes of the given polynomial.
(iii) $p(x) = x^2$
⇒ $p(0) = 0^2 = 0$
∴ $x = 0$ is a zero of the given polynomial
(iv) $p(x) = lx + m$
$\Rightarrow p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m$
$= - m + m = 0$
∴ $x = -\frac{m}{l}$ is a zero of the given polynomial.
(v) $p(x) = 2x + 1$
$\Rightarrow p\left(\frac{1}{2}\right) = 2 \times \frac{1}{2} + 1$
$= 1 + 1 = 2 \neq 0$
∴ $x = \frac{1}{2}$ is not a zero of the given polynomial.

Example 2: Find the zero of the polynomial in each of the following cases :

(i) $p(x) = x + 5$
(ii) $p(x) = 2x + 5$
(iii) $p(x) = 3x - 2$

Sol.

To find the zero of a polynomial p(x) means to solve the polynomial equation $p(x) = 0$.
(i) For the zero of polynomial $p(x) = x + 5$
$p(x) = 0 \quad \Rightarrow \quad x + 5 = 0 \quad \Rightarrow \quad x = -5$
∴ $x = -5$ is a zero of the polynomial.
$p(x) = x + 5$.
(ii) $p(x) = 0 \Rightarrow 2x + 5 = 0$
⇒ $2x = -5$ and $x = -\frac{5}{2}$
∴ $-\frac{5}{2}$ is a zero of $p(x) = 2x + 5$.
(iii) $p(x) = 0 \Rightarrow 3x - 2 = 0$
⇒ $3x = 2$ and $x = \frac{2}{3}$.
∴ $x = \frac{2}{3}$ is zero of $p(x) = 3x - 2$.

Factors And Coefficients Of A Polynomial

Factor:

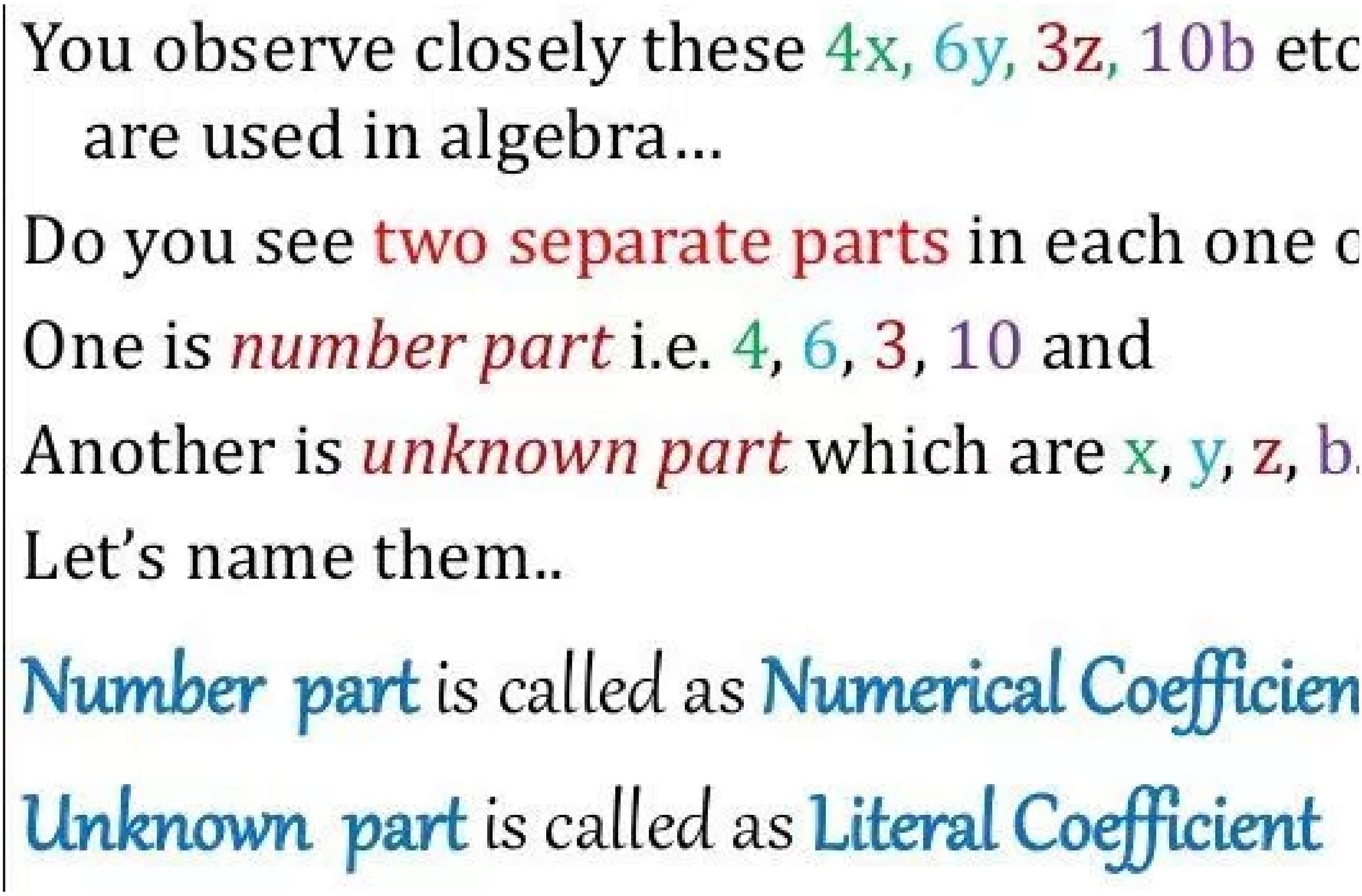
When numbers (constants) and variables are multiplied to form a term, then each quantity multiplied is called a **factor** of the term. A constant factor is called a numerical factor while a variable factor is called a literal factor.

For Example:

(i) 7, x and $7x$ are factors of $7x$, in which
7 is constant (numerical) factor and x is variable (literal) factor.
(ii) In $5x^2y$, the numerical factor is –5 and literal factors are : x, y, xy, x^2 and x^2y.

Coefficient:

Any factor of a term is called the **coefficient** of the product of the remaining factors.



You observe closely these 4x, 6y, 3z, 10b etc
are used in algebra...

Do you see two separate parts in each one c

One is *number part* i.e. 4, 6, 3, 10 and

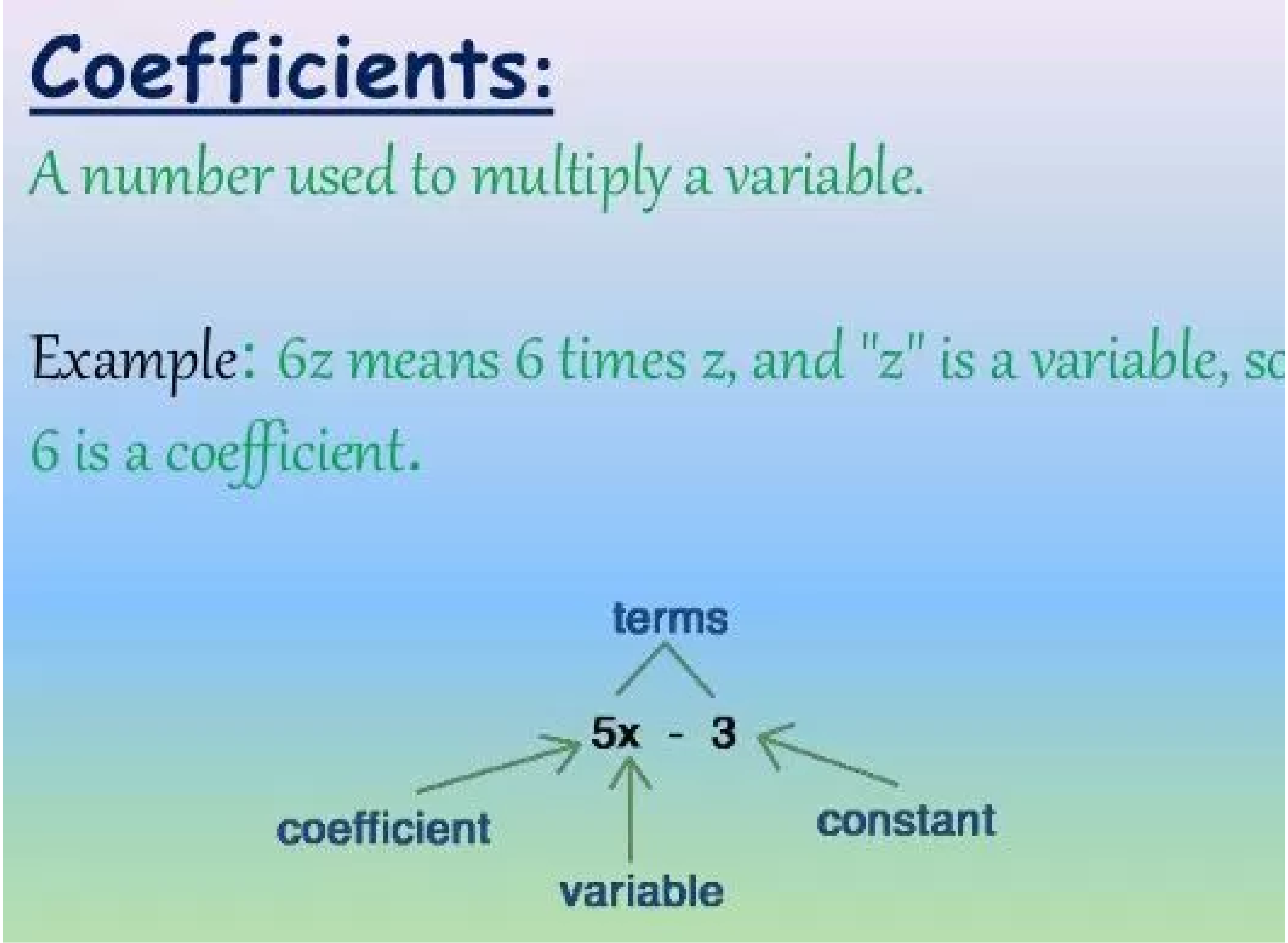
Another is *unknown part* which are x, y, z, b.

Let's name them..

Number part is called as Numerical Coefficien

Unknown part is called as Literal Coefficient

There are two types of coefficients:
1. Numerical coefficient or simply coefficient
2. Literal coefficient



Coefficients:

A number used to multiply a variable.

Example: 6z means 6 times z, and "z" is a variable, sc

6 is a coefficient.

For Example:

(i) In 7x ; 7 is coefficient of x

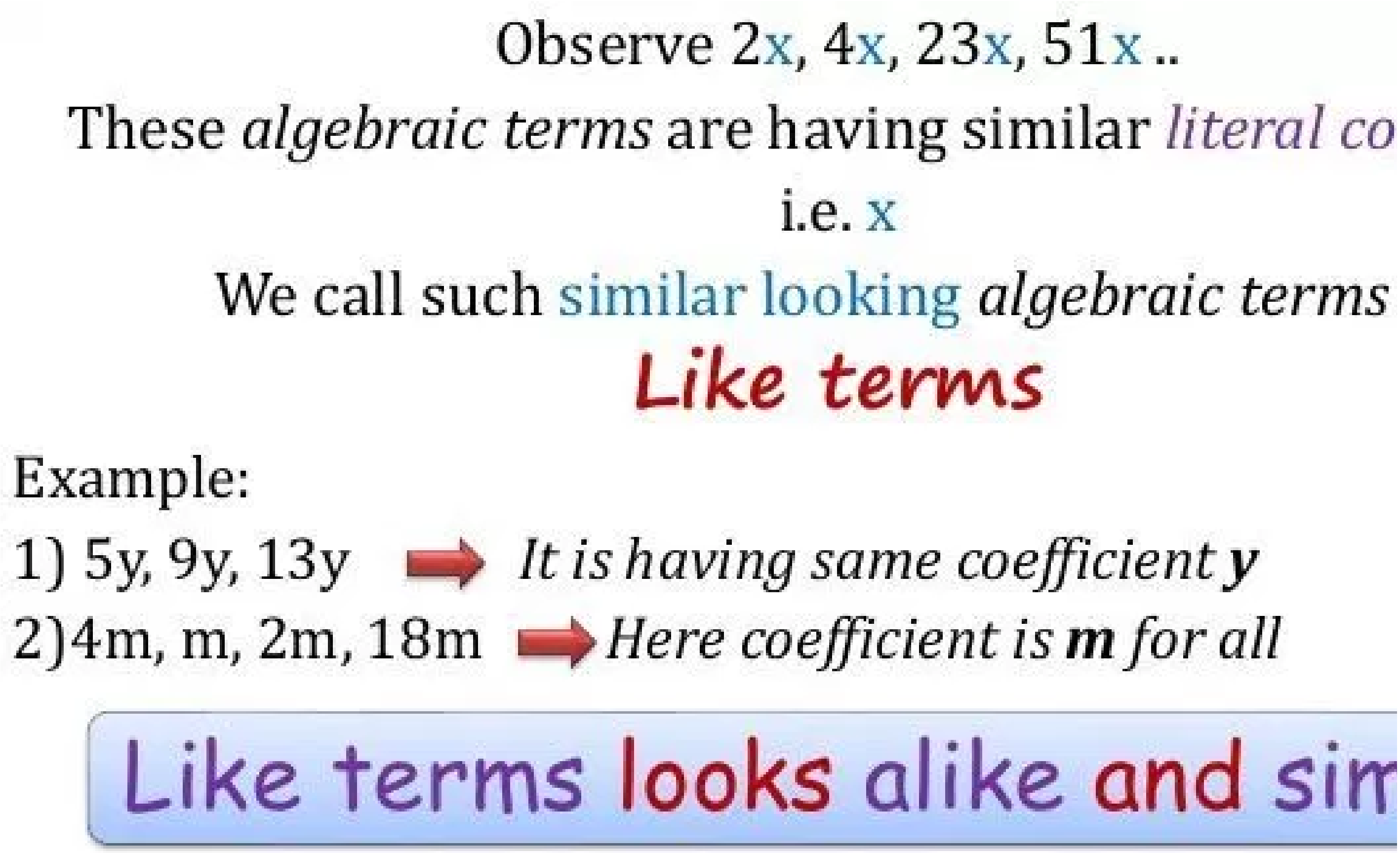
(ii) In 7xy, the numerical coefficient of the term 7xy is 7 and the literal coefficient is xy.

In a more general way,
Coefficient of xy = 7
Coefficient of 7x = y
Coefficient of 7y = x

(iii) In $(- mn^2)$, the numerical coefficient of the term is (- 1) and the literal coefficient is mn^2.
In a more general way,
Coefficient of $mn^2 = - 1$
Coefficient of $(-n^2) = m$
Coefficient of $m = (- n^2)$

(iv) In $-5x^2y$; 5 is coefficient of $-x^2y$; -5 is coefficient of x^2y.

Like and unlike terms: Two or more terms having the same algebraic factors are called like terms, and two or more terms having different algebraic factors are called unlike terms.



Observe 2x, 4x, 23x, 51x ..

These *algebraic terms* are having similar *literal co*

i.e. x

We call such similar looking *algebraic terms*

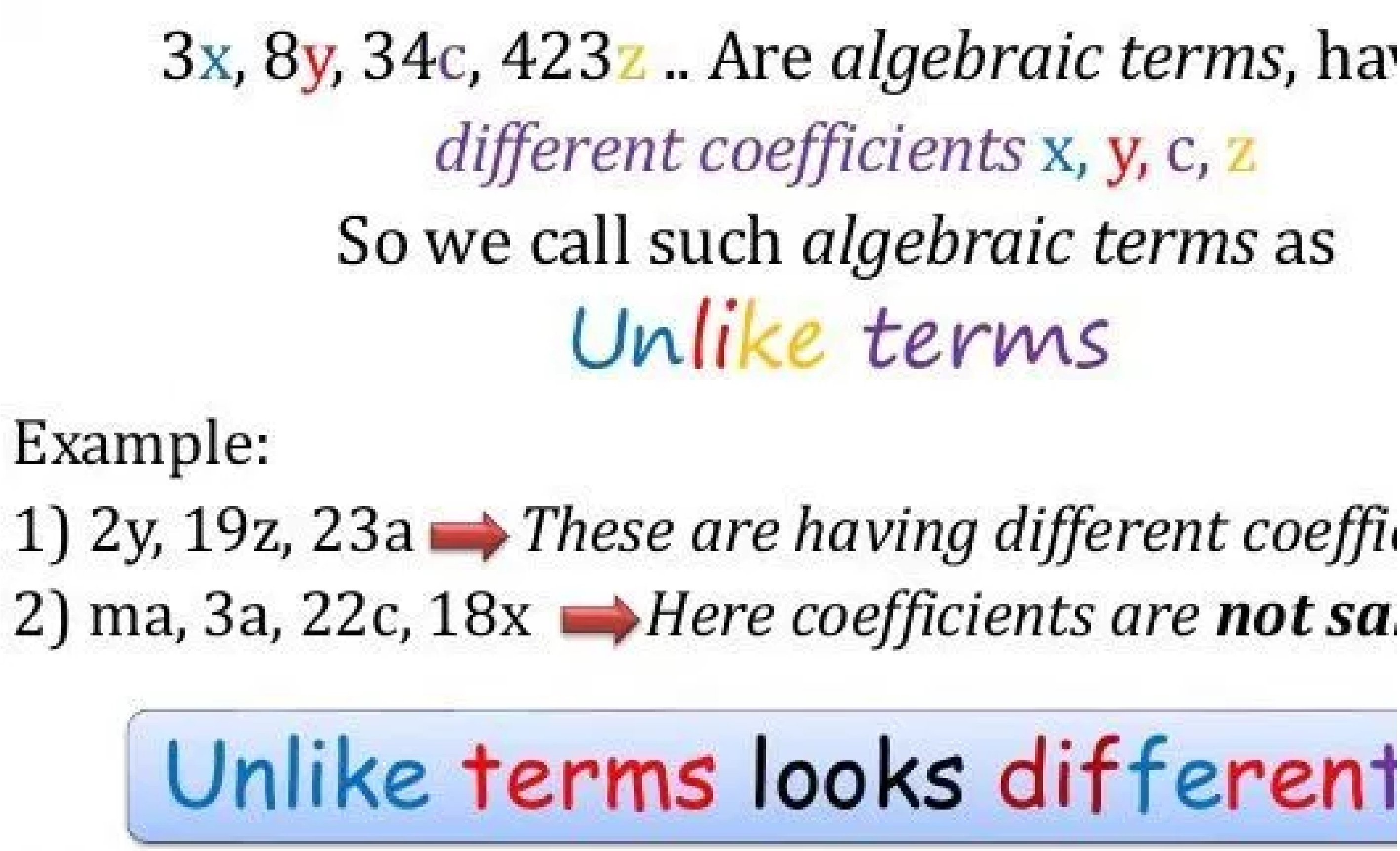
Like terms

Example:

1) 5y, 9y, 13y → *It is having same coefficient **y***

2)4m, m, 2m, 18m → *Here coefficient is **m** for all*

Like terms looks alike and sim



3x, 8y, 34c, 423z .. Are *algebraic terms*, hav

different coefficients x, y, c, z

So we call such *algebraic terms* as

Unlike terms

Example:

1) 2y, 19z, 23a → *These are having different coeffi*

2) ma, 3a, 22c, 18x → *Here coefficients are **not sa***

Unlike terms looks different

Example: In the expression $5x^2 + 7xy - 7y - 5xy$, look at the terms 7xy and (- 5xy). The factors of 7xy are 7, x, and y and the factors of (- 5xy) are (- 5), x, and y. The algebraic factors (which contain variables) of both terms are x and y. Hence, they are like terms. Other terms $5x^2$ and (- 7y) have different algebraic factors [5 × x × x and (- 7y)]. Hence, they are unlike terms.

Factors And Coefficients Of A Polynomial With Examples

Example 1: Write the coefficient of:

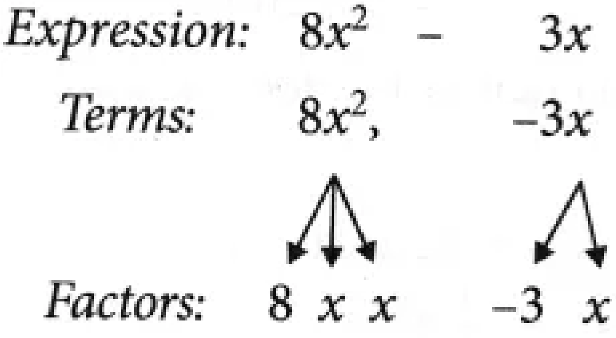
(i) x^2 in $3x^3 – 5x^2 + 7$
(ii) xy in 8xyz
(iii) $−y$ in $2y^2 – 6y + 2$
(iv) x^0 in $3x + 7$

Solution:

(i) −5
(ii) 8z
(iii) 6
(iv) Since $x^0 = 1$,
Therefore $3x + 7 = 3x + 7x^0$
coefficient of x^0 is 7.

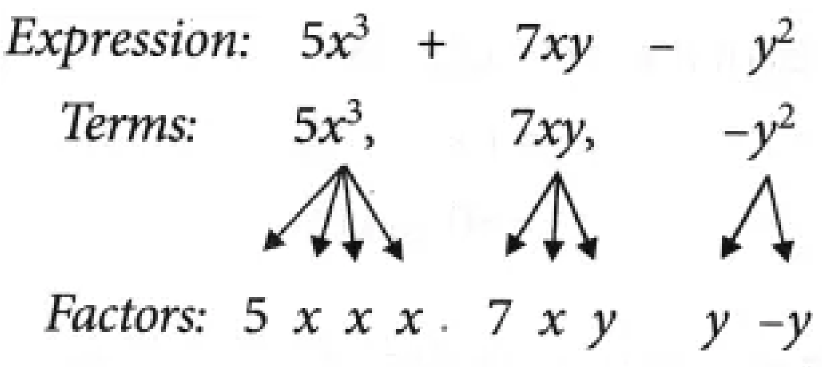
Example 2: Find the terms and factors of algebraic expression $8x^2 – 3x$.

Solution:



Example 3: Find the terms and factors of algebraic expression $5x^3 + 7xy – y^2$.

Solution:



This is called the tree diagram and it is the best way to represent expression, terms, and factors.

Example 4: Identify like terms in the following:
2xy, -xy^2, x^{2}y, 5y, 8yx, 12yx^2, -11xy

Solution: 2xy, 8yx, -11xy are like terms having the same algebraic factors x and y.
x^{2}y and 12yx^2 are also like terms having the same algebraic factors x, x and y.

Example 5: State whether the given pairs of terms are like or unlike terms:
(a) 19x, 19y (b) 4m^{2}p, 7pm^2

Solution:

(a) 19x and 19y are unlike terms having different algebraic factors, i.e., x and y.
(b) 4m^{2}p, 7pm^2 are like terms having the same algebraic factors, i.e., m, m, p.

Relationship Between Zeros And Coefficients Of A Polynomial

Consider quadratic polynomial
P(x) = 2x^2 – 16x + 30.
Now, 2x^2 – 16x + 30 = (2x – 6) (x – 3)
= 2 (x – 3) (x – 5)
The zeros of P(x) are 3 and 5.
Sum of the zeros = 3 + 5 = 8 = $\frac{-(-16)}{2}$ = - $\left[\frac{\text{coefficient of x}}{\text{coefficient of x}^2}\right]$

Product of the zeros = 3 × 5 = 15 = $\frac{30}{2}$ = $\left[\frac{\text{constant term}}{\text{coefficient of } x^2}\right]$

So if $ax^2 + bx + c$, a ≠ 0 is a quadratic polynomial and α, β are two zeros of polynomial then

$\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

In general, it can be proved that if α, β, γ are the zeros of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$\alpha + \beta + \gamma = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-d}{a}$

Note: $\frac{b}{a}$, $\frac{c}{a}$ and $\frac{d}{a}$ are meaningful because a ≠ 0.

Relationship Between Zeros And Coefficients Of A Polynomial Example Problems With Solutions

Example 1: Find the zeros of the quadratic polynomial $6x^2 - 13x + 6$ and verify the relation between the zeros and its coefficients.

Sol. We have, $6x^2 - 13x + 6 = 6\times2 - 4x - 9x + 6$

= 2x (3x – 2) –3 (3x – 2)

= (3x – 2) (2x – 3)

So, the value of $6x^2 - 13x + 6$ is 0, when

(3x – 2) = 0 or (2x – 3) = 0 i.e.,

When x = $\frac{2}{3}$ or $\frac{3}{2}$

Therefore, the zeros of $6x^2 - 13x + 6$ are

$\frac{2}{3}$ and $\frac{3}{2}$

Sum of the zeros

= $\frac{2}{3} + \frac{3}{2} = \frac{13}{6} = -\frac{(-13)}{6}$ = - $\left[\frac{\text{coefficient of x}}{\text{coefficient of } x^2}\right]$

Product of the zeros

= $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = \left[\frac{\text{constant term}}{\text{coefficient of } x^2}\right]$

Example 2: Find the zeros of the quadratic polynomial 4×2 – 9 and verify the relation between the zeros and its coefficients.

Sol. We have,

$4x^2 - 9 = (2x)^2 - 3^2 = (2x - 3)(2x + 3)$

So, the value of $4x^2 - 9$ is 0, when

2x – 3 = 0 or 2x + 3 = 0

i.e., when $x = \frac{3}{2}$ or $x = \frac{-3}{2}$.

Therefore, the zeros of $4x^2 - 9$ are $\frac{3}{2}$ & $\frac{-3}{2}$.

Sum of the zeros

$= \frac{3}{2} - \frac{3}{2} = 0 = -\frac{(0)}{4} = -\left[\frac{\text{coefficient of x}}{\text{coefficient of x}^2}\right]$

Product of the zeros

$= \frac{3}{2} \times \frac{-3}{2} = \frac{-9}{4} = \left[\frac{\text{constant term}}{\text{coefficient of x}^2}\right]$

Example 3: Find the zeros of the quadratic polynomial $9x^2 - 5$ and verify the relation between the zeros and its coefficients.

Sol. We have,

$9x^2 - 5 = (3x)^2 - (\sqrt{5})^2 = (3x - \sqrt{5})(3x + \sqrt{5})$

So, the value of $9x^2 - 5$ is 0,

when $3x - \sqrt{5} = 0$ or $3x + \sqrt{5} = 0$

i.e., when $x = \frac{\sqrt{5}}{3}$ or $x = \frac{-\sqrt{5}}{3}$.

Sum of the zeros

$= \frac{\sqrt{5}}{3} - \frac{\sqrt{5}}{3} = 0 = -\frac{(0)}{9} = -\left[\frac{\text{coefficient of x}}{\text{coefficient of x}^2}\right]$

Product of the zeros

$= \left(\frac{\sqrt{5}}{3}\right) \times \left(\frac{-\sqrt{5}}{3}\right) = \frac{-5}{9} = \left[\frac{\text{constant term}}{\text{coefficient of x}^2}\right]$

Example 4: If α and β are the zeros of ax2 + bx + c, a ≠ 0 then verify the relation between the zeros and its coefficients.

Sol. Since a and b are the zeros of polynomial $ax^2 + bx + c$.

Therefore, (x – α), (x – β) are the factors of the polynomial $ax^2 + bx + c$.

$\Rightarrow ax^2 + bx + c = k(x - \alpha)(x - \beta)$

$\Rightarrow ax^2 + bx + c = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

$\Rightarrow ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + k\alpha\beta$ …(1)

Comparing the coefficients of x^2, x and constant terms of (1) on both sides, we get

$a = k$, $b = -k(\alpha + \beta)$ and $c = k\alpha\beta$

$\Rightarrow \alpha + \beta = \frac{-b}{k}$ and $\alpha\beta = \frac{c}{k}$

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ [∵ k = a]

Sum of the zeros $= \frac{-b}{a} = \frac{\text{- coefficient of x}}{\text{coefficient of x}^2}$

Product of the zeros $= \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of x}^2}$

Example 5: Prove relation between the zeros and the coefficient of the quadratic polynomial
$ax^2 + bx + c$.

Sol. Let a and b be the zeros of the polynomial $ax^2 + bx + c$
$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ ….(1)
$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$ ….(2)
By adding (1) and (2), we get
$\alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$
$= \frac{-2b}{2a} = \frac{-b}{a} = \frac{\text{- coefficient of x}}{\text{coefficient of x}^2}$
Hence, sum of the zeros of the polynomial
$ax^2 + bx + c$ is $\frac{-b}{a}$
By multiplying (1) and (2), we get
$\alpha\beta = \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a}$
$= \frac{b^2-b^2+4ac}{4a^2}$
$= \frac{4ac}{4a^2} = \frac{c}{a}$
$= \frac{\text{constant term}}{\text{coefficient of x}^2}$
Hence, product of zeros $= \frac{c}{a}$

Example 6: find the zeroes of the quadratic polynomial $x^2 – 2x – 8$ and verify a relationship between zeroes and its coefficients.

Sol. $x^2 – 2x – 8 = x^2 – 4x + 2x – 8$
$= x (x – 4) + 2 (x – 4) = (x – 4) (x + 2)$
So, the value of $x^2 – 2x – 8$ is zero when
$x – 4 = 0$ or $x + 2 = 0$ i.e., when $x = 4$ or $x = – 2$.
So, the zeroes of $x^2 – 2x – 8$ are 4, – 2.
Sum of the zeroes
$= 4 – 2 = 2 = -\frac{(-2)}{1} = \frac{\text{- coefficient of x}}{\text{coefficient of x}^2}$
Product of the zeroes
$= 4 (–2) = –8 = \frac{-8}{1} = \frac{\text{constant term}}{\text{coefficient of x}^2}$

Example 7: Verify that the numbers given along side of the cubic polynomials are their zeroes. Also verify the relationship between the zeroes and the coefficients. $2x^3 + x^2 – 5x + 2$; , 1, – 2

Sol. Here, the polynomial p(x) is $2x^3 + x^2 – 5x + 2$
Value of the polynomial $2x^3 + x^2 – 5x + 2$

when x = 1/2
= $2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$ = $\frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$ = 0
So, 1/2 is a zero of p(x).
On putting x = 1 in the cubic polynomial
$2x^3 + x^2 - 5x + 2$
= 2(1)3 + (1)2 −¬ 5(1) + 2 = 2 + 1 – 5 + 2 = 0
On putting x = – 2 in the cubic polynomial
$2x^3 + x^2 - 5x + 2$
= 2(–2)3 + (–2)2 – 5 (–2) + 2
= – 16 + 4 + 10 + 2 = 0
Hence, $\frac{1}{2}$, 1, – 2 are the zeroes of the given polynomial.
Sum of the zeroes of p(x)
= $\frac{1}{2}$ + 1 – 2 = $\frac{-1}{2}$ = $\frac{-\text{ coefficient of } x^2}{\text{coefficient of } x^3}$
Sum of the products of two zeroes taken at a time
= $\frac{1}{2}$ × 1 + $\frac{1}{2}$ × (–2) + 1 × (–2)
= $\frac{1}{2}$ – 1 – 2 = $\frac{-5}{2}$ = $\frac{\text{coefficient of } x}{\text{coefficient of } x^3}$
Product of all the three zeroes
= $\frac{1}{2}$ × (1) × (–2) = –1
= $\frac{-2}{2}$ = $\frac{-\text{ constant term}}{\text{coefficient of } x^3}$

How To Form A Polynomial With The Given Zeroes

Let zeros of a quadratic polynomial be α and β.
x = β,   x = β
x – α = 0,   x – β = 0
The obviously the quadratic polynomial is
(x – α) (x – β)
i.e., $x^2 - (\alpha + \beta)\, x + \alpha\beta$
x^2 – (Sum of the zeros)x + Product of the zeros

Form A Polynomial With The Given Zeros Example Problems With Solutions

Example 1: Form the quadratic polynomial whose zeros are 4 and 6.

Sol. Sum of the zeros = 4 + 6 = 10
Product of the zeros = 4 × 6 = 24
Hence the polynomial formed

= x^2 – (sum of zeros) x + Product of zeros
= $x^2 - 10x + 24$

Example 2: Form the quadratic polynomial whose zeros are –3, 5.

Sol. Here, zeros are – 3 and 5.
Sum of the zeros = – 3 + 5 = 2
Product of the zeros = (–3) × 5 = – 15
Hence the polynomial formed
= x^2 – (sum of zeros) x + Product of zeros
= $x^2 - 2x - 15$

Example 3: Find a quadratic polynomial whose sum of zeros and product of zeros are respectively $\frac{1}{2}$, – 1

Sol. Let the polynomial be $ax^2 + bx + c$ and its zeros be α and β.
(i) Here, $\alpha + \beta = \frac{1}{4}$ and $\alpha.\beta = -1$
Thus the polynomial formed
= x^2 – (Sum of zeros) x + Product of zeros
$= x^2 - \left(\frac{1}{4}\right) x - 1 = x^2 - \frac{x}{4} - 1$
The other polynomial are $k\left(x^2 - \frac{x}{4} - 1\right)$
If k = 4, then the polynomial is $4x^2 - x - 4$.

Example 4: Find a quadratic polynomial whose sum of zeros and product of zeros are respectively $\sqrt{2}$, $\frac{1}{3}$

Sol. Here, $\alpha + \beta = \sqrt{2}$, $\alpha\beta = \frac{1}{3}$
Thus the polynomial formed
= x^2 – (Sum of zeroes) x + Product of zeroes
= $x^2 - \sqrt{2}\, x + \frac{1}{3}$
Other polynomial are $k\left(x^2 - \frac{x}{3} - 1\right)$
If k = 3, then the polynomial is
$3x^2 - 3\sqrt{2}x + 1$

Example 5: Find a quadratic polynomial whose sum of zeros and product of zeros are respectively 0, √5

Sol. Here, $\alpha + \beta = 0$, $\alpha\beta = \sqrt{5}$
Thus the polynomial formed

= x^2 – (Sum of zeroes) x + Product of zeroes
= x^2 – (0) x + √5 = x2 + √5

Example 6: Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time, and product of its zeroes as 2, – 7 and –14, respectively.

Sol. Let the cubic polynomial be $ax^3 + bx^2 + cx + d$
⇒ $x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}$ …(1)
and its zeroes are α, β and γ then
$\alpha + \beta + \gamma = 2 = \frac{-b}{a}$
$\alpha\beta + \beta\gamma + \gamma\alpha = -7 = \frac{c}{a}$
$\alpha\beta\gamma = -14 = \frac{-d}{a}$
Putting the values of $\frac{b}{a}$, $\frac{c}{a}$, and $\frac{d}{a}$ in (1), we get
$x^3 + (-2)x^2 + (-7)x + 14$
⇒ $x^3 - 2x^2 - 7x + 14$

Example 7: Find the cubic polynomial with the sum, sum of the product of its zeroes taken two at a time and product of its zeroes as 0, –7 and –6 respectively.

Sol. Let the cubic polynomial be $ax^3 + bx^2 + cx + d$
⇒ $x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}$ …(1)
and its zeroes are α, β and γ then
$\alpha + \beta + \gamma = 0 = \frac{-b}{a}$
$\alpha\beta + \beta\gamma + \gamma\alpha = -7 = \frac{c}{a}$
$\alpha\beta\gamma = -6 = \frac{-d}{a}$
Putting the values of $\frac{b}{a}$, $\frac{c}{a}$, and $\frac{d}{a}$ in (1), we get
$x^3 - (0)x^2 + (-7)x + (-6)$
⇒ $x^3 - 7x + 6$

Example 8: If α and β are the zeroes of the polynomials $ax^2 + bx + c$ then form the polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$
Since α and β are the zeroes of $ax^2 + bx + c$
So $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$
Sum of the zeroes = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$
$= \frac{\frac{-b}{c}}{\frac{c}{a}} = \frac{-b}{c}$

Product of the zeroes

$= \frac{1}{\alpha} . \frac{1}{\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$

But required polynomial is

x^2 – (sum of zeroes) x + Product of zeroes

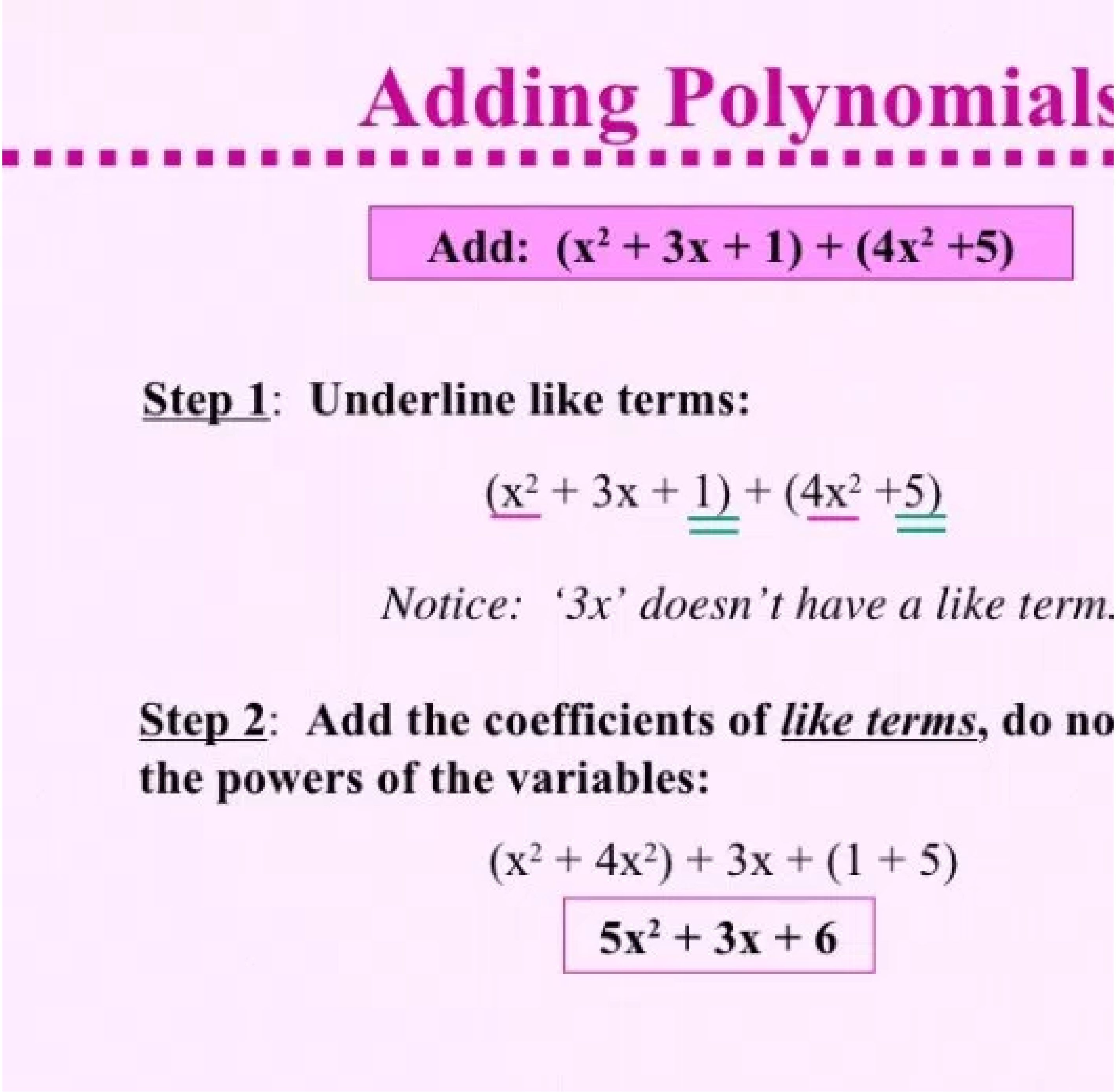
$\Rightarrow x^2 - \left(\frac{-b}{c}\right) x + \left(\frac{a}{c}\right)$

$\Rightarrow x^2 + \frac{b}{c} x + \frac{a}{c}$

$\Rightarrow c \left(x^2 + \frac{b}{c} x + \frac{a}{c}\right)$

$\Rightarrow cx^2 + bx + a$

Adding Polynomials



Adding Polynomials

Add: $(x^2 + 3x + 1) + (4x^2 + 5)$

Step 1: Underline like terms:

$(\underline{x^2} + 3x + \underline{\underline{1}}) + (\underline{4x^2} + \underline{\underline{5}})$

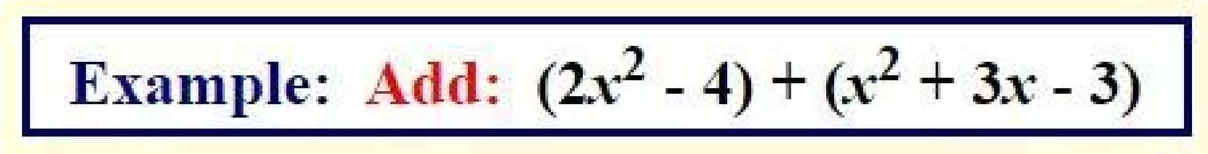
Notice: '3x' doesn't have a like term.

Step 2: Add the coefficients of *like terms*, do no the powers of the variables:

$(x^2 + 4x^2) + 3x + (1 + 5)$

$\boxed{5x^2 + 3x + 6}$

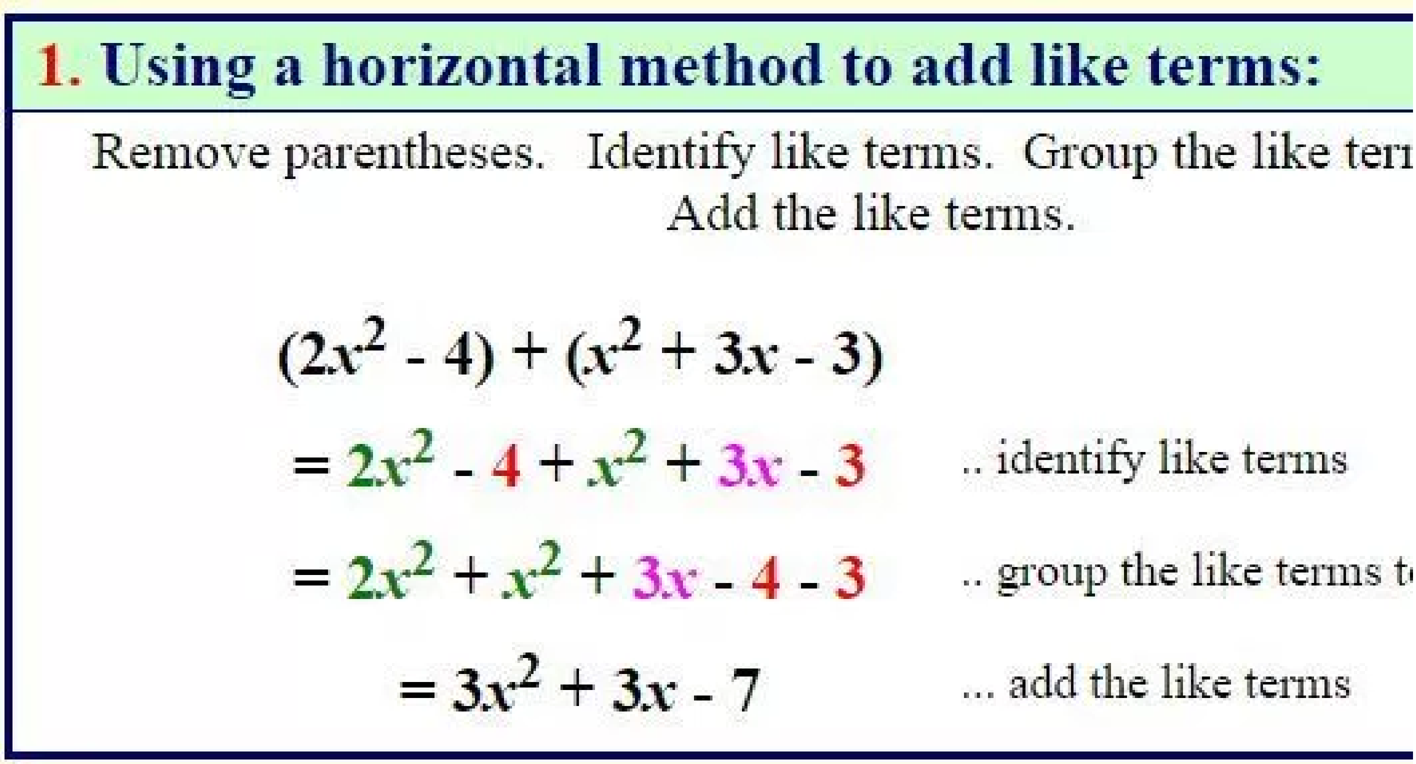
Add like terms by adding the numerical portion of the terms, following the rules for adding signed numbers.
(The numerical portion of an expression is called the coefficient.)



Example: Add: $(2x^2 - 4) + (x^2 + 3x - 3)$

Below are several different ways to attack this example:

- Using a Horizontal Method to add like terms:



1. Using a horizontal method to add like terms:

Remove parentheses. Identify like terms. Group the like ter
Add the like terms.

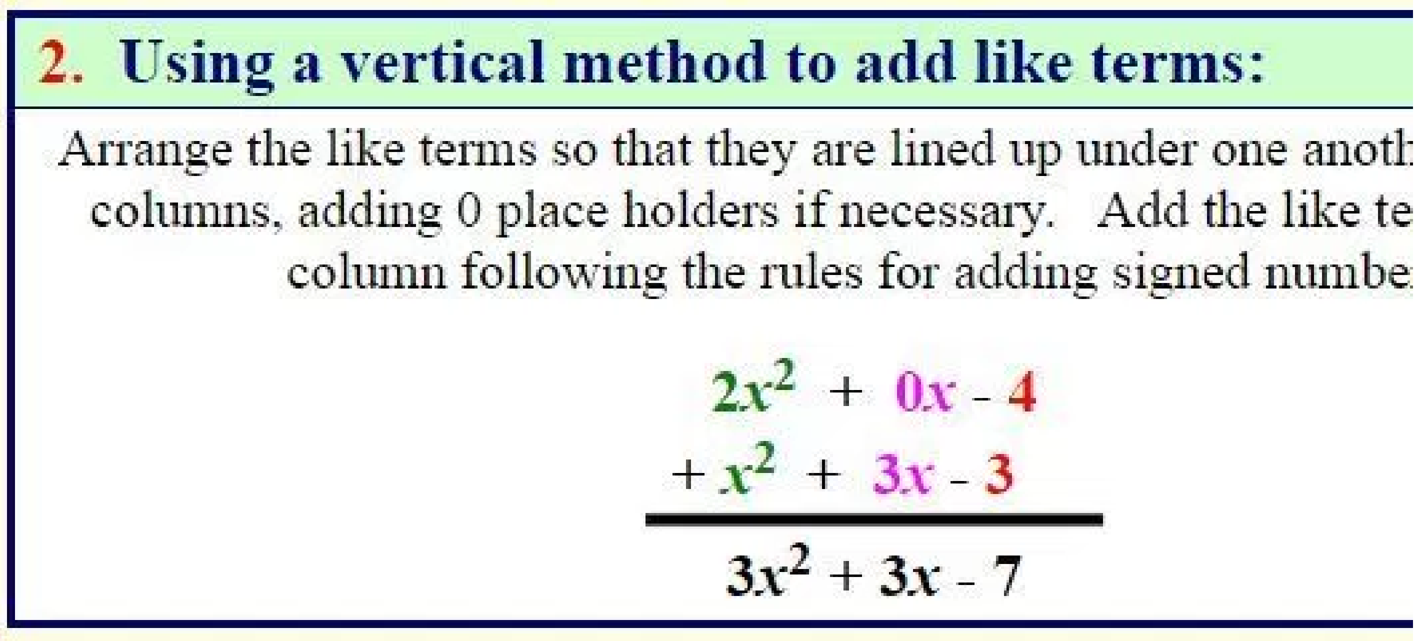
$$(2x^2 - 4) + (x^2 + 3x - 3)$$

$$= 2x^2 - 4 + x^2 + 3x - 3$$ .. identify like terms

$$= 2x^2 + x^2 + 3x - 4 - 3$$ .. group the like terms t

$$= 3x^2 + 3x - 7$$ ... add the like terms

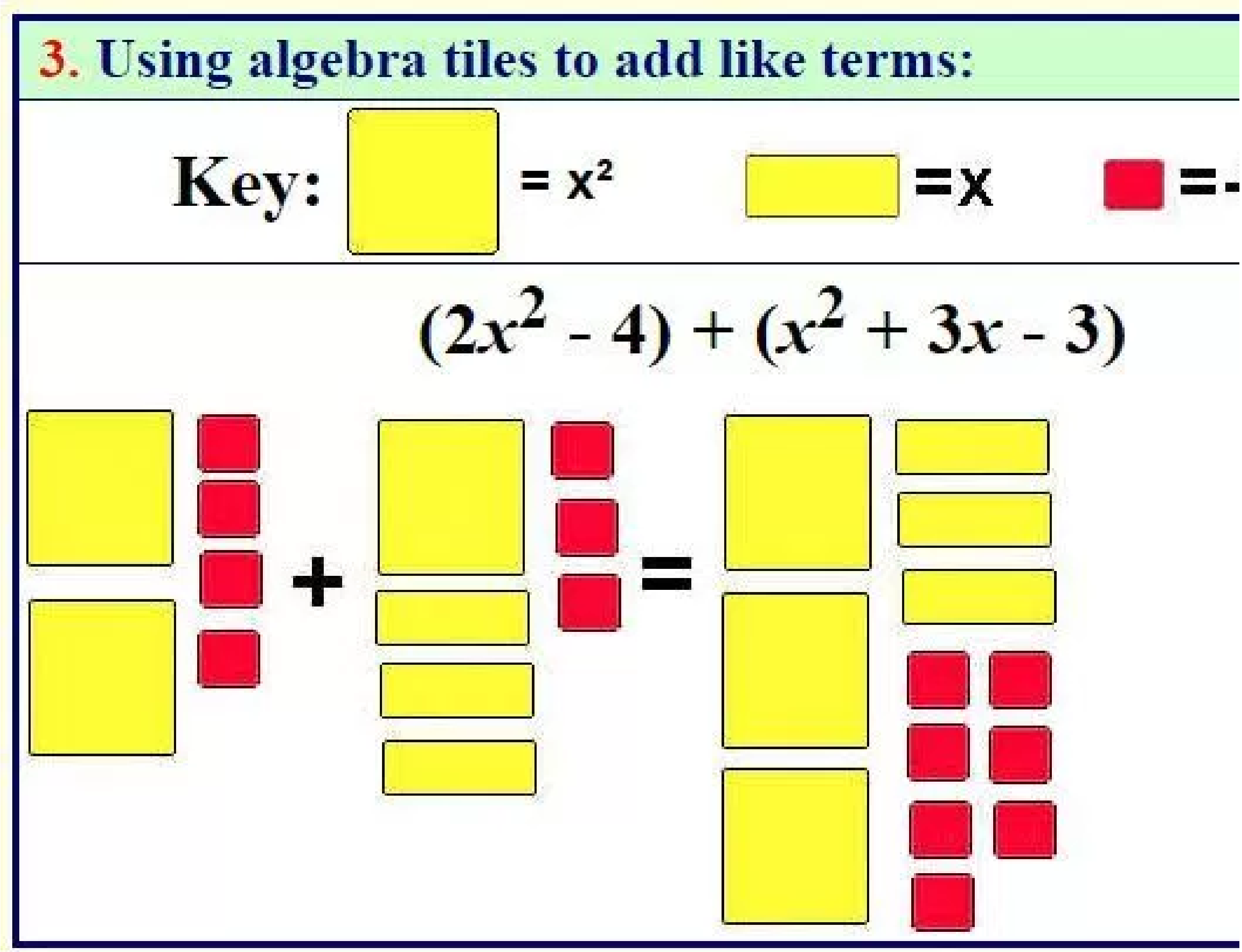
- Using a Vertical Method to add like terms:



2. Using a vertical method to add like terms:

Arrange the like terms so that they are lined up under one anoth
columns, adding 0 place holders if necessary. Add the like te
column following the rules for adding signed numbe

$$\begin{array}{r} 2x^2 + 0x - 4 \\ + x^2 + 3x - 3 \\ \hline 3x^2 + 3x - 7 \end{array}$$

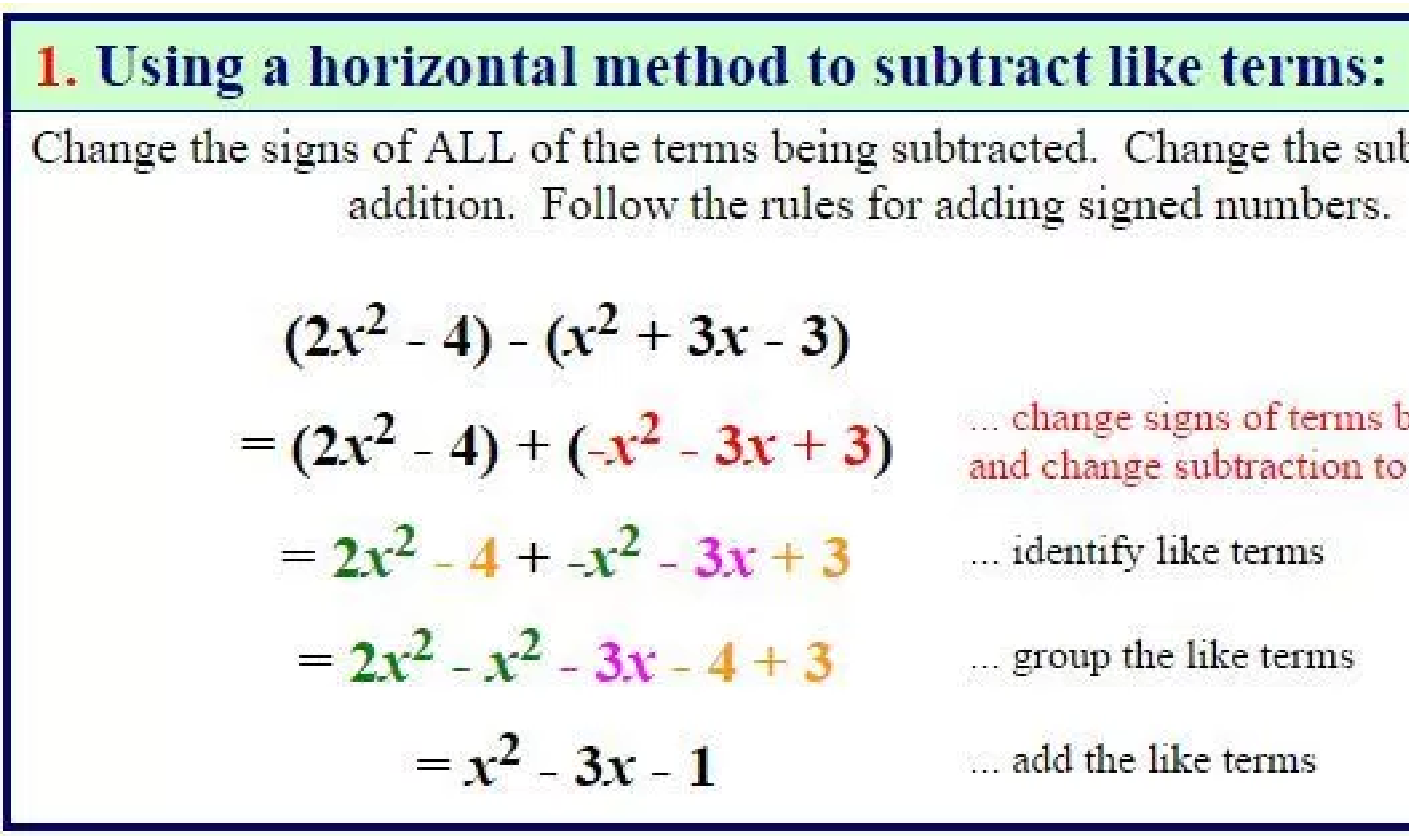


Subtracting Polynomials

Subtract like terms by changing the signs of the terms being subtracted, and following the rules for adding polynomials.

Example: **Simplify:** $(2x^2 - 4) - (x^2 + 3x - 3)$

Below are several different ways to attack this example:



1. Using a horizontal method to subtract like terms:

Change the signs of ALL of the terms being subtracted. Change the sub
addition. Follow the rules for adding signed numbers.

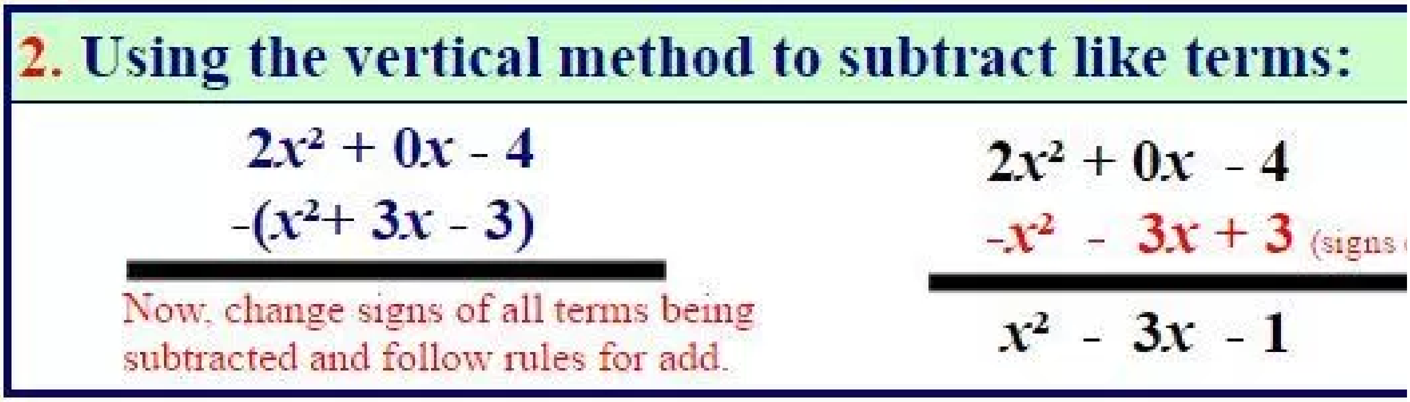
$$(2x^2 - 4) - (x^2 + 3x - 3)$$

$$= (2x^2 - 4) + (-x^2 - 3x + 3)$$ ... change signs of terms b
and change subtraction to

$$= 2x^2 - 4 + -x^2 - 3x + 3$$ ... identify like terms

$$= 2x^2 - x^2 - 3x - 4 + 3$$ ... group the like terms

$$= x^2 - 3x - 1$$ ... add the like terms

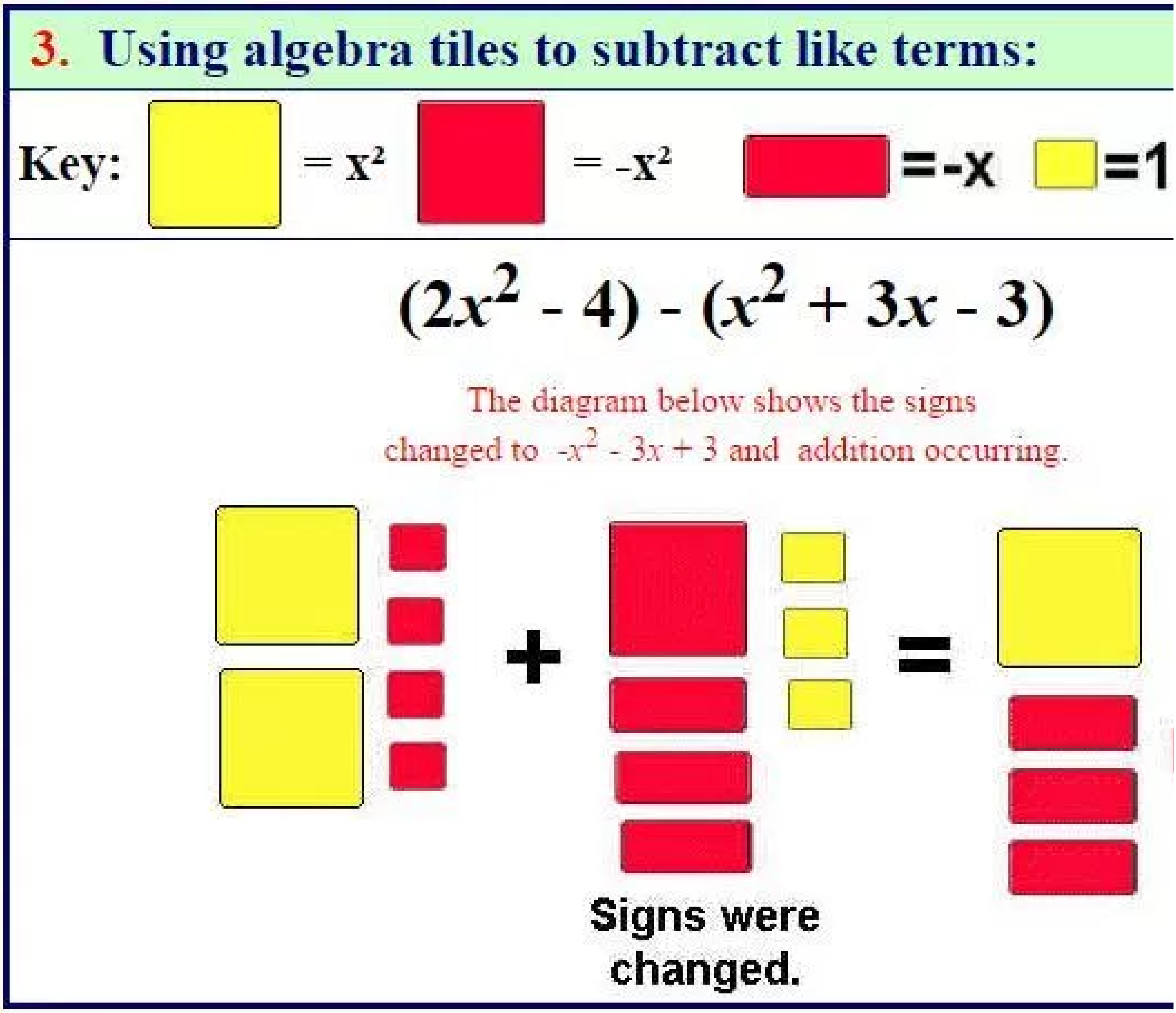


2. Using the vertical method to subtract like terms:

$$\begin{array}{r} 2x^2 + 0x - 4 \\ -(x^2 + 3x - 3) \\ \hline \end{array}$$

Now, change signs of all terms being subtracted and follow rules for add.

$$\begin{array}{r} 2x^2 + 0x - 4 \\ -x^2 - 3x + 3 \\ \hline x^2 - 3x - 1 \end{array}$$ (signs



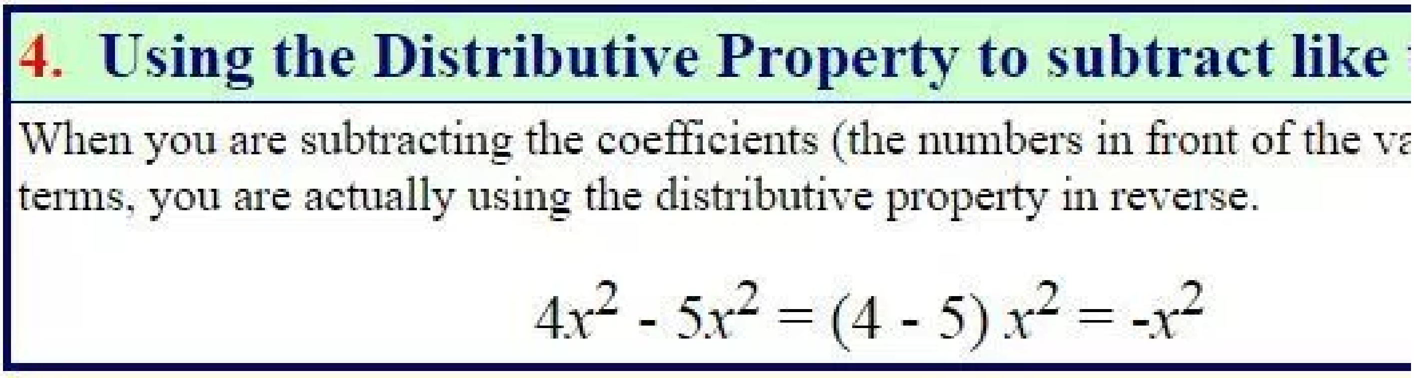
3. Using algebra tiles to subtract like terms:

Key: 🟨 = x^2 🟥 = $-x^2$ ▬ =-x ▫ =1

$$(2x^2 - 4) - (x^2 + 3x - 3)$$

The diagram below shows the signs changed to $-x^2 - 3x + 3$ and addition occurring.

Signs were changed.



4. Using the Distributive Property to subtract like

When you are subtracting the coefficients (the numbers in front of the va
terms, you are actually using the distributive property in reverse.

$$4x^2 - 5x^2 = (4 - 5)\,x^2 = -x^2$$

Dividing Polynomials

We will be examining polynomials divided by monomials and by binomials.

Steps for Dividing a Polynomial by a Monomial:

1. Divide each term of the polynomial by the monomial.
   a) Divide numbers (coefficients)
   b) Subtract exponents
   * The number of terms in the polynomial equals the number of terms in the answer when dividing by a monomial.
2. Remember that numbers do not cancel and disappear! A number divided by itself is **1.** It reduces to the number 1.
3. Remember to write the appropriate sign in between the terms.

Example:

$$\frac{16x^6 - 12x^4 + 4x^2}{4x^2}$$

$$\frac{\overset{4}{\cancel{16}}x^6}{\underset{1}{\cancel{4}}\,x^2} - \frac{\overset{3}{\cancel{12}}x^4}{\underset{1}{\cancel{4}}x^2} + \frac{\overset{1}{\cancel{4x^2}}}{\underset{1}{\cancel{4x^2}}}$$

Notice how the numbers (the coefficients) were divided.

Answer:

$$4x^4 - 3x^2 + 1$$

Notice how the exponents were subtracted.

Notice how the last term reduced to one.

The polynomial on the top has 3 terms and the answer has 3 terms.

Think about it:

$$\frac{16x^6 - 12x^4 + 4x^2}{4x^2}$$

Dividing by a number is the same process as multiplying by the reciprocal of that number.

$$\frac{1}{4x^2}\left(16x^6 - 12x^4 + 4x^2\right)$$

Notice how we used the reciprocal of $4x^2$

$$\frac{16x^6}{4x^2} - \frac{12x^4}{4x^2} + \frac{4x^2}{4x^2}$$

Now the reciprocal was distributed across the parentheses and the problem proceeds as in the example above.

Answer:

$$4x^4 - 3x^2 + 1$$

Steps for Dividing a Polynomial by a Binomial:

1. Remember that the terms in a binomial cannot be separated from one another when reducing. For example, in the binomial 2x + 3, the 2x can never be reduced unless the entire expression 2x + 3 is reduced.
2. Factor completely both the numerator and denominator before reducing.
3. Divide both the numerator and denominator by their greatest common factor.

Example 1:

$$\frac{2x+2}{x+1} = \frac{2(\cancel{x+1})}{\cancel{x+1}} = 2$$

Notice that the x+1 was reduced as a “set”.

Example 2:

$$\frac{x^2-16}{x-4} = \frac{(x+4)(\cancel{x-4})}{\cancel{x-4}} = x+4$$

Example 3:

$$\frac{x^2+7x+12}{x^2-9} = \frac{(\cancel{x+3})(x+4)}{(\cancel{x+3})(x-3)} = \frac{x+4}{x-3}$$

Example 4:

$$\frac{2-x}{4x-8} = \frac{2-x}{4(x-2)} = \frac{-1(\cancel{x-2})}{4(\cancel{x-2})} = \frac{-1}{4}$$

Tricky strategy: Notice that the -1 was factored out of the numerator to create a binomial compatible with the one in the denominator.
2 – x = -1(x – 2)

Multiplying Binomials

There are numerous ways to set up the multiplication of two binomials. The first three methods shown here work for multiplying **ALL** polynomials, not just binomials. All methods, of course, give the same answer.

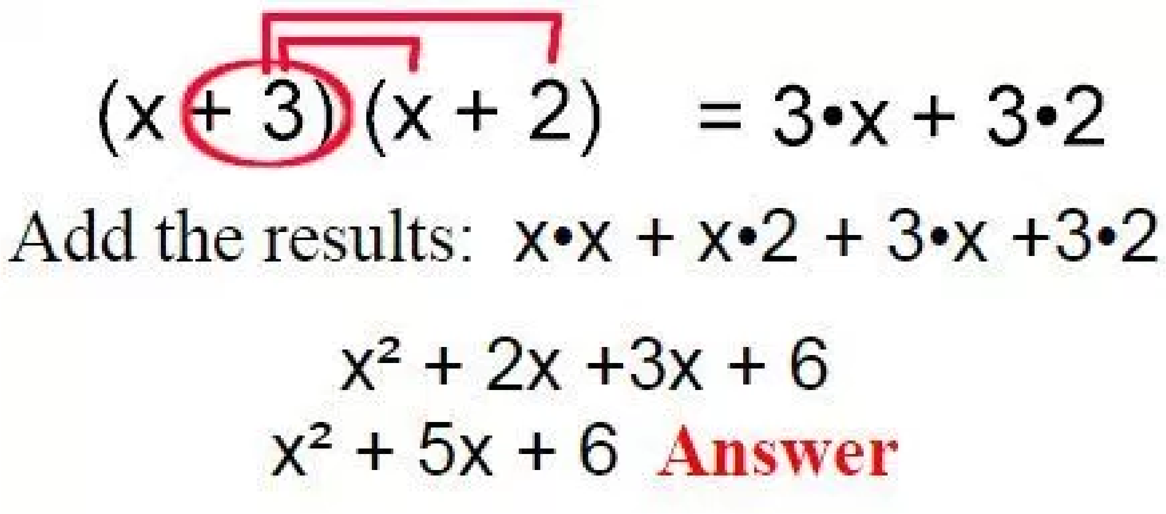
Multiply $(x + 3)(x + 2)$

1. “Distributive” Method:

The most universal method. Applies to all polynomial multiplications, not just to binomials.
Start with the first term in the first binomial – the circled blue X. Multiply (distribute) this term times EACH of the terms in the second binomial.

$$(x + 3)\ (x + 2) \quad = x\bullet x + x\bullet 2$$

Now, take the second term in the first binomial – the circled red +3 (notice we take the sign also). Multiply this term times EACH of the terms in the second binomial.

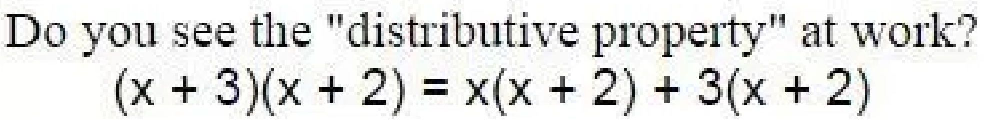


$$(x + 3)\ (x + 2) \quad = 3\bullet x + 3\bullet 2$$

Add the results: $x\bullet x + x\bullet 2 + 3\bullet x + 3\bullet 2$

$$x^2 + 2x + 3x + 6$$

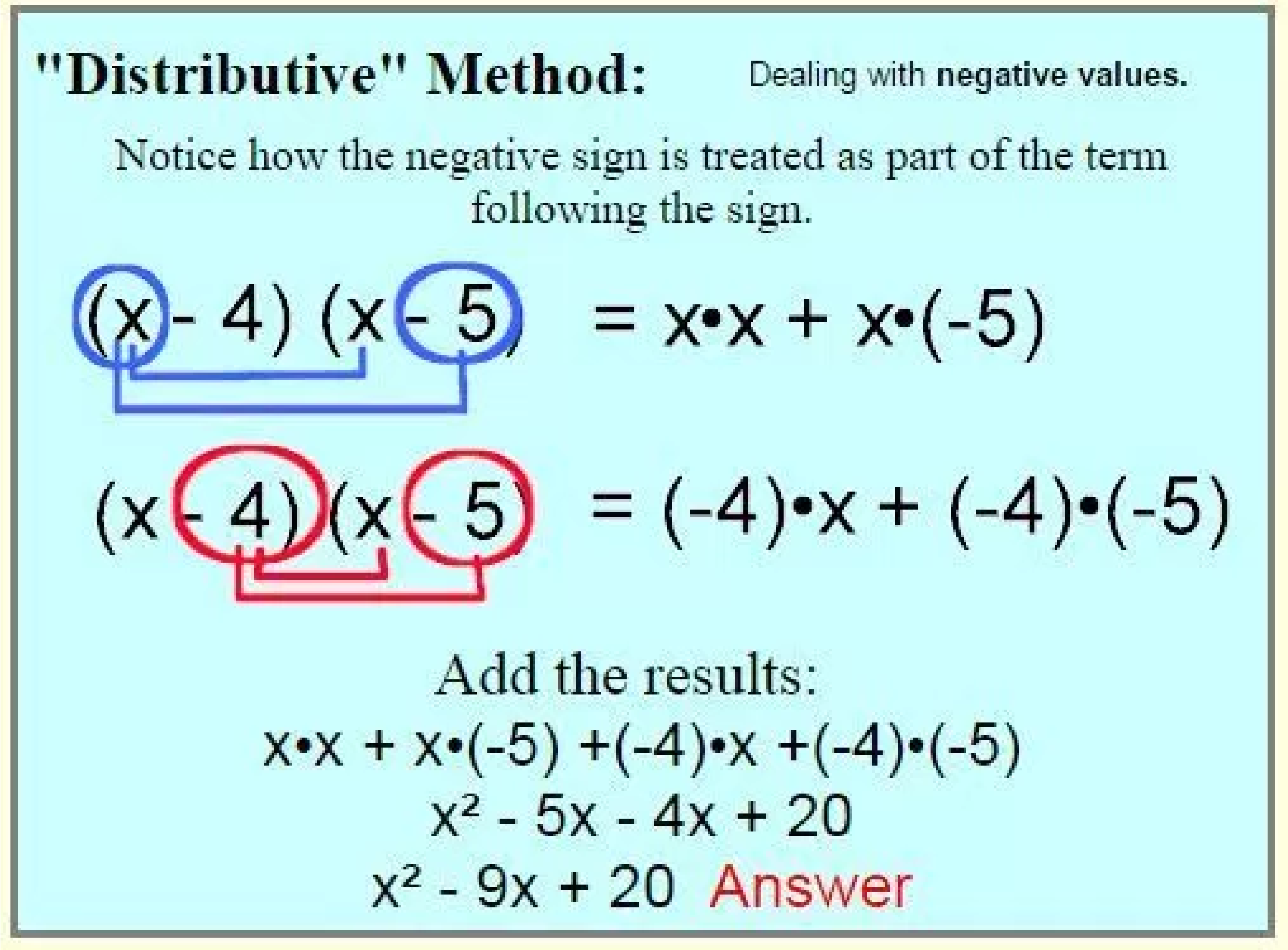
$x^2 + 5x + 6$ Answer



Do you see the "distributive property" at work?

$$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$$

Before we move on to the next set up method, let’s look at an example of the “distributive” method involving negative values.



"Distributive" Method: Dealing with **negative values.**

Notice how the negative sign is treated as part of the term following the sign.

$(x - 4)(x - 5) = x\bullet x + x\bullet(-5)$

$(x - 4)(x - 5) = (-4)\bullet x + (-4)\bullet(-5)$

Add the results:

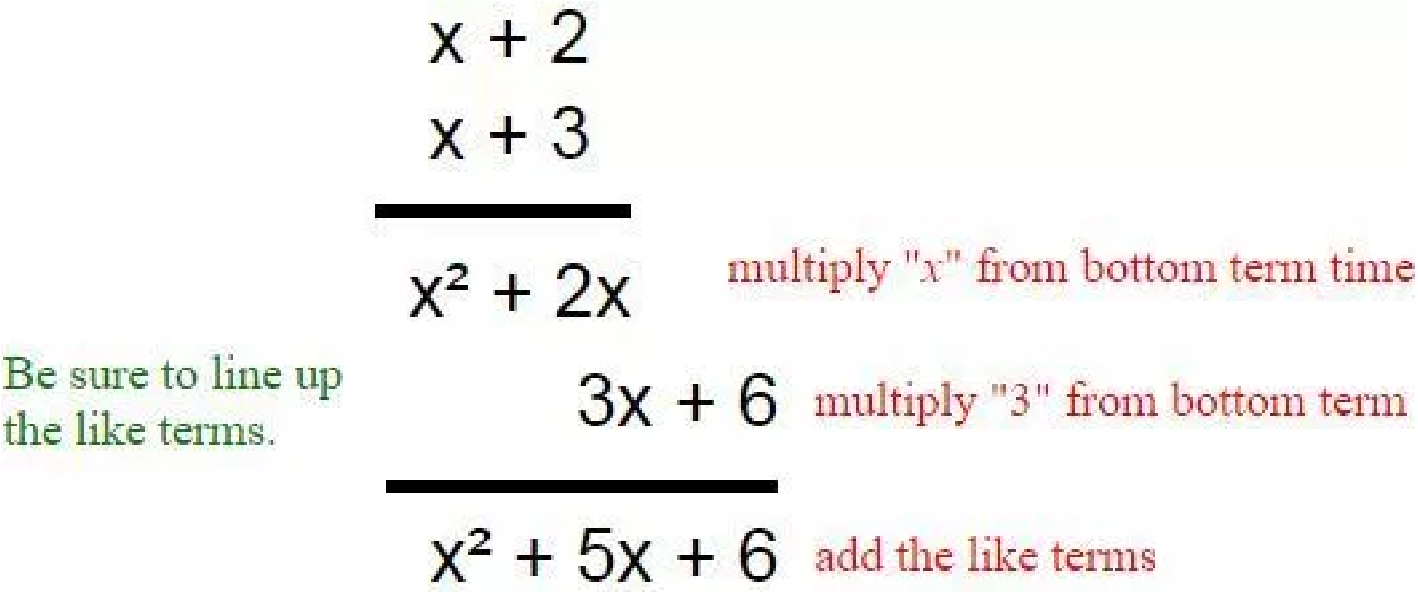
$x\bullet x + x\bullet(-5) + (-4)\bullet x + (-4)\bullet(-5)$

$x^2 - 5x - 4x + 20$

$x^2 - 9x + 20$ Answer

2. "Vertical" Method:

This is a vertical "picture" of the distributive method.
This style applies to all polynomial multiplications.



$$\begin{array}{r}
x + 2 \\
x + 3 \\
\hline
x^2 + 2x \phantom{{}+6} \\
3x + 6 \\
\hline
x^2 + 5x + 6
\end{array}$$

multiply "x" from bottom term time

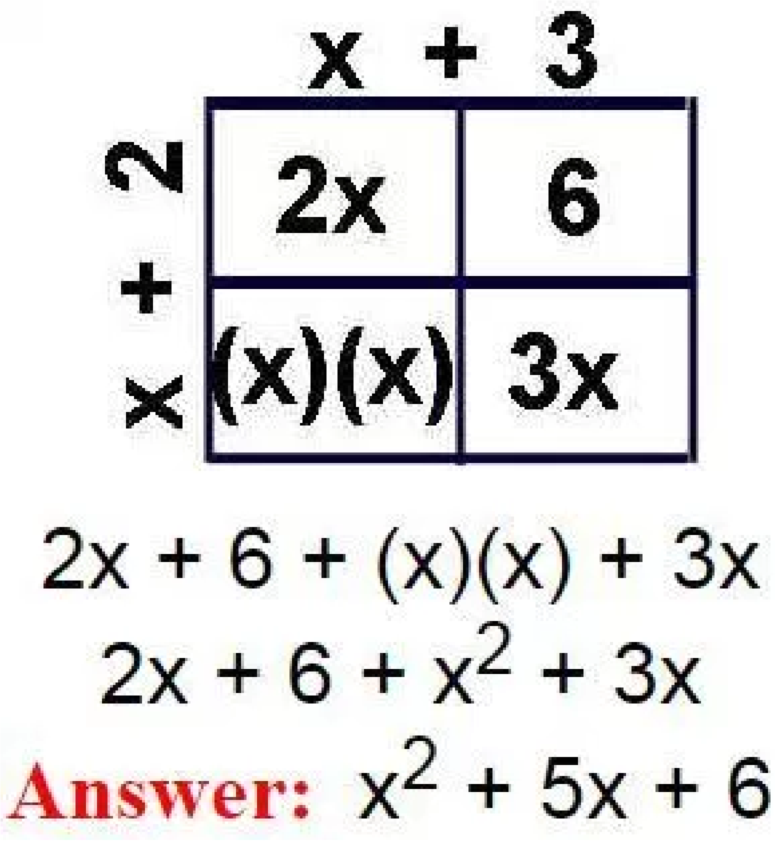
Be sure to line up the like terms.

multiply "3" from bottom term

add the like terms

3. "Grid" Method

This is a “table” version of the distributive method.
This style applies to all polynomial multiplications.
To multiply by the grid method, place one binomial at the top of a 2×2 grid (for binomials) and the second binomial on the side of the grid. Place the terms such that each term with its sign lines up with a row or column of the grid. Multiply the rows and columns of the grid to complete the interior of the grid. Finish by adding together the entries inside the grid.



	x	+ 3
2	2x	6
x +	(x)(x)	3x

$$2x + 6 + (x)(x) + 3x$$

$$2x + 6 + x^2 + 3x$$

Answer: $x^2 + 5x + 6$

C A U T I O N !!!

There are set up methods that work **ONLY** for binomials. While these set ups may be helpful to understanding binomial multiplication, you must remember that they do not extend to other types of multiplications, such as a binomial times a trinomial. You will have to go back to the “distributive method” for these other polynomial multiplications.

4. “FOIL” Method: multiply First Outer Inner Last

For Binomial Multiplication ONLY!

The words/letters used to describe the FOIL process pertain to the distributive method for multiplying two binomials. These words/letters do not apply to other multiplications such as a binomial times a trinomial.

$$\textbf{F:}\ (x+3)(x+2)$$

$$\textbf{O:}\ (x+3)(x+2)$$

$$\textbf{I:}\ (x+3)(x+2)$$

$$\textbf{L:}\ (x+3)(x+2)$$

$$\begin{aligned}(x+3)(x+2) &= x^2+2x+3x+6\\ &= x^2+5x+6\end{aligned}$$

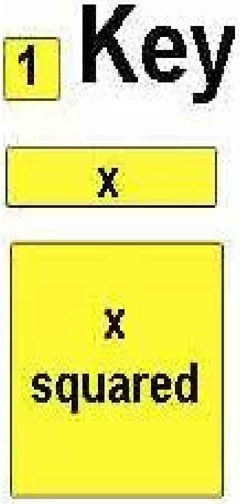
The drawback to using the FOIL lettering is that it ONLY WORKS on binomial multiplicatior

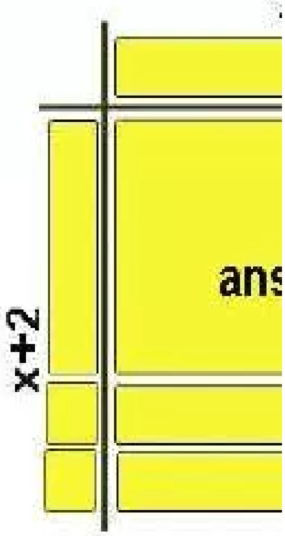
5. “Algebra Tile” Method

While this method is helpful for understanding how binomials are multiplied, it is not easily applied to ALL multiplications and may not be practical for overall use.

The example shown here is for binomial multiplication only!
To multiply binomials using algebra tiles, place one expression at the top of the grid and the second expression on the side of the grid. You MUST maintain straight lines when you are filling in the center of the grid. The tiles needed to complete the inner grid will be your answer.

answer.





Answer: x^2+5x+6

Division Algorithm For Polynomials

If p(x) and g(x) are any two polynomials with
$g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that
$p(x) = q(x) \times g(x) + r(x)$
where $r(x) = 0$ or degree of r(x) < degree of g(x).
The result is called Division Algorithm for polynomials.

Dividend = Quotient × Divisor + Remainder

Polynomials – Long Division

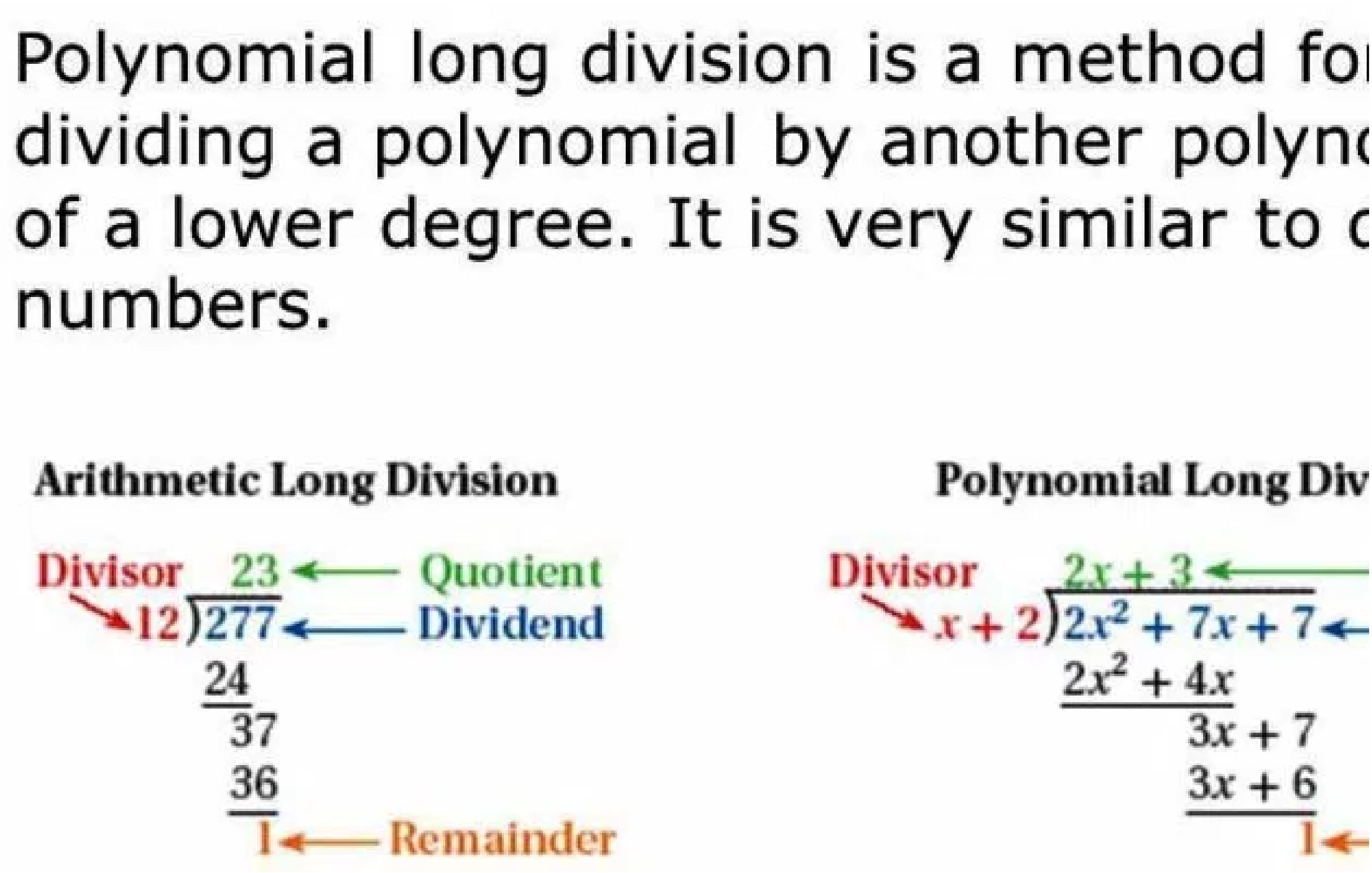
Working rule to Divide a Polynomial by Another Polynomial:

Step 1: First arrange the term of dividend and the divisor in the decreasing order of their degrees.

Step 2: To obtain the first term of quotient divide the highest degree term of the dividend by the highest degree term of the divisor.

Step 3: To obtain the second term of the quotient, divide the highest degree term of the new dividend obtained as remainder by the highest degree term of the divisor.

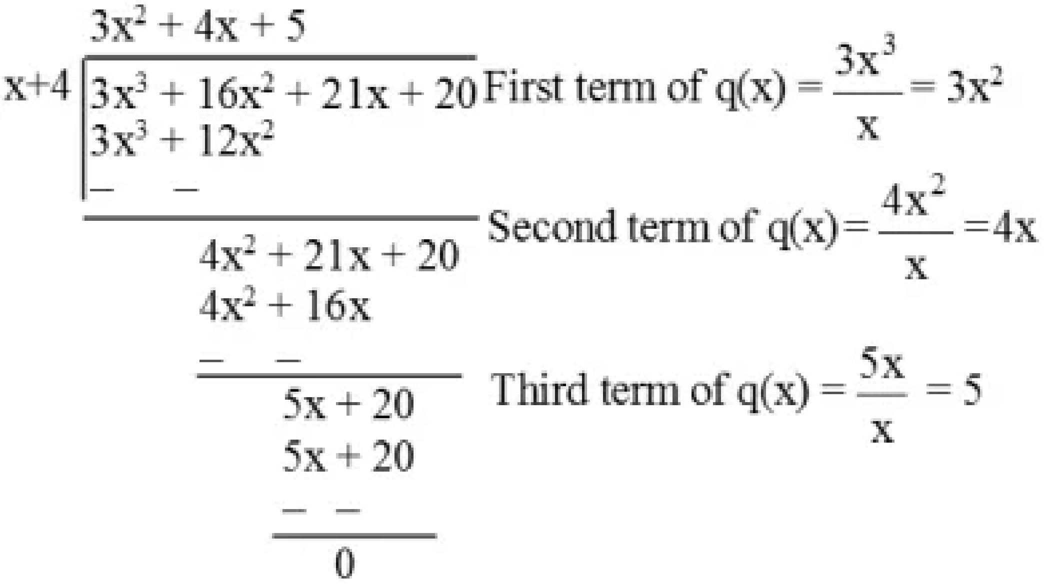
Step 4: Continue this process till the degree of remainder is less than the degree of divisor.



Polynomial long division is a method fo
dividing a polynomial by another polyn
of a lower degree. It is very similar to
numbers.

Division Algorithm For Polynomials With Examples

Example 1: Divide $3x^3 + 16x^2 + 21x + 20$ by $x + 4$.
Sol.



First term of q(x) $= \frac{3x^3}{x} = 3x^2$

Second term of q(x)$= \frac{4x^2}{x} = 4x$

Third term of q(x) $= \frac{5x}{x} = 5$

Quotient = $3x^2 + 4x + 5$
Remainder = 0

Example 2: Apply the division algorithm to find the quotient and remainder on dividing p(x) by g(x) as given below :

$p(x) = x^3 – 3x^2 + 5x – 3$ and $g(x) = x^2 – 2$

Sol. We have,
$p(x) = x^3 – 3x^2 + 5x – 3$ and $g(x) = x^2 – 2$

$$\begin{array}{r|l} & x-3 \\ \hline x^2-2 & x^3-3x^2+5x-3 \\ & x^3 \qquad\quad -2x \\ & - \qquad\quad + \\ \hline & \quad -3x^2+7x-3 \\ & \quad -3x^2 \qquad +6 \\ & \quad + \qquad\quad - \\ \hline & \qquad\qquad 7x-9 \end{array}$$

First term of quotient is $\frac{x^3}{x^2} = x$

Second term of quotient is $\frac{-3x^2}{x^2} = -3$

We stop here since
degree of $(7x – 9)$ < degree of $(x^2 – 2)$
So, quotient = $x – 3$, remainder = $7x – 9$
Therefore,
Quotient × Divisor + Remainder
= $(x – 3) (x^2 – 2) + 7x – 9$
= $x3 – 2x – 3x^2 + 6 + 7x – 9$
= $x^3 – 3x^2 + 5x – 3$ = Dividend
Therefore, the division algorithm is verified.

Example 3: Apply the division algorithm to find the quotient and remainder on dividing p(x) by g(x) as given below
$p(x) = x^4 – 3x^2 + 4x + 5$, $g (x) = x^2 + 1 – x$

Sol. We have,
$p(x) = x^4 – 3x^2 + 4x + 5$, $g (x) = x^2 + 1 – x$

$$\begin{array}{r|l} & x^2+x-3 \\ \hline x^2-x+1 & x^4-3x^2+4x+5 \\ & x^4-x^3+x^2 \\ & - \quad + \quad - \\ \hline & \quad x^3-4x^2+4x+5 \\ & \quad x^3-x^2+x \\ & \quad - \quad + \qquad - \\ \hline & \qquad -3x^2+3x+5 \\ & \qquad -3x^2+3x-3 \\ & \qquad + \qquad - \qquad + \\ \hline & \qquad\qquad\qquad 8 \end{array}$$

We stop here since
degree of (8) < degree of $(x^2 - x + 1)$.
So, quotient = $x^2 + x - 3$, remainder = 8
Therefore,
Quotient × Divisor + Remainder
$= \ (x^2 + x - 3)(x^2 - x + 1) + 8$
$= \ x^4 - x^3 + x^2 + x^3 - x^2 + x - 3x^2 + 3x - 3 + 8$
$= \ x^4 - 3x^2 + 4x + 5 \qquad = \text{Dividend}$
Therefore the Division Algorithm is verified.

Example 4: Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm. $t^2 - 3$; $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

Sol. We divide $2t^4 + 3t^3 - 2t^2 - 9t - 12$ by $t^2 - 3$

$$\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 \qquad\quad - 6t^2 \\
 & - \qquad\qquad + \\
\hline
 & \qquad 3t^3 + 4t^2 + 9t - 12 \\
 & \qquad 3t^3 \qquad\quad - 9t \\
 & \qquad - \qquad\quad + \\
\hline
 & \qquad\quad 4t^2 \qquad\quad - 12 \\
 & \qquad\quad 4t^2 \qquad\quad - 12 \\
 & \qquad\quad - \qquad\qquad + \\
\hline
 & \qquad\qquad\qquad 0
\end{array}$$

Here, remainder is 0, so $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.
$2t^4 + 3t^3 - 2t^2 - 9t - 12 = (2t^2 + 3t + 4)(t^2 - 3)$

Example 5: Obtain all the zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$
$x = \sqrt{\frac{5}{3}}$, $x = -\sqrt{\frac{5}{3}}$
$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ Or $3x^2 - 5$ is a factor of the given polynomial.
Now, we apply the division algorithm to the given polynomial and $3x^2 -$

5.

$$\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4 \qquad\;\; -5x^2 \\
 & - \qquad\quad\;\; + \\
\hline
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3 \qquad\;\; -10x \\
 & \qquad - \qquad\quad\;\; + \\
\hline
 & \qquad\qquad 3x^2 \qquad -5 \\
 & \qquad\qquad 3x^2 \qquad -5 \\
 & \qquad\qquad - \qquad\quad\; + \\
\hline
 & \qquad\qquad\qquad 0
\end{array}$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$= (3x^2 - 5)(x^2 + 2x + 1) + 0$

Quotient $= x^2 + 2x + 1 = (x + 1)^2$

Zeroes of $(x + 1)^2$ are $-1, -1$.

Hence, all its zeroes are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, $-1, -1$.

Example 6: On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

Sol. $p(x) = x^3 - 3x^2 + x + 2$ $q(x) = x - 2$ and $r(x) = -2x + 4$

By Division Algorithm, we know that

$p(x) = q(x) \times g(x) + r(x)$

Therefore,

$x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = (x - 2) \times g(x)$

$\Rightarrow g(x) = \frac{x^3-3x^2+3x-2}{x-2}$

On dividing $x^3 - 3x^2 + x + 2$ by $x - 2$,

we get g(x)

$$
\begin{array}{r|l l}
 & x^2 - x + 1 & \\
\cline{2-2}
x-2 & x^3 - 3x^2 + 3x - 2 & \text{First term of quotient is } \frac{x^3}{x} = x \\
 & x^3 - 2x^2 & \\
 & - \quad + & \\
\cline{2-2}
 & \quad -x^2 + 3x - 2 & \text{Second term of quotient is } \frac{-x^2}{x} = -x \\
 & \quad -x^2 + 2x & \\
 & \quad + \quad - & \\
\cline{2-2}
 & \qquad x - 2 & \text{Third term of quotient is } \frac{x}{x} = 1 \\
 & \qquad x - 2 & \\
 & \qquad - \quad + & \\
\cline{2-2}
 & \qquad 0 &
\end{array}
$$

Hence, $g(x) = x^2 - x + 1$.

Example 7: Give examples of polynomials p(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)
(ii) deg q(x) = deg r(x)
(iii) deg q(x) = 0

Sol.

(i) Let $q(x) = 3x^2 + 2x + 6$, degree of q(x) = 2
$p(x) = 12x^2 + 8x + 24$, degree of p(x) = 2
Here, deg p(x) = deg q(x)

(ii) $p(x) = x^5 + 2x^4 + 3x^3 + 5x^2 + 2$
$q(x) = x^2 + x + 1$, degree of q(x) = 2
$g(x) = x^3 + x^2 + x + 1$
$r(x) = 2x^2 - 2x + 1$, degree of r(x) = 2
Here, deg q(x) = deg r(x)

(iii) Let $p(x) = 2x^4 + x^3 + 6x^2 + 4x + 12$
q(x) = 2, degree of q(x) = 0
$g(x) = x^4 + 4x^3 + 3x^2 + 2x + 6$
r(x) = 0
Here, deg q(x) = 0

Example 8: If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are a – b, a , a + b. Find a and b.

Sol. ∵ $a - b, a, a + b$ are zeros

∴ product $(a - b)\,a(a + b) = -1$

$\Rightarrow (a^2 - b^2)\,a = -1$ …(1)

and sum of zeroes is $(a - b) + a + (a + b) = 3$

$\Rightarrow 3a = 3 \Rightarrow a = 1$ …(2)

by (1) and (2)

$(1 - b^2)1 = -1$

$\Rightarrow 2 = b^2 \Rightarrow b = \pm\sqrt{2}$

∴ $a = -1$ & $b = \pm\sqrt{2}$

Example 9: If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Sol. ∵ $2 \pm \sqrt{3}$ are zeroes.

∴ $x = 2 \pm \sqrt{3}$

$\Rightarrow x - 2 = \pm$(squaring both sides)

$\Rightarrow (x - 2)^2 = 3 \quad \Rightarrow \quad x^2 + 4 - 4x - 3 = 0$

$\Rightarrow x^2 - 4x + 1 = 0$, is a factor of given polynomial

∴ other factors $= \frac{x^4 - 6x^3 - 26x^2 + 138x - 35}{x^2 - 4x + 1}$

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & \underset{-}{x^4} \underset{+}{- 4x^3} \underset{-}{+ x^2} \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & \underset{+}{-2x^3} \underset{-}{+ 8x^2} \underset{+}{- 2x} \\
\hline
 & -35x^2 + 140x - 35 \\
 & \underset{+}{-35x^2} \underset{-}{+ 140x} \underset{+}{- 35} \\
\hline
 & \times
\end{array}
$$

∴ other factors $= x^2 - 2x - 35$

$= x^2 - 7x + 5x - 35 = x(x - 7) + 5(x - 7)$

$= (x - 7)(x + 5)$

∴ other zeroes are $(x - 7) = 0 \Rightarrow x = 7$

$x + 5 = 0 \Rightarrow x = -5$

Example 10: If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k & a.

Sol.

$$
\begin{array}{rl}
 & x^2 - 4x + (8-k) \\
x^2 - 2x + k\,\big) & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + x^2k \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + x^2(16-k) - 25x + 10 \\
 & -4x^3 + x^2(8) \quad - 4xk \\
 & + \quad - \quad + \\
\hline
 & x^2[8-k] + x[4k-25] + 10 \\
 & x^2[8-k] - 2x[8-k] + k(8-k) \\
 & - \quad + \quad - \\
\hline
 & x[4k - 25 + 16 - 2k] + 10 - 8k + k^2
\end{array}
$$

According to questions, remainder is x + a

∴ coefficient of x = 1

⇒ $2k - 9 = 1$

⇒ $k = (10/2) = 5$

Also constant term = a

⇒ $k^2 - 8k + 10 = a$ ⇒ $(5)^2 - 8(5) + 10 = a$

⇒ $a = 25 - 40 + 10$

⇒ $a = -5$

∴ $k = 5, a = -5$