CBSE Test Paper 04

CH-12 Three Dimensional Geometry

1. The area of the triangle whose vertices are (1 , 2 , 3) , (2 , 5 , - 1) , (- 1 , 1 , 2) is (sq. units)
   a. 145
   b. 165
   c. 150
   d. $\frac{\sqrt{155}}{2}$
2. The centre of the sphere through the points (0 , 3 , 4) , (0 , 5 , 0) , (4 , 0 , 3) and (- 3 , 4 , 0) is
   a. none of these
   b. (- 4 , 3 , 0)
   c. (0 , 2 , 3)
   d. (0 , 0 , 0)
3. The plane x + y = 0 is
   a. none of these
   b. passes through z – axis
   c. parallel to z – axis
   d. perpendicular to z – axis
4. The angle between the lines x = 1 , y = 2 and y = - 1 , z = 0 is
   a. 0^0
   b. 30^0
   c. 60^0
   d. 90^0
5. Volume of a tetrahedron is k X area of one face X length of perpendicular from the opposite vertex upon it, where k is
   a. it is 1/6
   b. it is 1/3
   c. it is 1/4
   d. it is 1/2
6. Fill in the blanks:

If the distance between the point (a, 2, 1) and (1, -1, 1) is 5, then a = ________.

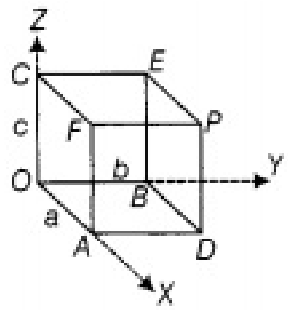
7. Fill in the blanks:

   The equation of z-axis, are ________.

8. A point is in the XZ-plane. What can you say about its y-coordinate?

9. Name the octants in which the following points lie
   (1, 2, 3), (4, -2, 3), (4, -2, -5), (-4, 2, -5), (-4, 2, 5), (-4, 2, 5), (-3, -1, 6), (2, -4, -7)

10. In the adjoining figure, if the coordinates of point P are (a, b, c), then write the coordinates of A, B, C, D and E.



11. If the points A (3, 2, - 4), B (9, 8, -10) and C (5, 4, - 6) are collinear, find the ratio in which C divides AB.

12. What is the length of foot of perpendicular drawn from the point P (3,4,5) on Y-axis.

13. Verify the following: (0, 7, -10), (1, 6, -6) and (4, 9, -6) are the vertices of an isosceles triangle.

14. Using section formula, show that the points A(2, -3, 4)b(-1, 2, 1) and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

15. Find the coordinates of the points which trisect the line segment joining the points P(4, 2, - 6) and Q(10, -16, 6).

Solution

1. (d) $\frac{\sqrt{155}}{2}$

   Explanation:

   let $\vec{a} = (1\hat{i} + 2\hat{j} + 3\hat{k}), \vec{b} = (2\hat{i} + 5\hat{j} - 1\hat{k}), \vec{c} = (-1\hat{i} + 1\hat{j} + 2\hat{k})$

   Ares of triangle $= \frac{1}{2}[\vec{a} * \vec{b} + \vec{b} * \vec{c} + \vec{c} * \vec{a}]$

   $$= \frac{1}{2}\left[\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 5 & -1 \end{vmatrix}\right] + \left[\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -1 \\ -1 & 1 & 2 \end{vmatrix}\right] + \left[\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}\right]$$

   $= \frac{1}{2}[-17\hat{i} + 7\hat{j} + 1\hat{k} + 11\hat{i} - 3\hat{j} + 7\hat{k} - 1\hat{i} + 5\hat{j} - 3\hat{k}]$

   $= \frac{1}{2}[-7\hat{i} + 9\hat{j} + 5\hat{k}]$

   Area $= \frac{1}{2}\sqrt{49 + 81 + 25}$

   Area $= \sqrt{155}/2$

2. (d) (0 , 0 , 0) Explanation:
   Let the equation of the required sphere be
   $\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 + 2\mathrm{ux} + 2\mathrm{vy} + 2\mathrm{wz} + \mathrm{d} = 0$ ....(1)
   since it passes through (0,3,4),(0,5,0),(4,0,3) and (-3,4,0) therefore
   $0^2 + 3^2 + 4^2 + 2u(0) + 2v(3) + 2w(4) + d = 0$
   $4^2 + 0^2 + 3^2 + 2u(4) + 2v(0) + 2w(3) + d = 0$
   $(-3)^2 + 4^2 + 0^2 + 2u(-3) + 2v(4) + 2w(0) + d = 0$
   $\Rightarrow 6v + 8w + 25 + d = 0 \ldots\ldots\ldots.(2)$
   $\Rightarrow 8u + 6w + 25 + d = 0 \ldots\ldots\ldots(3)$
   $\Rightarrow -6u + 8v + 25 + d = 0 \ldots\ldots\ldots.(4)$

$\Rightarrow 10\mathrm{v}+25+\mathrm{d}=0 \ldots \ldots \ldots \ (5)$

Subtracting (5) from (2),(3),(4) respectively, we get

$\Rightarrow -4\mathrm{v}+8\mathrm{w}=0 \ldots \ldots . (6)$

$\Rightarrow 8u+6w-10v=0$...........(7)

$\Rightarrow -6u-2v=0 \ldots \ldots . (8)$

Solving these, we get, u=v=w=0

$\therefore$ Centre of sphere (-u,-v,-w)=(0,0,0)

3. (b) passes through z – axis Explanation:

   Co - ordinates of any point on z-axis are (0,0, z)

   $\therefore$ The plane x+y=0 will pass through z-axis if(0,0,z) stisfies x+y = 0 ....(1)

   Now L.H.S of (1)=x+y

   For (0,0,z), L.H.S of (1) = 0+0=0=R.H.S of (1)

   Hence the result
4. (d) 90^0

   Explanation: x = 1 , y = 2 represents XY plane and y = - 1 , z = 0 represents YZ plane.Since XY perpendicular to YZ .hence angle is 90 degrees
5. (a) it is 1/6

   Explanation:

   Volume of tetrahedron =$\frac{1}{6}[\bar{a}\bar{b}\bar{c}]$ where a,b,c are co-terminus edges of tetrahedron.a X b is area of one face and c is the perpendicular from the opposite vertex

6. a = 5 or -3

7. x = 0, y = 0

8. Any point on the XZ-plane will have the coordinate (x, 0, z), so its y-coordinate is 0.

9. Point (1, 2, 3) lies in Ist Octant.

   Point (4, - 2, 3) lies in IVth Octant. Point (4, -2, -5) lies in VIIIth Octant.

   Point (4, 2, -5) lies in Vth Octant. Point (-4, 2, -5) lies in VIth octant.

   Point (-4, 2, 5) lies in IInd Octant. Point (-3, -1, 6) lies in VIIIth octant.

   Point (2, -4, -7) lies in VIIIth Octant.

10. Given, the coordinates of point P are (a, b, c).

    Which shows that, OA = a, OB = b and OC = c.

Now, point A lies on X-axis, so its coordinates is (a, 0, 0).

Point D lies in XY-plane, so its coordinates is (a, b, 0).

Point B lies on Y-axis, so its coordinates is (0, b, 0).

Point C lies on X-axis, so its coordinates is (0, 0, c) and point E lies in YZ-plane, so its coordinates is (0, b, c).

Hence, the coordinates are A(a, 0, 0), D(a, b, 0), B(0, b, 0), C(0, 0, c) and E(0, b, c).

11. A (3,2,−4), B (9,8,−10) and C (5,4,−6)

AC = $\sqrt{4+4+4} = 2\sqrt{3}$

AB = $\sqrt{36+36+36} = 6\sqrt{3}$

BC = $\sqrt{16+16+16} = 4\sqrt{3}$

AC : BC = 1 : 2

12. When we draw perpendicular from the point P(3, 4, 5) on Y-axis, the x and z-coordinates will be zero and y-coordinate will be 4

i.e., coordinate on Y -axis is Q (0, 4, 0).

Length of foot of perpendicular form P to Q = Distance between points P and Q

$= \sqrt{(3-0)^2 + (4-4)^2 + (5-0)^2}$

[$\because$ distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$]

$= \sqrt{3^2 + 0^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$

13. Let A(0, 7, -10), B(1, 6, -6) and C(4, 9, -6) be three vertices of triangle ABC. Then

$$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(4-0)^2 + (9-7)^2 + (-6+10)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$$

Now AB = BC

Thus, ABC is an isosceles triangle.

14. Let the points b(-1, 2, 1) divides the join of A(2, -3, 4) and $C\left(0, \frac{1}{3}, 2\right)$ in the ratio k : 1 internally.

Then coordinates of B are $\left(\frac{2}{k+1}, \frac{\frac{1}{3}k-3}{k+1}, \frac{2k+4}{k+1}\right)$

Now, $\frac{2}{k+1} = -1 \Rightarrow 2 = -k - 1 \Rightarrow k = -3$

Thus the point B divides the join of A and C in the ratio -3:1 so points A, B, C are collinear.

15. Let R and S be two point which trisect the join of PQ.

PR = RS = SQ

$\therefore$ Point R divides the join of PQ in the ratio 1 : 2

$\therefore$ Coordinates of R is $\left(\frac{1\times10+2\times4}{1+2}, \frac{1\times-16+2\times2}{1+2}, \frac{2\times6+2\times-6}{1+2}\right)$

= (6, -4, - 2)

Also point S divides the join of PQ in the ratio 2 : 1.

$\therefore$ Coordinates of S is $\left(\frac{2\times10+1\times4}{1+2}, \frac{2\times-16+1\times2}{1+2}, \frac{2\times6+1\times-6}{1+2}\right)$

= (8, -10, 2)