EXERCISE 2.3

Q No 1: Which of the following relations are functions? Give reasons. If it is a function, Determine its domain & Range.

(i) $\{(2,1),(5,1),(8,1),(11,1)(14,1)(17,1)\}$

(ii) $\{(2,1),(4,2),(6,3)(8,4),(10,5)(12,6)(14,7)\}$

(iii) $\{(1,3)(1,5)(2,5)\}$

Sol: (i) $f = \{(2,1),(5,1),(8,1),(11,1)(14,1)(17,1)\}$

Here No two ordered pairs have same first component.

$\therefore$ f is a function.

and D_f = Set of first components $= \{2,5,8,11,14,17\}$

R_f = Set of Second Component - $\{1\}$.

(ii) $f = \{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

Here No two ordered pairs have same first component.

$\therefore$ f is a function.

and $D_f = \{2,4,6,8,10,12,14\}$; $R_f = \{1,2,3,4,5,6,7\}$

(iii) $f = \{(1,3)(1,5)(2,5)\}$

Here 2 ordered pairs have same first component.

ie 1 has two images 3 and 5, which is not possible for f to be function.

$\therefore$ f is not a function.

Q No. 2 Find the range and domain of following real fns.:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9-x^2}$

Sol. (i) $f(x) = -|x| = \begin{cases} -(x) \ ; & x \geq 0 \\ -(-x) \ ; & x < 0 \end{cases}$

$\therefore$ $D_f = (-\infty, \infty)$ and $R_f = (-\infty, 0]$

(ii) $f(x) = \sqrt{9-x^2}$

For f to be define $9-x^2 \geq 0$ or $9 \geq x^2$

i.e. $x^2 \leq 9 \Rightarrow |x|^2 \leq 9$

$\Rightarrow |x| \leq 3 \Rightarrow -3 \leq x \leq 3$

$\therefore$ $D_f = \{x; -3 \leq x \leq 3\}$ is $D_f = [-3, 3]$

For Range: Put $f(x) = y$.

$\therefore$ $y = \sqrt{9-x^2}$

Clearly $y \geq 0$

Now $y^2 = 9-x^2 \Rightarrow x^2 = 9-y^2$

But $x^2 \geq 0 \Rightarrow 9-y^2 \geq 0 \Rightarrow 9 \geq y^2 \Rightarrow y^2 \leq 9$

$\therefore$ $|y|^2 \leq 9 \Rightarrow |y| \leq 3 \Rightarrow -3 \leq y \leq 3$ But $y \geq 0$

$\Rightarrow 0 \leq y \leq 3$

$\therefore$ $R_f = \{y; y \in R; 0 \leq y \leq 3\} = [0, 3]$

Q No 3: A function is defined by $f(x) = 2x-5$, Write down the value of (i) $f(0)$ (ii) $f(7)$ (iii) $f(-3)$

Sol:

Here $f(x) = 2x-5$

$\therefore$ (i) $f(0) = 2(0)-5 = 0-5 = -5$

(ii) $f(7) = 2(7)-5 = 14-5 = 9$

(iii) $f(-3) = 2(-3)-5 = -6-5 = -11$

Q No 4: The function 't' which maps the temp in degree Celsius in to temp. in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C when $t(C) = 212$

Sol.

Here $t(c) = \frac{9c}{5} + 32$

(i) $t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9 \times 28}{5} + 32 = \frac{252}{5} + 32 = 50\frac{2}{5} + 32 = 82\frac{2}{5}$

(iii) $t(-10) = \frac{9(-10)}{5} + 32 = -18 + 32 = 14$

(iv) $t(c) = 212 \Rightarrow \frac{9c}{5} + 32 = 212 \Rightarrow \frac{9}{5}c = 180 \Rightarrow c = \frac{180 \times 5}{9} = 100$

QNo.5: Find the Range of each of following function;
(i) $f(x) = 2 - 3x$; $x \in R$, $x > 0$ (ii) $f(x) = x^2 + 2$; $x \in R$ (iii) x, $x \in R$.

Sol: (i) Here $f(x) = 2 - 3x$; $x \in R$, $x > 0$.

Put $f(x) = y$. $\therefore$ $y = 2 - 3x \Rightarrow x = \frac{y-2}{-3} = \frac{2-y}{3}$

Now $x > 0 \Rightarrow \frac{2-y}{3} > 0 \Rightarrow 2 - y > 0 \Rightarrow 2 > y \Rightarrow y < 2$

$\therefore R_f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2$; $x \in R$.

Now We know that $x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$ ie $x^2 + 2 \geq 2$ ie $f(x) \geq 2$

$\Rightarrow f(x) \in [2, \infty) \Rightarrow R_f = [2, \infty)$

(iii) Here $f(x) = x$

Put $y = f(x)$

$\therefore y = x$

Now $x \in R \Rightarrow y \in R \Rightarrow f(x) \in R$

$\Rightarrow R_f = R$.

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