APPLICATIONS OF DERIVATIVES

True/False

1) Slope of the tangent to the curve $y = x^3 - 5$ at the point $x = 1$ is 3 .
2) Function f decreases where $f'(x) > 0$.
3) Slope of the normal to the curve $y = x^2 + 10$ at the point $x = -1$ is $\frac{1}{2}$.
4) Function f decreases where $f'(x) < 0$.
5) $f(x) = \sin x$ is strictly decreasing function in $\left[0, \frac{\pi}{2}\right]$.
6) The value of function f is maximum at a if $f'(a) = 0$ and $f''(a) < 0$.
7) Logarithmic function $f(x) = \log x$ is a strictly increasing function.
8) Velocity of a moving particle cannot be expressed as derivative of displacement function of the particle.
9) The value of function f is maximum or minimum at a if $f'(a) = 0$.
10) Derivative of a function is the slope of the normal to the curve represented by that function.

2 and 6 Marks Questions

1. The volume of spherical balloon is increasing at the rate of 25 c.c./s . Find the rate of change of its surface area at the instant when its radius is 5 cm.
2. A man 160 cm tall, walks away from a source of light at the top of the pole 6 m high at the rate of 1.1 m/s. How fast is the length of his shadow increasing when he is 1 m away from the pole?
3. The side of square sheet is increasing at the rate of 3 cm/s. At what rate is the area increasing when the side is 10 cm long?
4. The length 'x' of rectangle is decreasing at the rate of 3 cm/min and the width 'y' is increasing at the rate of 2 cm/min. Find the rates of change of the perimeter and the area of rectangle when x = 8 cm and y = 6 cm.
5. Water is dripping out of conical funnel at the uniform rate of 2 cc/s through a tiny hole at the vertex at the bottom. When the slant height of water is 5 cm, find the rate of decrease of the slant height of the water.
6. The radius of spherical soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of change of its volume and surface area when its radius is 4 cm.
7. Determine the intervals in which the following functions are increasing or decreasing :

(a) $f(x) = x^3 + 2x^2 - 1$ (b) $f(x) = 30 - 24x + 15x^2 - 2x^3$ (c) $f(x) = 20 - 12x + 9x^2 - 2x^3$
(d) $f(x) = 17 - 18x + 12x^2 - 2x^3$ (e) $f(x) = 20 - 9x + 6x^2 - x^3$ (f) $f(x) = 6 + 12x + 3x^2 - 2x^3$
(g) $f(x) = 2x^3 - 15x^2 + 36x + 1$ (h) $f(x) = x^3 - 6x^2 + 9x + 8$ (i) $f(x) = 2x^3 - 12x^2 + 18x + 5$

8. Find the equation of tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y = 9$.
9. Find the equation of normal to the curve $3x^2 - y^2 = 8$ parallel to the line $x + 3y = 4$.
10. Find the point(s) on the curve $y = 3x^2 - 12x + 6$ at which the tangent is parallel to $x -$ axis.
11. Find the point(s) on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to $x -$ axis.
12. Find the point(s) on the curve $y = \frac{1}{4}x^2$ where the slope of tangent is $\frac{16}{3}$.
13. Find the point(s) on the curve $y = x^2 - 4x + 2$ where the slope of tangent is 0 .
14. Find the point on the curve $y = x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line $y = 3x - 2$.
15. Find the point on the curve $y = 5x^2 - 2x^3$ at which the tangent lines are parallel to the line $y = 4x + 5$.
16. Prove that the curves $4x = y^2$ and $4xy = k$ cut at right angles if $k^2 = 512$.

17. **Using differentials, find the approximate value of the following :**

    $\sqrt{50}$, $\sqrt{80}$, $\sqrt{0.26}$, $\sqrt{0.82}$, $(127)^{1/3}$, $\sqrt[3]{0.007}$

18. **If the sum of the lengths of hypotenuse and a side of a right-angled triangle is given, show that the area is maximum, when the angle between them is 60^0.**
19. **A wire of length 25 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What could be the lengths of the two pieces so that the combined area of the square and circle is minimum?**
20. **A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. What could be the lengths of the two pieces so that the combined area of the square and equilateral triangle is minimum?**
21. **Prove that the perimeter of a right angled triangle of given hypotenuse equal to 5 cm is maximum when the triangle is isosceles.**
22. **Of all rectangles with perimeter 40 cm find the one having maximum area. Also find the area.**
23. **Find the volume of the largest cylinder that can be inscribed in a sphere of radius R.**
24. **Find the volume of largest cone that can be inscribed in a sphere of radius R.**
25. **Show that height of the cylinder of maximum volume that can be inscribed in a sphere of 30 cm is $\frac{60}{\sqrt{3}}$ cm.**
26. **A window is in the form of rectangle surmounted by a semi-circle opening. If the perimeter of window is 10 cm, find the dimensions of the window so as to admit maximum possible light through the whole opening.**
27. **Show that the height of a closed cylinder of given volume and least surface area is equal to its diameter.**

28. **Find the point on the following curves which is nearest to the given points :**
    - **(i)** $y^2 = 4x$ **; given point is** $(2, -8)$
    - **(ii)** $y^2 = 2x$ **; given point is** $(1, 4)$
    - **(iii)** $x^2 = 8y$ **; given point is** $(2, 4)$ **.**

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