# Chapter—10

# POLYGON

Rani made five diagrams with scale and pencil.

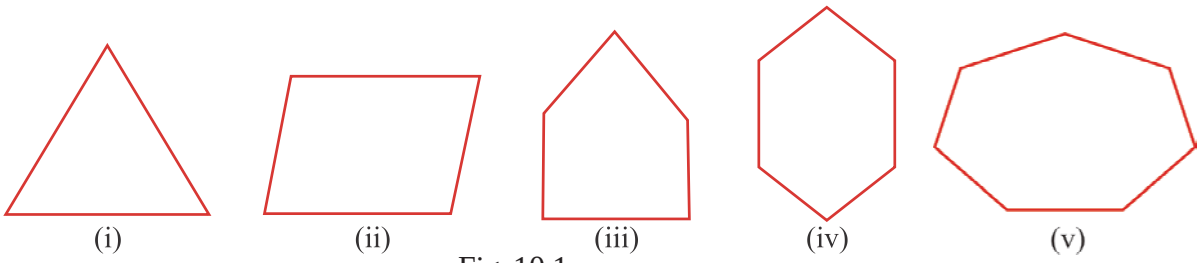


Fig. 10.1

She asked her friends Mary, “Do you recognise these figures ?” Mary said, “In the last class we have learnt that we name figure on the basis of the number of arms or sides.”

In figure 10.1(i), the figure obtained by three sides a tringle; figure 10.1(ii) is made by four sides and is a Quadrilateral figure 10.1(iii) is made up of five sides and is a pentagon".

Rani said to Mary, "Right, you have correctly identified the figures but what name will you give for figure no. 10.1 (iv) and figure 10.1(v) ? Rani said, “You know, fig. no. 10.1(iv) is known as a hexagon and fig. no. 10.1(v) is called a septagon”. All these closed fig. are made up of many sides, therefore closed fig. with three or more than three sides are known as polygons.

## Activity 1

Look of the figure below carefully and fill in the blanks.

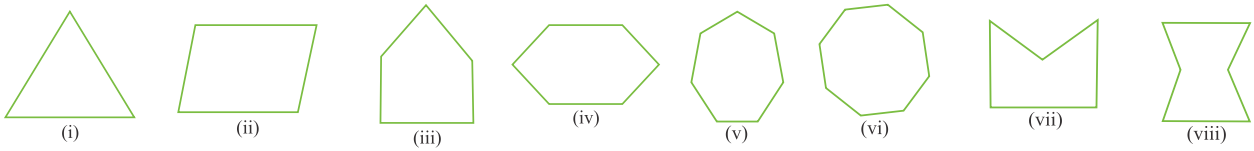


Fig. 10.2

**Table 10.1**

| S. No. | Fig. No. | Name of the fig. | No. of Vertices | No of sides | No. of angles |
|---|---|---|---|---|---|
| 1. | 10.2(i) | triangle | 3 | 3 | 3 |
| 2. | 10.2(ii) | | | | |
| 3. | 10.2(iii) | | | | |
| 4. | 10.2(iv) | | | | |
| 5. | 10.2(v) | | | | |
| 6. | 10.2(vi) | | | | |
| 7. | 10.2(vii) | | | | |
| 8. | 10.2(viii) | | | | |

In this activity, we saw that the ploygons (iii) and (vii) are both pentagons and fig. no. (iv) and (viii) are both hexagons, but is there any difference between the polygons (iii) and (vii) and in fig. (iv) and (viii) ?

Think about it, discuss with your friends and your teachers and write down the conclusions.

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In fig. (vii) and (viii) some vertices are projected inwards whereas in all other fig. all the vertices are projected outwards, Thus we get two types of polygons.

## 1-Convex polygons

Those polygons whose vertex are projected outwards and each internal angle is less than $180^0$ are called convex polygons like.

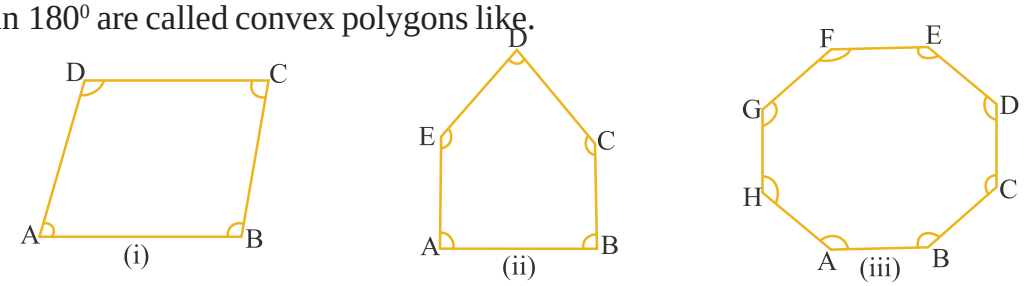



fig. 10.3

## 2-Concave polygon

Polygons in which at least one vertex is projected inwards and at least one angle is more than 180° are known as concave polygons.

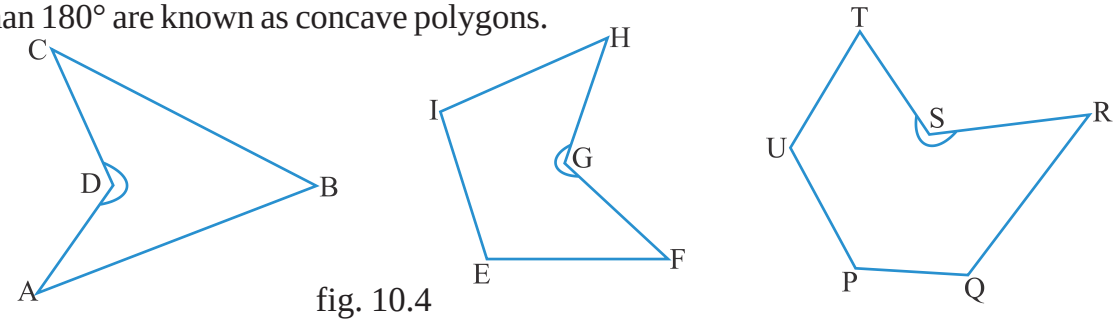



fig. 10.4

What conclusions can we draw from activity (i) observe than draw conclusions.

After observation we get the result that in any polygon the number of vertices, the number of arms or sides and the number of angles are equal to each other.

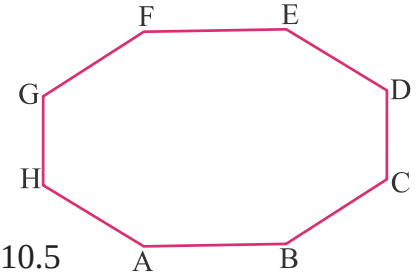



fig. 10.5

The polygons in fig 10.5 is a octagon. Try to identify the number of arms and the diagonals that can be drawn from vertex A with respect to the vertices.

Complete the following table -

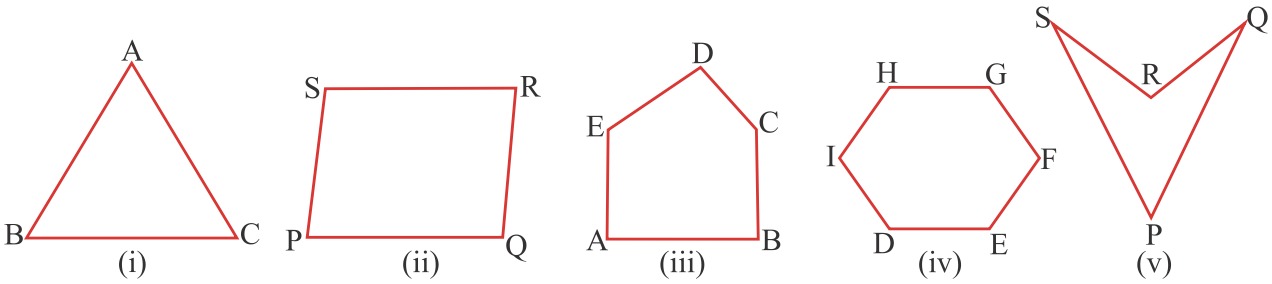
**Table 10.2**

| S.No. | The Name of the vetex to which other vertices have been joined | Name of the line segments | |
|---|---|---|---|
| | | arms/sides | diagonals |
| 1. | A | AB, AH | AC, AD, AE, AF, AG |
| 2. | B | BC, BA | BD, BE, BF, BG, BH |
| 3. | C | -------- | ---------- |

On the basis of this activity write the definition of diagonal of a polygon :

## Activity 2

In the fig. given below, draw diagonals from any one vertex and write the numbers of diagonals drawn in the blanks.




_____ _____ _____ _____ _____

fig. 10.6

## Activity 3

Below are given some quadrilaterals, draw the diagonals and find the answers of the following questions :

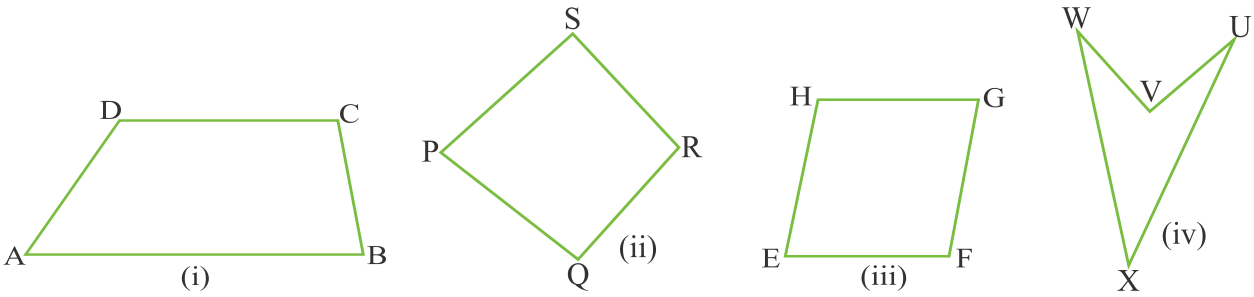



fig. 10.7

1. In how many triangles did the quadrilateral get divided ?
2. Is the sum of the internal angles of the two triangles equal to the sum of internal angles of the quadrilateral ?
3. What is the value of the sum of internal angles of a triangle ?
4. What is the sum of internal angles of a quadrilateral ?

## Activity 4

In the fig. given below, diagonals have been drawn.

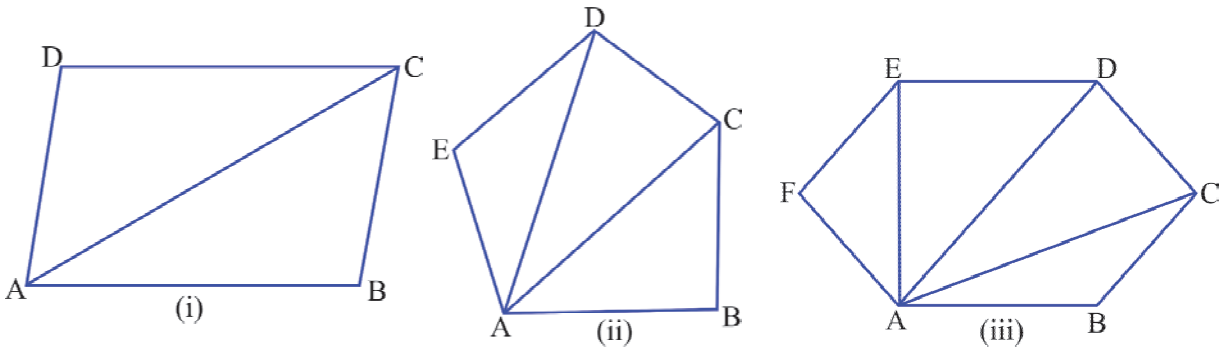



Fig. 10.8

Look at the fig. carefully complete the table given below.

**Table 10.3**

| S.No. | Name of Polygons | No. of sides of a polygons | No of triangles obtained in a polygon | Sum of the internal angles of a polygon= No. of tringles x $180^0$ | Sum of internal angles |
|---|---|---|---|---|---|
| 1. | quadrilateral | 04 | 02 or (4 - 2) | 2 x $180^0$ | $360^0$ |
| 2. | pentagon | 05 | 03 or (5 - 2) | 3 x $180^0$ | $540^0$ |
| 3. | hexagon | ---------- | ---------- | ---------- | ---------- |
| 4. | heptagon | ---------- | ---------- | ---------- | ---------- |
| 5. | octagon | ---------- | ---------- | ---------- | ---------- |

While doing the activity 4, Mary said that the number of triangles in a polygon is two less than the number of sides.

So, if there are n number of sides in a polygon, then the number of triangles in the polygon would be (n - 2).

Rani said, for (n - 2) triangles, the sum internal angles of a polygon would be (n -2) x $180^0$.

### Therefore

*The sum of internal angles in a polygon with n sides = (n - 2) x $180^0$.*

### Example - 1

If the number of sides in a ploygon is 15, find the sum of its intenal angles.

### Solution

The sum of internal angles of a polygon = (n - 2) ×180° where 'n' is the no of sides of the polygons

= (15 - 2) × 180°

= 13 × 180° = 2340° Ans.

## Regular and Irregular polygons

You know that the triangle in which all the sides are equal is called an equilateral triangle when each angle in a quadrilateral is of 90° and all the sides are equal, what is that fig. known as ? Can the sides and angles be equal in other polygons also ?

**When a polygon has equal sides and its angles are equal, it is known as a equilateral polygon or a regular polygon.**

When a pentagon has equal sides and angles it is known as an equipentagon. Will the internal angles of an equipentagon also be equal? Is this a Regular polygons?

**Those polygons in which the measures of sides and angles are different are known as irregular polygons.**

Different kinds of quadrilateral on the basis of characteristics.

In class VII you have studied about kinds of quadrilaterals, sides of a quadrilateral, External angles, adjacent angles opposite angles and the sum of internal angles.

Let us try to identify the different types of quadrilaterals through an activity.

## Activity 5

Complete the table as per the first example .

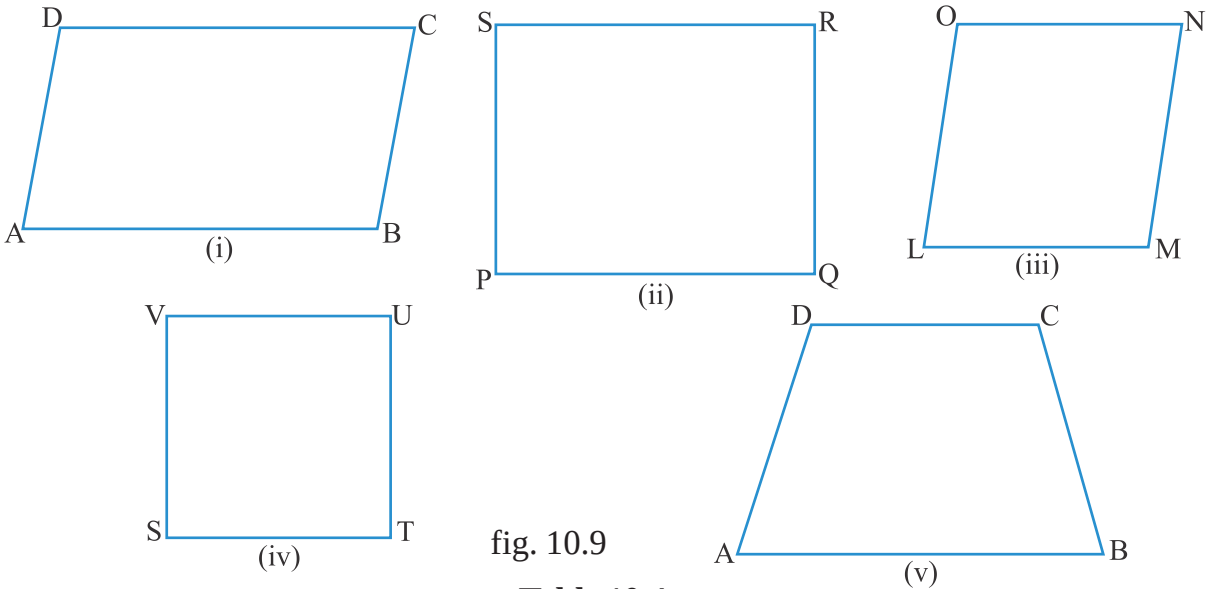



fig. 10.9

**Table 10.4**

| Fig. | Name of Parallel sides | Name of equal sides | Type of quadrilateral |
|---|---|---|---|
| 10.9(i) | AB//CD, BC//AD | AB = CD, BC = AD | Parallelogram |
| 10.9(ii) | | | |
| 10.9(iii) | | | |
| 10.9(iv) | | | |

## Activity 6

Given below are three fig. of quadrilateral. Fill in the blanks by measuring the fig. as shown in the example.

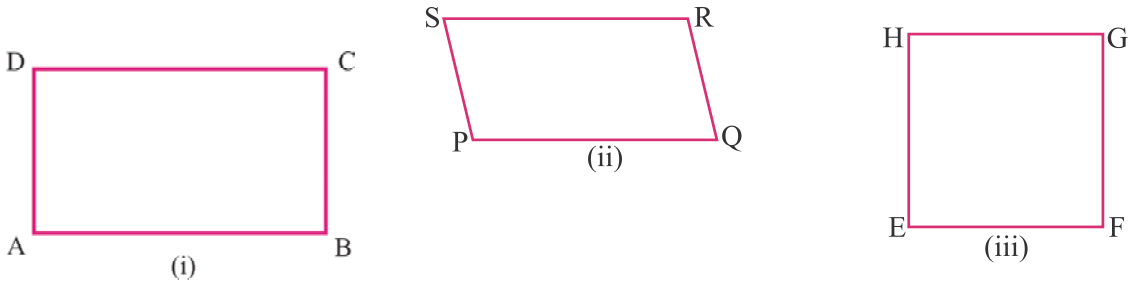



fig. 10.10

**Table 10.5**

| S. No. | Name of Parallelogram | Measures of the sides (In cms.) | Measures of angles (in degrees) |
|---|---|---|---|
| 1. | Quadrilateral ABCD | AB=-----cm., BC=.....cm.<br>CD=-----cm., DA=.....cm | $\angle A =$ ___ $\angle B =$ ___<br>$\angle C =$ ___ $\angle D =$ ___ |
| 2. | Quadrilateral PQRS | PQ=-----cm., QR=.....cm<br>RS=-----cm., SP =.....cm | $\angle P =$ ___ $\angle Q =$ ___<br>$\angle R =$ ___ $\angle S =$ ___ |
| 3. | Quadrilateral EFGH | EF=-----cm., FG=.....cm<br>GH=-----cm., HE =.....cm | $\angle E =$ ___ $\angle F =$ ___<br>$\angle G =$ ___ $\angle H =$ ___ |

On the basis of the results obtained in the fig. you have seen that in a parallelogram, the parallel sides and the opposite angles are equal in measurement.

So we can conclude that.

The measurement of opposite sides and opposite angles of a parallelogram are equal.

## Practice - 1

1. Draw any two parallelograms and write the measurement of the opposite sides and opposite angles. Are they equal ?

## Example - 2

ABCD is a parallelogram in which $\angle c = 75^0$. Find the other angles.

### Solution

Given : $\angle c = 75^0$

Therefore $\angle A = 75^0$ (because opposite angle are equal)

Since $\angle A + \angle B + \angle C + \angle D = 360^0$ (why ?)

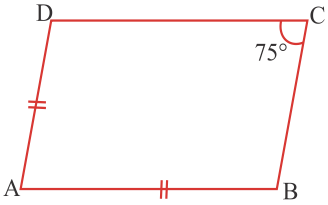



Fig. 10.11

$\Rightarrow \quad 75^0 + 75^0 + \angle B + \angle D = 360^0$

$\Rightarrow \quad 150^0 + \angle B + \angle D = 360^0$

$\Rightarrow \quad \angle B + \angle D = 360^0 - 150^0$

$\Rightarrow \quad \angle B + \angle D = 210^0$

$\Rightarrow \quad \angle B + \angle B = 210^0 \qquad [\because \angle D = \angle B]$

$\Rightarrow \quad \angle B = \frac{210^0}{2}$

or

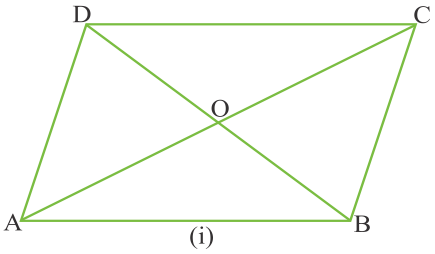
$\angle B = 105^0$

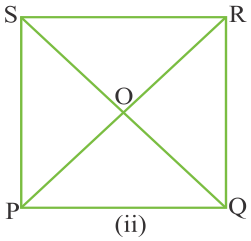
Thus, $\angle D = 105^0$

## Characteristics of Diagonals drawn in a parallelogram.

You have already learnt how to draw diagonals in a parallelogram. You also known that the diagonals drawn in a quadritateral intersect each other. Let us examine the features of the diagonals of a quadrilateral.

### Activity 7



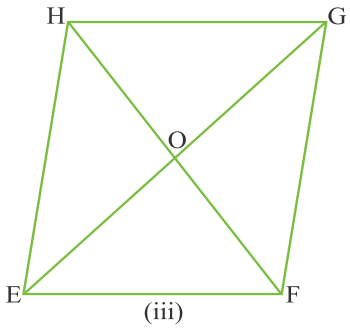




fig. 10.12

**Table 10.6**

| S.No | Name of the parallelogram | Measurement of the line segments in cm. | Is the point of intersection 'O' the midpoint of the diagonals? |
|---|---|---|---|
| 1. | Parallelogram ABCD | AO =___ cm. OC=___cm<br>OB =___ cm. OD=___cm | |
| 2. | Parallelogram PQRS | OP =___ cm. OR=___cm<br>OQ =___ cm. OS=___cm | |
| 3. | Parallelogram EFGH | OE =___ cm. OG=___cm<br>OF =___ cm. OH=___cm | |

## Conclusion

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Draw many such parallelograms and examine your conclusions whether the diagonals of the parallelogram bisect each other.

Give reasons for the questions below -

1. Is every square a parallelogram ?

   ---

2. Is every rectangle a parallelogram ?

   ---

3. Is every quadrilateral a parallelogram ?

   ---

4. Then in all the above cases do the diagonals bisect each other ?

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## Practice - 2

Draw a quadrilateral is which the diagonals bisect each other.

## Activity 8

Given below are some figures of rectangle and squares. Measures the diagonals in these figures and fill in the blanks.

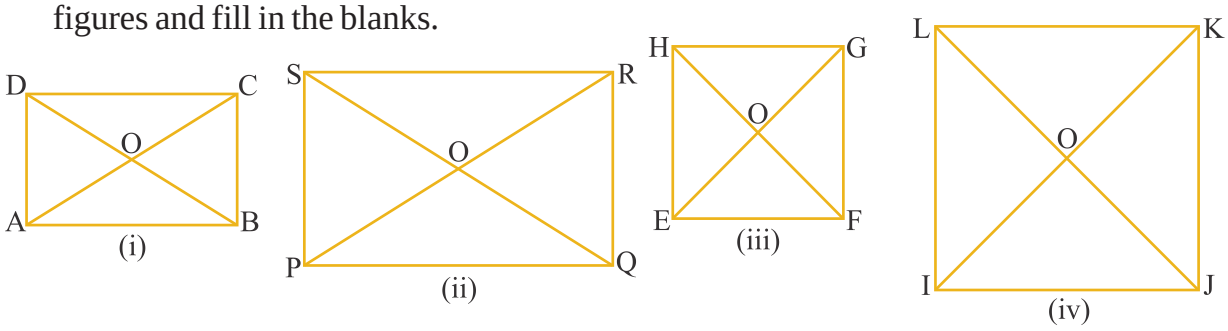



Fig. 10.13

**Table : 10.7**

| Fig No. | Name of the quadrilateral | Measurement of diagonals (in cms.) | Mesurement of line segments (in cms) | Are diagonal equal to each other Yes or No | The diagonals bisect each other Yes or No |
|---|---|---|---|---|---|
| 10.13(i) | Rectangle ABCD | AC=.....BD=.... | OA=.....OC=.... OB=.....OD=.... | | |
| 10.13(ii) | Rectangle PQRS | PR=.....QS=.... | OP=.....OR=.... OQ=.....OS=.... | | |
| 10.13(iii) | Square EFGH | EF=.....FH=.... | OE=.....OG=.... OF=.....OH=.... | | |
| 10.13(iv) | Square IJKL | JK=.....JL=.... | OI=.....OJ=.... OJ=.....OL=.... | | |

In every situation, we conclude that " The length of the diagonals in a square or a rectangle are equal to each other and they bisect each other."

## Practice - 3

Draw different measurement of squares and rectangles with their diagonals and verify their characteristics.

## Activity 9

Below are given some squares and rectagles. Draw line segments to join their diagonals and measure the angles at their points of inter section.

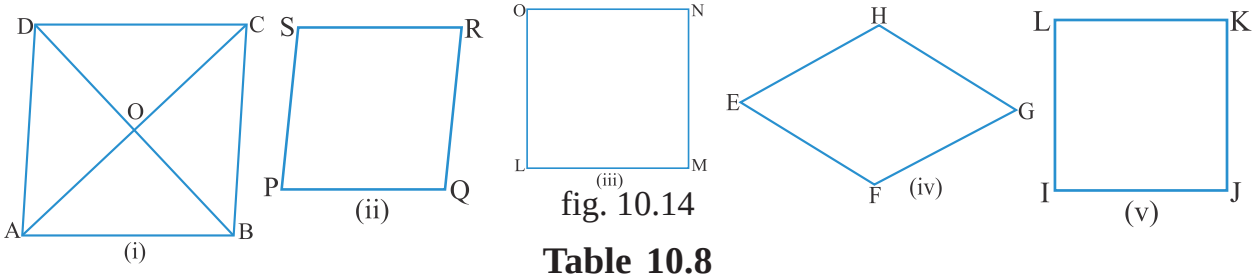



fig. 10.14

**Table 10.8**

| S. No. | Figure No | Name of the quadrilateral | Measurement of the angle at the point of inter section of the diagonals |
|---|---|---|---|
| 1. | 10.14(i) | parallelogram ABCD | $\angle AOB =$ ____ $\angle BOC =$ ____ $\angle COD$ ____ $\angle DOA =$ ____ |
| 2. | 10.14(ii) | | |
| 3. | 10.14(iii) | | |
| 4. | 10.14(iv) | | |
| 5. | 10.14(v) | | |

What is the value of the angles at the point of intersection of the diagonals ?

Thus, we find that “The diagonals of parallelograms and squares intersect each other at right angles”.

**Table 10.9**

| S. No. | Type of quadrilateral | Arms/sides | Angles | Diagonals |
|---|---|---|---|---|
| 1. | Parallelogram | Opposite sides are parallel and equal. | Opposite angles are equal. | Diagonals bisect each other at the point of intersection. |
| 2. | Rectangle | Opposite sides are parallel and equal. | Opposite angles are equal. (each angle measures $90^0$) | Diagonals are equal and bisect each other at the point of intersection |

| | | | | |
|---|---|---|---|---|
| 3. | Square | All sides are equal opposite sides are parallel | Opposite angles are equal (each angle measures 90°) | Diagonals bisect each other at right angles & the diagonals are also equal to each other |
| 4. | Rhombus | All sides are equal opposite sides are parallel | Opposite angles are equal. | Diagonals bisect each other at right angles. |

## Exercise 10.1

1. Each question has four options. Choose the correct alternative answer.

   a. The two diagonals are equal in

   (i) a parallelogram (ii) quadrilateral

   (iii) rectangle (iv) trapezium

   b. In which quadrilateral are all angles equal to each other ?

   (i) parallelogram (ii) trapezium

   (iii) rectangle (iv) rhombus

   c. If one of the angles of a parallelogram is $60^0$ then value of opposite angle are:

   (i) $60^0$ (ii) $120^0$

   (iii) $90^0$ (iv) $180^0$

   d . The adjacent sides of a parallelogram are of 8cms. and 6 cms. respectively. What would be its perimeter ?

   (i) 14cm. (ii) 28cm.

   (iii) 56cm. (iv) 60cm.

2. Fill in the blanks

   (1) A quadrilateral where the diagonals are equal and intersect each other at right angle, is known as a .....

   (2) The quadrilateral in which only one pair of opposite sides are parallel is known as an .....

   (3) If a polygon has 6 vertices, then the number of sides in the polygon would be .....

   (4) If the number of sides in a polygon is “n” then the number of diagonals that can be drawn from one of the vertices would be .....

3. The four vertices of a square piece of paper have been cut off with the help of a pair of seissiors. What kind of polygonic figure has been obtained ? Discuss with your friends.

4. The sides of a polygon are 8 in numbers what would be the sum of its internal angles?

5. If one of the angles of a parallelogram is $120^0$, find the measurement of the other angles?

6- The adjacent sides of a parallelogram are in the ratio 3 : 4, If the perimeter of a parallelogram is 84cm, find out its sides.

7- Which of these statments are not true. Correct them and rewrite.

1. All squares are rhombus.
2. The diagonals of a rectangle are equal.
3. All rectagles are not parallalograms.
4. The diagonals of a parallelogram bisect each other at right angles.
5. All squares are rectangles.

## Symmetry in polygons

You have studied about quadrilaterals and are now well aquainted with four sided figures. You have learnt in class VII that if any figure can be folded into two in such a way that half completely covers the other half, then the line along which this figure is folded is known as the line of synmetry. For example : Cut square from a piece of paper and if you fold it along the dotted lines as shown in fig. 10, you can get four lines of symmetry.

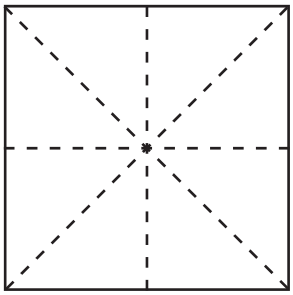
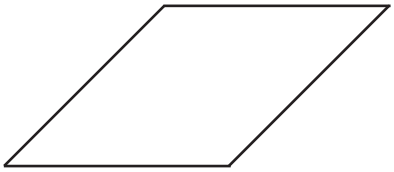


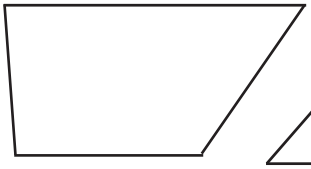
fig. 10

Similary, cut papers like the figures given below and try to find the lines of symmetry for these figures. With a pencil, now draw dotted lines in the book for these figures. Also write the number of such lines you have drawn in each figure.

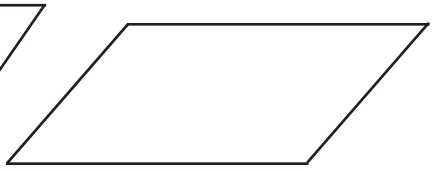
Rectangle



trapezium



rhombus



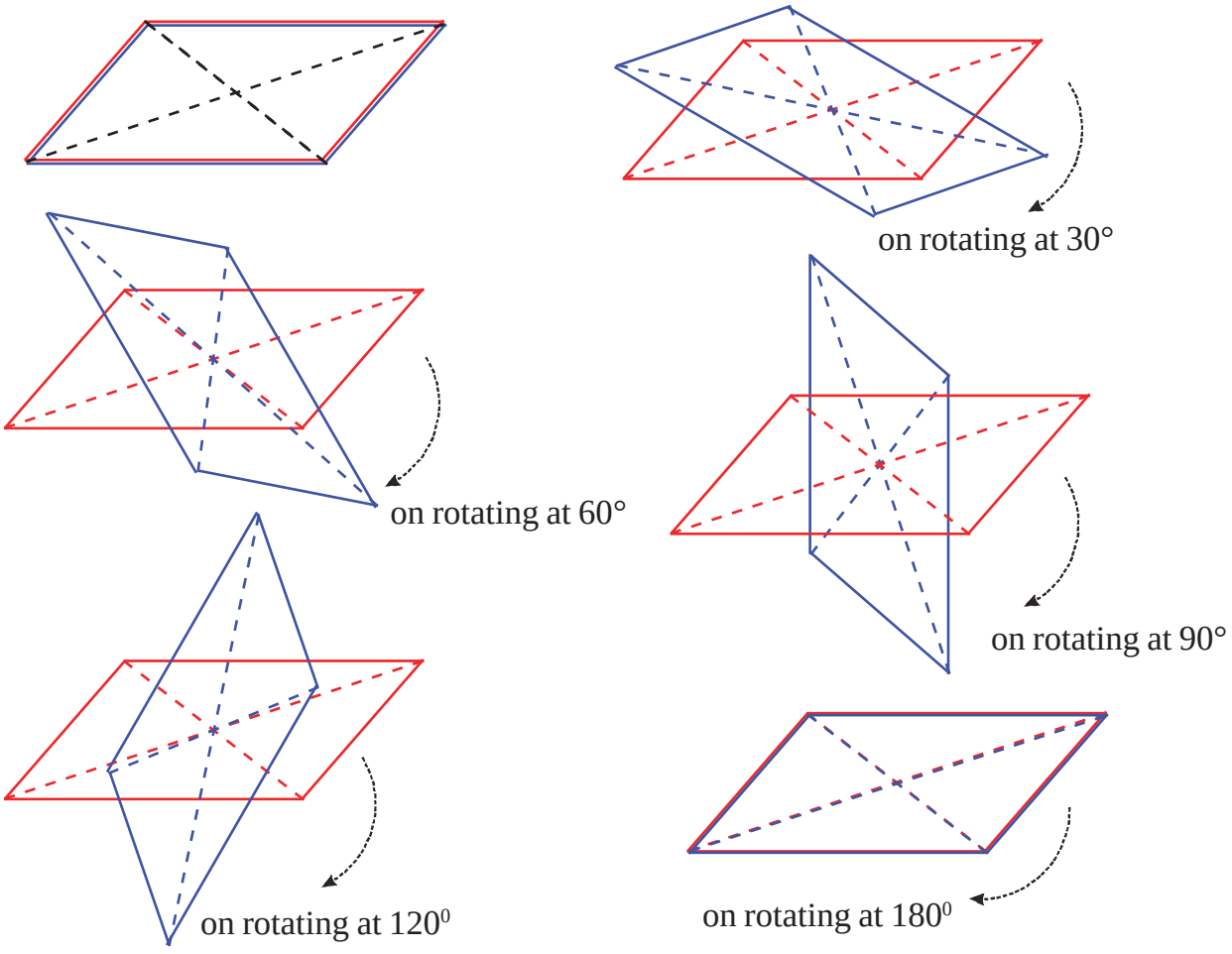
Parallelogram

Rhombus with two equal sides

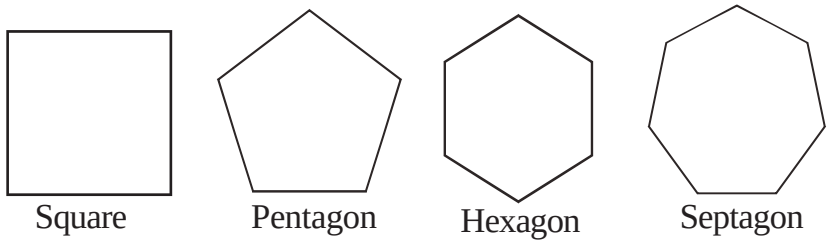
A parallelogram which is neither a square nor a rectangle and has no line of symmetry but still it seems that there is some kind of a equilibrium in these figures also, Let us find out.

Fold a piece of paper in two equal part and make a parallelogram. Cut the paper into two equal shapes in such a way that two parallelograms of the same size are obtained.

Draw diagonals in the two parallelograms to get their intersecting point. On the point of intersection intersect a pin and rotate the apparatus in such a way that the upper parallelograms rotates on the lower one. Notice at which points does the upper parallelogram completely overlaps the lower figure.




Thus, we see that first at $180^0$ angle and later at $360^0$ angle, the parallelogram on top rotated on the lower one completely overlaps the parallelogram. The symmetry of this kind is known as rotating symmetry and the point at which the figures are rotated is known as the centre of rotation. On completing a full rotation around the centre, the number of times the figures overlap each other is called the rotational sequence. Can you find out the rotational squence for the figures ?



## Activity 10

Below are given some figures complete the table by determining the rotational sequence and axis of symmetry.

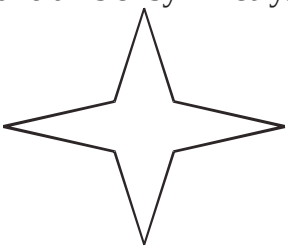


fig 10 (i)

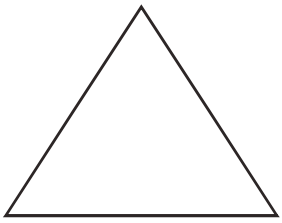


fig. 10(ii)

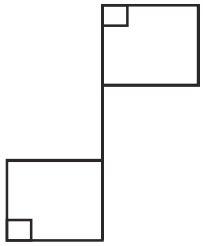


fig. 10(iii)

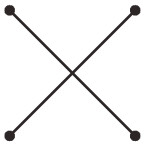


fig. 10(iv)

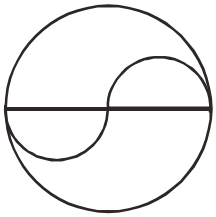


fig. 10 (v)

fig. 10 (vi)

| Fig. No. | Rotational squence | No. of symmetrical axes |
|---|---|---|
| fig 10 (i) | | |
| fig 10 (ii) | | |
| fig 10 (iii) | | |
| fig 10 (v) | | |
| fig 10 (v) | | |
| fig 10 (vi) | | |

Is there any relationship between the rotational sequence and the number of symmetrical axes ? What conclusions do you draw from this ?

## Practice

1. Where do you find examples of rotational symmetry in your daily life ? Give any five examples.
2. Can you think of any kind of symmetry ? Think and discuss with your friends.

## Exercise 10.2

Q. 1- Three angles of a quadrilateral are $80^0$, $110^0$ and $120^0$respectively. Determine the fourth angle.

Q. 2- If three angles of any quadrilateral are equal and fourth angle is $75^0$ then determine the measure of each of the three equal angles.

Q. 3- Ratio of the adjacent angles of a parallelogram is 2 : 3 . Determine the measure of each angle.

Q. 4- One angle of a quadrilateral is a right angle and remaining (three) angle are in the ratio of 2 : 3 : 4 determine the measure of each of them.

Q. 5- Angles of a quadrilateral are in the ratio of 1 : 2 : 3 : 4, determine the measures of all the angles.

Q. 6 - Adjacent angles of a parallelogram are $x^0$ and $(x + 20)^0$ then determine the measure of each angle.

## We have learnt

1. A figure circumscribed by three or more sides is called a polygon.
2. When all the arms or sides of a polygon are equal, then it is known as a regular polygonal area or a equi-polygonal area.
3. Polygons in which the lengths of the sides are different are known as irregular polygonal areas.
4. The measures of each angle of a equi-polygonal area or a regular polygon are also equal.
5. If the number of sides in a polygon is ‘n’ then on joining all the opposite vertices from one vertex, the area gets divided into (n-2) triangles.
6. The value of the sum of all the internal angles of a prolygonal area is (n-2)×180°.
7. The diagonals of a parallelogram, rectangle, square and rhombus bisect each other at the point of intersection.
8. The two diagonals of a square and a rectangle are equal to one other.
9. The diagonals of a square and a rhombus bisect each other at right angles.
10. The polygonal area in which each angle is less than 180° is known as a convex polygon.
11. The polygonal area in which at least one internal angle is more than 180° is known as a concave polygonal area.