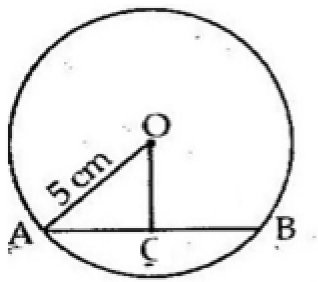
EXERCISE 17 (A)

Question 1:

A chord of length 6 cm is drawn in a circle of radius 5 cm. Calculate its distance from the centre of the circle.

Solution 1:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



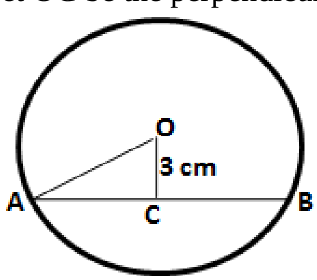
We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
$\therefore$ AC = CB = 3 cm
In ΔOCA,
$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)
$\Longrightarrow OC^2 = (5)^2 - (3)^2 = 16$
$\Longrightarrow OC = 4$ cm

Question 2:

A chord of length 8 cm is drawn at a distance of 3 cm from the centre of a circle. Calculate the radius of the circle.

Solution 2:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

∴ AB = 8 cm

$\Rightarrow$ AC = CB = $\dfrac{AB}{2}$

$\Rightarrow$ AC = CB = $\dfrac{8}{2}$

$\Rightarrow$ AC = CB = 4 cm

In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow OA^2 = (4)^2 + (3)^2 = 25$

$\Rightarrow OA = 5 \text{ cm}$

Hence, radius of the circle is 5 cm.

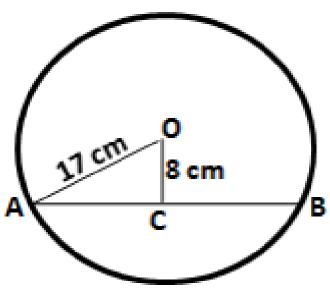
Question 3:

The radius of a circle is 17.0 cm and the length of perpendicular drawn from its centre to a chord is 8.0 cm. Calculate the length of the chord.

Solution 3:

Let AB be the chord and O be the centre of the circle.

Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

∴ AC = CB

In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow AC^2 = (17)^2 - (8)^2 = 225$

$\Rightarrow AC = 51 \text{ cm}$

∴ AB = 2 AC = 2 × 15 = 30 cm

Question 4:

A chord of length 24 cm is at a distance of 5 cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 12 cm from the centre.

Solution 4:

Let AB be the chord of length 24 cm and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.
We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
$\therefore$ AC = CB = 12 cm
In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$= (5)^2 + (12)^2 = 169$

$\Rightarrow OA = 13$ cm

$\therefore$ radius of the circle = 13 cm
Let $A'B'$ be new chord at a distance of 12 cm from the centre.

$\therefore (OA')^2 = (OC')^2 + (A'C')^2$

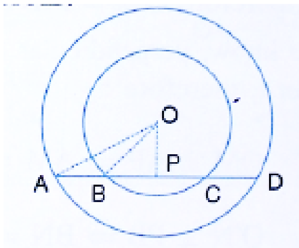
$\Rightarrow (A'C')^2 = (13)^2 - (12)^2 = 25$

$\therefore A'C' = 5\text{cm}$

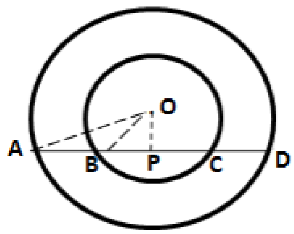
Hence, length of the new chord = 2 × 5 = 10 cm

Question 5:

In the following figure, AD is a straight line. OP $\perp$ AD and O is the centre of both the circles. If OA = 34 cm. OB = 20 cm and OP = 16cm; find the length of AB.

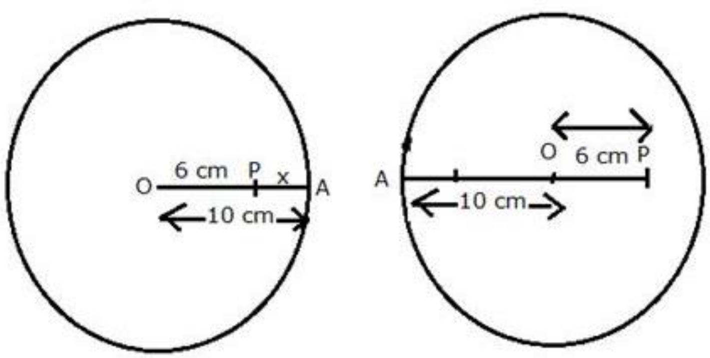


Solution 5:



For the inner circle, BC is a chord and $OP \perp BC$.
We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.
∴ BP = PC
By Pythagoras theorem,
$OA^2 = OP^2 + BP^2$
$\Rightarrow BP^2 = (20)^2 - (16)^2 = 144$
$\therefore BP = 12cm$
For the outer circle, AD is the chord and $OP \perp AD$.
We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.
∴ AP = PD
By Pythagoras Theorem,
$OA^2 = OP^2 + AP^2$
=> $AP^2 = (34)^2 - (16)^2 = 900$
=> AP = 30 cm
AB = AP − BP = 30 − 12 = 18 cm

Question 6:
O is the centre of a circle of radius 10 cm. P is any point in the circle such that OP = 6 cm. A is the point travelling along the circumference. x is the distance from A to P. what are the least and the greatest values of x in cm? what is the position of the points O, P and A at these values?
Solution 6:
The least value of x will be when A is on OP produced, i.e. O, P and A are collinear.



∴ AP = OA - OP = 10 - 6 = 4 cm.
The maximum value of x will be when A is on PO produced, i.e. A, O and P are collinear.
∴ AP = OA + OP = 10 + 6 = 16 cm.

Question 7:

In a circle of radius 17 cm, two parallel chords of lengths 30 cm and 16 cm are drawn. Find the distance between the chords, if both the chords are

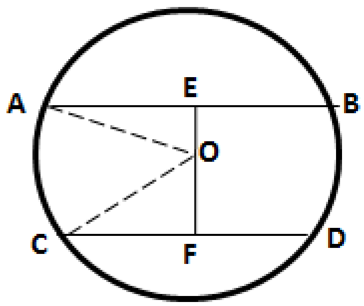
(i) on the opposite sides of the centre,

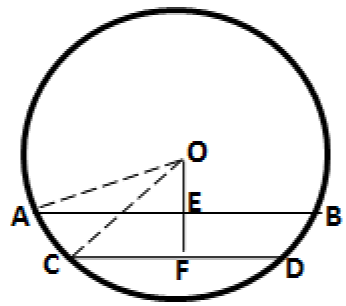
(ii) on the same side of the centre.

Solution 7:

Let O be the centre of the circle and AB and CD be the two parallel chords of length 30 cm and 16 cm respectively.

Drop OE and OF perpendicular on AB and CD from the centre O.





$\text{OP} \perp \text{AB}$ and $\text{OF} \perp \text{CD}$

$\therefore$ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \text{AE} = \frac{30}{2} = 15\text{cm}; \quad \text{CF} = \frac{16}{2} = 8\text{cm}$

In right ΔOAE,

$\text{OA}^2 = \text{OE}^2 + \text{AE}^2$

$\Rightarrow \text{OE}^2 = \text{OA}^2 - \text{AE}^2 = (17)^2 - (15)^2 = 64$

$\therefore \text{OE} = 8\text{cm}$

In right ΔOCF,

$\text{OC}^2 = \text{OF}^2 + \text{CF}^2$

$\Rightarrow \text{OF}^2 = \text{OC}^2 - \text{CF}^2 = (17)^2 - (8)^2 = 225$

$\therefore \text{OF} = 15 \text{ cm}$

(i) The chords are on the opposite sides of the centre:

$\therefore \text{EF} = \text{EO} + \text{OF} = (8 + 15) = 23 \text{ cm}$

(ii) The chords are on the same side of the centre:

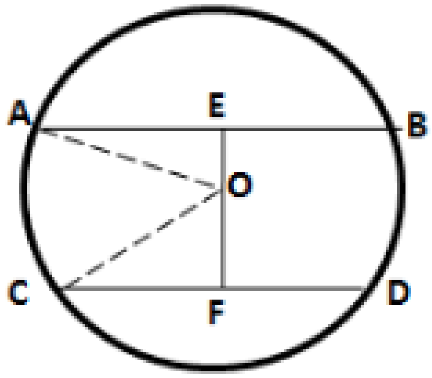
$\therefore \text{EF} = \text{OF} - \text{OE} = (15 - 8) = 7 \text{ cm}$

Question 8:

Two parallel chords are drawn in a circle of diameter 30.0 cm. The length of one chord is 24.0 cm and the distance between the two chords is 21.0 cm; find the length of another chord.

Solution 8:

Since the distance between the chords is greater than the radius of the circle (15 cm), so the chords will be on the opposite sides of the centre.



Let O be the centre of the circle and AB and CD be the two parallel chords such that AB = 24 cm.

Let length of CD be 2x cm.

Drop OE and OF perpendicular on AB and CD from the centre O.

$OE \perp AB$ and $OF \perp CD$

$\therefore$ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$$\Rightarrow AE = \frac{24}{2} = 12cm; \quad CF = \frac{2x}{2} = x \text{ cm}$$

In right ΔOAE,

$$OA^2 = OE^2 + AE^2$$

$$\Rightarrow OE^2 = OA^2 - AE^2 = (15)^2 - (12)^2 = 81$$

$$\therefore OE = 9 \text{ cm}$$

$$\therefore OF = EF - OE = (21 - 9) = 12 \text{ cm}$$

In right ΔOCF,

$$OC^2 = OF^2 + CF^2$$

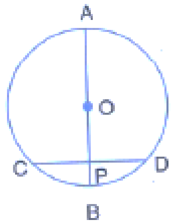
$$\Rightarrow x^2 = OC^2 - OF^2 = (15)^2 - (12)^2 = 81$$

$$\therefore x = 9 \text{ cm}$$

Hence, length of chord $CD = 2x = 2 \times 9 = 18$ cm

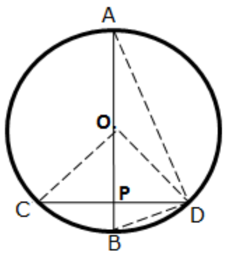
Question 9:

A chord CD of a circle whose centre is O, is bisected at P by a diameter AB.



Given OA = OB = 15 cm and OP = 9 cm. calculate the length of:
(i) CD (ii) AD (iii) CB

Solution 9:



$OP \perp CD$
$\therefore$ OP bisects CD
(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow CP = \dfrac{CD}{2}$

In right ΔOPC,

$OC^2 = OP^2 + CP^2$

$\Rightarrow CP^2 = OC^2 - OP^2 = (15)^2 - (9)^2 = 144$

$\therefore CP = 12 \text{ cm}$

$\therefore CD = 12 \times 2 = 24 \text{ cm}$

(ii) Join BD

$\therefore BP = OB - OP = 15 - 9 = 6 \text{ cm}$

In right ΔBPD,

$BD^2 = BP^2 + PD^2$

$= (6)^2 + (12)^2 = 180$

In ΔADB, $\angle$ADB = 90°
(Angle in a semicircle is a right angle)

$\therefore AB^2 = AD^2 + BD^2$

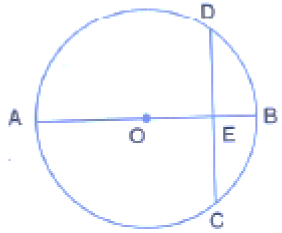
$\Rightarrow AD^2 = AB^2 - BD^2 = (30)^2 - 180 = 720$

$\therefore AD = \sqrt{720} = 26.83 \text{ cm}$

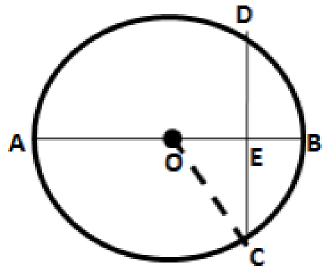
(iii) Also, $BC = BD = \sqrt{180} = 13.42 \text{ cm}$

Question 10:

The figure given below, shows a circle with centre O in which diameter AB bisects the chord CD at point E. If CE = ED = 8 cm and EB = 4cm, find the radius of the circle.



Solution 10:



Let the radius of the circle be r cm.

$\therefore OE = OB - EB = r - 4$

Join OC

In right ΔOEC,

$OC^2 = OE^2 + CE^2$

$\Rightarrow r^2 = (r-4)^2 + (8)^2$

$\Rightarrow r^2 = r^2 - 8r + 16 + 64$

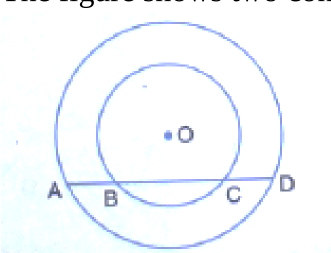
$\Rightarrow 8r = 80$

$\therefore r = 10 \text{ cm}$

Hence, radius of the circle is 10 cm.

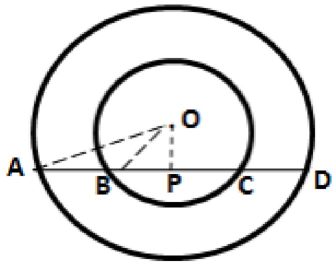
Question 11:

The figure shows two concentric circles and AD is a chord of larger circle.



Prove that: AB = CD

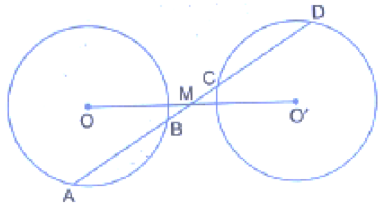
Solution 11:



Drop $OP \perp AD$
$\therefore$ OP bisects AD
(Perpendicular drawn from the centre of a circle to a chord bisects it)
$\Rightarrow AP = PD$ ……………. (i)
Now, BC is a chord for the inner circle and $OP \perp BC$
$\therefore$OP bisects BC
(Perpendicular drawn from the centre of a circle to a chord bisects it)
$\Rightarrow BP = PC$ ……………. (ii)
Subtracting (ii) from (i),
$AP - PB = PD - PC$
$\Rightarrow AB = CD$

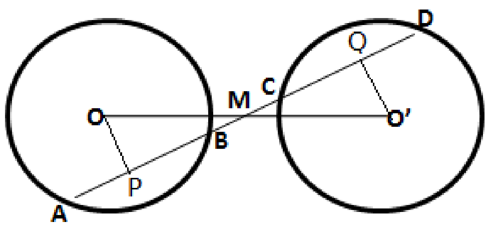
Question 12:

A straight line is drawn cutting two equal circles and passing through the mid-point M of the line joining their centres O and O'



Prove that the chords AB and CD, which are intercepted by the two circles are equal.

Solution 12:

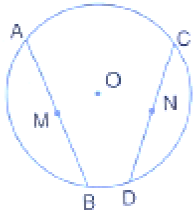


Given: A straight line Ad intersects two circles of equal radii at A, B, C and D.

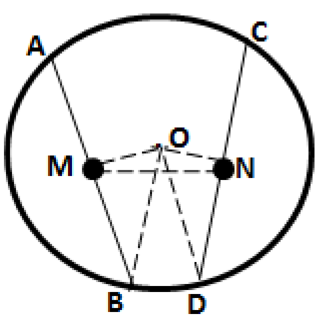
The line joining the centres OO' intersect AD at M
And M is the midpoint of OO'.
To prove: AB = CD
Construction: From O, draw $OP \perp AB$ and from O', draw $O'Q \perp CD$.
Proof:
In ΔOMP and $\Delta O'MQ$,
$\angle OMP = \angle O'MQ$ (vertically opposite angles)
$\angle OPM = \angle O'QM$ (each = 90°)
$OM = O'M$ (Given)
By Angle – Angle – Side criterion of congruence,
$\therefore \Delta OMP \cong \Delta O'MQ$, (by AAS)
The corresponding parts of the congruent triangle are congruent
$\therefore OP = O'Q$ (c.p.ct)
We know that two chords of a circle or equal circles which are equidistant from the centre are equal.
$\therefore AB = CD$

Question 13:

M and N are the mid-points of two equal chords AB and CD respectively of a circle with centre O. prove that:
(i) $\angle BMN = \angle DNM$.
(ii) $\angle AMN = \angle CNM$.



Solution 13:



Drop $OM \perp AB$ and $ON \perp CD$

$\therefore$ OM bisects AB and ON bisects CD
(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow BM = \frac{1}{2}AB = \frac{1}{2}CD = DN$ ……………(1)

Applying Pythagoras theorem,

$$OM^2 = OB^2 - BM^2$$
$$= OD^2 - DN^2 \quad \text{(by (1))}$$
$$= ON^2$$

$\therefore OM = ON$

$\Rightarrow \angle OMN = \angle ONM$ ……………(2)

(Angles opp to equal sides are equal)

(i) $\angle OMB = \angle OND$ (both 90°)

Subtracting (2) from above,

$\angle BMN = \angle DNM$

(ii) $\angle OMA = \angle ONC$ (both 90°)

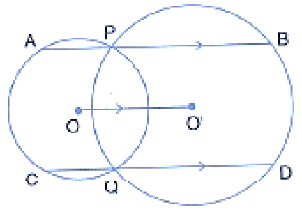
Adding (2) to above,

$\angle AMN = \angle CNM$

Question 14:

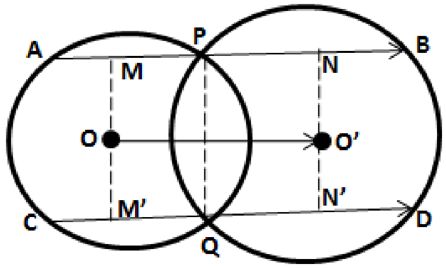
In the following figure; P and Q are the points of intersection of two circles with centres O and O'. If straight lines APB and CQD are parallel to OO'; prove that:

(i) $OO' = \frac{1}{2}AB$, (ii) AB = CD



Solution 14:

Drop OM and O'N perpendicular on AB and OM' and O'N' perpendicular on CD.



$\therefore$ $MP = \frac{1}{2}AP,\ PN = \frac{1}{2}BP,\ M'Q = \frac{1}{2}CQ,\ \ QN' = \frac{1}{2}QD$

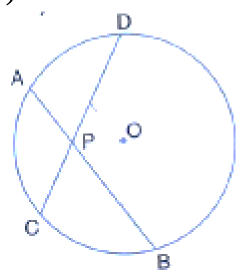
Now, $OO' = MN = MP + PN = \frac{1}{2}(AP + BP) = \frac{1}{2}AB$ ---------(i)

And $OO' = M'N' = M'Q + QN' = \frac{1}{2}(CQ + QD) = \frac{1}{2}CD$ -------(ii)

By (i) and (ii)
AB = CD

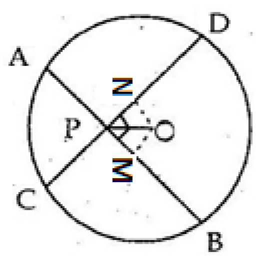
Question 15:

Two equal chords AB and CD of a circle with centre O, intersect each other at point P inside the circle, prove that:
(i) AP = CP,
(ii) BP = DP



Solution 15:

Drop OM and ON perpendicular on AB and CD.
Join OP, OB and OD.



$\therefore$ OM and ON bisect AB and CD respectively
(Perpendicular drawn from the centre of a circle to chord bisects it)

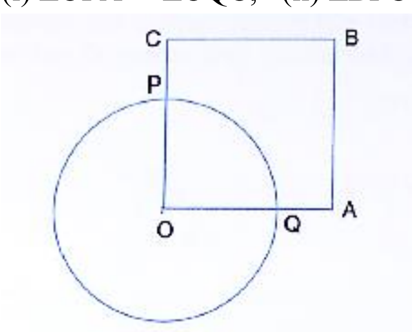
$\therefore MP = \frac{1}{2}AB = \frac{1}{2}CD = ND$ ……….(i)

In rt Δ OMB, $OM^2 = OB^2 - MB^2$ ………………..(ii)
In rt Δ OND, $ON^2 = OD^2 - ND^2$ ………………..(iii)

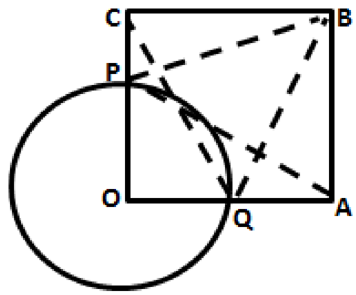
From (i),(ii) and (iii)
OM = ON
In ΔOPM and ΔOPN,
$\angle OMP = \angle ONP$ (both 90°)
OP = OP (Common)
OM = ON (Proved above)
By Right Angle – Hypotenuse – Side criterion of congruence,
$\therefore$ $\Delta OPM \cong \Delta OPN$ (by RHS)
The corresponding parts of the congruent triangles are congruent.
$\therefore$ PM = PN (c.p.c.t)
Adding (i) to both sides,
MB + PM = ND + PN
$\Longrightarrow$ BP = DP
Now, AB = CD
$\therefore$ AB – BP = CD – DP ($\because$ BP = DP)
$\Longrightarrow$ AP = CP

Question 16:

In the following figure, OABC is a square. A circle is drawn with O as centre which meets OC at P and OA at Q. Prove that:
(i) $\Delta OPA \cong \Delta OQC$, (ii) $\Delta BPC \cong \Delta BQA$.



Solution 16:

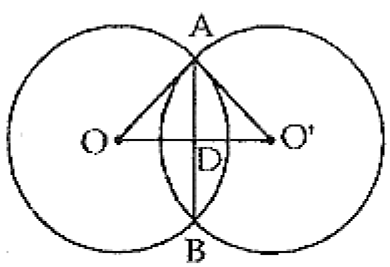


(i)
In ΔOPA and ΔOQC,
OP = OQ (radii of same circle)
$\angle AOP = \angle COQ$ (both 90°)
OA = OC (Sides of the square)
By Side – Angle – Side criterion of congruence,
$\therefore \Delta OPA \cong \Delta OQC$ (by SAS)
(ii)
Now, OP = OQ (radii)
And OC = OA (sides of the square)
$\therefore$ OC – OP = OA – OQ
$\Rightarrow$ CP = AQ …………… (1)
In ΔBPC and ΔBQA,
BC = BA (Sides of the square)
$\angle PCB = \angle QAB$ (both 90°)
PC = QA (by (1))
By Side – Angle – Side criterion of congruence,
$\therefore \Delta BPC \cong \Delta BQA$ (by SAS)

Question 17:

The length of common chord of two intersecting circles is 30 cm. If the diameters of these two circles be 50 cm and 34 cm, calculate the distance between their centres.

Solution 17:



OA = 25 cm and AB = 30 cm

$$\therefore\ AD = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 30\right) cm = 15\ cm$$

Now in right angled ΔADO,

$$OA^2 = AD^2 + OD^2$$

$$\Rightarrow OD^2 = OA^2 - OD^2 = 25^2 - 15^2$$

$$= 625 - 225 = 400$$

$\therefore\ OD = \sqrt{400} = 20$ cm

Again, we have O'A = 17 cm

In right angle $\Delta ADO'$

$O'A^2 = AD^2 + O'D^2$

$\Rightarrow O'D^2 = O'A^2 - AD^2 = 17^2 - 15^2$

$= 289 - 225 = 64$

$\therefore O'D = 8$ cm

$\therefore OO' = (OD + O'D)$

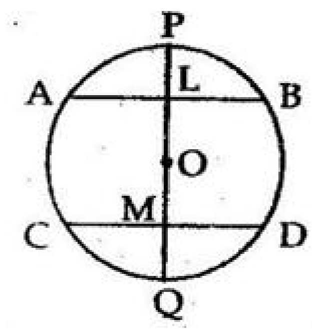
$= (20 + 8) = 28$ cm

∴ the distance between their centres is 28 cm

Question 18:

The line joining the mid-points of two chords of a circle passes through its centre. Prove that the chords are parallel.

Solution 18:



Given: AB and CD are the two chords of a circle with centre O.

L and M are the midpoints of AB and CD and O lies in the line joining ML

To prove: AB ∥ CD

Proof: AB and CD are two chords of a circle with centre O.

Line LOM bisects them at L and M

Then, OL ⊥ AB

And, OM ⊥ CD

∴ $\angle ALM = \angle LMD = 90°$

But they are alternate angles

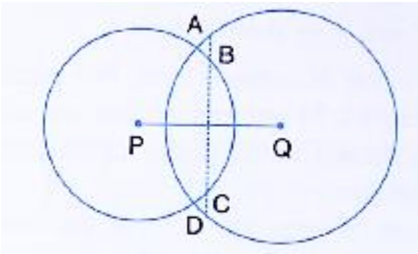
∴ AB ∥ CD.

Question 19:

In the following figure, the line ABCD is perpendicular to PQ; where P and Q are the centres of the circles. Show that:

(i) AB = CD,

(ii) AC = BD.



Solution 19:

In the circle with centre Q, $QO \perp AD$

$\therefore \quad OA = OD \quad \ldots\ldots\ldots\ldots\ldots (1)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

In the circle with centre P, $PO \perp BC$

$\therefore \; OB = OC \quad \ldots\ldots\ldots\ldots\ldots..(2)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

(i)

(1) – (2) Gives,

$AB = CD \quad \ldots\ldots\ldots\ldots\ldots(3)$

(ii)

Adding BC to both sides of equation (3)

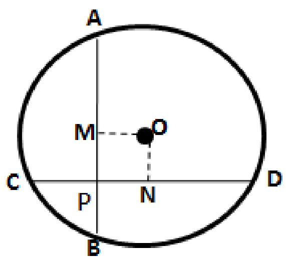
$AB + BC = CD + BC$

$\Rightarrow AC = BD$

Question 20:

AB and CD are two equal chords of a circle with centre O which intersect each other at right angle at point P. If $OM \perp AB$ and $ON \perp CD$; show that OMPN is a square.

Solution 20:



Clearly, all the angles of OMPN are 90°

$OM \perp AB$ and $ON \perp CD$

$\therefore$ $BM = \frac{1}{2}AB = \frac{1}{2}CD = CN$ ………….. (i)

(Perpendicular drawn from the centre of a circle to a chord bisects it)

As the two equal chords AB and CD intersect at point P inside

The circle,

$\therefore$ $AP = DP$ and $CP = BP$ ……………. (ii)

Now, $CN - CP = BM - BP$ (by (i) and (ii))

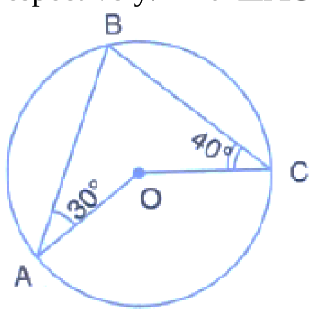
$\Rightarrow PN = MP$

$\therefore$ Quadrilateral OMPN is a square

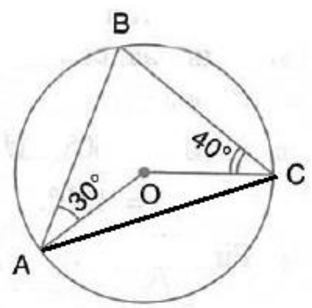
EXERCISE. 17 (B)

Question 1:

In the given figure, O is the centre of the circle. $\angle OAB$ and $\angle OCB$ are 30° and 40° respectively. Find $\angle AOC$. Show your steps of working.



Solution 1:



Join AC,

Let $\angle OAC = \angle OCA = x$ (say)

$\therefore \angle AOC = 180° - 2x$

Also, $\angle BAC = 30° + x$

$\angle BCA = 40° + x$

In ΔABC,

$\angle ABC = 180° - \angle BAC - \angle BCA$

$= 180° - (30° + x) - (40° + x) = 110° - 2x$

Now, $\angle AOC = 2\angle ABC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow 180° - 2x = 2(110° - 2x)$

$\Rightarrow 2x = 40°$

$\therefore x = 20°$

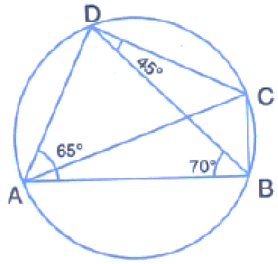
$\therefore \angle AOC = 180° - 2 \times 20° = 140°$

Question 2:

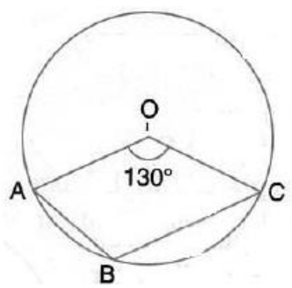
In the figure, $\angle BAD = 65°$, $\angle ABD = 70°$, $\angle BDC = 45°$

(i) Prove that AC is a diameter of the circle

(ii) Find $\angle ACB$



Solution 2:



Here, Reflex $\angle AOC = 2\angle ABC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow 360° - 130° = 2\angle ABC$

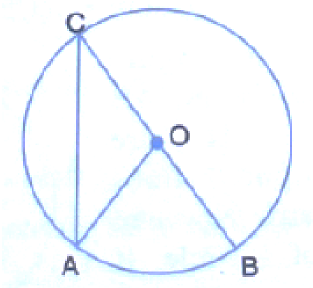
$\Rightarrow \angle ABC = \frac{230°}{2} = 115°$

Question 3:

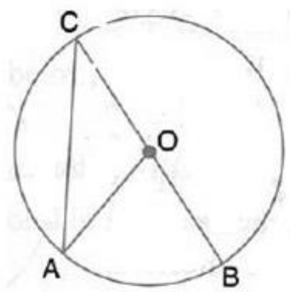
Given O is the centre of the circle and $\angle AOB = 70°$. Calculate the value of:

(i) $\angle OCA$,

(ii) $\angle OCA$.



Solution 3:



Here, $\angle AOB = 2\angle ACB$

(Angle at the center is double the angle at the circumference by the same chord)

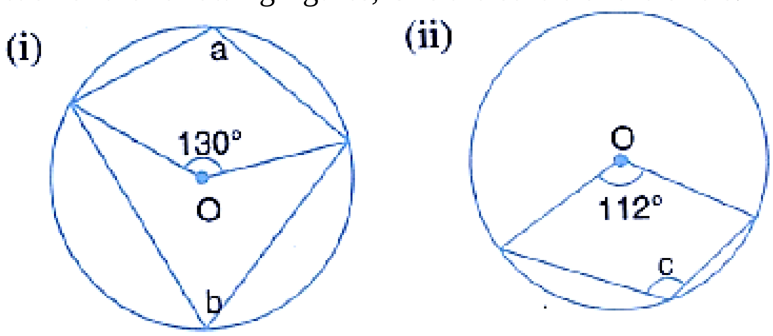
$$\Rightarrow \angle ACB = \frac{70°}{2} = 35°$$

Now, $OC = OA$ (radii of same circle)

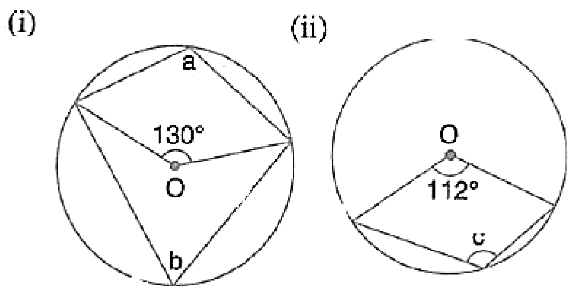
$\Rightarrow \angle OCA = \angle OAC = 35°$

Question 4:

In each of the following figures, O is the centre of the circle. Find the values of a, b and c.



Solution 4:



(i) Here, $b = \frac{1}{2} \times 130°$

(Angle ate he centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow b = 65°$

Now, $a + b = 180°$

(Opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow a = 180° - 65° = 115°$

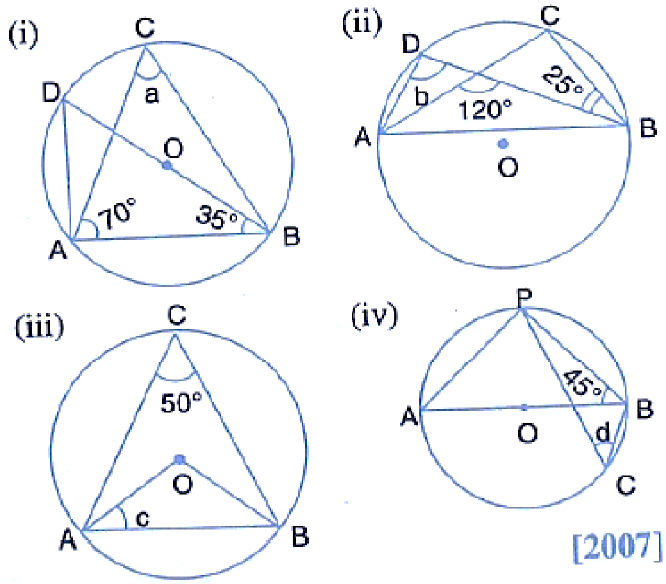
(ii) Here, $c = \frac{1}{2}$ Reflex $(112°)$

(Angle at the cente is double the angle at the circumference subtended by the same chord)

$\Rightarrow c = \frac{1}{2} \times (360° - 112°) = 124°$

Question 5:

In each of the following figures, O is the centre of the circle. Find the values of a, b, c and d.



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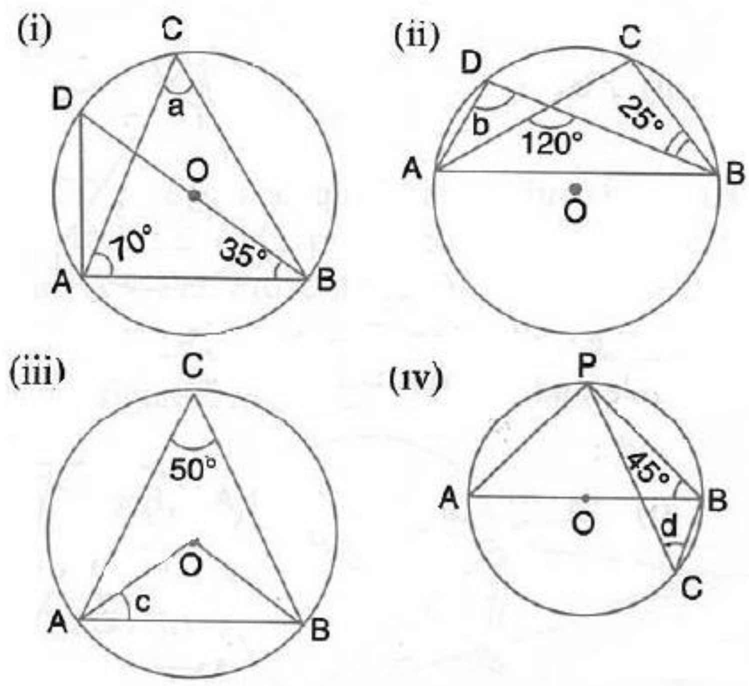
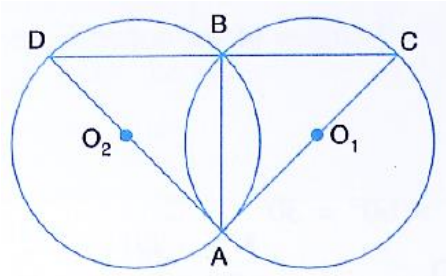
Solution 5:



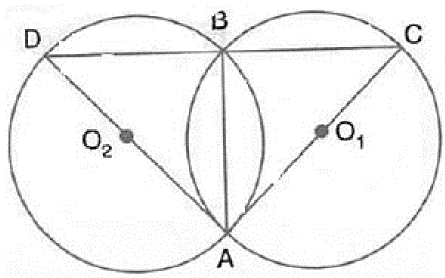
(i) Here, $\angle BAD = 90°$(Angle in a semicircle)
$\therefore \angle BDA = 90° - 35° = 55°$
Again, $a = \angle ACB = \angle BDA = 55°$
(Angle subtended by the same chord on the circle are equal)
(ii) Here, $\angle DAC = \angle CBD = 25°$
(Angle subtended by the same chord on the circle are equal)
Again, $120° = b + 25°$
(In a triangle, measure of exterior angle is equal to the sum of pair of opposite interior angles)
$\Rightarrow b = 95°$
(iii) $\angle AOB = 2\angle AOB = 2 \times 50° = 100°$
(Angle at the centre is double the angle at the circumference subtended by the same chord)
Also, $OA = OB$
$\Rightarrow \angle OBA = \angle OAB = C$
$\therefore c = \dfrac{180° - 100°}{2} = 40°$
(iv) $\angle APB = 90°$ (Angle in a semicircle)
$\therefore \angle BAP = 90° - 45° = 45°$
Now, $d = \angle BCP = \angle BAP = 45°$
(Angle subtended by the same chord on the circle are equal)

Question 6:

In the figure, AB is common chord of the two circle. If AC and AD are diameters; prove that D, B and C are in a straight line. O_1 and O_2 are the centres of two circles.



Solution 6:



$\angle DBA = 90°$ and $\angle CBA = 90°$

(Angles in a semicircle is a right angle)

Adding both we get,

$\angle DBC = 180°$

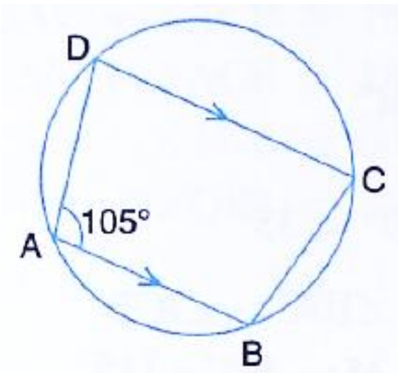
$\therefore$ D, B and C form a straight line.

Question 7:

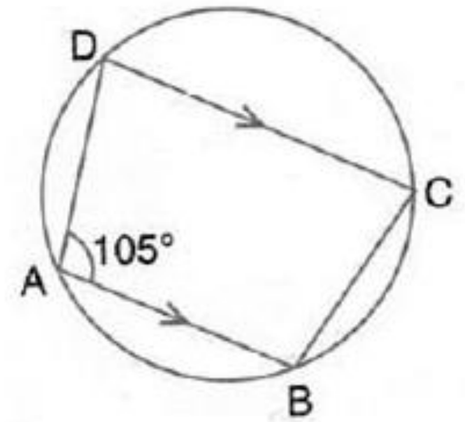
In the figure, given below, find:

(i) $\angle BCD$, (ii) $\angle ADC$, (iii) $\angle ABC$,

Show steps of your working



Solution 7:



(i) $\angle BCD + \angle BAD = 180°$

(Sum of opposite angles of a cyclic quadrilateral is $180°$)

$\Rightarrow \angle BCD = 180° - 105° = 75°$

(ii) Now, AB || CD

$\therefore \angle BAD + \angle ADC = 180°$

(Interior angles on the same side of parallel lines is $180°$)

$\Rightarrow \angle ADC = 180° - 105° = 75°$

(iii) $\angle ADC + \angle ABC = 180°$

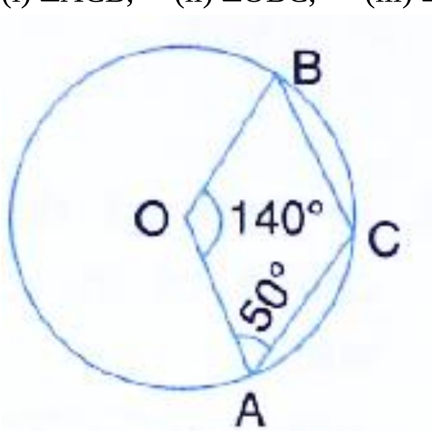
(Sum of opposite angles of a cyclic quadrilateral is $180°$)

$\Rightarrow \angle ABC = 180° - 75° = 105°$

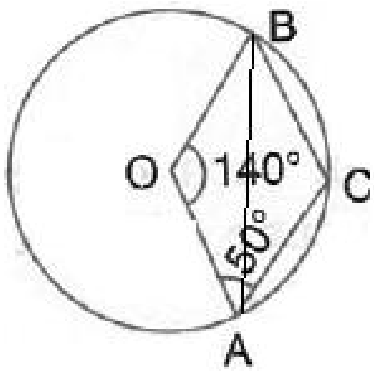
Question 8:

In the given figure, O is the centre of the circle. If $\angle AOB = 140°$ and $\angle OAC = 50°$; Find:

(i) $\angle ACB$, (ii) $\angle OBC$, (iii) $\angle OAB$, (iv) $\angle CBA$.



Solution 8:



Here, $\angle ACB = \frac{1}{2}$ Reflex $(\angle AOB) = \frac{1}{2}(360° - 140°) = 110°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

Now, OA = OB (Radii of same circle)

$\therefore \angle OBA = \angle OAB = \frac{180° - 140°}{2} = 20°$

$\therefore \angle CAB = 50° - 20° = 30°$

$\Delta CAB,$

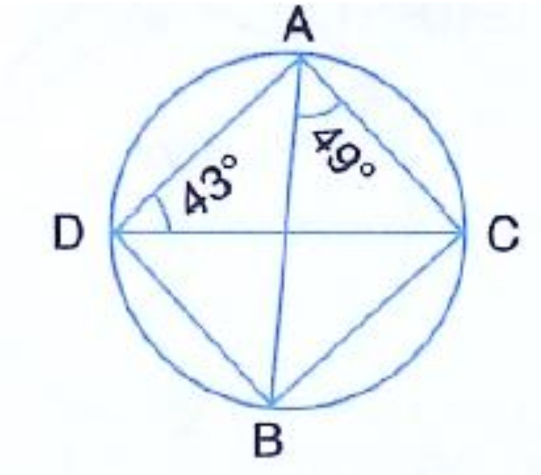
$\angle CBA - 180° - 110° - 30° = 40°$

$\therefore \angle OBC = \angle CBA + \angle OBA = 40° + 20° = 60°$

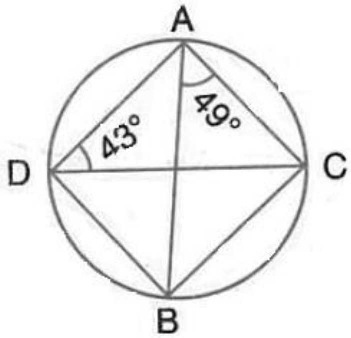
Question 9:

Calculate:

(i) $\angle CDB$, (ii) $\angle ABC$, (iii) $\angle ACB$



Solution 9:



Here,

$\angle CDB = \angle BAC = 49°$

$\angle ABC = \angle ADC = 43°$

(Angle subtend by the same chord on the circle are equal)

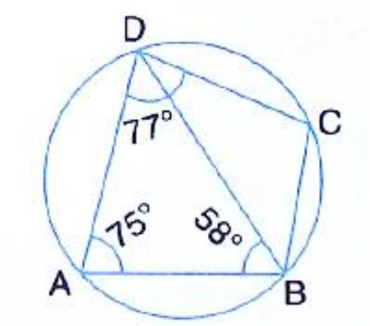
By angle – sum property of a triangle,

$\angle ACB = 180° - 49° - 43° = 88°$

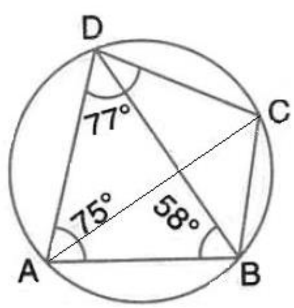
Question 10:

In the figure, given below, ABCD is a cyclic quadrilateral in which ∠BAD = 75°; ∠ABD = 58° and ∠ADC = 77°. Find:

(i) ∠BDC, (ii) ∠BCD, (iii) ∠BCA.



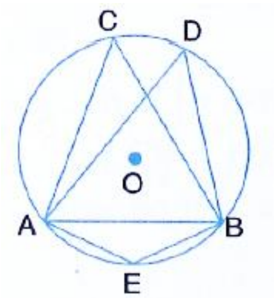
Solution 10:



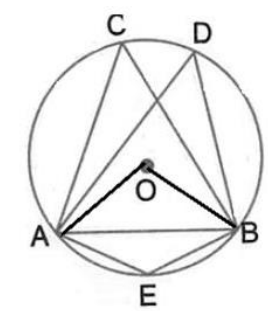
(i) By angle – sum property of triangle ABD,
$\angle ADB = 180° - 75° - 58° = 47°$
$\therefore \angle BDC = \angle ADC - \angle ADB = 77° - 47° = 30°$
(ii) $\angle BAD + \angle BCD = 180°$
(Sum of opposite angles of a cyclic quadrilateral is $180°$)
$\Rightarrow \angle BCD = 180° - 75° = 105°$
(iii) $\angle BDA = \angle ADB = 47°$
(Angle subtended by the same chord on the circle are equal)

Question 11:

In the following figure, O is centre of the circle and ΔABC is equilateral.
Find:
(i) ∠ADB, (ii) ∠AEB.



Solution 11:

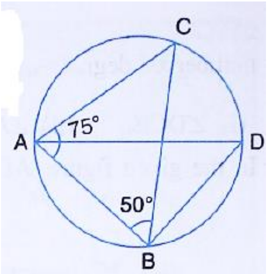


Since $\angle ACB$ and $\angle ADB$ are in the same segment,
$\angle ADB = \angle ACB = 60°$
Join OA and OB
Here, $\angle AOB = 2\angle ACB = 2 \times 60° = 120°$

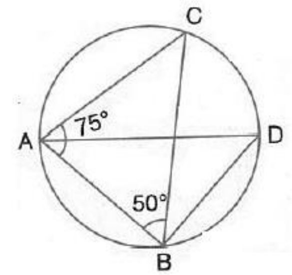
$$\angle AEB = \frac{1}{2} \text{Reflex}(\angle AOB) = \frac{1}{2}(360° - 120°) = 120°$$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

Question 12:
Given: $\angle CAB = 75°$ and $\angle CBA = 50°$. Find the value of $\angle DAB + \angle ABD$.



Solution 12:



In $\Delta ABC, \angle CBA = 50°, \angle CAB = 75°$

$$\angle ACB = 180° - (\angle CBA + \angle CAB)$$
$$= 180° - (50° + 75°)$$
$$= 180° - 125°$$
$$= 55°$$

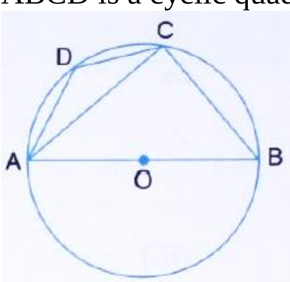
But $\angle ADB = \angle ACB = 55°$

(Angle subtended by the same chord on the circle are equal)

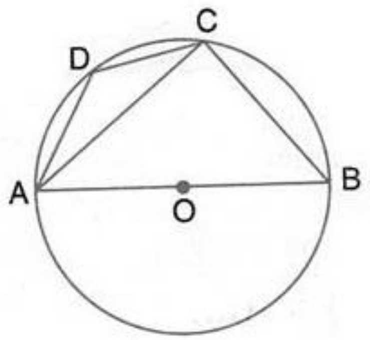
Now consider ΔABD,

$$\angle DAB + \angle ABD + \angle ADB = 180°$$
$$\Rightarrow \angle DAB + \angle ABD + 55° = 180°$$
$$\Rightarrow \angle DAB + \angle ABD = 180° - 55°$$
$$\Rightarrow \angle DAB + \angle ABD = 125°$$

Question 13:
ABCD is a cyclic quadrilateral in a circle with centre O. If $\angle ADC = 130°$; find $\angle BAC$.



Solution 13:



Here $\angle ACB = 90°$

(Angle in a semicircle is right angle)

Also, $\angle ABC = 180° - \angle ADC = 180° - 130° = 50°$

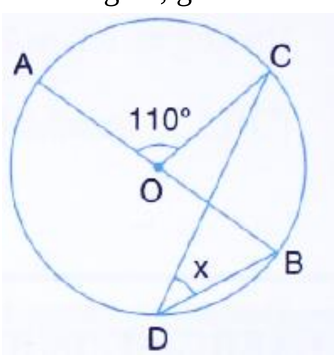
(pair of opposite angles in a cy clic quadrilateral are supplementary)

By angle sum property of right triangle ACB,

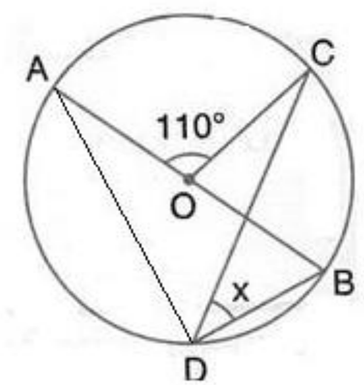
$\angle BAC = 90° - \angle ABC = 90° - 50° = 40°$

Question 14:

In the figure, given below, AOB is a diameter of the circle and ∠AOC = 110°. Find ∠BDC.



Solution 14:



Join AD.

Here, $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}\times 110° = 55°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

Also, $\angle ADB = 90°$

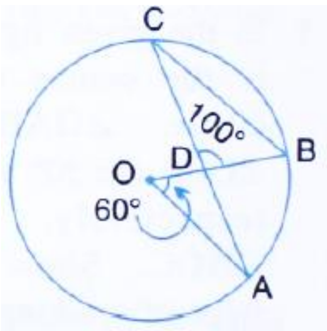
(Angle in a semicircle is a right angle)

$\therefore \angle BDC = 90° - \angle ADC = 90° - 55° = 35°$

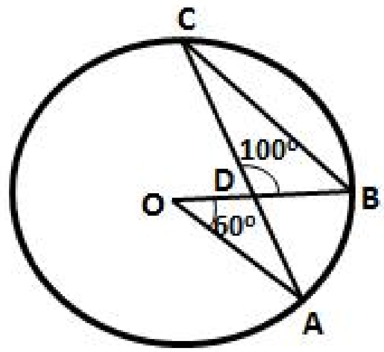
Question 15:

In the following figure, O is the centre of the circle,

∠AOB = 60° and ∠BDC = 100° Find ∠OBC.



Solution 15:



Here, $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2}\times 60° = 30°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

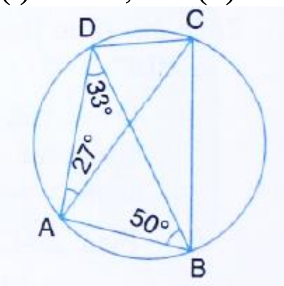
By angle sum property of ΔBDC,

$\therefore \angle DBC = 180° - 100° - 30° = 50°$

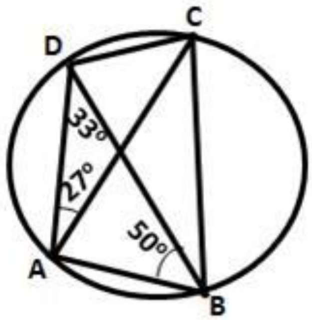
Hence, $\angle OBC = 50°$

Question 16:

ABCD is a cyclic quadrilateral in which ∠DAC = 27°; ∠DBA = 50° and ∠ADB = 33°. Calculate:
(i) ∠DBC, (ii) ∠DCB, (iii) ∠CAB



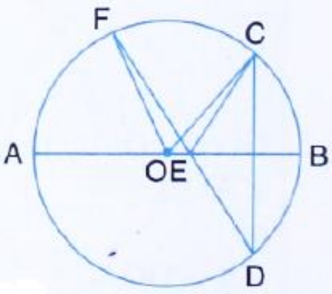
Solution 16:



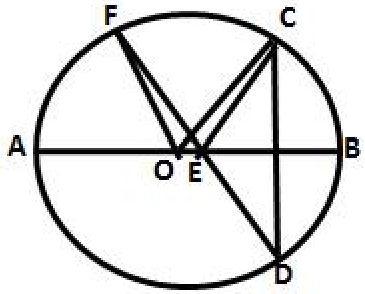
(i) $\angle DBC = \angle DAC = 27°$
(Angle subtended by the same chord on the circle are equal)
(ii) $\angle ACB = \angle ADB = 33°$
$\angle ACD = \angle ABD = 50°$
(Angle subtended by the same chord on the circle are equal)
$\therefore \angle DCB = \angle ACD + \angle ACB = 50° + 33° = 83°$
(iii) $\angle DAB + \angle DCB = 180°$
(Pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow 27° + \angle CAB + 83° = 180°$
$\Rightarrow \angle CAB = 180° - 110° = 70°$

Question 17:

In the figure given below, AB is diameter of the circle whose centre is O. given that: ∠ECD = ∠EDC = 32°. Show that ∠COF = ∠CEF.



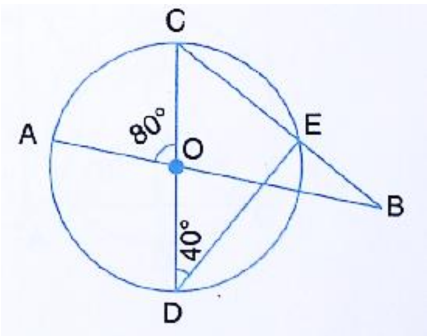
Solution 17:



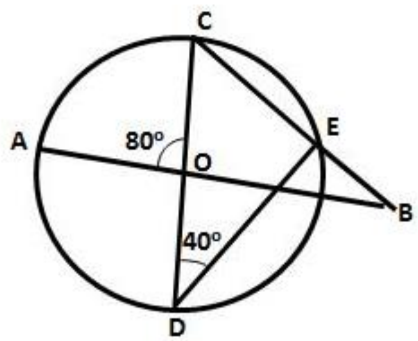
Here, $\angle COF = 2\angle CDF = 2\times 32° = 64°$ ……………… (i)
(Angle at the centre is double the angle at the circumference subtended by the same chord)
In ΔECD,
$\angle CEF = \angle ECD + \angle EDC = 32° + 32° = 64°$ …………(ii)
(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)
From (i) and (ii), we get
$\angle COF = \angle CEF$

Question 18:

In the figure given below, AB and CD are straight lines through the centre O of a circle. If ∠AOC = 80° and ∠CDE = 40°, Find the number of degrees in:
(i) ∠ DCE, (ii) ∠ABC.



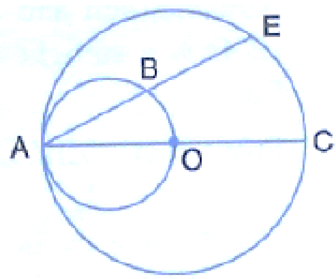
Solution 18:



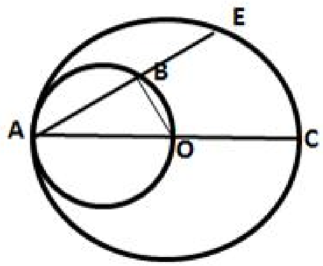
(i) Here, $\angle CED = 90°$
(Angle in a semicircle is a right angle)
$\therefore \angle DCE = 90° - \angle CDE = 90° - 40° = 50°$
$\therefore \angle DCE = \angle OCB = 50°$
(ii) In ΔBOC,
$\angle AOC = \angle OCB + \angle OBC$
(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)
$\Rightarrow \angle OBC = 80° - 50° = 30° \quad [\angle AOC = 80°, \text{given}]$
Hence, $\angle ABC = 30°$

Question 19:

In the given figure, AC is a diameter of a circle, whose centre is O. A circle is described on AO as diameter. AE, a chord of the larger circle, intersects the smaller circle at B. prove that AB = BE

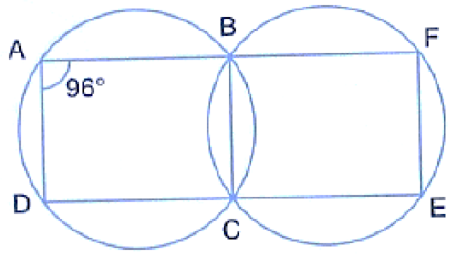


Solution 19:

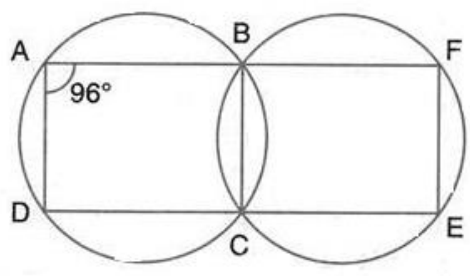


Join OB,
Then $\angle OBA = 90°$
(Angle in a semicircle is a right angle)
i.e. $OB \perp AE$
We know the perpendicular drawn from the centre to a chord bisects the chord.
$\therefore AB = BE$

Question 20:
In the following figure,
(i) If ∠BAD = 96°, find ∠BCD and ∠BFE.
(ii) Prove that AD is parallel to FE.



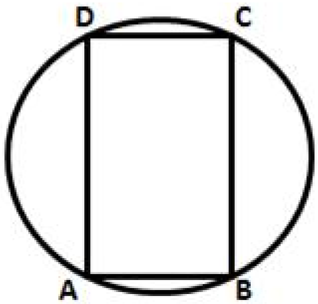
Solution 20:



(i) ABCD is a cyclic quadrilateral
$\therefore \angle BAD + \angle BCD = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle BCD = 180° - 96° = 84°$
$\therefore \angle BCE = 180° - 84° = 96°$
Similarly, BCEF is a cyclic quadrilateral
$\therefore \angle BCE + \angle BFE = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\therefore \angle BFE = 180° - 96° = 84°$
(ii) Now, $\angle BAD + \angle BFE = 96° + 84° = 180°$
But these two are interior angles on the same side
of a pair of lines AD and FE
$\therefore AD \parallel FE$

Question 21:
Prove that:
(i) the parallelogram, inscribed in a circle, is a rectangle.
(ii) the rhombus, inscribed in a circle, is a square.

Solution 21:



(i) Let ABCD be a parallelogram, inscribe in a circle,

Now, $\angle BAD = \angle BCD$

(Opposite angles of a parallelogram are equal)

And $\angle BAD = \angle BCD = 180°$

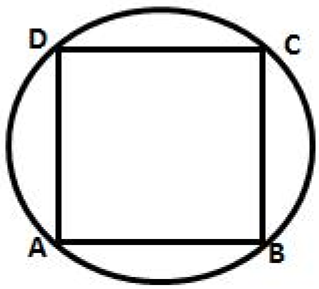
(pair of opposite angles in a cyclic quadrilateral are supplementary)

$$\therefore \angle BAD = \angle BCD = \frac{180°}{2} = 90°$$

∥y, the other two angles are $90°$ and opposite pair of sides

Are equal.

∴ ABCD is a rectangle.



(ii) Let ABCD be a rhombus, inscribed in a circle

Now, $\angle BAD = \angle BCD$

(Opposite angles of a parallelogram are equal)

And $\angle BAD = \angle BCD = 180°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

$$\therefore \angle BAD = \angle BCD = \frac{180°}{2} = 90°$$

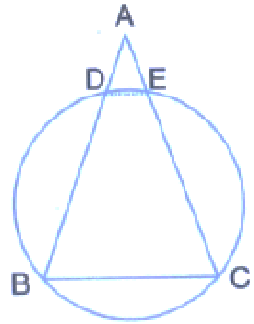
∥y, the other two angles are $90°$ and all the sides

are equal.

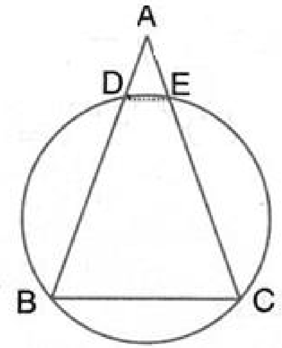
∴ ABCD is a square.

Question 22:

In the following figure AB = AC. Prove that DECB is an isosceles trapezium.



Solution 22:



Here, AB = AC

$\Rightarrow \angle B = \angle C$

$\therefore$ DECB is a cyclic quadrilateral

(Ina triangle, angles opposite to equal sides are equal)

Also, $\angle B + \angle DEC = 180°$ ………. (1)

(pair of opposite angles in a cyclic quadrilateral are supplementary

$\Rightarrow \angle C + \angle DEC = 180°$ [from (1)]

But this is the sum of interior angles

On one side of a transversal.

$\therefore$ DE || BC

But $\angle ADE = \angle B$ and $\angle AED = \angle C$ [Corresponding angles]

Thus, $\angle ADE = \angle AED$

$\Rightarrow AD = AE$

$\Rightarrow AB - AD = AC - AE (\because AB = AC)$

$\Rightarrow BD = CE$

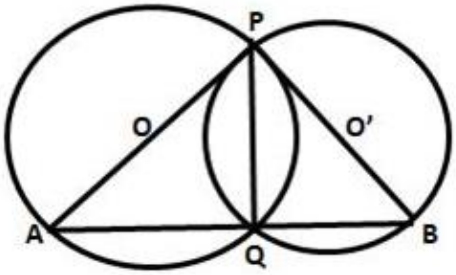
Thus, we have, $DE \parallel BC$ and $BD = CE$

Hence, DECB is an isosceles trapezium

Question 23:

Two circles intersect at P and Q. through P diameter PA and PB of the two circles are drawn. Show that the points A, Q and B are collinear.

Solution 23:

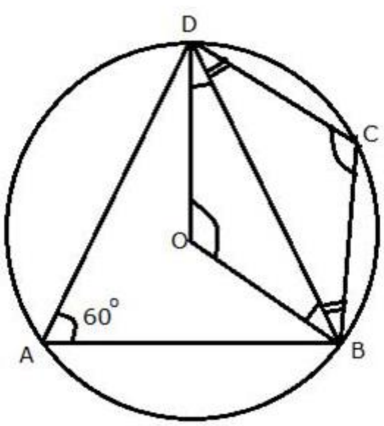


Let O and O' be the centres of two intersecting circle, where
Points of intersection are P and Q and PA and PB are their diameter respectively.
Join PQ, AQ and QB.
∴ $\angle AQP = 90°$ and $\angle BQP = 90°$
(Angle in a semicircle is a right angle)
Adding both these angles,
$\angle AQP + \angle BQP = 180°$ $\Rightarrow \angle AQB = 180°$
Hence, the points A, Q and B are collinear.

Question 24:

ABCD is a quadrilateral inscribed in a circle, having $\angle = 60°$; O is the center of the circle. Show that:
$\angle OBD + \angle ODB$
$= \angle CBD + \angle CDB$

Solution 24:



$\angle BOD = 2\angle BAD = 2\times 60° = 120°$

And $\angle BCD = \frac{1}{2} \text{Refelx}\left(\angle BOD\right) = \frac{1}{2}\left(360° - 120°\right) = 120°$

(Angle at the centre is double the angle at the circumference subtended by the same chord

$\therefore \angle CBD + \angle CDB = 180° - 120° = 60°$

(By angle sum property of triangle CBD)

Again, $\angle OBD + \angle ODB = 180° - 120° = 60°$

(By angle sum property of triangle OBD)

$\therefore \angle OBD + \angle ODB = \angle CBD + \angle CDB$

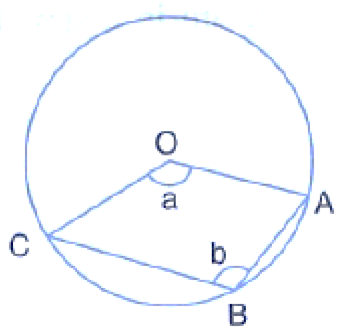
Question 25:

The figure given below, shows a Circle with centre O.

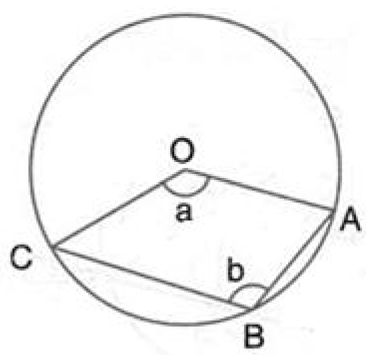
Given: $\angle AOC = a$ and $\angle ABC = b$.

(i) Find the relationship between a and b

(ii) Find the measure of angle OAB, is OABC is a parallelogram



Solution 25:



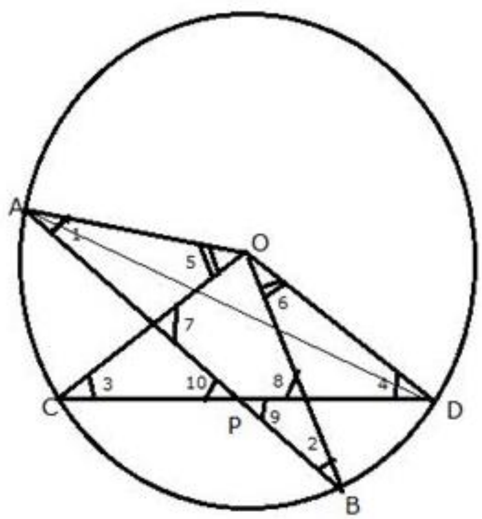
(i) $\angle ABC = \frac{1}{2} \text{Reflex } \left(\angle COA\right)$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow b = \frac{1}{2}(360° - a)$
$\Rightarrow a + 2b = 180°$
(ii) Since OABC is a parallelogram, so opposite angles are equal
$\therefore a = b$
Using relationship in (i)
$3a = 180°$
$\therefore a = 60°$
Also, $OC \parallel BA$
$\therefore \angle COA + \angle OAB = 180°$
$\Rightarrow 60° + \angle OAB = 180°$
$\Rightarrow \angle OAB = 120°$

Question 26:
Two chords AB and CD intersect at P inside the circle. Prove that the sum of the angles subtended by the arcs AC and BD at the centre O is equal to twice the angle APC.

Solution 26:



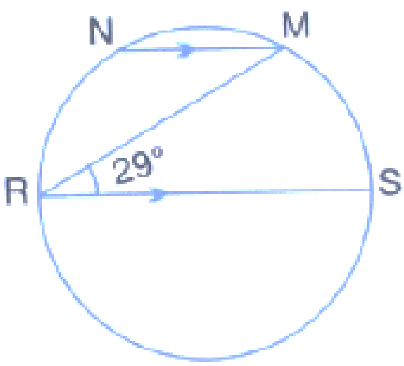
Given: two chords AB and CD intersect each other at P inside the circle. OA, OB, OC and OD are joined.
To prove: $\angle AOC + \angle BOD = 2\angle APC$
Construction: Join AD.
Proof: Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining Part of the circle.
$\angle AOC = 2\angle ADC$ ....(1)

Similarly,
$\angle BOD = 2\angle BAD \quad \ldots\ldots\ldots\ldots(2)$
Adding (1) and (2),
$\angle AOC + \angle BOD = 2\angle ADC + 2\angle BAD$
$\quad\quad = 2(\angle ADC + \angle BAD) \quad \ldots\ldots\ldots(3)$

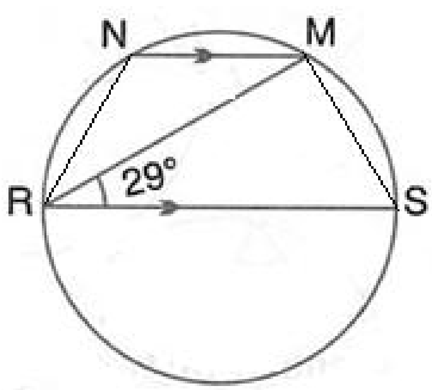
But ΔPAD,
Ext. $\angle APC = \angle PAD + \angle ADC$
$\quad\quad = \angle BAD + \angle ADC \quad \ldots\ldots\ldots\ldots\ldots(4)$
From (3) and (4),
$\angle AOC + \angle BOD = 2\angle APC$

Question 27:

In the given figure, RS is a diameter of the circle. NM is parallel to RS and ∠MRS = 29°. Calculate:
(i) ∠RNM,
(ii) ∠NRM.



Solution 27:

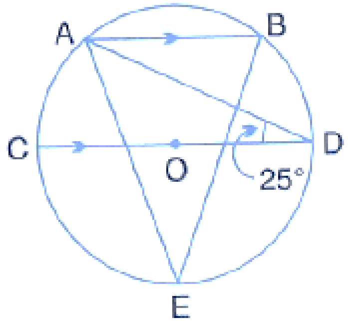


(i) Join RN and MS.

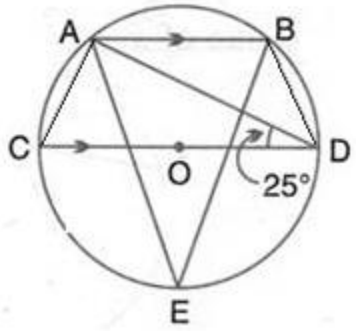
$\therefore \angle RMS = 90°$
(Angle in a semicircle is a right angle)
$\therefore \angle RSM = 90° - 29° = 61°$
(By angle sum property of triangle RMS)
$\therefore \angle RNM = 180° \angle RSM = 180° - 61° = 119°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
(ii) Also, $RS \parallel NM$
$\therefore \angle NMR = \angle MRS = 29°$ (Alternate angles)
$\therefore \angle NMS = 90° + 29° = 119°$
Also, $\angle NRS + \angle MS = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle NMR + 29° + 119° = 180°$
$\Rightarrow \angle NRM = 180° - 148°$
$\therefore \angle NRM = 32°$

Question 28:

In the figure, given alongside, AB ∥ CD and O is the centre of the circle. If ∠ADC = 25°; find the angle AEB give reasons in support of your answer.



Solution 28:

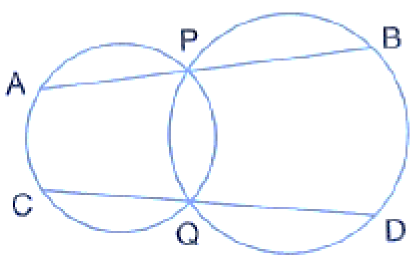


Join AC and BD

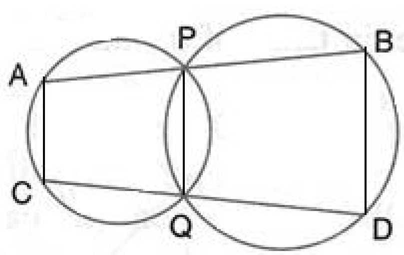
$\therefore \angle CAD = 90°$ and $\angle CBD = 90°$
(Angle in a semicircle is a right angle)
Also, $AB \parallel CD$
$\therefore \angle BAD = \angle ADC = 25°$ (alternate angles)
$\angle BAC = \angle BAD + \angle CAD = 25° + 90° = 115°$
$\therefore \angle ADB = 180° - 25° - \angle BAC = 180° - 25° - 115° = 40°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
Also, $\angle AEB = \angle ADB = 40°$
(Angle subtended by the same chord on the circle are equal)

Question 29:

Two circle intersect at P and Q. through P, a straight line APB is drawn to meet the circles in A and B. Through Q, a straight line is drawn to meet the circles at C and D. prove that AC is parallel to BD.



Solution 29:



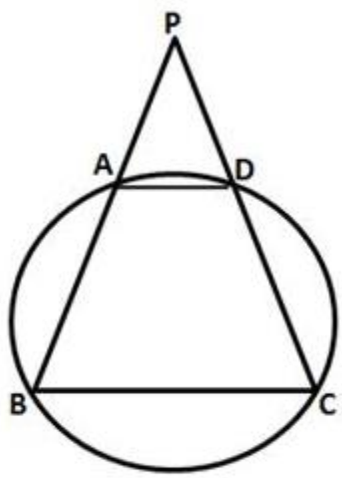
Join AC, PQ and BD
ACQP is a cyclic quadrilateral
$\therefore \angle CAP + \angle PQC = 180°$ ………….(i)
(pair of opposite in a cyclic quadrilateral are supplementary)
PQDB is a cyclic quadrilateral
$\therefore \angle PQD + \angle DBP = 180°$ ………….(ii)
(pair of opposite angles in a cyclic quadrilateral are supplementary)
Again, $\angle PQC + \angle PQD = 180°$ …………. (iii)
(CQD is a straight line)

Using (i), (ii) and (iii)
$\therefore \angle CAP + \angle DBP = 180°$
Or $\therefore \angle CAB + \angle DBA = 180°$
We know, if a transversal intersects two lines such
That a pair of interior angles on the same side of the
Transversal is supplementary, then the two lines are parallel
$\therefore AC \parallel BD$

Question 30:

ABCD is a cyclic quadrilateral in which AB and DC on being produced, meet at P such that PA = PD. Prove that AD is parallel to BC.

Solution 30:

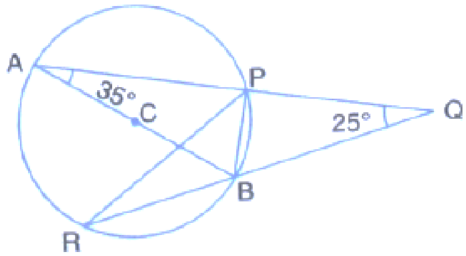


Let ABCD be the given cyclic quadrilateral
Also, PA = PD (Given)
$\therefore \angle PAD = \angle PDA$ ……..(1)
$\therefore \angle BAD = 180° - \angle PAD$
And $\angle CDA = 180° - \angle PDA = 180° - \angle PAD$ (From (1))
We know that the opposite angles of a cyclic quadrilateral are supplementary
$\therefore \angle ABC = 180° - \angle CDA = 180° - (180° - \angle PAD) = \angle PAD$
And $\angle DCB = 180° - \angle BAD = 180° - (180° - \angle PAD) = \angle PAD$
$\therefore \angle ABC = \angle DCB = \angle PAD = \angle PAD$
That means $AD \parallel BC$

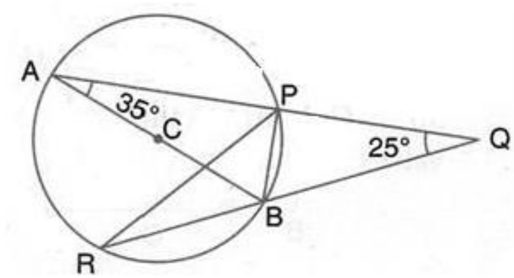
Question 31:

AB is a diameter of the circle APBR as shown in the figure. APQ and RBQ are straight lines. Find:

(i) ∠PRB, (ii) ∠PBR, (iii) ∠BPR.



Solution 31:



(i) $\angle PRB = \angle PAB = 35°$

(Angles subtended by the same chord on the circle are equal)

(ii) $\angle BPA = 90°$

(angle in a semicircle is a right angle)

$\therefore \angle BPQ = 90°$

$\therefore \angle PBR = \angle BQP + \angle BPQ = 25° + 90° = 115°$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

(iii) $\angle ABP = 90° - \angle BAP = 90° - 35° = 55°$

$\therefore \angle ABR = \angle PBR = \angle ABP = 115° - 55° = 60°$

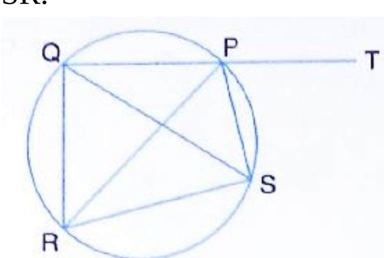
$\therefore \angle APR = \angle ABR = 60°$

(Angles subtended by the same chord on the circle are equal)

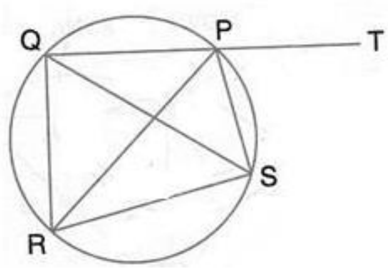
$\therefore \angle BPR = 90° - \angle APR = 90° - 60° = 30°$

Question 32:

In the given figure, SP is bisector of ∠RPT and PQRS is a cyclic quadrilateral. Prove that SQ = SR.



Solution 32:



PQRS is a cyclic quadrilateral

$\therefore \angle QRS + \angle QPS = 180°$ ....(i)

(pair of opposite angles in a cyclic quadrilateral are supplementary)

Also, $\angle QPS + \angle SPT = 180°$ ......(ii)

(Straight line QPT)

From (i) and (ii)

$\angle QRS = \angle SPT$ .... (iii)

Also, $\angle RQS = \angle RPS$ ......(iv)

(Angle subtended by the same chord on the circle are equal)

And $\angle RPS = \angle SPT$ (PS bisects $\angle RPT$) ...... (v)

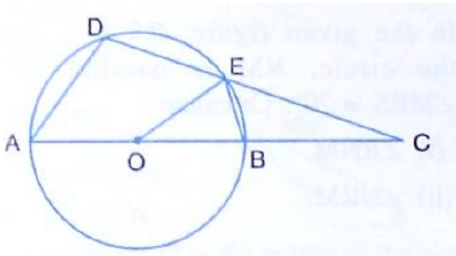
From (iii), (iv) and (v)

$\angle QRS = \angle RQS$

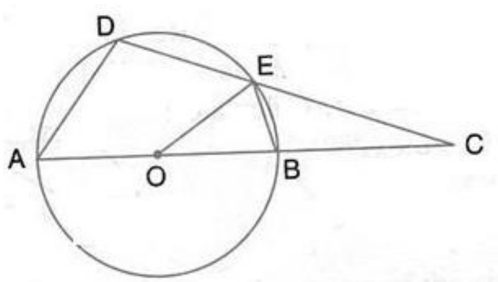
$\Rightarrow SQ = SR$

Question 33:

In the figure, O is the centre of the circle, ∠AOE = 150°, ∠DAO = 51°. Calculate the sizes of the angles CEB and OCE.



Solution 33:



$\angle ADE = \frac{1}{2} \text{Reflex}(\angle AOE) = \frac{1}{2}(360° - 150°) = 105°$

(Angle at the center is double the angle ate the circumference subtended by the same chord)

$\angle DAB + \angle BED = 180°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle BED = 180° - 51° = 129°$

$\therefore \angle CEB = 180° - \angle BED$ (straight line)

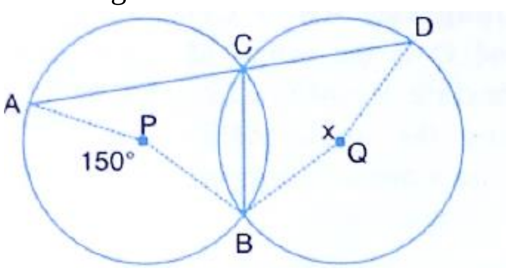
$= 180° - 129° = 51°$

Also, by angle sum property of ΔADC,

$\angle OCE = 180° - 51° - 105° = 24°$

Question 34:

In the figure, given below, P and Q are the centres of two circles intersecting at B and C ACD is a straight line. Calculate the numerical value of x.



Solution 34:

$\angle ACB = \frac{1}{2}\angle APB = \frac{1}{2} \times 150° = 75°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\angle ACB + \angle BCD = 180°$

(Straight line)

$\Rightarrow \angle BCD = 180° - 75° = 105°$

Also, $\angle BCD = \frac{1}{2}$ reflex $\angle BQD = \frac{1}{2}(360° - x)$

(Angle at the center is double the angle at the circumference subtended by the same chord)

$\Rightarrow 105° = 180° - \frac{x}{2}$

$\therefore x = 2(180° - 105°) = 2 \times 75° = 150°$

Question 35:

The figure shows two circles which intersect at A and B. The centre of the smaller circle is O and lies on the circumference of the larger circle. Given ∠APB = a°.

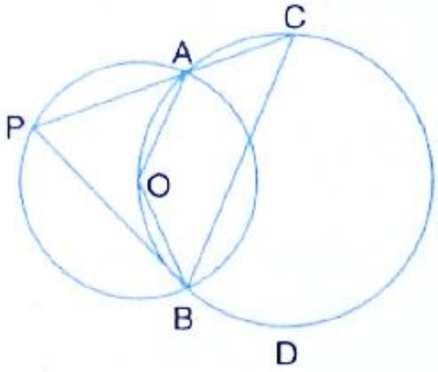
Calculate, in terms of a°, the value of:

(i) obtuse ∠AOB,

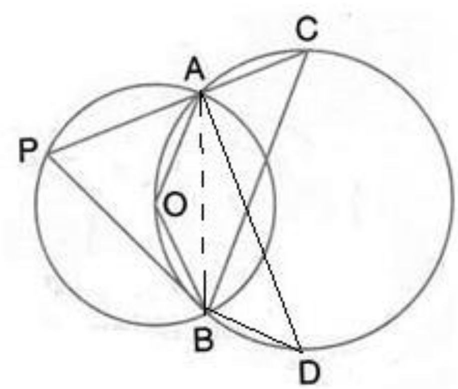
(ii) ∠ACB

(iii) ∠ADB.

Give reasons for your answers clearly.



Solution 35:



(i) obtuse $\angle AOB = 2\angle APB = 2a°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) OABC is a cyclic quadrilateral

$\therefore \angle AOB + \angle ACB = 180°$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow ACB = 180° - 2a°$

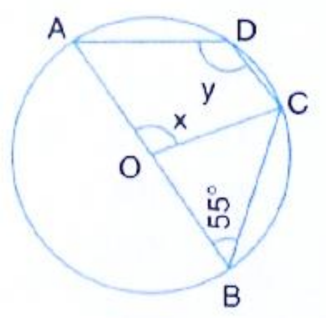
(iii) Join AB.

$\angle ADB = \angle ACB = 180° - 2a°$

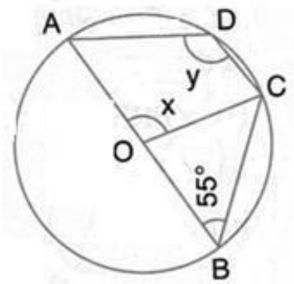
(Angle subtended by the same arc on the circle are equal)

Question 36:

In the given figure, O is the centre of the circle and ∠ABC = 55°. Calculate the values of x and y.



Solution 36:



$\angle AOC = 2\angle ABC = 2\times 55°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\therefore x = 110°$

ABCD is cyclic quadrilateral

$\therefore \angle ADC + \angle ABC = 180°$

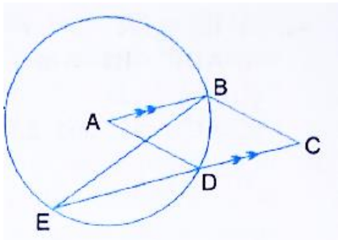
(pair of opposite angles in a cyclic quadrilateral are supplementary

$\Rightarrow y = 180° - 55° = 125°$

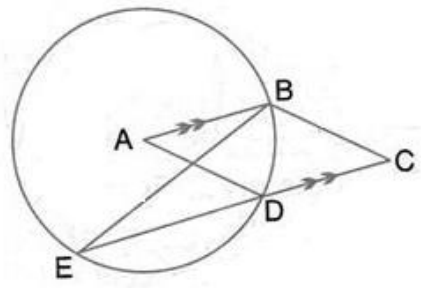
Question 37:

In the given figure, A is the centre of the circle, ABCD is a parallelogram and CDE is a straight line.

Prove that: ∠BCD = 2∠ABE.



Solution 37:



$\angle BAD = 2\angle BED$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

And $\angle BED = \angle ABE$ (alternate angles)

$\therefore \angle BAD = 2\angle ABE$ .... (i)

ABCD is a parallelogram

$\therefore \angle BAD = \angle BCD$ .... (ii)

(opposite angles in a parallelogram are equal)

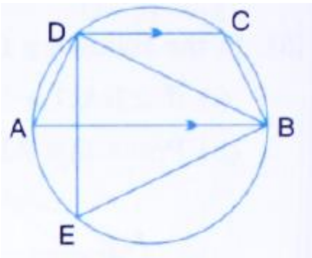
From (i) and (ii),

$\angle BCD = 2\angle ABE$

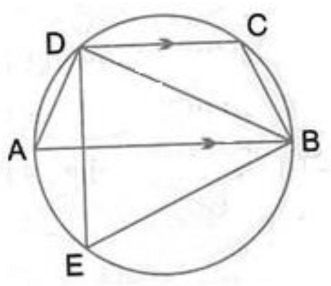
Question 38:

ABCD is a cyclic quadrilateral in which AB is parallel to DC and AB is a diameter of the circle. Given $\angle BED = 65°$; Calculate:

(i) $\angle DAB$, (ii) $\angle BDC$



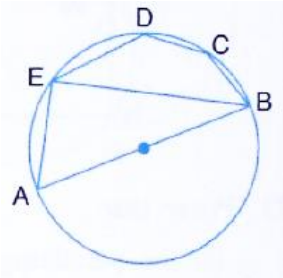
Solution 38:



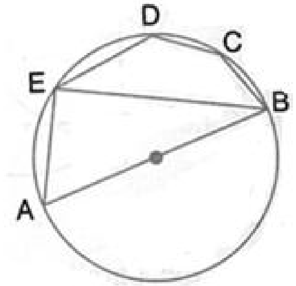
(i) $\angle DAB = \angle BED = 65°$
(Angle subtended by the same chord on the circle are equal)
(ii) $\angle ADB = 90°$
(Angle in a semicircle is a right angle)
$\therefore \angle ABD = 90° - \angle DAB = 90° - 65° = 25°$
$AB \parallel DC$
$\therefore \angle BDC = \angle ABD = 25°$ (Alternate angles)

Question 39:

In the given figure, AB is a diameter of the circle. Chord ED is parallel to AB and ∠EAB = 63°. Calculate
(i) ∠EBA (ii) ∠BCD



Solution 39:

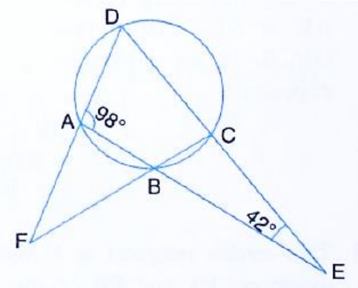


(i) $\angle AEB = 90°$
(Angle in a semicircle is a right angle)
Therefore $\angle EBA = 90° - \angle EAB = 90° - 63° = 27°$
(ii) $AB \parallel ED$
Therefore $\angle DEB = EBA = 27°$ (Alternate angles)
Therefore BCDE is a cyclic quadrilateral
Therefore $\angle DEB + \angle BCD = 180°$
[pair of opposite angles in a cyclic quadrilateral are supplementary]
Therefore $\angle BCD = 180° - 27° = 153°$

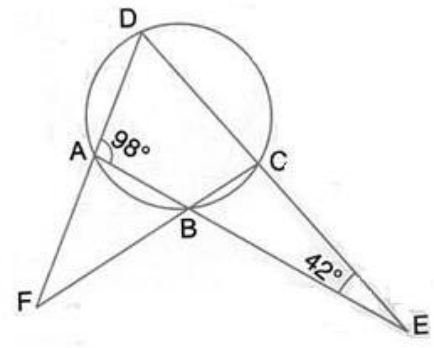
Question 40:

The sides AB and DC of a cyclic quadrilateral ABCD are produced to meet at E; the sides DA and CB are produced to meet at F. If ∠BEC = 42° and ∠BAD = 98°; Calculate:

(i) ∠AFB (ii) ∠ADC



Solution 40:



By angle sum property of ΔADE,

$\angle ADC = 180° - 98° - 42° = 40°$

Also, $\angle ADC + \angle ABC = 180°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\therefore \angle ABC = 180° - 40° = 140°$

Also, $\angle BAF = 180° - \angle BAD = 180° - 98° = 82°$

$\therefore \angle ABC = \angle AFB + \angle BAF$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

$\Rightarrow \angle AFB = 140° - 82° = 58°$

Thus, $\angle AFB = 58°$ and $\angle ADC = 40°$

Question 41:

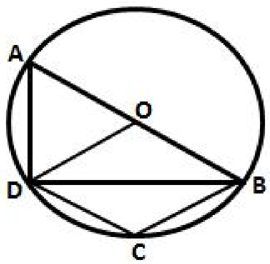
In the given figure, AB is a diameter of the circle with centre O. DO is parallel to CB and ∠DCB = 120°.

Calculate:

(i) ∠DAB, (ii) ∠DBA, (iii) ∠DBC, (iv) ∠ADC.

Also show that the ΔAOD is an equilateral triangle.

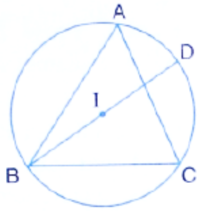
Solution 41:



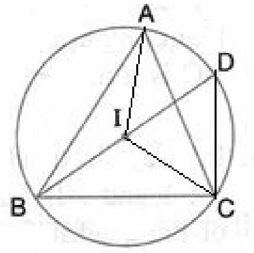
(i) ABCD is a cyclic quadrilateral
$\therefore \angle DCB + \angle DAB = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle DAB = 180° - 120° = 60°$
(ii) $\angle ADB = 90°$
(Angle in a semicircle is a right angle)
$\therefore \angle DBA = 90° - \angle DAB = 90° - 60° = 30°$
(iii) $OD = OB$
$\therefore \angle ODB = \angle OBD$
Or $\angle ABD = 30°$
Also, $AB \parallel ED$
$\therefore \angle DBC = \angle ODB = 30°$ (Alternate angles)
(iv) $\angle ABD + \angle DBC = 30° + 30° = 60°$
$\Rightarrow \angle ABC = 60°$
In cyclic quadrilateral ABCD,
$\angle ADC + \angle ABC = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle ADC = 180° - 60° = 120°$
In ΔAOD, OA = OD (radii of the same circle)
$\angle AOD = \angle DAO$ Or $\angle DAB = 60°$ [proved in (i)]
$\Rightarrow \angle AOD = 60°$
$\angle ADO = \angle AOD = \angle DAO = 60°$
$\therefore \Delta AOD$ is an equilateral triangle.

Question 42:

In the given figure, I is the incentre of ΔABC. BI when produced meets the circumcircle of ΔABC at D. $\angle BAC = 55°$ and $\angle ACB = 65°$; calculate:
(i) $\angle DCA$, (ii) $\angle DAC$, (iii) $\angle DCI$, (iv) AIC



Solution 42:



Join IA, IC and CD

(i) IB is the bisector of $\angle ABC$

$$\Rightarrow \angle ABD = \frac{1}{2}\angle BC = \frac{1}{2}\left(180° - 65° - 55°\right) = 30°$$

$\angle DCA = \angle ABD = 30°$

(Angle in the same segment)

(ii) $\angle DAC = \angle CBD = 30°$

(Angle in the same segment)

(iii) $\angle ACI = \frac{1}{2}\angle ACB = \frac{1}{2} \times 65° = 32.5°$

(CI is the angular bisector of $\angle ACB$)

$\therefore DCI = \angle DCA + \angle ACI = 30° + 32.5° = 62.5°$

(iv) $\angle IAC = \frac{1}{2}\angle BAC = \frac{1}{2} \times 55° = 27.5°$

(AI is the angular bisector of $\angle BAC$)

$\therefore \angle AIC = 180° - \angle IAC - \angle ICA = 180° - 27.5° - 32.5° = 120°$

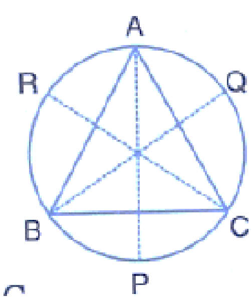
Question 43:

A triangle ABC is inscribed in a circle. The bisectors of angles BAC, ABC and ACB meet the circumcircle of the triangle at points P, Q and R respectively. Prove that:

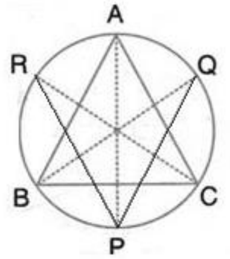
(i) $\angle ABC = 2\angle APQ$,

(ii) $\angle ACB = 2\angle APR$,

(iii) $\angle QPR = 90° - \frac{1}{2}\angle BAC$



Solution 43:



Join PQ and PR

(i) BQ is the bisector of $\angle ABC$

$\Rightarrow \angle ABQ = \frac{1}{2}\angle ABC$

Also, $\angle APQ = \angle ABQ$

(Angle in the same segment)

$\therefore \angle ABC = 2\angle APQ$

(ii) CR is the bisector of $\angle ACB$

$\Rightarrow ACR = \frac{1}{2}\angle ACB$

Also, $\angle ACR = \angle APR$

(Angle in the same segment)

$\therefore \angle ACB = 2\angle APR$

(iii) Adding (i) and (ii)

We get

$\angle ABC + \angle ACB\angle = 2(\angle APR + \angle APQ) = 2\angle QPR$

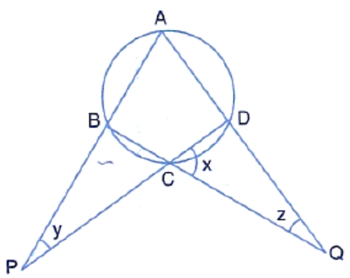
$\Rightarrow 180^\circ - \angle BAC = 2\angle QPR$

$\Rightarrow \angle QPR = 90^\circ - \frac{1}{2}\angle BAC$

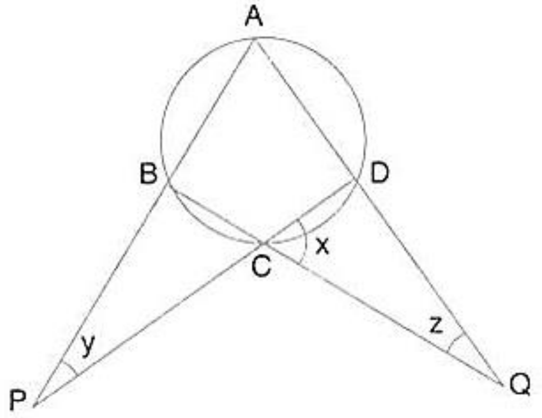
Question 44:

Calculate the angles x, y and z if:

$\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$



Solution 44:



Let x = 3k, y = 4k and z = 5k

$\angle ADB = x + z = 8k$ and $\angle ABC = x + y = 7k$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

Also, $\angle ABC + \angle ADC = 180°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow 8k + 7k = 180°$

$\Rightarrow 15k = 180°$

$\therefore k = \dfrac{180°}{15} = 12°$

$\therefore x = 3 \times 12° = 36°$

$y = 4 \times 12° = 48°$

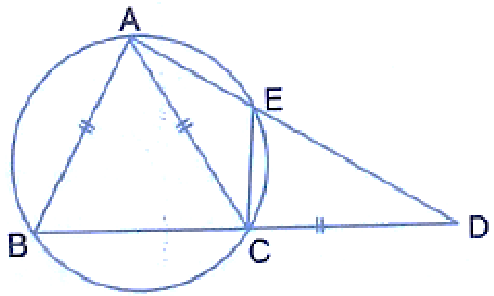
$z = 5 \times 12° = 60°$

Question 45:

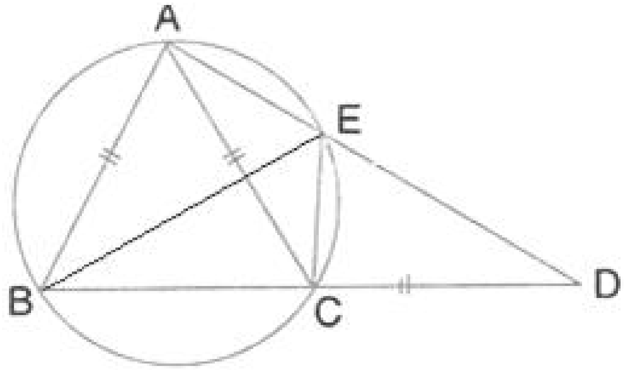
In the given figure, AB = AC = CD and ∠ADC = 38°. Calculate:

(i) Angle ABC

(ii) Angle BEC



Solution 45:



(i) AC = CD

$\therefore \angle CAD = \angle CDA = 38°$

$\therefore \angle ACD = 180° - 2 \times 38° = 104°$

$\therefore \angle ACB = 180° - 104° = 76°$ (Straight line)

Also, AB = AC

$\therefore \angle ABC = \angle ACB = 76°$

(ii) By angle sum property,

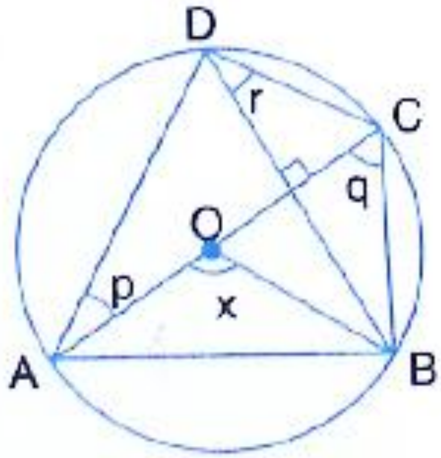
$\angle BAC = 180° - 2 \times 76° = 38°$

$\therefore \angle BEC = \angle BAC = 38°$

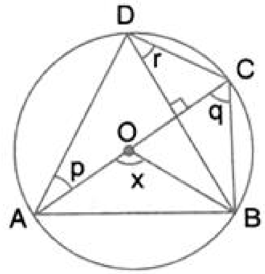
(Angles in the same chord)

Question 46:

In the given figure, AC is a diameter of circle, centre O. Chord BD is perpendicular to AC. Write down the angles p, q and r in terms of x.



Solution 46:



$\angle AOB = 2\angle ACB = 2ADB$
(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow x = 2q$ and $\angle ADB = \frac{x}{2} \therefore q = \frac{x}{2}$

Also, $\angle ADC = 90°$
(Angle in a semicircle)

$\Rightarrow r + \frac{x}{2} = 90°$

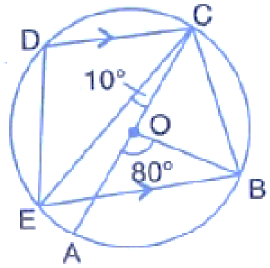
$\Rightarrow r = 90° - \frac{x}{2}$

Again, $\angle DAC = \angle DBC$
(Angle in the same segment)
$\Rightarrow p = 90° - q$

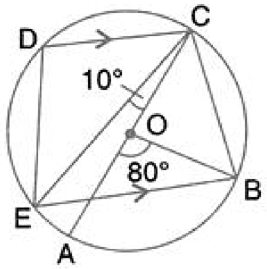
$\Rightarrow p = 90° - \frac{x}{2}$

Question 47:

In the given figure, AC is the diameter of the circle with centre O. CD and BE are parallel. Angle $\angle AOB = 80°$ and $\angle ACE = 10°$.
Calculate:
(i) Angle BEC,
(ii) Angle BCD,
(iii) Angle CED



Solution 47:



(i) $\angle BOC = 180° - 80° = 100°$ (Straight line)

And $\angle BOC = 2\angle BEC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow \angle BEC = \frac{100°}{2} = 50°$

(ii) $DC \parallel EB$

$\therefore DCE = \angle BEC = 50°$ (Alternate angles)

$\because \angle AOB = 80°$

$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB = 40°$

(Angle at the center is double the angle at the circumference subtended by the same chord)

We have,

$\angle BCD = \angle ACB + \angle ACE + \angle DCE = 40° + 10° + 50° = 100°$

(iii) $\angle BED = 180° - \angle BCD = 180° - 100° = 80°$

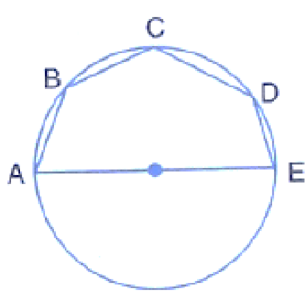
(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle CED + 50° = 80°$

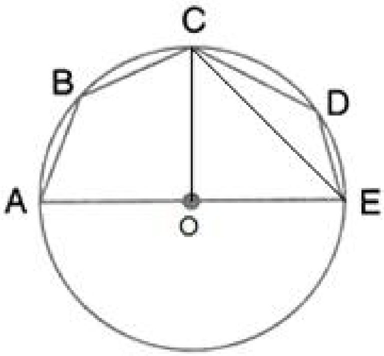
$\Rightarrow \angle CED = 30°$

Question 48:

In the given figure, AE is the diameter of the circle. Write down the numerical value of ∠ABC + ∠CDE. Give reasons for your answer.



Solution 48:



Join centre O and C and EC.

$$\angle AOC = \frac{180^\circ}{2} = 90^\circ$$

And $\angle AOC = 2\angle AEC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$$\Rightarrow \angle AEC = \frac{90^\circ}{2} = 45^\circ$$

Now, ABCE is a cyclic quadrilateral

$\therefore \angle ABC + \angle AEC = 180^\circ$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

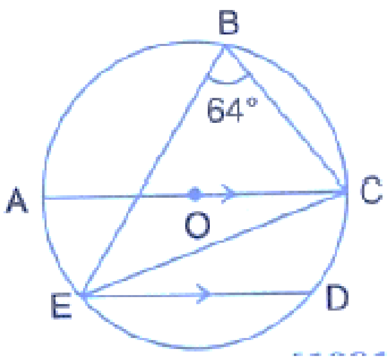
$\Rightarrow \angle ABC = 180^\circ - 45^\circ = 135^\circ$

Similarly, $\angle CDE = 135^\circ$

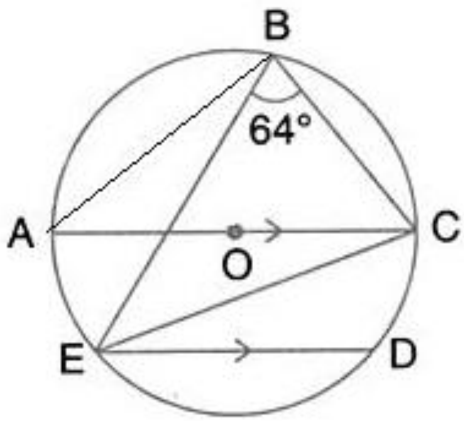
$\therefore \angle ABC + \angle CDE - 135^\circ + 135^\circ = 270^\circ$

Question 49:

In the given figure, AOC is a diameter and AC is parallel to ED. If ∠CBE = 64°, Calculate ∠DEC.



Solution 49:



Join AB,

$\angle ABC = 90°$

(Angle in a semi circle)

$\therefore \angle ABE = 90° - 64° = 26°$

Now, $\angle ABE = \angle ACE = 26°$

(Angle in the same segment)

Also, $AC \parallel ED$

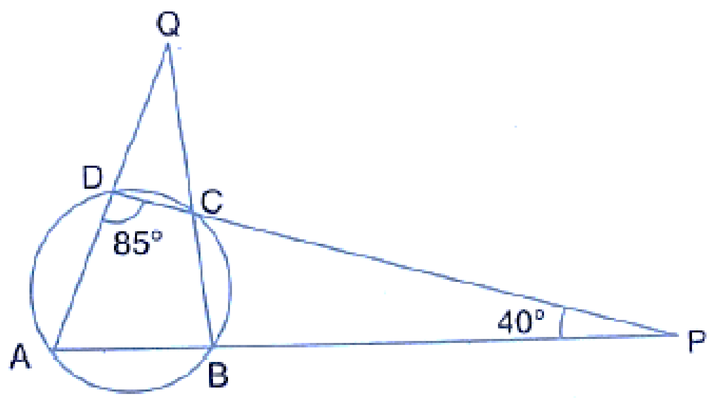
$\therefore \angle DEC = \angle ACE = 26°$ (Alternate angles)

Question 50:

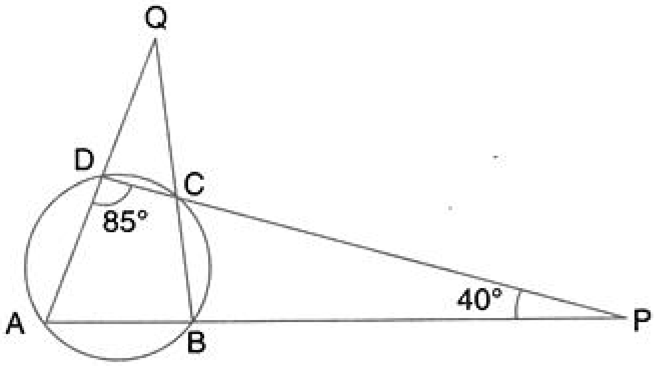
Use the given figure to find:

(i) $\angle BAD$,

(ii) $\angle DQB$



Solution 50:



(i) By angle sum property of ΔADP,

$\angle BAD = 180° - 85° - 40° = 55°$

(ii) $\angle ABC = 180° - \angle ADC = 180° - 85° = 95°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

By angle sum property,

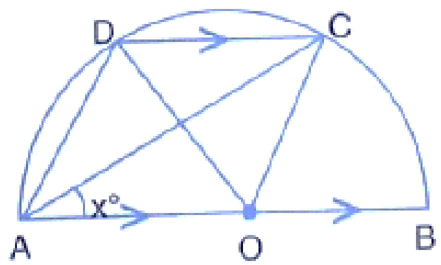
$\angle AQB = 180° - 95° - 55°$

$\Rightarrow \angle DQB = 30°$

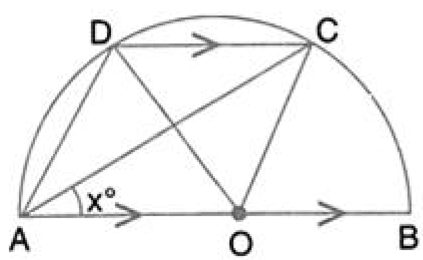
Question 51:

In the given figure, AOB is a diameter and DC is parallel to AB. If ∠CAB = x°; find (in terms of x) the values of ;

(i) ∠COB, (ii) ∠DOC, (iii) ∠DAC (iv) ∠ADC.



Solution 51:



(i) $\angle COB = 2\angle CAB = 2x$

(Angle ate the centre is double the angler at the circumference subtended by the same order)

(ii) $\angle OCD = \angle COB = 2x$ (Alternate angles)

In ΔOCD, $OC = OD$

$\therefore \angle ODC = \angle OCD = 2x$

By angle sum property of ΔOCD,

$\angle DOC = 180° - 2x - 2x = 180° - 4x$

(iii) $\angle DAC = \frac{1}{2}\angle DOC = \frac{1}{2}(180° - 4x) = 90° - 2x$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(iv) $DC \parallel AO$

$\therefore \angle ACD = \angle OAC = x$ (Alternate angles)

By angle sum property,

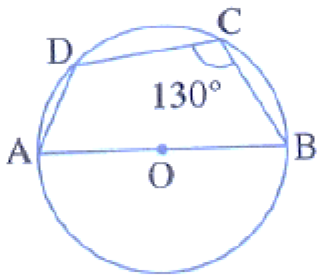
$\angle ADC = 180° - \angle DAC - \angle ACD = 180° - (90° - 2x) - x = 90° + x$

Question 52:

In the given figure, AB is the diameter of a circle with centre O. ∠BCD = 130°. Find:

(i) ∠DAB

(ii) ∠DBA



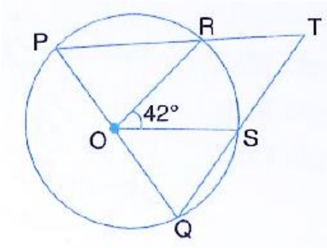
Solution 52:

i. ABCD is a cyclic quadrilateral
   m∠DAB = 180° − ∠DCB
   = 180° - 130°
   = 50°

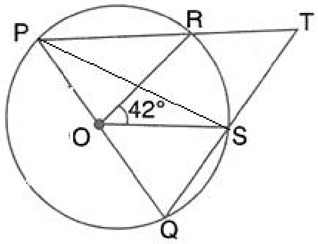
ii. In ΔADB,
   m∠DAB + m∠ADB + m∠DBA = 180°
   ⇒ 50° + 90° + m∠DBA = 180°
   ⇒ m∠DBA = 40°

Question 53:

In the given figure, PQ is the diameter of the circle whose centre is O. Given ∠ROS = 42°, Calculate ∠RTS.



Solution 53:



Join PS.

$\angle PSQ = 90°$

(Angle in a semicircle)

Also, $\angle SPR = \frac{1}{2}\angle ROS$

(Angle ate the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow SPT = \frac{1}{2} \times 42° = 21°$

∴ In right triangle PST,

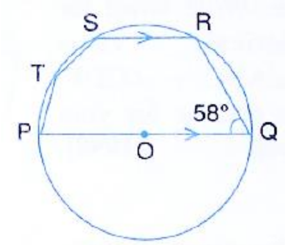
$\angle PTS = 90° - \angle SPT$

$\Rightarrow \angle RTS = 90° - 21° = 69°$

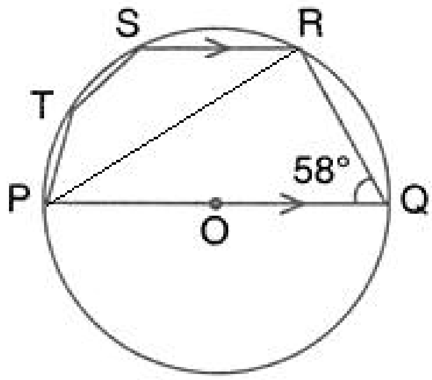
Question 54:

In the given figure, PQ is a diameter. Chord SR is parallel to PQ. Given that ∠PQR = 58°, Calculate:

(i) ∠RPQ, (ii) ∠STP



Solution 54:



Join PR.

(i) $\angle PRQ = 90°$

(Angle in a semicircle)

$\therefore$ In right triangle PQR,

$\angle RPQ = 90° - \angle PQR = 90° - 58° = 32°$

(ii) Also, $SR \parallel PQ$

$\therefore \angle PRS = \angle RPQ = 32°$ (Alternate angles)

In cyclic quadrilateral PRST,

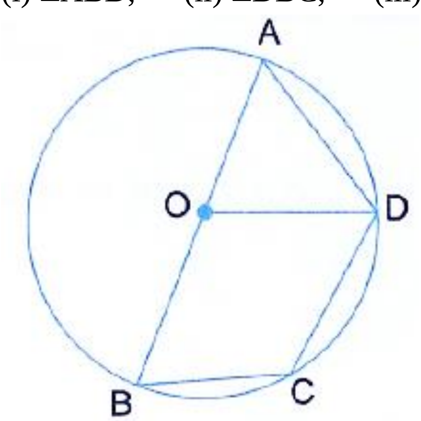
$\angle STP = 180° - \angle PRS = 180° - 32° = 148°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

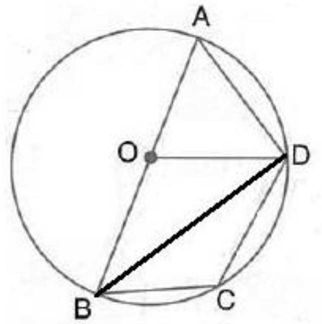
Question 55:

AB is the diameter of the circle with centre O. OD is parallel to BC and $\angle AOD = 60°$. Calculate the numerical values of:

(i) $\angle ABD$, (ii) $\angle DBC$, (iii) $\angle ADC$.



Solution 55:



Join BD.

(i) $\angle ABD = \frac{1}{2}\angle AOD = \frac{1}{2} \times 60° = 30°$

(Angle at the first is double the angle at the circumference subtended by the same chord)

(ii) $\angle BDA = 90°$

(Angle in a semicircle)

Also, ΔOAD is equilateral $\left(\because \angle OAD = 60°\right)$

$\therefore \angle ODB = 90° - \angle ODA = 90° - 60° = 30°$

Also, $OD \parallel BC$

$\therefore \angle DBC = \angle ODB = 30°$ (Alternate angles)

(iii) $\angle ABC = \angle ABD + \angle DBC = 30° + 30° = 60°$

In cyclic quadrilateral ABCD,

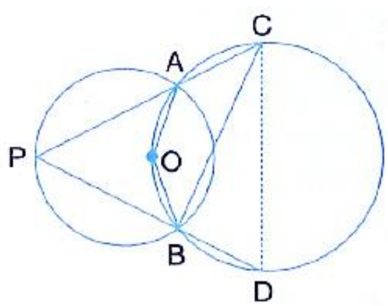
$\angle ADC = 180° - \angle ABC = 180° - 60° = 120°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

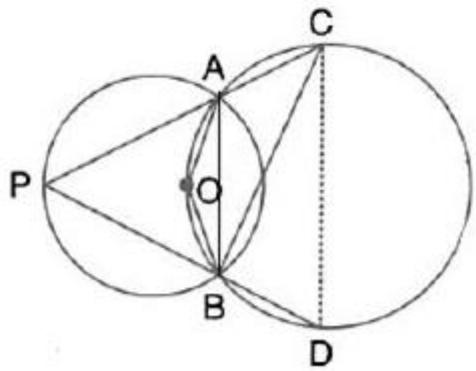
Question 56:

In the given figure, the centre O of the small circle lies on the circumference of the bigger circle. If ∠APB = 75° and ∠BCD = 40°, find:

(i) ∠AOB, (ii) ∠ACB, (iii) ∠ABD, (iv) ∠ADB



Solution 56:



Join AB and AD

(i) $\angle AOB = 2\angle APB = 2\times 75° = 150°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) In cyclic quadrilateral AOBC,

$\angle ACB = 180° - \angle AOB = 180° - 150° = 30°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

(iii) In cyclic quadrilateral ABDC

$\angle ABD = 180° - \angle ACD = 180° - (40° + 30°) = 110°$

(pair of opposite angles in a cyclic quadrilateral are supplementary

(iv) In cyclic quadrilateral AOBD,

$\angle ADB = 180° - \angle AOB = 180° - 150° = 30°$

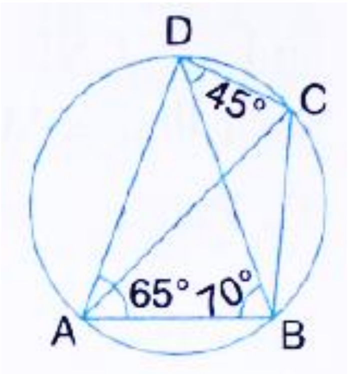
(pair of opposite angles in a cyclic quadrilateral are supplementary

Question 57:

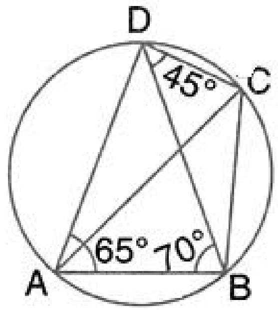
In the given figure, ∠BAD = 65°, ∠ABD = 70° and ∠BDC = 45°. Find:

(i) ∠BCD (ii) ∠ACB

Hence, show that AC is a diameter



Solution 57:



(i) In cyclic quadrilateral ABCD,

$\angle BCD = 180° - \angle BAD = 180° - 65° = 115°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

(ii) By angle sum property of ΔABD,

$\angle ADB = 180° - 65° - 70° = 45°$

Again, $\angle ACB = \angle ADB = 45°$

(Angle in the same segment)

$\therefore \angle ADC = \angle ADB + \angle BDC = 45° + 45° = 90°$

Hence, AC is a semicircle.

(since angle in a semicircle is a right angle)

Question 58:

In a cyclic quadrilateral ABCD, ∠A : ∠C = 3 : 1 and ∠B : ∠D = 1: 5; Find each angle of the quadrilateral.

Solution 58:

Let $\angle A$ and $\angle C$ be 3x and x respectively

In cyclic quadrilateral ABCD,

$\angle A + \angle C = 180°$

(pairs of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow 3x + x = 180°$

$$\Rightarrow x = \frac{180°}{4} = 45°$$

$\therefore \angle A = 135°$ and $\angle C = 45°$

Let the measure of $\angle B$ and $\angle D$ be y and 5y respectively

In cyclic quadrilateral ABCD,

$\angle B + \angle D = 180°$

(pair of opposite angles in a cyclic quadrilateral are supplementary are supplementary)

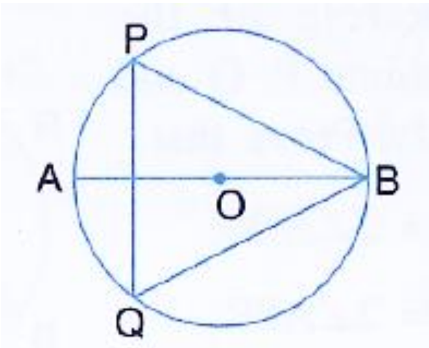
$\Rightarrow y + 5y = 180°$

$\Rightarrow y = \frac{180^\circ}{6} = 30°$

$\therefore \angle B = 30^\circ$ and $\angle D = 150^\circ$

Question 59:

The given figure shows a circle with centre O and ∠ABP = 42°

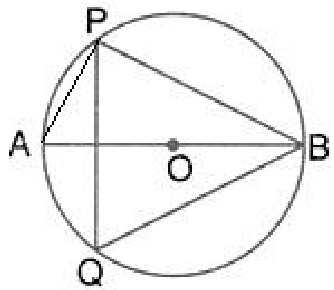


Calculate the measure of:

(i) ∠PQB

(ii) ∠QPB + ∠PBQ

Solution 59:



Join AP.

(i) $\angle APB = 90°$

(Angle in a semicircle)

$\therefore \angle BAP = 90° - \angle ABP = 90° - 42° = 48°$

Now, $\angle PQB = \angle BAP = 48°$

(Angle in the same segment)

(ii) By angle sum property of ΔBPQ,

$\angle QPB + \angle PBQ = 180° - \angle PQB = 180° - 48° = 132°$

Question 60:

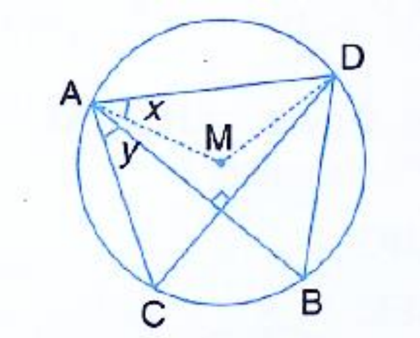
In the given figure, M is the centre of the circle. Chords AB and CD are perpendicular to each other.

If $\angle MAD = x$ and $\angle BAC = y$:

(i) express $\angle AMD$ in terms of x.

(ii) express $\angle ABD$ in terms of y.

(iii) prove that: $x = y$.



Solution 60:

In the figure, M is the centre of the circle.

Chords AB and CD are perpendicular to each other at L.

$\angle MAD = x$ and $\angle BAC = y$

(i) In ΔAMD,

$MA = MD$

$\therefore \angle MAD = \angle MDA = x$

But in ΔAMD,

$\angle MAD + \angle MDA + \angle AMD = 180°$

$\Rightarrow x + x + \angle AMD = 180°$

$\Rightarrow 2x + \angle AMD = 180°$

$\Rightarrow \angle AMD = 180° - 2x$

(ii) $\therefore$Arc $AD \angle AMD$ at the centre and $\angle ABD$ at the remaining

(Angle in the same segment)

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow \angle AMD = 2\angle ABD$

$\Rightarrow \angle ABD = \frac{1}{2}(180° - 2x)$

$\Rightarrow \angle ABD = 90° - x$

$AB \perp CD$, $\angle ALC = 90^{\circ}°$

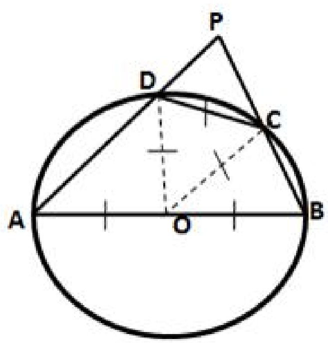
In ΔALC,

$\therefore \angle LAC + \angle LCA = 90°$
$\Rightarrow \angle BAC + \angle DAC = 90°$
$\Rightarrow y + \angle DAC = 90°$
$\therefore \angle DAC = 90° - y$
We have, $\angle DAC = \angle ABD$ [Angles in the same segment]
$\therefore \angle ABD = 90° - y$
(iii) we have, $\angle ABD = 90° - y$ and $\angle ABD = 90° - x$ [proved]
$\therefore 90° - x = 90° - y$
$\Rightarrow x = y$

Question 61:

In a circle, with centre O, a cyclic quadrilateral ABCD is drawn with AB as a diameter of the circle and CD equal to radius of the circle. If AD and BC produced meet at point P; show that $\angle APB = 60°$.

Solution 61:

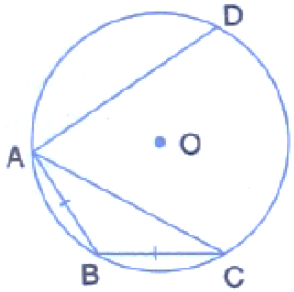


Join OD and OC.
ΔOCD, $OD = OD = CD$
$\therefore$ ΔOCD is an equilateral triangle
$\therefore \angle ODC = 60°$
Also, in cyclic quadrilateral ABCD
$\angle ADC + \angle ABC = 180°$
(pair of opposite angles in cyclic quadrilateral are supplementary)
$\Rightarrow \angle ODA + 60° + \angle ABP = 180°$
$\Rightarrow \angle OAD + \angle ABP = 90°$ $(\because OA = OD)$
$\Rightarrow \angle PAB + \angle ABP = 120°$
By angle sum property of ΔPAB,
$\therefore \angle APB = 180° - \angle PAB - \angle ABP = 180° - 120° = 60°$

EXERCISE. 17 (C)

Question 1:

In the given diagram, chord AB = chord BC.



(i) what is the relation between arcs AB and BC?

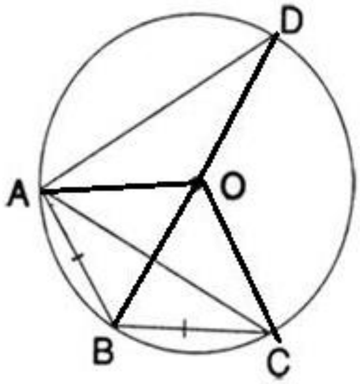
(ii) what is the relation between ∠AOB and ∠BOC?

(iii) If arc AD is greater than arc ABC, then what is the relation between chords AD and AC?

(iv) If ∠AOB = 50°, find the measure of angle BAC.

Solution 1:

Join OA, OB, OC and OD.



(i) Arc AB = Arc BC [∵ Equal chords subtends equal arcs]

(ii) $\angle AOB = \angle BOC$ [∵ Equal chords subtends equal arcs]

(iii) If arc AD > arc ABC, then chord AD > AC

(iv) $\angle AOB = 50°$

But $\angle AOB = \angle BOC$ [from (ii) above]

$\therefore \angle BOC = 50°$

Now arc BC subtends $\angle BOC$ at the center and $\angle BAC$ at

The remaining part of the circle.

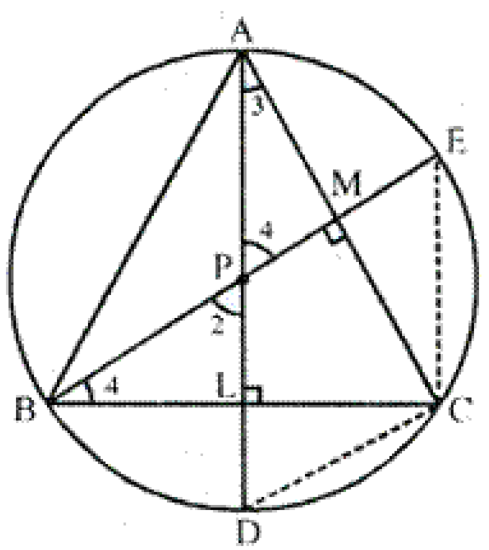
$$\therefore \angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 50° = 25°$$

Question 2:

In Δ ABC, the perpendicular from vertices A and B on their opposite sides meet (when produced) the circumcircle of the triangle at points D and E respectively.
Prove that: arc CD = arc CE

Solution 2:

Given: In ΔABC, the perpendiculars from vertices A and B on their opposite sides meet (when produced) the circumcircle of the triangle at points D and E respectively.
To prove: Arc CD = Arc CE
Construction: Join CE and CD
Proof:

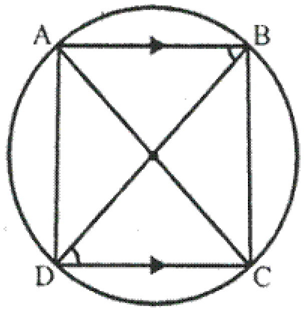


In ΔAPM and ΔBPL
$\angle AMP = \angle BLP$ [each = $90°$]
$\angle 1 = \angle 2$ [vertically opposite angles]
$\Delta APM \sim \Delta BPL$ [AA postulate]
∴ Third angle = Third angle
$\therefore \angle 3 = \angle 4$
∴ Arc which subtends equal angle at the
Circumference of the circle are also equal.
∴ Arc CD = Arc CE

Question 3:

In a cyclic-trapezium, the non-parallel sides are equal and the diagonals are also equal. Prove it.

Solution 3:

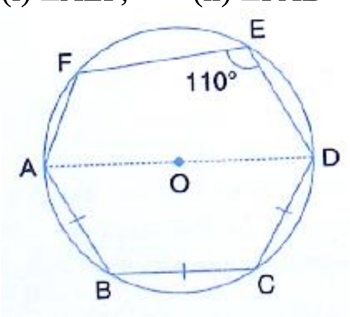


A cyclic trapezium ABCD in which AB ∥ DC and AC and BD are joined.
To prove-
(i) AD = BC
(ii) AC = BD
Proof:
∵ chord AD subtends $\angle ABD$ and chord BC subtends $\angle BDC$
At the circumference of the circle.
But $\angle ABD = \angle BDC$ [proved]
Chord AD = Chord BC
$\Rightarrow AD = BC$
Now in ΔADC and Δ BCD
$DC = DC$ [Common]
$\angle CAD = \angle CBD$ [angles in the same segment]
And $AD = BC$ [proved]
By Side – Angle – Side criterion of congruence, we have
$\therefore \Delta ADC \cong \Delta BCD$ [SAS axion]

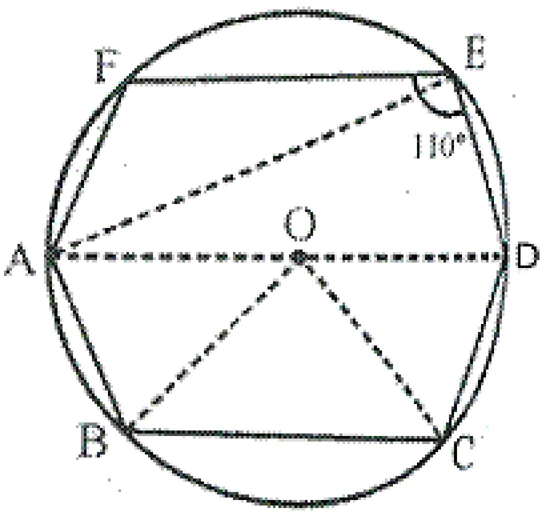
The corresponding parts of the congruent triangle are congruent
$\therefore AC = BD$ [c.p.c.t]

Question 4:

In the following figure, AD is the diameter of the circle with centre O. chords AB, BC and CD are equal. If ∠DEF = 110°, Calculate:
(i) ∠AEF, (ii) ∠FAB



Solution 4:



Join AE, OB and OC

(i) $\because$ AOD is the diameter,

$\therefore \angle AED = 90°$ [Angle in a semi-circle]

But $\angle DEF = 110°$ [given]

$\therefore \angle AEF = \angle DEF - \angle AED$

$= 110° - 90° = 20°$

(ii) $\because$ Chord AB = Chord BC = Chord CD [given]

$\therefore \angle AOB = \angle BOC = \angle COD$ (Equal chords subtends equal angles at the centre)

But $\angle AOB + \angle BOC + \angle COD = 180°$ [AOD is a straight line]

$\therefore \angle AOB = \angle BOC = \angle COD = 60°$

In $\Delta OAB,\ OA = OB$

$\therefore \angle OAB = \angle OBA$ [radii of the same circle]

But $\angle OAB + \angle OBA = 180° - \angle AOB$

$= 180° - 60°$

$= 120°$

$\therefore \angle OAB = \angle OBA = 60°$

In cyclic quadrilateral ADEF,

$\angle DEF + \angle DAF = 180°$

$\Rightarrow \angle DAF = 180° - \angle DEF$

$= 180° - 110°$

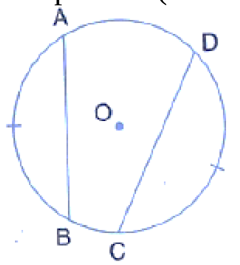
$= 70°$

Now, $\angle FAB = \angle DAF + \angle OAB$

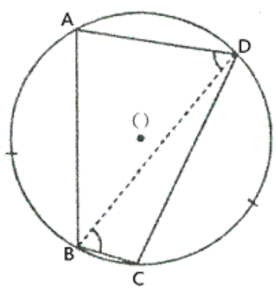
$= 70° + 60° = 130°$

Question 5:

In the given figure, if arc AB = arc CD, then prove that the quadrilateral ABCD is an isosceles – trapezium (O is the centre of the circle).



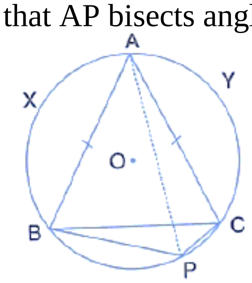
Solution 5:



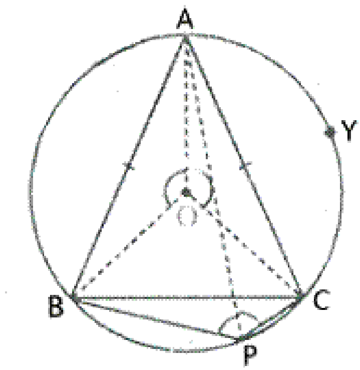
Given –
In the figure, O is the centre of a circle and arc AB = arc CD
To prove –
ABCD is an isosceles trapezium.
Construction – Join BD, AD and BC.
Proof – Since equal arcs subtends equal angles at the circumference of a circle.
$\therefore \angle ADB = \angle DBC$ [∵ arc AB = arc CD]
But, these are alternate angles.
$\therefore AD \parallel BC$
∴ ABCD is a trapezium
∵ Arc AB = Arc CD [Given]
∴ Chord AB = Chord CD
∴ ABCD is an isosceles trapezium

Question 6:

In the given figure ABC is an isosceles triangle and O is the centre of its circumcircle. Prove that AP bisects angle BPC.



Solution 6:



Given –

ΔABC is an isosceles triangle and O is the centre of its circumcircle

To prove –

AP bisects $\angle BPC$

Proof –

Chord AB subtends $\angle APB$ and chord AC subtends $\angle APC$ at the Circumference of the circle.

But chord AB = Chord AC

$\therefore \angle APB = \angle APC$

$\therefore$ AP is the bisector of $\angle BPC$

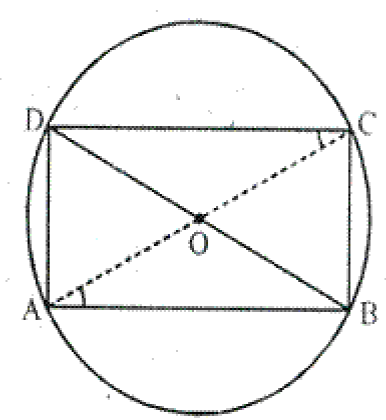
Question 7:

If two sides of a cyclic quadrilateral are parallel; prove that:

(i) its other two sides are equal.

(ii) its diagonals are equal.

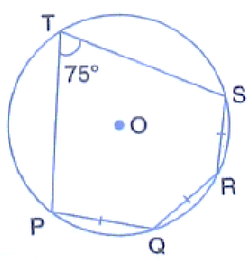
Solution 7:



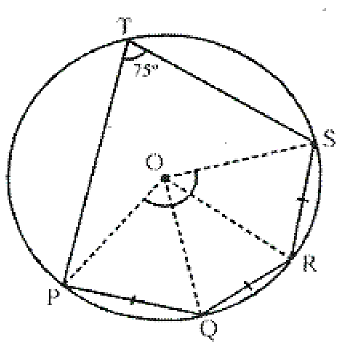
Given –
ABCD is a cyclic quadrilateral in which AB ∥ DC. AC and BD are its diagonals.
To prove –
(i) $AD = BC$
(ii) $AC = BD$
Proof –
(i) $AB \parallel DC \Rightarrow \angle DCA = \angle CAB$ [Alternate angles]
Now, chord AD subtends $\angle DCA$ and chord BC subtends $\angle CAB$
At the circumference of the circle.
$\therefore \angle DCA = \angle CAB$ [proved]
∴ Chord AD = Chord BC or AD = BC
(ii) Now in ΔABC and ΔADB,
AB = AB [Common]
$\angle ACB = \angle ADB$ [Angles in the same segment]
$BC = AD$ [Proved]
By Side – Angle – Side criterion of congruence, we have
$\Delta ACB \cong \Delta ADB$ [SAS postulate]
The corresponding parts of the congruent triangles are congruent.
$\therefore AC = BD$ [c.p.c.t]

Question 8:

The given figure shows a circle with centre O. Also, PQ = QR = RS and ∠PTS = 75°. Calculate:
(i) ∠POS, (ii) ∠QOR, (iii) ∠PQR.



Solution 8:



Join OP, OQ and OS.

$\because PQ = QR = RS,$

$\angle POQ = \angle QOR = \angle ROS$ [Equal chords subtends equal angles at the centre]

Arc PQRS subtends $\angle POS$ at the center and $\angle PTS$ at the remaining
Parts of the circle.

$\therefore \angle POS = 2\angle PTS = 2 \times 75° = 150°$

$\Rightarrow \angle POQ + \angle QOR + \angle ROS = 150°$

$\Rightarrow \angle POQ = \angle QOR = \angle ROS = \dfrac{150°}{3} = 50°$

In ΔOPQ, $OP = OQ$ [radii of the same circle]

$\therefore \angle OPQ = \angle OQP$

But $\angle OPQ + \angle OQP + \angle POQ = 180°$

$\therefore \angle OPQ + \angle QP = 50° = 180°$

$\Rightarrow \angle OPQ + \angle OQP = 180° - 50°$

$\Rightarrow \angle OPQ + \angle OPQ = 130°$

$\Rightarrow 2\angle OPQ = 130°$

$\Rightarrow \angle OPQ = \angle OQP = \dfrac{130°}{2} = 65°$

Similarly, we can prove that

In $\Delta OQR, \angle OQR = \angle ORQ = 65°$

And in $\Delta ORS, \angle ORS = \angle OSR = 65°$

(i) Now $\angle POS = 150°$

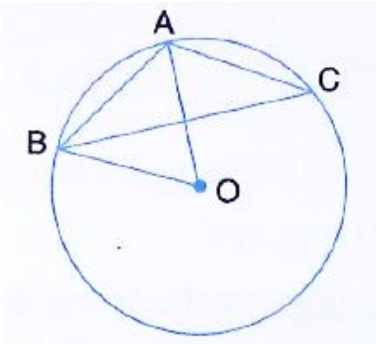
(ii) $\angle QOR = 50°$ and

(iii) $\angle PQR = \angle PQO + \angle OQR = 65° + 65° = 130°$

Question 9:

In the given figure, AB is a side of a regular six-sided polygon and AC is a side of a regular eight sided polygon inscribed in the circle with centre O. Calculate the sizes of:

(i) ∠AOB, (ii) ∠ACB (iii) ∠ABC



Solution 9:

(i) Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$\therefore \angle ACB = \frac{1}{2}\angle AOB$

Since AB is the side of a regular hexagon,
$\angle AOB = 60°$

(ii) $\angle AOB = 60° \Rightarrow \angle ACB = \frac{1}{2} \times 60° = 30°$

(iii) Since AC is the side of a regulare octagon,

$\angle AOC = \frac{360}{8} = 45°$

Again, Arc AC subtends $\angle AOC$ at the center and $\angle ABC$ at the remaining part of the circle.

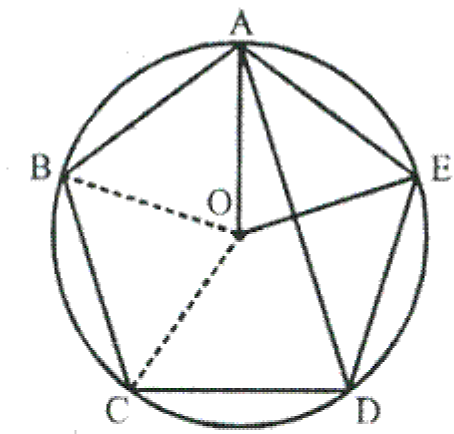
$\Rightarrow \angle ABC = \frac{1}{2}\angle AOC$

$\Rightarrow \angle ABC = \frac{45°}{2} = 22.5°$

Question 10:

In a regular pentagon ABCDE, Inscribed in a circle; find ratio between angle EDA and angle ADC.

Solution 10:



Arc AE subtends $\angle AOE$ at the centre and
$\angle ADE$ at the remaining part of the circle.

$\therefore \angle ADE = \frac{1}{2}\angle AOE$

$= \frac{1}{2} \times 72°$

$= 36°$ [central angle is a regular pentagon at O]

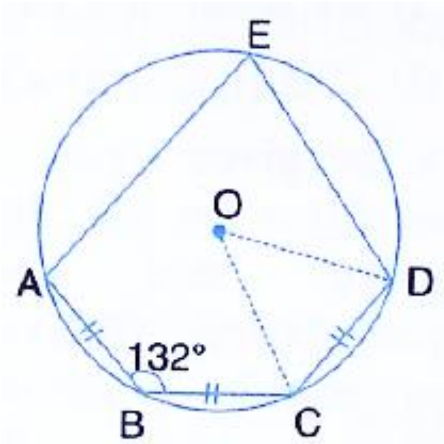
$\angle ADC = \angle ADB + \angle BDC$

$= 36° + 36° + 72°$

$\therefore \angle ADE : \angle ADC = 36° : 72° = 1 : 2$

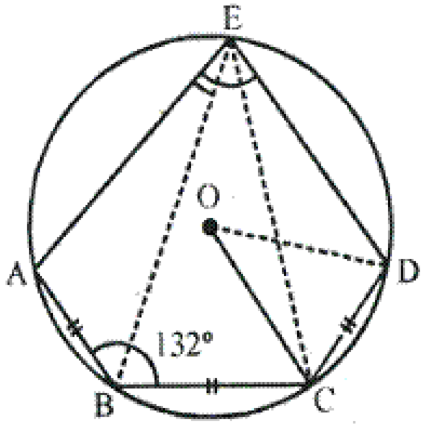
Question 11:

In the given figure, AB = BC = CD and $\angle ABC = 132°$. Calcualte:



(i) $\angle AEB$, (ii) $\angle AED$, (iii) $\angle COD$.

Solution 11:



In the figure, O is the centre of circle, with AB = BC = CD.

Also, $\angle ABC = 132°$

(i) In cyclic quadrilateral ABCE

$\angle ABC + \angle AEC = 180°$ [sum of opposite angles]

$\rightarrow 132° + \angle AEC = 180°$

$\rightarrow \angle AEC = 180° - 132°$

$\rightarrow \angle AEC = 48°$

Since, $AB = BC$, $\angle AEB = \angle BEC$ [equal chords subtends equal angles]

$\therefore \angle AEB = \frac{1}{2} \angle AEC$

$= \frac{1}{2} \times 48°$

$= 24°$

(ii) Similarly, $AB = BC = CD$

$\angle AEB = \angle BEC = \angle CED = 24°$

$\angle AED = \angle AEB + \angle BEC + \angle CED$

$= 24° + 24° + 24° = 72°$

(iii) Arc CD subtends $\angle COD$ at the centre and

$\angle CED$ at the remaining part of the circle.

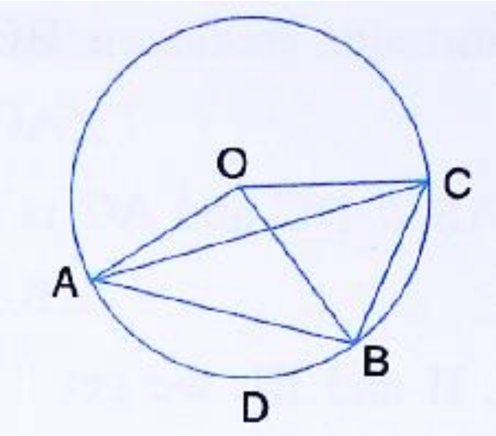
$\therefore COD = 2\angle CED$

$= 2 \times 24°$

$= 48°$

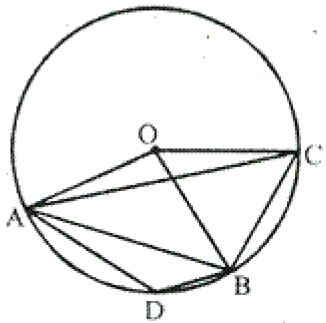
Question 12:

In the figure, O is the centre of the circle and the length of arc AB is twice the length of arc BC. If angle AOB = 108°, find:



(i) $\angle CAB$ (ii) $\angle ADB$

Solution 12:



(i) Join AD and DB

Arc B = 2 arc BC and $\angle AOB = 180°$

$$\therefore \angle BOC = \frac{1}{2}\angle AOB$$
$$= \frac{1}{2}\times 108°$$
$$= 54°$$

Now, Arc BC subtends $\angle BOC$ at the centre and $\angle CAB$ at the remaining part of the circle.

$$\therefore CAB = \frac{1}{2}\angle BOC$$
$$= \frac{1}{2}\times 54°$$
$$= 27°$$

(ii) Again, Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$$\therefore ACB = \frac{1}{2}\angle AOB$$
$$= \frac{1}{2}\times 108°$$
$$= 54$$

In cyclic quadrilateral ADBC

$\angle ADB + \angle ACB = 180°$ [sum of opposite angles]

$\Rightarrow \angle ADB + 54° = 180°$

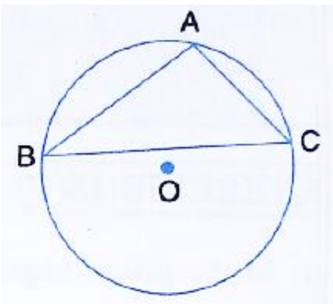
$\Rightarrow \angle ADB = 180° - 54°$

$\Rightarrow \angle ADB = 126°$

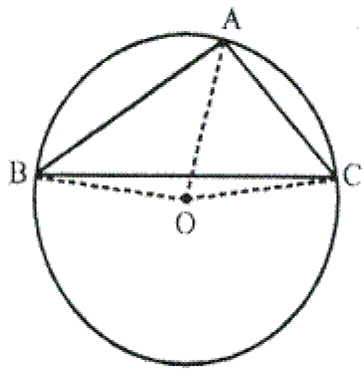
Question 13:

The figure shows a circle with centre O. AB is the side of regular pentagon and AC is the side of regular hexagon.

Find the angles of triangle ABC.



Solution 13:



Join OA, OB and OC

Since AB is the side of a regular pentagon,

$$\angle AOB = \frac{360°}{5} = 72°$$

Again AC is the side of a regular hexagon,

$$\angle AOC = \frac{360°}{6} = 60°$$

But $\angle AOB + \angle AOC + \angle BOC = 360°$ [Angles at a point]

$\Rightarrow 72° + 60° + \angle BOC = 360°$

$\Rightarrow 132° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 360° - 132°$

$\Rightarrow \angle BOC = 228°$

Now, Arc BC subtends $\angle BOC$ at the centre and

$\angle BAC$ at the remaining part of the circle.

$$\Rightarrow \angle BAC = \frac{1}{2}\angle BOC$$

$$\Rightarrow \angle BAC = \frac{1}{2} \times 228° = 114°$$

Similarly, we can prove that

$$\Rightarrow \angle ABC = \frac{1}{2} \angle AOC$$

$$\Rightarrow \angle ABC = \frac{1}{2} \times 60° = 30°$$

And

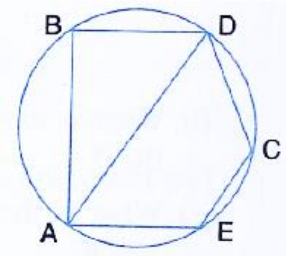
$$\Rightarrow \angle ACB = \frac{1}{2} \angle AOB$$

$$\Rightarrow \angle ACB = \frac{1}{2} \times 72° = 36°$$

Thus, angles of the triangle are, 114°, 30° and 36°

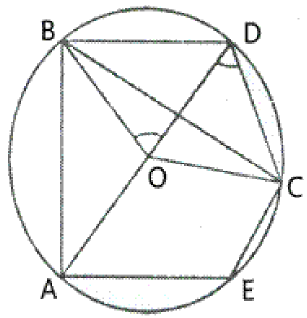
Question 14:

In the given figure, BD is a side of a regular hexagon, DC is a side of a regular pentagon and AD is a diameter. Calculate:



(i) ∠ADC (ii) ∠BDA, (iii) ∠ABC, (iv) ∠AEC.

Solution 14:



Join BC, BO, CO and EO

Since BD is the side of a regular hexagon,

$\angle BOD = \frac{360}{6} = 60°$

Since DC is the side of a regular pentagon,

$\angle COD = \frac{360}{5} = 72°$

In ΔBOD. $\angle BOD = 60°$ and OB = OD

$\therefore \angle OBD = \angle ODB = 60°$

(i) In ΔOCD, $\angle COD = 72°$ and OC = OD

$$\therefore \angle ODC = \frac{1}{2}(180° - 72°)$$
$$= \frac{1}{2} \times 108°$$
$$= 54°$$

Or, $\angle ADC = 54°$

(ii) $\angle BDO = 60°$ or $\angle BDA = 60°$

(iii) Arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$$\therefore ABC = \frac{1}{2}\angle AOC$$
$$= \frac{1}{2}[\angle AOD - \angle COD]$$
$$= \frac{1}{2} \times (180° - 72°)$$
$$= \frac{1}{2} \times 108°$$
$$= 54°$$

(iv) In cyclic quadrilateral AECD

$\angle AEC + \angle ADC = 180°$ [sum of opposite angles]

$\Rightarrow \angle AEC + 54° = 180°$

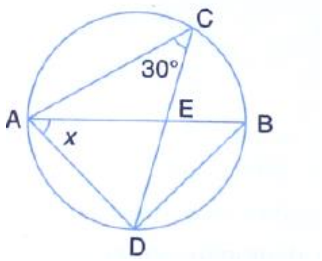
$\Rightarrow \angle AEC = 180° - 54°$

$\Rightarrow \angle AEC = 126°$

EXERCISE. 17 (D)

Question 1:

In the given circle with diameter AB, find the value of x.



Solution 1:

$\angle ABD = \angle ACD = 30°$ [Angle in the same segment]

Now in ΔADB,

$\angle BAD + \angle ADB + \angle ABD = 180°$ [angles of a triangle]

But, $\angle ADB = 90°$ [Angle in a semi-circle]

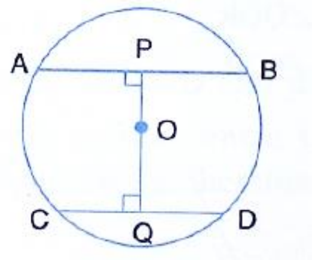
$\therefore x + 90° + 30° = 180°$

$\Rightarrow x = 180° - 120°$

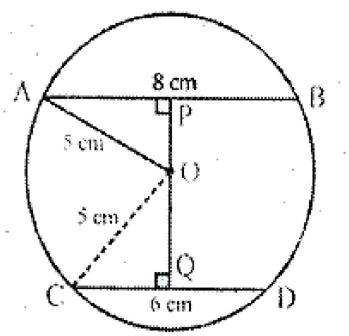
$\Rightarrow x = 60°$

Question 2:

In the given figure, O is the centre of the circle with radius 5 cm. OP and OQ are perpendicular to AB and CD respectively. AB = 8 cm and CD = 6 cm. determine the length of PQ.



Solution 2:



Radius of the circle whose centre is 0 = 5 cm
OP ⊥ AB and OQ ⊥ CD, AB = 8 cm and CD = 6 cm.
Join OA and OC, then OA = OC + 5 cm
Since OP ⊥ AB, P is the midpoint of AB
Similarly, Q is the midpoint of CD
In right ΔOAP,

$OA^2 = OP^2 + AP^2$ [Pythagoras Theorem]

$\Rightarrow (5)^2 = OP^2 + (4)^2$ $[\because AP = PB = \frac{1}{2} \times 8 = 4 \text{ cm}]$

$\Rightarrow 25 = OP^2 + 16$

$\Rightarrow OP^2 = 25 - 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow OP = 3 \text{ cm}$

Similarly, in right ΔOCQ,

$OC^2 = OQ^2 + CQ^2$ [Pythagoras theorem]

$\Rightarrow (5)^2 = OQ^2 + (3)^2$

$\Rightarrow 25 = OQ^2 + 9$

$\Rightarrow OQ^2 = 25 - 9$

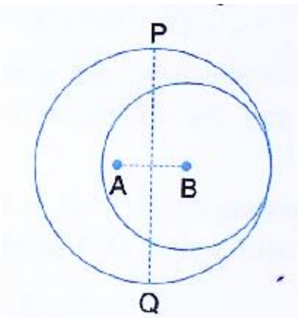
$\Rightarrow OQ^2 = 16$

$\Rightarrow OQ = 4\text{cm}$

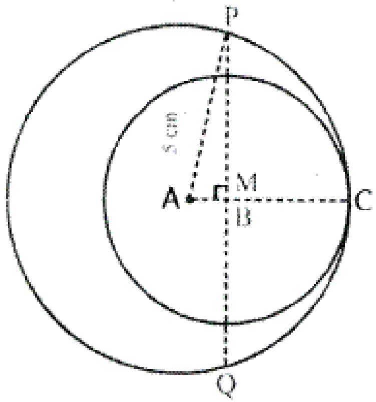
Hence, $PQ = OP + OQ = 3 + 4 = 7 \text{ cm}$

Question 3:

The given figure shows two circles with centres A and B; and radii 5 cm and 3 cm respectively, touching each other internally. If the perpendicular bisector of AB meets the bigger circle in P and Q, find the length of PQ.



Solution 3:



Join AP and produce AB to meet the bigger circle at C.
AB = AC – BC = 5 cm – 3 cm = 2 cm
But, M is the mid – point of AB.

$\therefore$ AM $= \frac{2}{2} = 1\text{cm}$

Now in right ΔAPM,

$AP^2 = MP^2 + AM^2$ [Pythagoras Theorem]

$\Rightarrow (5)^2 = MP^2 + 1^2$

$\Rightarrow 25 = MP^2 1$

$\Rightarrow MP^2 = 25 - 1$

$\Rightarrow MP^2 = 24$

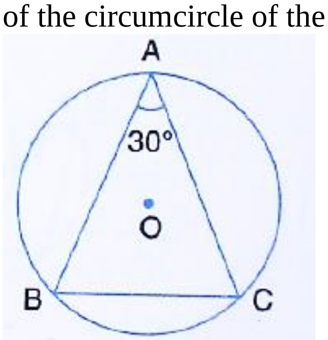
$\Rightarrow MP = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$ cm

$\therefore PQ = 2MP = 2 \times 2\sqrt{6} = 4\sqrt{6}$ cm

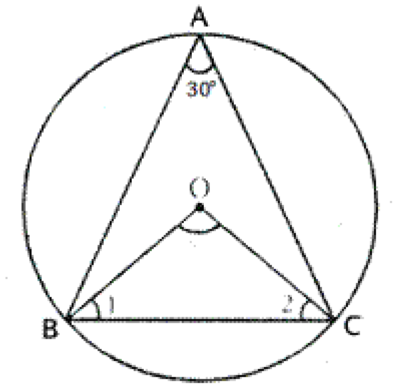
$\Rightarrow PQ = 4 \times 2.45 = 9.8$ cm

Question 4:

In the given figure, ABC is a triangle in which ∠BAC = 30°. Show that BC is equal to the radius of the circumcircle of the triangle ABC, whose centre is O.



Solution 4:



Given – In the figure ABC is a triangle in which $\angle A = 30°$

To prove – BC is the radius of circumcircle of ΔABC whose centre is O.

Construction – join OB and OC.

Proof:

$\angle BOC = 2\angle BAC = 2 \times 30° = 60°$

Now in ΔOBC,

$OB = OC$ [Radii of the same circle]

$\angle OBC = \angle OCB$

But, in ΔBOC,

$\angle OBC + \angle OCB + \angle BOC = 180°$ [Angles of a triangle]

$\Rightarrow \angle OBC + \angle OBC + 60° = 180°$

$\Rightarrow 2\angle OBC + 60° = 180°$

$\Rightarrow 2\angle OBC = 180° - 60°$

$\Rightarrow 2\angle OBC = 120°$

$\Rightarrow \angle OBC = \dfrac{120°}{2} = 60°$

$\Rightarrow \angle OBC = \angle OCB = \angle BOC = 60°$

$\Rightarrow \Delta BOC$ is an equilateral triangle

$\Rightarrow BC = OB = OC$

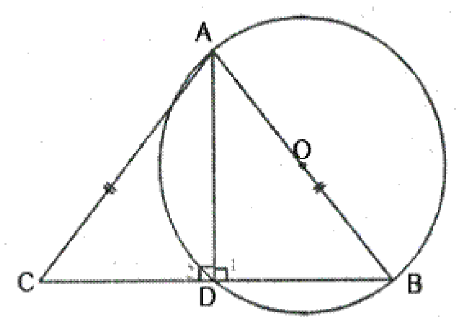
But, OB and OC are the radii of the circum – circle

$\therefore$ BC is also the radius of the circum – circle

Question 5:

Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.

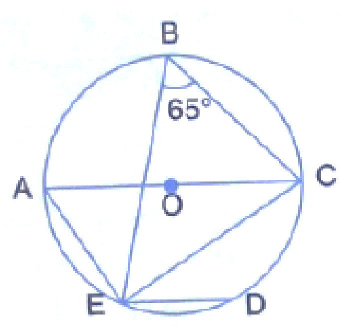
Solution 5:



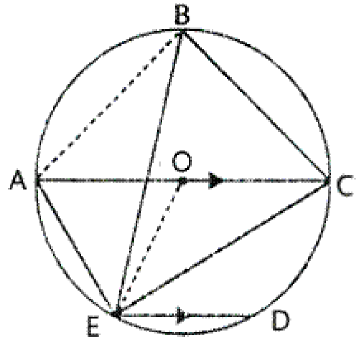
Given – In ΔABC, AB = AC and a circle with AB as diameter is drawn
Which intersects the side BC and D.
To prove – D is the mid point of BC
Construction – Join AD.
Proof - $\angle 1 = 90°$ [Angle in a semi circle]
But $\angle 1 + \angle 2 = 180°$ [Linear pair]
$\therefore \angle 2 = 90°$
Now in right ΔABD and ΔACD,
Hyp. AB = Hyp. AC [Given]
Side AD = AD [Common]
∴ By the right Angle – Hypotenuse – side criterion of congruence, we have
$\Delta ABD \cong \Delta ACD$ [RHS criterion of congruence]
The corresponding parts of the congruent triangle are congruent.
∴ BD = DC [c.p.c.t]
Hence D is the mid point of BC.

Question 6:

In the given figure, chord ED is parallel to diameter AC of the circle. Given ∠CBE = 65°, calculate ∠DEC.



Solution 6:



Join OE.

Arc EC subtends $\angle EOC$ at the centre and $\angle EBC$ at the remaining Part of the circle.

$\angle EOC = 2\angle EBC = 2 \times 65° = 130°$

Now in ΔOEC, $OE = OC$ [radii of the same circle]

$\therefore \angle OEC = \angle OCE$

But, in ΔEOC,

$\angle OEC + \angle OCE + \angle EOC = 180°$ [Angles of a triangle]

$\Rightarrow \angle OCE + \angle OCE + \angle EOC = 180°$

$\Rightarrow 2\angle OCE + 130° = 180°$

$\Rightarrow 2\angle OCE = 180° - 130°$

$\Rightarrow 2\angle OCE = 50°$

$\Rightarrow \angle OCE = \frac{50°}{2} = 25°$

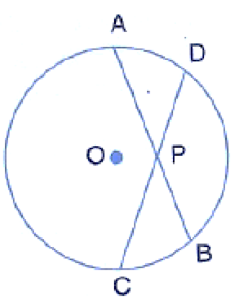
$\therefore AC \parallel ED$ [Given]

$\therefore \angle DEC = \angle OCE$

$\Rightarrow \angle DEC = 25°$

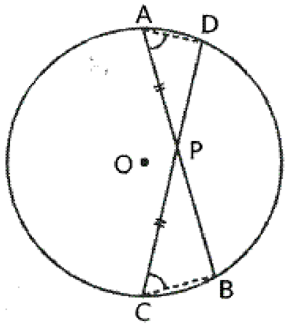
Question 7:

Chords AB and CD of a circle intersect each other at point P such that AP = CP.



Show that: AB = CD

Solution 7:



Given – two chords AB and CD intersect
Each other at P inside the circle
With centre O and AP = CP
To prove – AB = CD
Proof – Two chords AB and CD intersect each other inside the circle at P.

$\therefore AP \times PB = CP \times PD$

$\Rightarrow \frac{AP}{CP} = \frac{PD}{PB}$

But $AP = CP$ ....(1) [given]

$\therefore PD = PB$ or $PB = PD$ .... (2)

Adding (1) and (2)

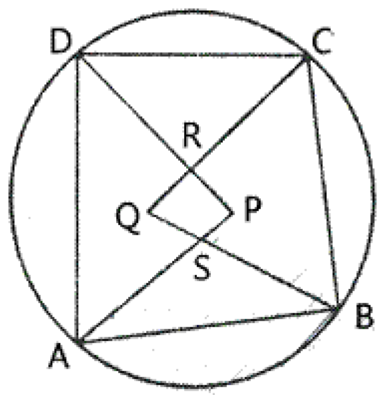
$AP + PB = CP + PD$

$\Rightarrow AB = CD$

Question 8:

The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic. Prove it.

Solution 8:



Given – ABCD is a cyclic quadrilateral and PQRS is a
Quadrilateral formed by the angle
Bisectors of angle $\angle A, \angle B, \angle C$ and $\angle D$

To prove – PQRS is a cyclic quadrilateral.

Proof – In ΔAPD,

$\angle PAD + \angle ADP + \angle APD = 180°$ .... (1)

Similarly, IN ΔBQC,

$\angle QBC + \angle BCQ + \angle BQC = 180°$ ....(2)

Adding (1) and (2), we get

$\angle PAD + \angle ADP + \angle APD + \angle QBC + \angle BCQ + \angle BQC = 180° + 180°$

$\Rightarrow \angle PAD + \angle ADP + \angle QBC + \angle BCQ + \angle APD + \angle BQC = 360°$

But $\angle PAD + \angle ADP + \angle QBC + \angle BCQ = \frac{1}{2}[\angle A + \angle B + \angle C + \angle D]$

$$= \frac{1}{2} \times 360° = 180°$$

$\therefore \angle APD + \angle BQC = 360° - 180° = 180°$ [from (3)]

But these are the sum of opposite angles of quadrilateral PRQS.

∴ Quad. PRQS is a cyclic quadrilateral.

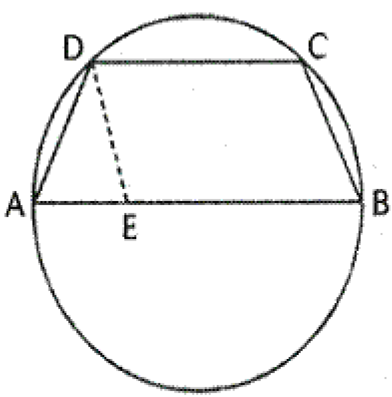
Question 9:

If two non-parallel sides of a trapezium are equal, it is cyclic. Prove it.

Or

An isosceles trapezium is always cyclic. Prove it.

Solution 9:



Given – ABCD is a trapezium in which AB ∥ CD and AD = BC

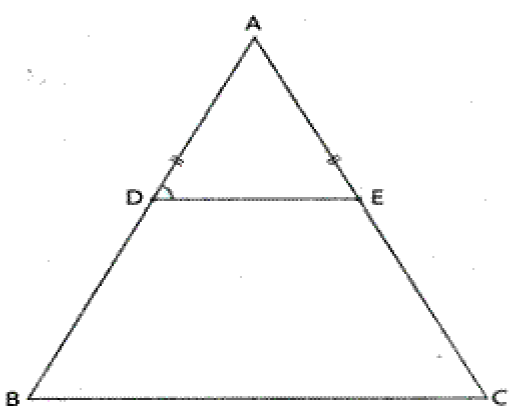
To prove – ABCD is cyclic

Construction – draw DE ∥ BC
Proof –
DCBE is a parallelogram [by construction]
$\angle DEB = \angle DCB$ [Opposite angles of parallelogram]
Also, $\angle DEB = \angle EDA + \angle DAE$ [Exterior angle property]
In ΔADE, $\angle DAE = \angle DAE$ ……. (1) [since AD = BC = DE Or AD = DE]
Also, $\angle DEB + \angle EDA = 180°$ ………. (2)
From (1) and (2),
$\angle DEB + \angle DAE = 180°$
$\Rightarrow \angle DCB + \angle DAE = 180°$
$\Rightarrow \angle C + \angle A = 180°$
Hence ABCD is cyclic trapezium

Question 10:

D and E are points on equal sides AB and AC of an isosceles triangle ABC such that AD = AE. Prove that the points B, C, E and D are concyclic.

Solution 10:

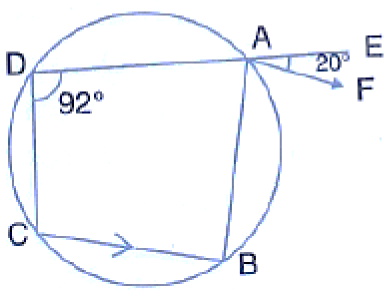


Given – In ΔABC, AB = AC and D and E are points on AB and AC
Such that AD = AE. DE is joined.
To prove B, C, E, D are concyclic.
Proof – In ΔABC, AB = AC
$\therefore \angle B = \angle C$ [Angles opposite to equal sides]
Similarly, In ΔADE, AD = AE [Given]
$\therefore \angle ADE = \angle AED$ [Angles opposite to equal sides]
In ΔABC,

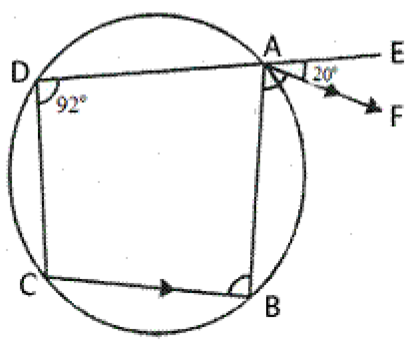
$\because \frac{AP}{AB} = \frac{AE}{AC}$
$\therefore DE \parallel BC$
$\therefore \angle ADE = \angle B$ [corresponding angles]
But $\angle B = \angle C$ [proved]
$\therefore$ Ext $\angle ADE =$ its interior opposite $\angle C$
$\therefore$ BCED is a cyclic quadrilateral
Hence B, C, E and D are concyclic.

Question 11:

In the given figure, ABCD is a cyclic quadrilateral. AF is drawn parallel to CB and DA is produced to point E. if ∠ADC = 92°, ∠FAE = 20°; determine ∠BCD. Give reason in support of your answer.



Solution 11:



In cyclic quad. ABCD,
$AF \parallel CB$ and DA is produced to E such that $\angle ADC = 92°$ and $\angle FAE = 20°$
Now we need to find the measure of $\angle BCD$
In cyclic quad. ABCD,
$\angle B + \angle = 180°$

$\Rightarrow \angle B + 92° = 180°$

$\Rightarrow \angle B = 180° - 92°$

$\Rightarrow \angle B = 88°$

Since $AF \parallel CB$, $\angle FAB = \angle B = 88°$

But, $\angle FAE = 20°$ [given]

Ext. $\angle BAE = \angle BAF + \angle FAE$

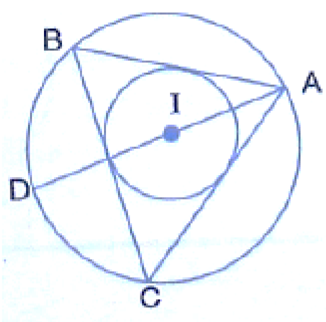
$= 88° + 22° = 108°$

But, Ext. $\angle BAE = \angle BCD$

$\therefore \angle BCD = 108°$

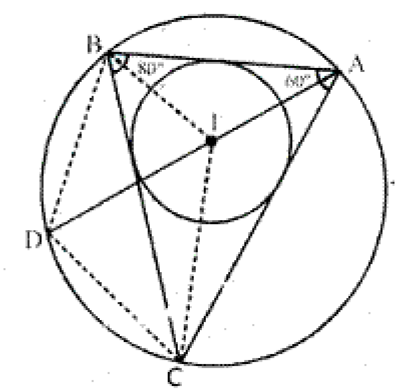
Question 12:

If I is the incentre of triangle ABC and AI when produced meets the circumcircle of triangle ABC in point D. If ∠BAC = 66° and ∠ABC = 80°. Calculate:



(i) ∠DBC (ii) ∠IBC (iii) BIC

Solution 12:



Join DB and DC, IB and IC

$\angle BAC = 66°$, $\angle ABC = 80°$, I is the incentre of the ΔABC,

(i) since $\angle DBC$ and $\angle DAC$ are in the same segment,

$\angle DBC = \angle DAC$

But, $\angle DAC = \frac{1}{2} \angle BAC = \frac{1}{2} \times 66° = 33°$

$\therefore \angle DBC = 33°$

(ii) Since I is the incentre of ΔABC, IB bisects $\angle ABC$

$\therefore \angle IBC = \frac{1}{2} \angle ABC = \frac{1}{2} \times 80° = 40°$

(iii) $\therefore \angle BAC = 66°$ and $\angle ABC = 80°$

In $\Delta ABC, \angle ACB = 180° - (\angle ABC + \angle BAC)$

$\Rightarrow \angle ACB = 180° - (80° + 66°)$

$\Rightarrow \angle ACB = 180° - (156°)$

$\Rightarrow \angle ACB = 34°$

Since IC bisects the $\angle C$

$\therefore \angle ICB = \frac{1}{2} \angle C = \frac{1}{2} \times 34° = 17°$

Now in ΔIBC

$\angle IBC + \angle ICB + \angle BIC = 180°$

$\Rightarrow 40° + 17° + \angle BIC = 180°$

$\Rightarrow 57° + \angle BIC = 180°$

$\Rightarrow \angle BIC = 180° - 57°$

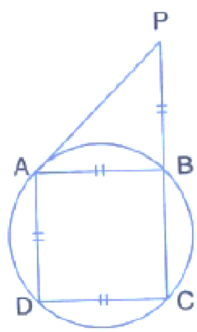
$\Rightarrow \angle BIC = 123°$

Question 13:

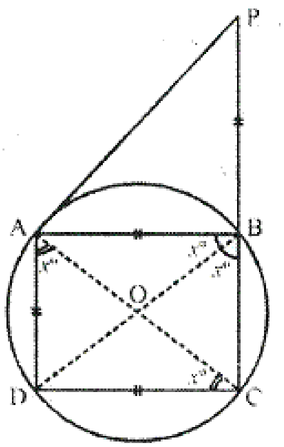
In the given figure, AB = AD = DC = PB and $\angle DBC = x°$. determine, in terms of x:

(i) $\angle ABD$, (ii) $\angle APB$

Hence or otherwise, prove that AP is parallel to DB.



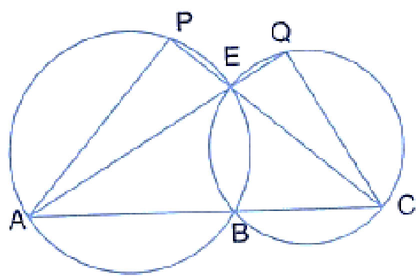
Solution 13:



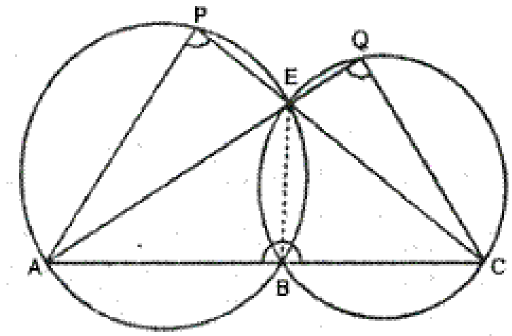
Given – In the figure, AB = AD = DC = PB and $DBC = X^{\circ}$
Join AC and BD
To find : the measure of $\angle ABD$ and $\angle APB$
Proof : $\angle DAC = \angle DBC = X$ [angels in the same segment]
But $\angle DCA = \angle DAC = X$ [$\because$ AD = DC]
Also, we have, $\angle ABD = \angle DAC$ [angles in the same segment]
In ΔABP, ext $\angle ABC = \angle BAP + \angle APB$
But, $\angle BAP = \angle APB$ [$\because$ AB = BP]
$2 \times X = \angle APB + \angle APB = 2\angle APB$
$\therefore 2\angle APB = 2X$
$\Rightarrow \angle APB = X$
$\therefore \angle APB = \angle DBC = X$,
But these are corresponding angles
$\therefore AP \parallel DB$

Question 14:

In the given figure; ABC, AEQ and CEP are straight lines. Show that ∠APE and ∠CQE are supplementary.



Solution 14:



Given – In the figure, ABC, AEQ and CEP are straight line

To prove - $\angle APE + \angle CQE = 180^\circ$

Construction – join EB

Proof – in cyclic quad ABEP,

$\angle APE + \angle ABE = 180^\circ$ .... (1)

Similarly, in cyclic quad BCQE

$\angle CQE + \angle CBE = 180^\circ$ .... (2)

Adding (1) and (2),

$\angle APE + \angle ABE + \angle CQE + \angle CBE = 180^\circ + 180^\circ = 360^\circ$

$\Rightarrow \angle APE + \angle ABE + \angle CBE = 360^\circ$

But, $\angle ABE + \angle CBE = 180^\circ$ [Linear pair]

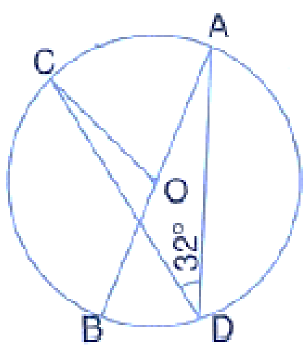
$\therefore \angle APE + \angle CQE + 180^\circ = 360^\circ$

$\Rightarrow \angle APE + \angle CQE = 360^\circ - 180^\circ = 180^\circ$

Hence $\angle APE$ and $\angle CQE$ are supplementary.

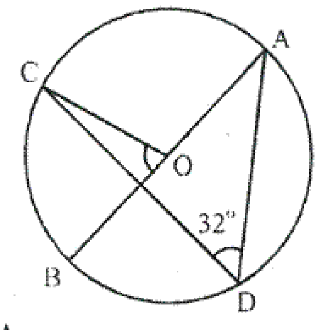
Question 15:

In the given figure, AB is the diameter of the circle with centre O.



If ∠ADC = 32°, find angle BOC

Solution 15:



Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining part
Of the circle

$\therefore \angle AOC = 2\angle ADC$

$\Rightarrow \angle AOC = 2 \times 32° = 64°$

Since $\angle AOC$ and $\angle BOC$ are linear pair, we have

$\angle AOC + \angle BOC = 180°$

$\Rightarrow 64° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 64°$

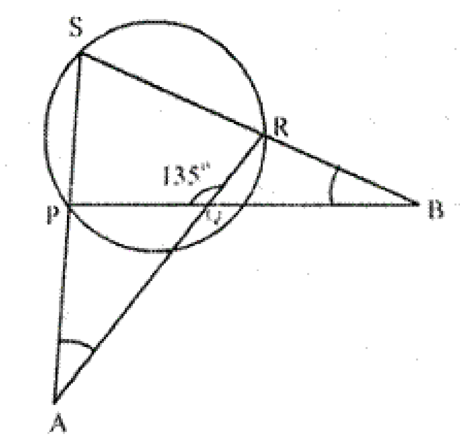
$\Rightarrow \angle BOC = 116°$

Question 16:

In a cyclic – quadrilateral PQRS, angle PQR = 135°. Sides SP and RQ produced meet at point A : whereas sides PQ and SR produced meet at point B.

If ∠A : ∠B = 2 : 1; find angles A and B.

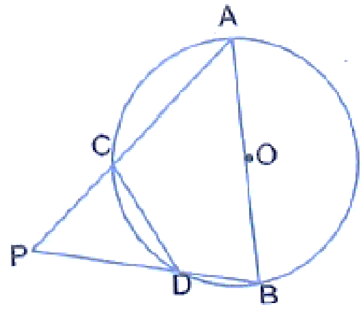
Solution 16:



PQRS is a cyclic quadrilateral in which $\angle PQR = 135°$
Sides SP and RQ are produced to meet at A and
Sides PQ and SR are produced to meet at B.
$\angle A = \angle B = 2:1$
Let $\angle A = 2X$, then $\angle B - X$
Now, in cyclic quad PQRS,
Since, $\angle PQR = 135°, \angle = 180° - 135°° = 45°$
[since sum of opposite angles of a cyclic quadrilateral are supplementary]
Since, $\angle PQR$ and $\angle PQA$ are linear pair,
$\angle PQR + \angle PQA = 180°$
$\Rightarrow 135° + \angle PQA = 180°$
$\Rightarrow \angle PQA = 180° - 135° = 45°$
Now, In ΔPBS,
$\angle P = 180° - (45° + x) = 180° - 45° - x = 135° - x$ .....(1)
Again, in ΔPQA,
EXT $\angle P = \angle PQA + \angle = 45° + 2X$ ....(2)
From (1) and (2),
$45° + 2x = 135° - x$
$\Rightarrow 2x + x = 135° - 45°$
$\Rightarrow 3x = 90°$
$\Rightarrow x = 30°$
Hence, $\angle A = 2x = 2 \times 30° = 60°$
And $\angle B = x = 30°$

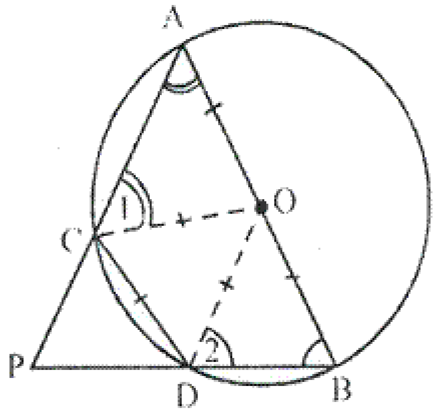
Question 17:

In the following figure, AB is the diameter of a circle with centre O and CD is the chord with length equal to radius OA.



Is AC produced and BD produced meet at point P; show that $\angle APB = 60°$

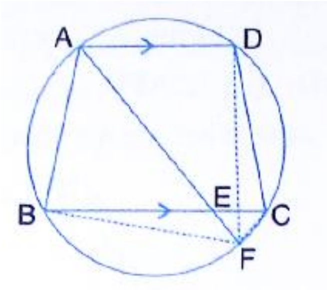
Solution 17:



Given – In the figure, AB is the diameter of a circle with centre O.
CD is the chord with length equal radius OA.
AC and BD produced meet at point P
To Prove : $\angle APB = 60°$
Construction – join OC and OD
Proof – We have CD = OC = OD [given]
Therefore, ΔOCD is an equilateral triangle
$\therefore \angle OCD = \angle ODC = \angle COD = 60°$
In ΔAOC, OA = OC [radii of the same circle]
$\therefore \angle A = \angle 1$
Similarly, in ΔBOD, OB = OD [radii of the same circle]
$\therefore \angle B = \angle 2$
Now, in cyclic quad ACBD
Since, $\angle ACD + \angle B = 180°$
[Since sum of opposite angles of a cyclic quadrilateral are supplementary]
$\Rightarrow 60° + \angle 1 + \angle B = 180°$
$\Rightarrow \angle 1 + \angle B = 180° - 60°$
$\Rightarrow \angle 1 + \angle B = 120°$
But, $\angle 1 = \angle A$
$\therefore \angle A + \angle B = 120°$ …..(1)
Now, in ΔAPB,
$\angle P + \angle A + \angle B = 180°$ [sum of angles of a triangles]
$\Rightarrow \angle P + 120° = 180°$
$\Rightarrow \angle P = 180° - 120°$ [from (1)]
$\Rightarrow \angle P = 60°$ or $\angle APB = 60°$

Question 18:

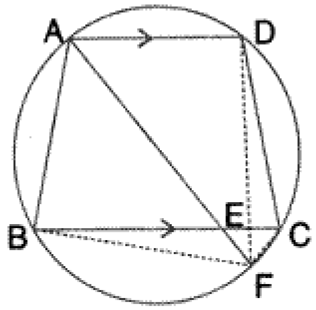
In the following figure, ABCD is a cyclic quadrilateral in which AD is parallel to BC.



If the bisector of angle A meets BC at point E and the given circle at point F, Prove that:

(i) EF = FC (ii) BF = DF

Solution 18:



Given – ABCD is a cyclic quadrilateral in which $AD \parallel BC$

Bisector of $\angle A$ meets BC at E and the given circle at F.

DF and BF are joined.

To prove –

(i) EF = FC

(ii) BF = DF

Proof – ABCD is a cyclic quadrilateral and AD ∥ BC

$\because$ AF is the bisector of $\angle A, \angle BAF = \angle DAF$

Also, $\angle DAE = \angle BAE$

$\angle DAE = \angle AEB$ [Alternate angles]

(i) In $\Delta ABE, \angle ABE = 180° - 2\angle AEB$

$\angle CEF = \angle AEB$ [vertically opposite angles]

$\angle ADC = 180° - \angle ABC = 180° - (180° - 2\angle AEB)$

$\angle ADC = 2\angle AEB$

$\angle AFC = 180° - \angle ADC$

$= 180° - 2\angle AEB$ [since ADFC is a cyclic quadrilateral]

$\angle ECF = 180° - (\angle AFC + \angle CEF)$

$= 180° - (180° - 2\angle AEB + \angle AEB)$

$= \angle AEB$

$\therefore EC = EF$

(ii) $\therefore$ Arc $BF =$ Arc DF [Equal arcs subtends equal angles]

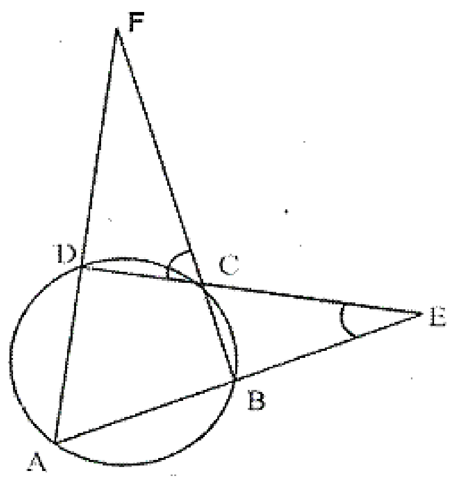
$\Rightarrow BF = DF$ [Equal arcs have equal chords]

Question 19:

ABCD is a cyclic quadrilateral. Sides AB and DC produced meet at point E; whereas sides BC and AD produced meet at point F.

If ∠DCF : ∠F : ∠E = 3 : 5 : 4, Find the angles of the cyclic quadrilateral ABCD.

Solution 19:



Given – In a circle, ABCD is a cyclic quadrilateral AB and DC

Are produced to meet at E and BC and AD are produced to meet at F.

$\angle DCF : \angle F : \angle E = 3 : 5 : 4$

Let $\angle DCF = 3X, \angle F = 5x, \angle E = 4x$

Now, we have to find, $\angle A, \angle B, \angle C$ and $\angle D$

In cyclic quad. ABCD, BC is produced.

$\therefore \angle A = \angle DCF = 3x$

In ΔCDF,

Ext $\angle CDA = \angle DCF + \angle = 3x + 5x = 8x$

In ΔBCE,

Ext $\angle ABC = \angle BCE + \angle E$ [$\angle BCE = \angle DCF$, Vertically opposite angles]

$= \angle DCF + \angle E$

$= 3x + 4x = 7x$

Now, in cyclic quad ABCD,

Since, $\angle B + \angle = 180°$

[since sum of opposite of a cyclic quadrilateral are supplementary]

$\Rightarrow 7x + 8x = 180°$

$\Rightarrow 15x = 180°$

$\Rightarrow x = \frac{180°}{15} = 12°$

$\therefore \angle A = 3x = 3 \times 12° = 36°$

$\angle B = 7x = 7 \times 12° = 84°$

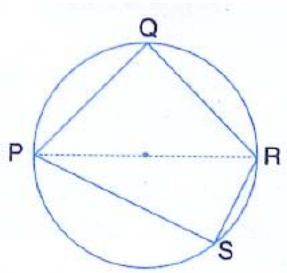
$\angle C = 180° - \angle A = 180° - 36° = 144°$

$\angle D = 8x = 8 \times 12° = 96°$

Question 20:

The following figure shows a circle with PR as its diameter.

If PQ = 7 cm and QR = 3RS = 6 cm, find the perimeter of the cyclic quadrilateral PQRS.



Solution 20:

In the figure, PQRS is a cyclic quadrilateral in which PR is a diameter

PQ = 7 cm

QR = 3 RS = 6 cm

3 RS = 6 cm $\Rightarrow$ RS = 2 cm

Now in ΔPQR,

$\angle Q = 90°$ [Angles in a semi circle]

$\therefore PR^2 = PQ^2 + QR^2$ [Pythagoras theorem]

$= 7^2 + 6^2$

$= 49 + 36$

$= 85$

Again in right ΔPSQ, $PR^2 = PS^2 + RS^2$

$\Rightarrow 85 = PS^2 + 2^2$

$\Rightarrow PS^2 = 85 - 4 = 81 = (9)^2$

$\therefore PS = 9\text{cm}$

Now, perimeter of quad $PQRS = PQ + QR + RS + SP$

$= (7 + 9 + 2 + 6)\text{cm}$

$= 24$

Question 21:

In the following figure, AB is the diameter of a circle with centre O.

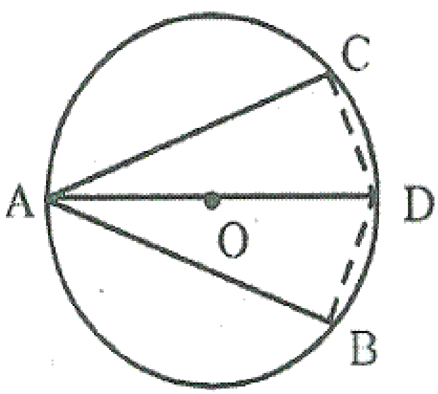
If chord AC = chord AD, Prove that:

(i) arc BC = arc DB

(ii) AB is bisector of ∠CAD.

Further, if the length of arc AC is twice the length of arc BC, find : (a) ∠BAC (b) ∠ABC

Solution 21:



Given – In a circle with centre O, AB is the diameter and AC and AD are two chords such that AC = AD

To prove: (i) arc BC = arc DB

(ii) AB is the bisector of $\angle CAD$

(iii) If arc AC = 2 arc BC, then find

(a) $\angle BAC$ (b) $\angle ABC$

Construction: Join BC and BD

Proof : In right angled ΔABC and ΔABD

Side AC = AD [Given]

Hyp. AB = AB [Common]

∴ By right Angle – Hypotenuse – Side criterion of congruence

$\Delta ABC \cong \Delta ABD$

(i) The corresponding parts of the congruent triangle are congruent.

$\therefore$ BC = BD [c.p.c.t]

$\therefore$Arc BC = Arc BD [equal chords have equal arcs]

(ii) $\angle BAC = \angle BAD$

$\therefore$ AB is the bisector of $\angle CAD$

(iii) If Arc AC = 2 arc BC,

Then $\angle ABC = 2\angle BAC$

But $\angle ABC + \angle BAC = 90°$

$\Rightarrow 2\angle BAC + \angle BAC = 90°$

$\Rightarrow 3\angle BAC = 90°$

$\Rightarrow \angle BAC = \frac{90°}{3} = 30°$

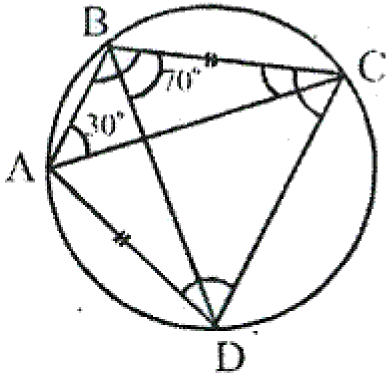
$\angle ABC = 2\angle BAC \Rightarrow \angle ABC = 2 \times 30° = 60°$

Question 22:

In cyclic quadrilateral ABCD; AD = BC, $\angle BAC = 30°$ and $\angle CBD = 70°$; find:

(i) $\angle BCD$ (ii) $\angle BCA$ (iii) $\angle ABC$ (iv) $\angle ADC$

Solution 22:



ABCD is a cyclic quadrilateral and AD = BC

$\angle BAC = 30°$, $\angle CBD = 70°$

$\angle DAC = \angle CBD$ [Angles in the same segment]

$\Rightarrow \angle DAC = 70°$ [$\because \angle CBD = 70°$]

$\Rightarrow \angle BAD = \angle BAC + \angle DAC = 30° + 70° = 100°$

Since the sum of opposite angles of cyclic quadrilateral is supplementary

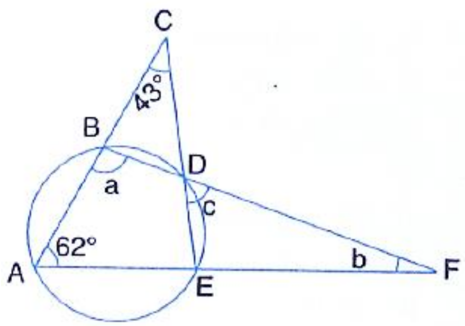
$\angle BAD + \angle BCD = 180°$

$\Rightarrow 100° + \angle BCD = 180°$ [From (1)]

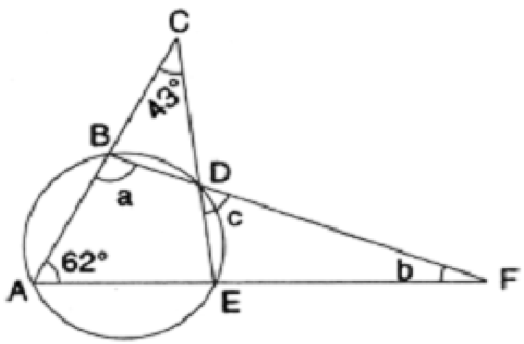
$\Rightarrow \angle BCD = 180° - 100° = 80°$
Since , $AD = BC$, $\angle ACD = \angle BDC$ [Equal chords subtends equal angles]
But $\angle ACB = \angle ADB$ [angles in the same segment]
$\therefore \angle ACD + \angle ACB = \angle BDC + \angle ADB$
$\Rightarrow \angle BCD = \angle ADC = 80°$
But in ΔBCD,
$\angle CBD + \angle BCD + \angle BDC = 180°$ [angles oaf a triangle]
$\Rightarrow 70° + 80° + \angle BDC = 180°$
$\Rightarrow 150° + \angle BDC = 180°$
$\therefore \angle BDC = 180° - 150° = 30°$
$\Rightarrow \angle ACD = 30°$ $[\because \angle ACD = \angle BDC]$
$\therefore \angle BCA = \angle BCD - \angle ACD = 80° - 30° = 50°$
Since the sum of opposite angles of cyclic quadrilateral is supplementary
$\angle ADC + \angle ABC = 180°$
$\Rightarrow 80° + \angle ABC = 180°$
$\Rightarrow \angle ABC = 180° - 80° = 100°$

Question 23:

In the given figure, ∠ACE = 43° and ∠CAF = 62° ; Find the values of a, b and c.



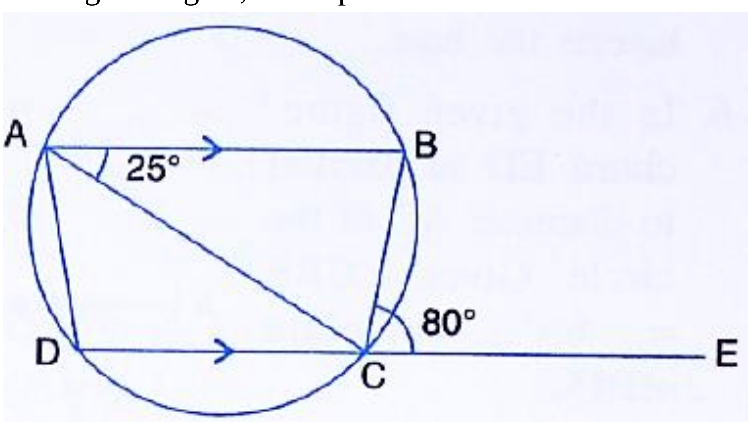
Solution 23:



Now, $\angle ACE = 43°$ and $\angle CAF = 62°$ [given]
In ΔAEC
$\therefore \angle ACE + \angle CAE + \angle AEC = 180°$
$\Rightarrow 43° + 62° + \angle AEC = 180°$
$\Rightarrow 105° + \angle AEC = 180°$
$\Rightarrow \angle AEC = 180° - 105° = 75°$
Now, $\angle ABD + \angle AED = 180°$
[Opposite angles of a cyclic quad and $\angle AED = \angle AEC$]
$\Rightarrow a + 75° = 180°$
$\Rightarrow a = 180° - 75°$
$\Rightarrow a = 105°$
$\angle EDF = \angle BAF$ [Angles in the alternate segments]
$\therefore c = 62°$
In $\Delta BAF, a + 62° + b = 180°$
$\Rightarrow 105° + 62° + b = 180°$
$\Rightarrow 167° + b = 180°$
$\Rightarrow b = 180° - 167° = 13°$
Hence, $a = 105°$, $b = 13°$ and $c = 62°$

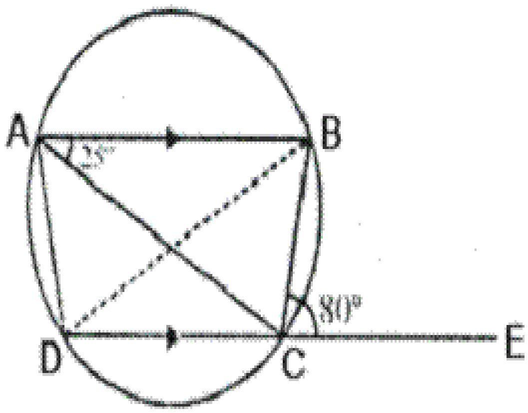
Question 24:

In the given figure, AB is parallel to DC ∠BCE = 80° and ∠BAC = 25°.



Find:
(i) ∠CAD (ii) ∠CBD (iii) ∠ADC

Solution 24:

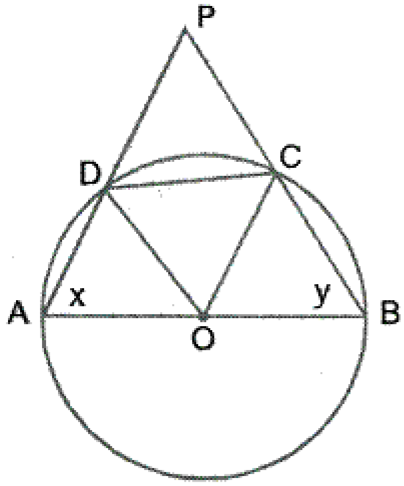


In the given figure,
ABCD is a cyclic quad in which AB ∥ DC
∴ ABCD is an isosceles trapezium
∴ AD = BC
Ext. $\angle BCE = \angle BAD$ [Exterior angle of a cyclic qud is equal to interior opposite angle]
$\therefore \angle BAD = 80°$ [$\because \angle BCE = 80°$]
But $\angle BAC = 25°$
$\therefore \angle CAD = \angle BAD - \angle BAC$
$= 80° - 25°$
$= 55°$
(ii) $\angle CBD = \angle CAD$ [Angle of the same segment]
$= 55°$
(iii) $\angle ADC = \angle BCD$ [Angles of the isosceles trapezium]
$= 180° - \angle BCE$
$= 180° - 80°$
$= 100°$

Question 25:

ABCD is a cyclic quadrilateral of a circle with centre O such that AB is a diameter of this circle and the length of the chord CD is equal to the radius of the circle. If AD and BC produced meet at P, Show that APB = 60°

Solution 25:



In a circle, ABCD is a cyclic quadrilateral in which

AB is the diameter and chord CD is equal to the radius of the circle

To prove - $\angle APB = 60°$

Construction – Join OC and OD

Proof – Since chord CD = CO = DO [radii of the circle]

$\therefore \Delta DOC$ is an equilateral triangle

$\therefore \angle DOC = \angle ODC = \angle DCO = 60°$

Let $\angle A = x$ and $\angle B = y$

Since OA =OB = OC = OD [radii of the same circle]

$\therefore \angle ODA = \angle OAD = x$ and

$\angle OCB = \angle OBC = y$

$\therefore \angle AOD = 180° - 2x$ and $\angle BOC = 180° - 2y$

But AOB is a straight line

$\therefore \angle AOD + \angle BOC + \angle COD = 180°$

$\Rightarrow 180° - 2x + 180° - 2y + 60° = 180°$

$\Rightarrow 2x + 2y = 240°$

$\Rightarrow x + y = 120°$

But $\angle A + \angle B + \angle P = 180°$ [Angles of a triangle]

$\Rightarrow 120° + \angle P = 180°$

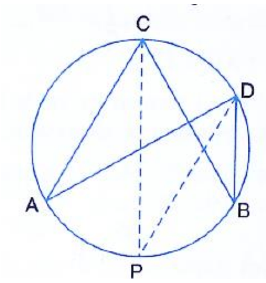
$\Rightarrow \angle P = 180° - 120°$

$\Rightarrow \angle P = 60°$

Hence $\angle APB = 60°$

Question 26:

In the figure, given alongside, CP bisects angle ACB.
Show that DP bisects angle ADB.

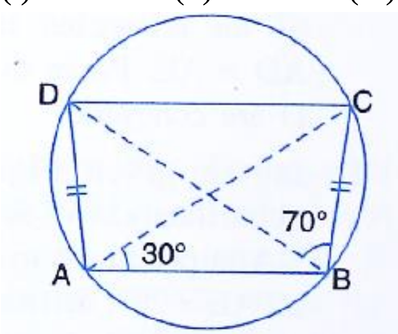


Solution 26:

Given – In the figure, CP is the bisector of $\angle ABC$
To prove – DP is the bisector of $\angle ADB$
Proof – Since CP is the bisector of $\angle ACB$
$\therefore \angle ACP = \angle BCP$
But $\angle ACP = \angle ADP$ [Angle in the same segment of the circle]
And $\angle BCP = \angle BDP$
But $\angle ACP = \angle BCP$
$\therefore \angle ADP = \angle BDP$
$\therefore$ DP is the bisector of $\angle ADB$

Question 27:

In the figure, given below, AD = BC, ∠BAC = 30° and ∠CBD = 70°.
Find:
(i) ∠BCD (ii) ∠BCA (iii) ∠ABC (iv) ∠ADB



Solution 27:

In the figure, ABCD is a cyclic quadrilateral
AC and BD are its diagonals
$\angle BAC = 30^\circ$ and $\angle CBD = 70^\circ$

Now we have to find the measure of

$\angle BCD, \angle BCA, \angle ABC$ and $\angle ADB$

We have $\angle CAD = \angle CBD = 70°$ [Angles in the same segment]

Similarly, $\angle BAD = \angle BDC = 30°$

$\therefore \angle BAD = \angle BAC + \angle CAD$

$= 30° + 70°$

$= 100°$

(i) Now $\angle BCD + \angle BAD = 180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow \angle BCD + \angle BAD\angle = 180°$

$\Rightarrow \angle BCD + 100° = 180°$

$\Rightarrow \angle BCD = 180° - 100°$

$\Rightarrow \angle BCD = 80°$

(ii) Since AD = BC, ABCD is an isosceles trapezium and AB ∥ DC

$\angle BAC = \angle DCA$ [Alternate angles]

$\Rightarrow \angle DCA = 30°$

$\therefore \angle ABD = \angle DAC = 30°$ [Angles in the same segment]

$\therefore \angle BCA = \angle BCD - \angle DAC$

$= 80° - 30°$

$= 50°$

(iii) $\angle ABC = \angle ABD + \angle CBD$

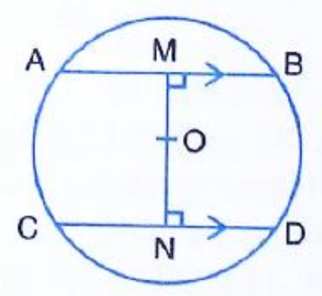
$= 30° + 70°$

$= 100°$

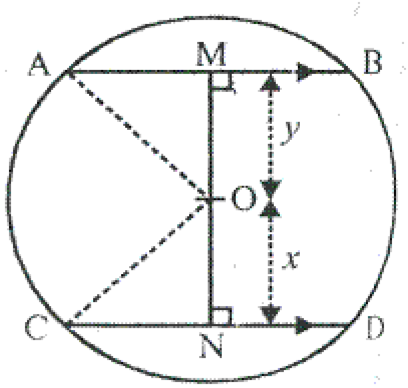
(iv) $\angle ADB = \angle BCA = 50°$ [Angles in the same segment]

Question 28:

In the figure, given below, AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, fins the distance MN between the two chords of lengths 24 cm and 18 cm respectively.



Solution 28:



Given – AB = 24 cm, cd = 18 cm

$\Rightarrow$ AM = 12 cm, CN = 9 cm

Also, OA = OC = 15 cm

Let MO = y cm, and ON = x cm

In right angled ΔAMO

$$(OA)^2 = (AM)^2 + (OM)^2$$

$$\Rightarrow 15^2 = 12^2 + y^2$$

$$\Rightarrow y^2 = 15^2 - 12^2$$

$$\Rightarrow y^2 = 225 - 144$$

$$\Rightarrow y^2 = 81$$

$$\Rightarrow y = 9\,\text{cm}$$

In right angled ΔCON

$$(OC)^2 = (ON)^2 + (CN)^2$$

$$\Rightarrow 15^2 = x^2 + 9^2$$

$$\Rightarrow x^2 = 15^2 - 9^2$$

$$\Rightarrow x^2 = 225 - 81$$

$$\Rightarrow x^2 = 144$$

$$\Rightarrow y = 12\,\text{cm}$$

Now, $MN = MO + ON$

$$= y + x$$

$$= 9\text{ cm} + 12\text{ cm}$$

$$= 21\text{ cm}$$