

# Mathematics

## (Chapter - 9)(Straight Lines)

(Class - XI)

### Exercise 9.1

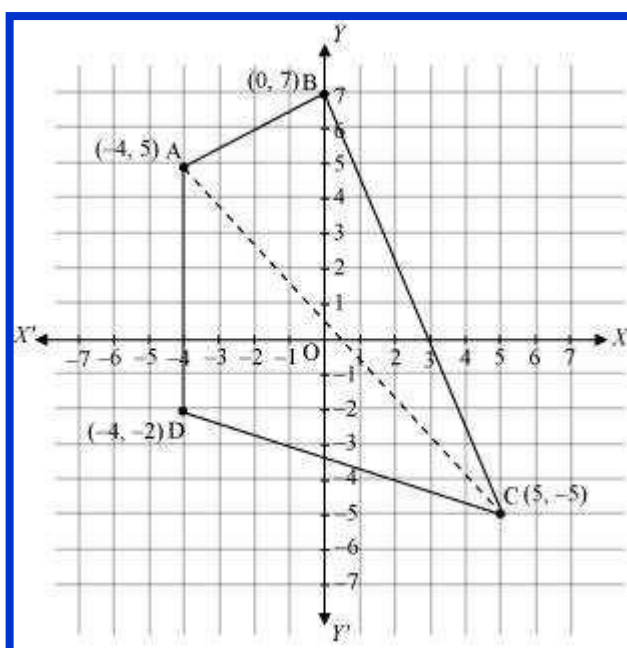
#### Question 1:

Draw a quadrilateral in the Cartesian plane, whose vertices are  $(-4, 5)$ ,  $(0, 7)$ ,  $(5, -5)$  and  $(-4, -2)$ . Also, find its area.

#### Answer 1:

Let ABCD be the given quadrilateral with vertices A  $(-4, 5)$ , B  $(0, 7)$ , C  $(5, -5)$ , and D  $(-4, -2)$ .

Then, by plotting A, B, C, and D on the Cartesian plane and joining AB, BC, CD, and DA, the given quadrilateral can be drawn as



To find the area of quadrilateral ABCD, we draw one diagonal, say AC.

Accordingly,  $\text{area (ABCD)} = \text{area } (\triangle ABC) + \text{area } (\triangle ACD)$

We know that the area of a triangle whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  is

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Therefore, area of  $\triangle ABC$

$$\begin{aligned}
&= \frac{1}{2} |-4(7+5) + 0(-5-5) + 5(5-7)| \text{ unit}^2 \\
&= \frac{1}{2} |-4(12) + 5(-2)| \text{ unit}^2 \\
&= \frac{1}{2} |-48 - 10| \text{ unit}^2 \\
&= \frac{1}{2} |-58| \text{ unit}^2 \\
&= \frac{1}{2} \times 58 \text{ unit}^2 \\
&= 29 \text{ unit}^2
\end{aligned}$$

Area of  $\Delta ACD$

$$\begin{aligned}
&= \frac{1}{2} |-4(-5+2) + 5(-2-5) + (-4)(5+5)| \text{ unit}^2 \\
&= \frac{1}{2} |-4(-3) + 5(-7) - 4(10)| \text{ unit}^2 \\
&= \frac{1}{2} |12 - 35 - 40| \text{ unit}^2 \\
&= \frac{1}{2} |-63| \text{ unit}^2 \\
&= \frac{63}{2} \text{ unit}^2
\end{aligned}$$

$$\text{Thus, area (ABCD)} = \left(29 + \frac{63}{2}\right) \text{ unit}^2 = \frac{58+63}{2} \text{ unit}^2 = \frac{121}{2} \text{ unit}^2$$

### Question 2:

The base of an equilateral triangle with side  $2a$  lies along the  $y$ -axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

### Answer 2:

Let ABC be the given equilateral triangle with side  $2a$ .

Accordingly,  $AB = BC = CA = 2a$

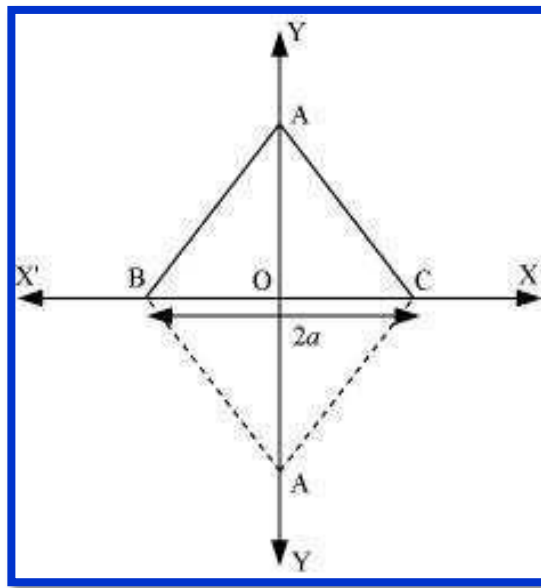
Assume that base BC lies along the  $y$ -axis such that the mid-point of BC is at the origin.

i.e.,  $BO = OC = a$ , where O is the origin.

Now, it is clear that the coordinates of point C are  $(0, a)$ , while the coordinates of point B are  $(0, -a)$ .

It is known that the line joining a vertex of an equilateral triangle with the mid-point of its opposite side is perpendicular.

Hence, vertex A lies on the y-axis.



On applying Pythagoras theorem to  $\triangle AOC$ , we obtain

$$(AC)^2 = (OA)^2 + (OC)^2$$

$$\Rightarrow (2a)^2 = (OA)^2 + a^2$$

$$\Rightarrow 4a^2 - a^2 = (OA)^2$$

$$\Rightarrow (OA)^2 = 3a^2$$

$$\Rightarrow OA = \sqrt{3}a$$

$$\therefore \text{Coordinates of point A} = (\pm\sqrt{3}a, 0)$$

Thus, the vertices of the given equilateral triangle are  $(0, a)$ ,  $(0, -a)$ , and  $(\sqrt{3}a, 0)$

or  $(0, a)$ ,  $(0, -a)$ , and  $(-\sqrt{3}a, 0)$  .

**Question 3:**

Find the distance between  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  when:

- (i) PQ is parallel to the y-axis,
- (ii) PQ is parallel to the x-axis.

Answer 3:

The given points are  $P(x_1, y_1)$  and  $Q(x_2, y_2)$ .

- (i) When PQ is parallel to the y-axis,  $x_1 = x_2$ .

$$\begin{aligned}\text{In this case, distance between P and Q} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(y_2 - y_1)^2} \\ &= |y_2 - y_1|\end{aligned}$$

- (ii) When PQ is parallel to the x-axis,  $y_1 = y_2$ .

$$\begin{aligned}\text{In this case, distance between P and Q} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x_2 - x_1)^2} \\ &= |x_2 - x_1|\end{aligned}$$

**Question 4:**

Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

**Answer 4:**

Let  $(a, 0)$  be the point on the x axis that is equidistant from the points (7, 6) and (3, 4).

$$\begin{aligned}\text{Accordingly, } \sqrt{(7-a)^2 + (6-0)^2} &= \sqrt{(3-a)^2 + (4-0)^2} \\ \Rightarrow \sqrt{49 + a^2 - 14a + 36} &= \sqrt{9 + a^2 - 6a + 16} \\ \Rightarrow \sqrt{a^2 - 14a + 85} &= \sqrt{a^2 - 6a + 25}\end{aligned}$$

On squaring both sides, we obtain  $a^2 - 14a + 85 = a^2 - 6a + 25$

$$\Rightarrow -14a + 6a = 25 - 85$$

$$\Rightarrow -8a = -60$$

$$\Rightarrow a = \frac{60}{8} = \frac{15}{2}$$

Thus, the required point on the x-axis is  $\left(\frac{15}{2}, 0\right)$ .

### Question 5:

Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points P (0, -4) and B (8, 0).

### Answer 5:

The coordinates of the mid-point of the line segment joining the points

$$P (0, -4) \text{ and } B (8, 0) \text{ are } \left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)$$

It is known that the slope ( $m$ ) of a non-vertical line passing through the points ( $x_1, y_1$ )

and ( $x_2, y_2$ ) is given by  $m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$ .

Therefore, the slope of the line passing through (0, 0) and (4, -2) is

$$\frac{-2-0}{4-0} = \frac{-2}{4} = -\frac{1}{2}.$$

Hence, the required slope of the line is  $-\frac{1}{2}$ .

### Question 6:

Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (-1, -1) are the vertices of a right angled triangle.

### Answer 6:

The vertices of the given triangle are A (4, 4), B (3, 5), and C (-1, -1).

It is known that the slope ( $m$ ) of a non-vertical line passing through the points ( $x_1, y_1$ )

and ( $x_2, y_2$ ) is given by  $m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$ .

$$\therefore \text{Slope of AB } (m_1) = \frac{5-4}{3-4} = -1$$

$$\text{Slope of BC } (m_2) = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$$

$$\text{Slope of CA } (m_3) = \frac{4+1}{4+1} = \frac{5}{5} = 1$$

It is observed that  $m_1 m_3 = -1$

This shows that line segments AB and CA are perpendicular to each other

i.e., the given triangle is right-angled at A (4, 4).

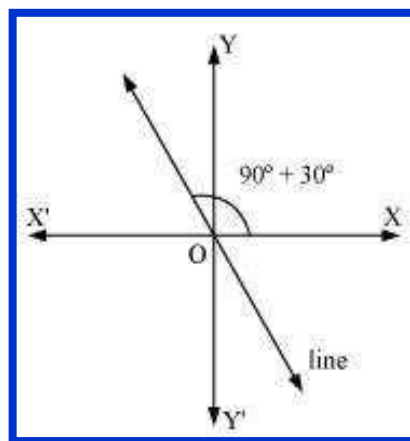
Thus, the points (4, 4), (3, 5), and (-1, -1) are the vertices of a right-angled triangle.

### Question 7:

Find the slope of the line, which makes an angle of  $30^\circ$  with the positive direction of  $y$ -axis measured anticlockwise.

### Answer 7:

If a line makes an angle of  $30^\circ$  with the positive direction of the  $y$ -axis measured anticlockwise, then the angle made by the line with the positive direction of the  $x$ -axis measured anticlockwise is  $90^\circ + 30^\circ = 120^\circ$ .



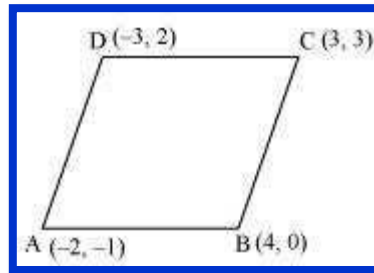
Thus, the slope of the given line is  $\tan 120^\circ = \tan (180^\circ - 60^\circ) = -\tan 60^\circ = -\sqrt{3}$

**Question 8:**

Without using distance formula, show that points  $(-2, -1)$ ,  $(4, 0)$ ,  $(3, 3)$  and  $(-3, 2)$  are vertices of a parallelogram.

**Answer 8:**

Let points  $(-2, -1)$ ,  $(4, 0)$ ,  $(3, 3)$ , and  $(-3, 2)$  be respectively denoted by A, B, C, and D.



$$\text{Slope of AB} = \frac{0+1}{4+2} = \frac{1}{6}$$

$$\text{Slope of CD} = \frac{2-3}{-3-3} = \frac{-1}{-6} = \frac{1}{6}$$

$$\Rightarrow \text{Slope of AB} = \text{Slope of CD}$$

$\Rightarrow$  AB and CD are parallel to each other.

$$\text{Now, slope of BC} = \frac{3-0}{3-4} = \frac{3}{-1} = -3$$

$$\text{Slope of AD} = \frac{2+1}{-3+2} = \frac{3}{-1} = -3$$

$$\Rightarrow \text{Slope of BC} = \text{Slope of AD}$$

$\Rightarrow$  BC and AD are parallel to each other.

Therefore, both pairs of opposite sides of quadrilateral ABCD are parallel. Hence, ABCD is a parallelogram.

Thus, points  $(-2, -1)$ ,  $(4, 0)$ ,  $(3, 3)$ , and  $(-3, 2)$  are the vertices of a parallelogram.

**Question 9:**

Find the angle between the x-axis and the line joining the points (3, -1) and (4, -2).

**Answer 9:**

The slope of the line joining the points (3, -1) and (4, -2) is  $m = \frac{-2 - (-1)}{4 - 3} = -2 + 1 = -1$

Now, the inclination ( $\theta$ ) of the line joining the points (3, -1) and (4, -2) is given by

$$\tan \theta = -1$$

$$\Rightarrow \theta = (90^\circ + 45^\circ) = 135^\circ$$

Thus, the angle between the x-axis and the line joining the points (3, -1) and (4, -2) is  $135^\circ$ .

**Question 10:**

The slope of a line is double of the slope of another line. If tangent of the angle between them is  $1/3$ , find the slopes of the lines.

**Answer 10:**

Let  $m_1$  and  $m$  be the slopes of the two given lines such that  $m_1 = 2m$ .

We know that if  $\theta$  is the angle between the lines  $l_1$  and  $l_2$  with slopes  $m_1$  and  $m_2$ , then

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

It is given that the tangent of the angle between the two lines is  $\frac{1}{3}$ .

$$\therefore \frac{1}{3} = \left| \frac{m - 2m}{1 + (2m) \cdot m} \right|$$

$$\Rightarrow \frac{1}{3} = \left| \frac{-m}{1 + 2m^2} \right|$$

$$\Rightarrow \frac{1}{3} = \frac{-m}{1 + 2m^2} \text{ or } \frac{1}{3} = -\left( \frac{-m}{1 + 2m^2} \right) = \frac{m}{1 + 2m^2}$$



**Case I**

$$\Rightarrow \frac{1}{3} = \frac{-m}{1+2m^2}$$

$$\Rightarrow 1+2m^2 = -3m$$

$$\Rightarrow 2m^2 + 3m + 1 = 0$$

$$\Rightarrow 2m^2 + 2m + m + 1 = 0$$

$$\Rightarrow 2m(m+1) + 1(m+1) = 0$$

$$\Rightarrow (m+1)(2m+1) = 0$$

$$\Rightarrow m = -1 \text{ or } m = -\frac{1}{2}$$

If  $m = -1$ , then the slopes of the lines are  $-1$  and  $-2$ .

If  $m = -\frac{1}{2}$ , then the slopes of the lines are  $-\frac{1}{2}$  and  $-1$ .

**Case II**

$$\frac{1}{3} = \frac{m}{1+2m^2}$$

$$\Rightarrow 2m^2 + 1 = 3m$$

$$\Rightarrow 2m^2 - 3m + 1 = 0$$

$$\Rightarrow 2m^2 - 2m - m + 1 = 0$$

$$\Rightarrow 2m(m-1) - 1(m-1) = 0$$

$$\Rightarrow (m-1)(2m-1) = 0$$

$$\Rightarrow m = 1 \text{ or } m = \frac{1}{2}$$

If  $m = 1$ , then the slopes of the lines are  $1$  and  $2$ .

If  $m = \frac{1}{2}$ , then the slopes of the lines are  $\frac{1}{2}$  and  $1$ .

Hence, the slopes of the lines are  $-1$  and  $-2$  or  $-\frac{1}{2}$  and  $-1$  or  $1$  and  $2$  or  $\frac{1}{2}$  and  $1$ .

**Question 11:**

A line passes through  $(x_1, y_1)$  and  $(h, k)$ . If slope of the line is  $m$ , show that  $k - y_1 = m(h - x_1)$ .

**Answer 11:**

The slope of the line passing through  $(x_1, y_1)$  and  $(h, k)$  is  $\frac{k - y_1}{h - x_1}$  .  
It is given that the slope of the line is  $m$ .

$$\therefore \frac{k - y_1}{h - x_1} = m$$

$$\Rightarrow k - y_1 = m(h - x_1)$$

Hence,  $k - y_1 = m(h - x_1)$

# Mathematics

## (Chapter - 9)(Straight Lines)

(Class – XI)

### Exercise 9.2

**In Exercises 1 to 8, find the equation of the line which satisfy the given conditions:**

**Question 1:**

Write the equations for the  $x$  and  $y$ -axes.

**Answer 1:**

The  $y$ -coordinate of every point on the  $x$ -axis is 0.

Therefore, the equation of the  $x$ -axis is  $y = 0$ .

The  $x$ -coordinate of every point on the  $y$ -axis is 0.

Therefore, the equation of the  $y$ -axis is  $x = 0$ .

**Question 2:**

Find the *equation* of the line which passes through the point  $(-4, 3)$  with slope  $\frac{1}{2}$ .

**Answer 2:**

We know that the equation of the line passing through point  $(x_0, y_0)$ , whose slope is  $m$ , is

$$(y - y_0) = m(x - x_0)$$

Thus, the equation of the line passing through point  $(-4, 3)$ , whose slope is  $\frac{1}{2}$ , is

$$(y - 3) = \frac{1}{2}(x + 4)$$

$$2(y - 3) = x + 4$$

$$2y - 6 = x + 4$$

$$\text{i.e., } x - 2y + 10 = 0$$

**Question 3:**

Find the equation of the line which passes through  $(0, 0)$  with slope  $m$ .

**Answer 3:**

We know that the equation of the line passing through point  $(x_0, y_0)$ , whose slope is  $m$ , is

$$(y - y_0) = m(x - x_0)$$

Thus, the equation of the line passing through point  $(0, 0)$ , whose slope is  $m$ , is

$$(y - 0) = m(x - 0)$$

$$\text{i.e., } y = mx$$

**Question 4:**

Find the equation of the line which passes through  $(2, 2\sqrt{3})$  and is inclined with the x-axis at an angle of  $75^\circ$ .

**Answer 4:**

The slope of the line that inclines with the x-axis at an angle of  $75^\circ$  is  $m$   
 $= \tan 75^\circ$

$$\Rightarrow m = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3} + 1}{\sqrt{3}}}{\frac{\sqrt{3} - 1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

We know that the equation of the line passing through point  $(x_0, y_0)$  ,

whose slope is  $m$ , is

$$(y - y_0) = m(x - x_0)$$

Thus, if a line passes through  $(2, 2\sqrt{3})$  and inclines with the x-axis at an angle of  $75^\circ$ ,

then the equation of the line is given as

$$(y - 2\sqrt{3}) = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}(x - 2)$$

$$(y - 2\sqrt{3})(\sqrt{3} - 1) = (\sqrt{3} + 1)(x - 2)$$

$$y(\sqrt{3} - 1) - 2\sqrt{3}(\sqrt{3} - 1) = x(\sqrt{3} + 1) - 2(\sqrt{3} + 1)$$

$$(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}$$

$$(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4\sqrt{3} - 4$$

$$\text{i.e., } (\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4(\sqrt{3} - 1)$$

**Question 5:**

Intersecting the  $x$ -axis at a distance of 3 units to the left of origin with slope  $-2$ .

**Answer 8:**

It is known that if a line with slope  $m$  makes  $x$ -intercept  $d$ , then the equation of the line is given as  $y = m(x - d)$

For the line intersecting the  $x$ -axis at a distance of 3 units to the left of the origin,  
 $d = -3$ .

The slope of the line is given as  $m = -2$

Thus, the required equation of the given line is  $y = -2 [x - (-3)]$   $y = -2x - 6$   
 i.e.,  $2x + y + 6 = 0$

**Question 6:**

Intersecting the  $y$ -axis at a distance of 2 units above the origin and makes an angle of  $30^\circ$  with the positive direction of the  $x$ -axis.

**Answer 6:**

It is known that if a line with slope  $m$  makes  $y$ -intercept  $c$ , then the equation of the line is given as  $y = mx + c$

Here,  $c = 2$  and  $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$  .

Thus, the required equation of the given line is

$$y = \frac{1}{\sqrt{3}}x + 2$$

$$y = \frac{x + 2\sqrt{3}}{\sqrt{3}}$$

$$\sqrt{3}y = x + 2\sqrt{3}$$

$$\text{i.e., } x - \sqrt{3}y + 2\sqrt{3} = 0$$

**Question 7:**

Passing through the points  $(-1, 1)$  and  $(2, -4)$ .

**Answer 7:**

It is known that the equation of the line passing through points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Therefore, the equation of the line passing through the points  $(-1, 1)$  and  $(2, -4)$  is

$$(y - 1) = \frac{-4 - 1}{2 - (-1)} (x + 1)$$

$$(y - 1) = \frac{-5}{3} (x + 1)$$

$$3(y - 1) = -5(x + 1)$$

$$3y - 3 = -5x - 5$$

$$\text{i.e., } 5x + 3y + 2 = 0$$

**Question 8:**

The vertices of  $\Delta PQR$  are P (2, 1), Q (-2, 3) and R (4, 5). Find equation of the median through the vertex R.

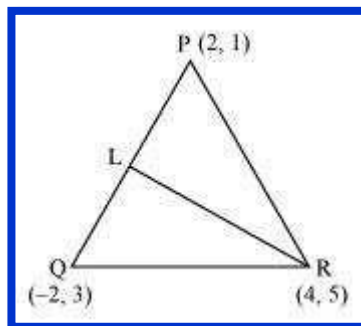
**Answer 8.**

Given that the vertices of  $\Delta PQR$  are P (2, 1), Q (-2, 3), and R (4, 5).

Let RL be the median through vertex R.

Accordingly, L is the mid-point of PQ.

By mid-point formula, the coordinates of point L are given by  $\left(\frac{2-2}{2}, \frac{1+3}{2}\right) = (0, 2)$



It is known that the equation of the line passing through points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Therefore, the equation of RL can be determined by substituting  $(x_1, y_1) = (4, 5)$  and  $(x_2, y_2) = (0, 2)$ .

$$\text{Hence, } y - 5 = \frac{2 - 5}{0 - 4} (x - 4)$$

$$\Rightarrow y - 5 = \frac{-3}{-4} (x - 4)$$

$$\Rightarrow 4(y - 5) = 3(x - 4)$$

$$\Rightarrow 4y - 20 = 3x - 12$$

$$\Rightarrow 3x - 4y + 8 = 0$$

Thus, the required equation of the median through vertex R is  $3x - 4y + 8 = 0$ .

**Question 9:**

Find the equation of the line passing through  $(-3, 5)$  and perpendicular to the line through the points  $(2, 5)$  and  $(-3, 6)$ .

**Answer 9:**

The slope of the line joining the points  $(2, 5)$  and  $(-3, 6)$  is  $m = \frac{6-5}{-3-2} = \frac{1}{-5}$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line perpendicular to the line through the points

$$(2, 5) \text{ and } (-3, 6) \quad = -\frac{1}{m} = -\frac{1}{\left(\frac{-1}{5}\right)} = 5$$

Now, the equation of the line passing through point  $(-3, 5)$ , whose slope is 5, is

$$\begin{aligned}(y-5) &= 5(x+3) \\ y-5 &= 5x+15 \\ \text{i.e., } 5x-y+20 &= 0\end{aligned}$$

**Question 10:**

A line perpendicular to the line segment joining the points  $(1, 0)$  and  $(2, 3)$  divides it in the ratio  $1:n$ . Find the equation of the line.

**Answer 10:**

According to the section formula, the coordinates of the point that divides the line segment joining the points  $(1, 0)$  and  $(2, 3)$  in the ratio  $1:n$  is given by

$$\left( \frac{n(1)+1(2)}{1+n}, \frac{n(0)+1(3)}{1+n} \right) = \left( \frac{n+2}{n+1}, \frac{3}{n+1} \right)$$

The slope of the line joining the points  $(1, 0)$  and  $(2, 3)$  is

$$m = \frac{3-0}{2-1} = 3$$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.



Therefore, slope of the line that is perpendicular to the line joining the points

$$(1, 0) \text{ and } (2, 3) = -\frac{1}{m} = -\frac{1}{3}$$

Now, the equation of the line passing through  $\left(\frac{n+2}{n+1}, \frac{3}{n+1}\right)$  and whose slope

is  $-\frac{1}{3}$  is given by

$$\begin{aligned}\left(y - \frac{3}{n+1}\right) &= -\frac{1}{3}\left(x - \frac{n+2}{n+1}\right) \\ \Rightarrow 3\left[(n+1)y - 3\right] &= -\left[x(n+1) - (n+2)\right] \\ \Rightarrow 3(n+1)y - 9 &= -(n+1)x + n + 2 \\ \Rightarrow (1+n)x + 3(1+n)y &= n + 11\end{aligned}$$

### Question 11:

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

### Answer 11:

The equation of a line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots \text{ (i)}$$

Here,  $a$  and  $b$  are the intercepts on  $x$  and  $y$  axes respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that  $a = b$ .

Accordingly, equation (i) reduces to

$$\begin{aligned}\frac{x}{a} + \frac{y}{a} &= 1 \\ \Rightarrow x + y &= a \quad \dots \text{ (ii)}\end{aligned}$$

Since the given line passes through point (2, 3), equation (ii) reduces to

$$2 + 3 = a \Rightarrow a = 5$$

On substituting the value of  $a$  in equation (ii), we obtain  $x + y = 5$ , which is the required equation of the line

**Question 12:**

Find equation of the line passing through the point  $(2, 2)$  and cutting off intercepts on the axes whose sum is 9.

**Answer 12:**

The equation of a line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots (i)$$

Here,  $a$  and  $b$  are the intercepts on  $x$  and  $y$  axes respectively.

It is given that  $a + b = 9 \Rightarrow b = 9 - a \dots (ii)$  From equations (i) and (ii), we obtain

$$\frac{x}{a} + \frac{y}{9-a} = 1 \quad \dots (iii)$$

It is given that the line passes through point  $(2, 2)$ . Therefore, equation (iii) reduces to

$$\frac{2}{a} + \frac{2}{9-a} = 1$$

$$\Rightarrow 2 \left( \frac{1}{a} + \frac{1}{9-a} \right) = 1$$

$$\Rightarrow 2 \left( \frac{9-a+a}{a(9-a)} \right) = 1$$

$$\Rightarrow \frac{18}{9a-a^2} = 1$$

$$\Rightarrow 18 = 9a - a^2$$

$$\Rightarrow a^2 - 9a + 18 = 0$$

$$\Rightarrow a^2 - 6a - 3a + 18 = 0$$

$$\Rightarrow a(a-6) - 3(a-6) = 0$$

$$\Rightarrow (a-6)(a-3) = 0$$

$$\Rightarrow a = 6 \text{ or } a = 3$$

If  $a = 6$  and  $b = 9 - 6 = 3$ , then the equation of the line is

$$\frac{x}{6} + \frac{y}{3} = 1 \Rightarrow x + 2y - 6 = 0$$

If  $a = 3$  and  $b = 9 - 3 = 6$ , then the equation of the line is

$$\frac{x}{3} + \frac{y}{6} = 1 \Rightarrow 2x + y - 6 = 0$$

**Question 13:**

Find equation of the line through the point  $(0, 2)$  making an angle  $\frac{2\pi}{3}$  with the positive

$x$ -axis. Also, find the equation of line parallel to it and crossing the  $y$ -axis at a distance of 2 units below the origin.

**Answer 13:**

The slope of the line making an angle  $\frac{2\pi}{3}$  with the positive  $x$ -axis is  $m = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

Now, the equation of the line passing through point  $(0, 2)$  and having a slope  $-\sqrt{3}$  is

$$(y - 2) = -\sqrt{3}(x - 0)$$

$$y - 2 = -\sqrt{3}x$$

$$\text{i.e., } \sqrt{3}x + y - 2 = 0$$

The slope of line parallel  $\sqrt{3}x + y - 2 = 0$  is  $-\sqrt{3}$  to line.

It is given that the line parallel to line  $\sqrt{3}x + y - 2 = 0$  crosses the  $y$ -axis 2 units below

the origin i.e., it passes through point  $(0, -2)$ .

Hence, the equation of the line passing through point  $(0, -2)$  and having a slope  $-\sqrt{3}$  is

$$y - (-2) = -\sqrt{3}(x - 0)$$

$$y + 2 = -\sqrt{3}x$$

$$\sqrt{3}x + y + 2 = 0$$

**Question 14:**

The perpendicular from the origin to a line meets it at the point  $(-2, 9)$ , find the equation of the line.

**Answer 14:**

The slope of the line joining the origin  $(0, 0)$  and point  $(-2, 9)$  is  $m_1 = \frac{9-0}{-2-0} = -\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and point

$$(-2, 9) \text{ is } m_2 = -\frac{1}{m_1} = -\frac{1}{\left(-\frac{9}{2}\right)} = \frac{2}{9}$$

Now, the equation of the line passing through point  $(-2, 9)$  and having a slope  $m_2$  is

$$(y-9) = \frac{2}{9}(x+2)$$

$$9y - 81 = 2x + 4$$

$$\text{i.e., } 2x - 9y + 85 = 0$$

**Question 15:**

The length  $L$  (in centimetre) of a copper rod is a linear function of its Celsius temperature  $C$ . In an experiment, if  $L = 124.942$  when  $C = 20$  and  $L = 125.134$  when  $C = 110$ , express  $L$  in terms of  $C$ .

**Answer 15:**

It is given that when  $C = 20$ , the value of  $L$  is 124.942, whereas when  $C = 110$ , the value of  $L$  is 125.134.

Accordingly, points  $(20, 124.942)$  and  $(110, 125.134)$  satisfy the linear relation between  $L$  and  $C$ .

Now, assuming  $C$  along the  $x$ -axis and  $L$  along the  $y$ -axis, we have two points i.e.,  $(20, 124.942)$  and  $(110, 125.134)$  in the  $XY$  plane.

Therefore, the linear relation between  $L$  and  $C$  is the equation of the line passing through points  $(20, 124.942)$  and  $(110, 125.134)$ .

$$(L - 124.942) = \frac{125.134 - 124.942}{110 - 20}(C - 20)$$

$$L - 124.942 = \frac{0.192}{90}(C - 20)$$

$$\text{i.e., } L = \frac{0.192}{90}(C - 20) + 124.942, \text{ which is the required linear relation}$$

### Question 16:

The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

### Answer 16:

The relationship between selling price and demand is linear.

Assuming selling price per litre along the x-axis and demand along the y-axis, we have two points i.e., (14, 980) and (16, 1220) in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the linear relationship between selling price per litre and demand is the equation of the line passing through points (14, 980) and (16, 1220).

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$y - 980 = \frac{240}{2}(x - 14)$$

$$y - 980 = 120(x - 14)$$

$$\text{i.e., } y = 120(x - 14) + 980$$

When  $x = \text{Rs } 17/\text{litre}$ ,

$$y = 120(17 - 14) + 980$$

$$\Rightarrow y = 120 \times 3 + 980 = 360 + 980 = 1340$$

Thus, the owner of the milk store could sell 1340 litres of milk weekly at Rs 17/litre.

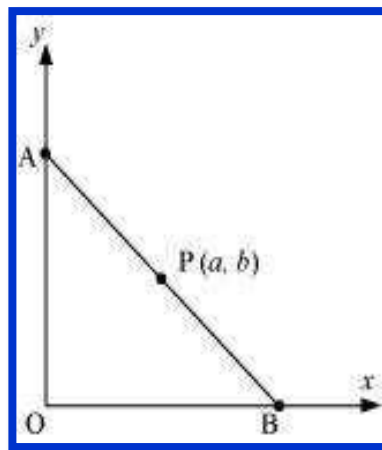
**Question 17:**

P ( $a$ ,  $b$ ) is the mid-point of a line segment between axes. Show that equation of the line

is  $\frac{x}{a} + \frac{y}{b} = 2$

**Answer 17:**

Let AB be the line segment between the axes and let P ( $a$ ,  $b$ ) be its mid-point.



Let the coordinates of A and B be ( $0$ ,  $y$ ) and ( $x$ ,  $0$ ) respectively.

Since P ( $a$ ,  $b$ ) is the mid-point of AB,

$$\left( \frac{0+x}{2}, \frac{y+0}{2} \right) = (a, b)$$

$$\Rightarrow \left( \frac{x}{2}, \frac{y}{2} \right) = (a, b)$$

$$\Rightarrow \frac{x}{2} = a \text{ and } \frac{y}{2} = b$$

$$\therefore x = 2a \text{ and } y = 2b$$

Thus, the respective coordinates of A and B are ( $0$ ,  $2b$ ) and ( $2a$ ,  $0$ ).

The equation of the line passing through points ( $0$ ,  $2b$ ) and ( $2a$ ,  $0$ ) is

$$(y - 2b) = \frac{(0 - 2b)}{(2a - 0)}(x - 0)$$

$$y - 2b = \frac{-2b}{2a}(x)$$

$$a(y - 2b) = -bx$$

$$ay - 2ab = -bx$$

$$\text{i.e., } bx + ay = 2ab$$

On dividing both sides by  $ab$ , we obtain

$$\frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$$

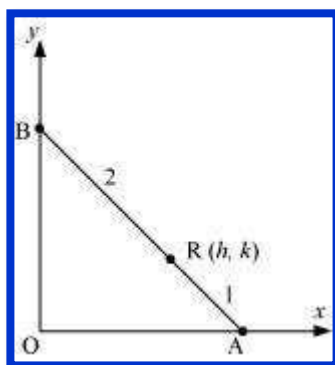
Thus, the equation of the line is  $\frac{x}{a} + \frac{y}{b} = 2$ .

### Question 18:

Point R ( $h$ ,  $k$ ) divides a line segment between the axes in the ratio 1:2. Find equation of the line.

### Answer 18:

Let AB be the line segment between the axes such that point R ( $h$ ,  $k$ ) divides AB in the ratio 1: 2.



Let the respective coordinates of A and B be ( $x$ , 0) and (0,  $y$ ).

Since point R ( $h$ ,  $k$ ) divides AB in the ratio 1: 2, according to the section formula,

$$\begin{aligned}
 (h, k) &= \left( \frac{1 \times 0 + 2 \times x}{1 + 2}, \frac{1 \times y + 2 \times 0}{1 + 2} \right) \\
 \Rightarrow (h, k) &= \left( \frac{2x}{3}, \frac{y}{3} \right) \\
 \Rightarrow h &= \frac{2x}{3} \text{ and } k = \frac{y}{3} \\
 \Rightarrow x &= \frac{3h}{2} \text{ and } y = 3k
 \end{aligned}$$

Therefore, the respective coordinates of A and B are  $\left(\frac{3h}{2}, 0\right)$  and  $(0, 3k)$ .

Now, the equation of line AB passing through points  $\left(\frac{3h}{2}, 0\right)$  and  $(0, 3k)$  is

$$\begin{aligned}
 (y - 0) &= \frac{3k - 0}{0 - \frac{3h}{2}} \left( x - \frac{3h}{2} \right) \\
 y &= -\frac{2k}{h} \left( x - \frac{3h}{2} \right) \\
 hy &= -2kx + 3hk \\
 \text{i.e., } 2kx + hy &= 3hk
 \end{aligned}$$

Thus, the required equation of the line is  $2kx + hy = 3hk$

### Question 19:

By using the concept of equation of a line, prove that the three points  $(3, 0)$ ,  $(-2, -2)$  and  $(8, 2)$  are collinear.

### Answer 20:

In order to show that points  $(3, 0)$ ,  $(-2, -2)$ , and  $(8, 2)$  are collinear, it suffices to show that the line passing through points  $(3, 0)$  and  $(-2, -2)$  also passes through point  $(8, 2)$ .

The equation of the line passing through points  $(3, 0)$  and  $(-2, -2)$  is



$$(y-0) = \frac{(-2-0)}{(-2-3)}(x-3)$$

$$y = \frac{-2}{-5}(x-3)$$

$$5y = 2x - 6$$

$$\text{i.e., } 2x - 5y = 6$$

It is observed that at  $x = 8$  and  $y = 2$ ,

$$\text{L.H.S.} = 2 \times 8 - 5 \times 2 = 16 - 10 = 6 = \text{R.H.S.}$$

Therefore, the line passing through points  $(3, 0)$  and  $(-2, -2)$  also passes through point  $(8, 2)$ . Hence, points  $(3, 0)$ ,  $(-2, -2)$ , and  $(8, 2)$  are collinear.

# Mathematics

## (Chapter - 9)(Straight Lines)

(Class – XI)

### Exercise 9.3

#### Question 1:

Reduce the following equations into slope-intercept form and find their slopes and the y intercepts.

(i)  $x + 7y = 0$

(ii)  $6x + 3y - 5 = 0$

(iii)  $y = 0$

#### Answer 1:

(i) The given equation is  $x + 7y = 0$ . It can be written as

$$y = -\frac{1}{7}x + 0 \quad \dots(1)$$

This equation is of the form  $y = mx + c$ , where  $m = -\frac{1}{7}$  and  $c = 0$ .

Therefore, equation (1) is in the slope-intercept form, where the slope and the y-

intercept are  $-\frac{1}{7}$  and 0 respectively.

(ii) The given equation is  $6x + 3y - 5 = 0$ .

It can be written as

$$y = \frac{1}{3}(-6x + 5)$$
$$y = -2x + \frac{5}{3} \quad \dots(2)$$

This equation is of the form  $y = mx + c$ , where  $m = -2$  and  $c = \frac{5}{3}$ .

Therefore, equation (2) is in the slope-intercept form, where the slope and the y-

intercept are  $-2$  and  $\frac{5}{3}$  respectively.

(iii) The given equation is  $y = 0$ .

It can be written as

$$y = 0.x + 0 \quad \dots(3)$$

This equation is of the form  $y = mx + c$ , where  $m = 0$  and  $c = 0$ .

Therefore, equation (3) is in the slope-intercept form, where the slope and the  $y$ -intercept are 0 and 0 respectively.

**Question 2:**

Reduce the following equations into intercept form and find their intercepts on the axes.

**(i)**  $3x + 2y - 12 = 0$

**(ii)**  $4x - 3y = 6$

**(iii)**  $3y + 2 = 0$ .

**Answer 2:**

**(i)** The given equation is  $3x + 2y - 12 = 0$ .

It can be written as

$$3x + 2y = 12$$

$$\frac{3x}{12} + \frac{2y}{12} = 1$$

$$\text{i.e., } \frac{x}{4} + \frac{y}{6} = 1 \quad \dots(1)$$

This equation is of the form  $\frac{x}{a} + \frac{y}{b} = 1$ ,

where  $a = 4$  and  $b = 6$ .

Therefore, equation (1) is in the intercept form, where the intercepts on the  $x$  and  $y$  axes are 4 and 6 respectively.

**(ii)** The given equation is  $4x - 3y = 6$ .

It can be written as

$$\frac{4x}{6} - \frac{3y}{6} = 1$$

$$\frac{2x}{3} - \frac{y}{2} = 1$$

$$\text{i.e., } \frac{x}{\left(\frac{3}{2}\right)} + \frac{y}{(-2)} = 1 \quad \dots(2)$$

This equation is of the form  $\frac{x}{a} + \frac{y}{b} = 1$ , where  $a = \frac{3}{2}$  and  $b = -2$ .

Therefore, equation (2) is in the intercept form, where the intercepts on the  $x$  and  $y$  axes are  $3/2$  and  $-2$  respectively.

**(iii)** The given equation is  $3y + 2 = 0$ .

It can be written as

$$3y = -2$$

$$\text{i.e., } \frac{y}{\left(-\frac{2}{3}\right)} = 1 \quad \dots(3)$$

The equation is of the form  $\frac{x}{a} + \frac{y}{b} = 1$ , where  $a = 0$  and  $b = -\frac{2}{3}$ .

Therefore, equation (3) is in the intercept form, where the intercept on the  $y$ -axis is  $-\frac{2}{3}$  and it has no intercept on the  $x$ -axis.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$ , we obtain  $\omega = 315^\circ$  and  $p = 2\sqrt{2}$ .

Thus, the perpendicular distance of the line from the origin is  $2\sqrt{2}$ , while the angle between the perpendicular and the positive x-axis is  $315^\circ$ .

### Question 3:

Find the distance of the point  $(-1, 1)$  from the line  $12(x + 6) = 5(y - 2)$ .

### Answer 3:

The given equation of the line is  $12(x + 6) = 5(y - 2)$ .

$$\Rightarrow 12x + 72 = 5y - 10$$

$$\Rightarrow 12x - 5y + 82 = 0 \dots (1)$$

On comparing equation (1) with general equation of line  $Ax + By + C = 0$ , we obtain  $A = 12$ ,  $B = -5$ , and  $C = 82$ .

It is known that the perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$  from a point

$$(x_1, y_1) \text{ is given by } d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

The given point is  $(x_1, y_1) = (-1, 1)$ .

Therefore, the distance of point  $(-1, 1)$  from the given line

$$= \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} \text{ units} = \frac{|-12 - 5 + 82|}{\sqrt{169}} \text{ units} = \frac{|65|}{13} \text{ units} = 5 \text{ units}$$

### Question 4:

Find the points on the x-axis, whose distances from the line  $\frac{x}{3} + \frac{y}{4} = 1$  are 4 units.

### Answer 4:

The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

$$\text{or, } 4x + 3y - 12 = 0 \dots (1)$$

On comparing equation (1) with general equation of line  $Ax + By + C = 0$ ,

we obtain  $A = 4$ ,  $B = 3$ , and  $C = -12$ .

Let  $(a, 0)$  be the point on the  $x$ -axis whose distance from the given line is 4 units.

It is known that the perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$  from a point

$(x_1, y_1)$  is given by  $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ .

Therefore,

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$\Rightarrow 4 = \frac{|4a - 12|}{5}$$

$$\Rightarrow |4a - 12| = 20$$

$$\Rightarrow \pm(4a - 12) = 20$$

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$

$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$

$$\Rightarrow a = 8 \text{ or } -2$$

Thus, the required points on the  $x$ -axis are  $(-2, 0)$  and  $(8, 0)$ .

### Question 5:

Find the distance between parallel lines

(i)  $15x + 8y - 34 = 0$  and  $15x + 8y + 31 = 0$

(ii)  $l(x + y) + p = 0$  and  $l(x + y) - r = 0$

### Answer 5:

It is known that the distance ( $d$ ) between parallel lines  $Ax + By + C_1 = 0$  and  $Ax + By + C_2 = 0$  is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

(i) The given parallel lines are  $15x + 8y - 34 = 0$  and  $15x + 8y + 31 = 0$ .

Here,  $A = 15$ ,  $B = 8$ ,  $C_1 = -34$ , and  $C_2 = 31$ .

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} \text{ units} = \frac{|-65|}{17} \text{ units} = \frac{65}{17} \text{ units}$$

(ii) The given parallel lines are  $l(x + y) + p = 0$  and  $l(x + y) - r = 0$ .

$lx + ly + p = 0$  and  $lx + ly - r = 0$  Here,

$A = l, B = l, C_1 = p$ , and  $C_2 = -r$ .

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|p + r|}{\sqrt{l^2 + l^2}} \text{ units} = \frac{|p + r|}{\sqrt{2l^2}} \text{ units} = \frac{|p + r|}{l\sqrt{2}} \text{ units} = \frac{1}{\sqrt{2}} \left| \frac{p + r}{l} \right| \text{ units}$$

### Question 6:

Find equation of the line parallel to the line  $3x - 4y + 2 = 0$  and passing through the point  $(-2, 3)$ .

### Answer 6:

The equation of the given line is

$$3x - 4y + 2 = 0$$

$$\text{or } y = \frac{3x}{4} + \frac{2}{4}$$

$$\text{or } y = \frac{3}{4}x + \frac{1}{2}$$

which is of the form  $y = mx + c$

∴ Slope of the given line =  $3/4$

It is known that parallel lines have the same slope.

$$\therefore \text{Slope of the other line} = m = \frac{3}{4}$$

Now, the equation of the line that has a slope of  $\frac{3}{4}$  and passes through the point  $(-2, 3)$  is

$$(y - 3) = \frac{3}{4}\{x - (-2)\}$$

$$4y - 12 = 3x + 6$$

$$\text{i.e., } 3x - 4y + 18 = 0$$

**Question 7:**

Find equation of the line perpendicular to the line  $x - 7y + 5 = 0$  and having x intercept 3.

**Answer 7:**

The given equation of line is  $x - 7y + 5 = 0$ . Or,  $y = \frac{1}{7}x + \frac{5}{7}$

which is of the form  $y = mx + c$

∴ Slope of the given line  $= \frac{1}{7}$

The slope of the line perpendicular to the line having a slope of  $\frac{1}{7}$  is  $m = -\frac{1}{\left(\frac{1}{7}\right)} = -7$

The equation of the line with slope  $-7$  and x-intercept 3 is given by y

$$= m(x - d)$$

$$\Rightarrow y = -7(x - 3)$$

$$\Rightarrow y = -7x + 21$$

$$\Rightarrow 7x + y = 21$$

**Question 8:**

Find angles between the lines  $\sqrt{3}x + y = 1$  and  $x + \sqrt{3}y = 1$

**Answer 8:**

The given lines are  $\sqrt{3}x + y = 1$  and  $x + \sqrt{3}y = 1$ .

$$y = -\sqrt{3}x + 1 \quad \dots(1) \quad \text{and} \quad y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}} \quad \dots(2)$$

The slope of line (1) is  $m_1 = -\sqrt{3}$ , while the slope of line (2) is  $m_2 = -\frac{1}{\sqrt{3}}$

The acute angle i.e.,  $\theta$  between the two lines is given by



$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan \theta = \left| \frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)} \right|$$

$$\tan \theta = \left| \frac{\frac{-3+1}{\sqrt{3}}}{1+1} \right| = \left| \frac{-2}{2 \times \sqrt{3}} \right|$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Thus, the angle between the given lines is either  $30^\circ$  or  $180^\circ - 30^\circ = 150^\circ$ .

### Question 9:

The line through the points  $(h, 3)$  and  $(4, 1)$  intersects the line  $7x - 9y - 19 = 0$ , at right angle. Find the value of  $h$ .

### Answer 9:

The slope of the line passing through points  $(h, 3)$  and  $(4, 1)$  is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of line  $7x - 9y - 19 = 0$  or  $y = \frac{7}{9}x - \frac{19}{9}$  is  $m_2 = \frac{7}{9}$

It is given that the two lines are perpendicular.

$$\begin{aligned}
\therefore m_1 \times m_2 &= -1 \\
\Rightarrow \left( \frac{-2}{4-h} \right) \times \left( \frac{7}{9} \right) &= -1 \\
\Rightarrow \frac{-14}{36-9h} &= -1 \\
\Rightarrow 14 &= 36-9h \\
\Rightarrow 9h &= 36-14 \\
\Rightarrow h &= \frac{22}{9}
\end{aligned}$$

Thus, the value of  $h$  is  $\frac{22}{9}$ .

### Question 10:

Prove that the line through the point  $(x_1, y_1)$  and parallel to the line  $Ax + By + C = 0$  is  $A(x - x_1) + B(y - y_1) = 0$ .

### Answer 10:

The slope of line  $Ax + By + C = 0$  or  $y = \left( \frac{-A}{B} \right)x + \left( \frac{-C}{B} \right)$  is  $m = -\frac{A}{B}$

It is known that parallel lines have the same slope.

$\therefore$  Slope of the other line =  $m = -\frac{A}{B}$

The equation of the line passing through point  $(x_1, y_1)$  and having a slope  $m = -\frac{A}{B}$  is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = -\frac{A}{B}(x - x_1)$$

$$B(y - y_1) = -A(x - x_1)$$

$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point  $(x_1, y_1)$  and parallel to line  $Ax + By + C = 0$  is  $A(x - x_1) + B(y - y_1) = 0$

**Question 11:**

Two lines passing through the point (2, 3) intersect each other at an angle of  $60^\circ$ . If slope of one line is 2, find equation of the other line.

**Answer 11:**

It is given that the slope of the first line,  $m_1 = 2$ .

Let the slope of the other line be  $m_2$ .

The angle between the two lines is  $60^\circ$ .

$$\begin{aligned}\therefore \tan 60^\circ &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ \Rightarrow \sqrt{3} &= \left| \frac{2 - m_2}{1 + 2m_2} \right| \\ \Rightarrow \sqrt{3} &= \pm \left( \frac{2 - m_2}{1 + 2m_2} \right) \\ \Rightarrow \sqrt{3} &= \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = - \left( \frac{2 - m_2}{1 + 2m_2} \right) \\ \Rightarrow \sqrt{3}(1 + 2m_2) &= 2 - m_2 \text{ or } \sqrt{3}(1 + 2m_2) = -(2 - m_2) \\ \Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 &= 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2 \\ \Rightarrow \sqrt{3} + (2\sqrt{3} + 1)m_2 &= 2 \text{ or } \sqrt{3} + (2\sqrt{3} - 1)m_2 = -2 \\ \Rightarrow m_2 &= \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} \text{ or } m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}\end{aligned}$$

**Case I:**  $m_2 = \left( \frac{2 - \sqrt{3}}{2\sqrt{3} + 1} \right)$

The equation of the line passing through point (2, 3) and having a slope of  $\frac{(2 - \sqrt{3})}{(2\sqrt{3} + 1)}$  is given by

$$(y-3) = \frac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$$

$$(2\sqrt{3}+1)y - 3(2\sqrt{3}+1) = (2-\sqrt{3})x - 2(2-\sqrt{3})$$

$$(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$

$$(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -1 + 8\sqrt{3}$$

In this case, the equation of the other line is  $(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -1 + 8\sqrt{3}$  .

**Case II:**  $m_2 = \frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}$

The equation of the line passing through point (2, 3) and having a slope of  $\frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}$  is

$$(y-3) = \frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}(x-2)$$

$$(2\sqrt{3}-1)y - 3(2\sqrt{3}-1) = -(2+\sqrt{3})x + 2(2+\sqrt{3})$$

$$(2\sqrt{3}-1)y + (2+\sqrt{3})x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3$$

$$(2+\sqrt{3})x + (2\sqrt{3}-1)y = 1 + 8\sqrt{3}$$

In this case, the equation of the other line is  $(2+\sqrt{3})x + (2\sqrt{3}-1)y = 1 + 8\sqrt{3}$

Thus, the required equation of the other line is  $(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -1 + 8\sqrt{3}$

or

$$(2+\sqrt{3})x + (2\sqrt{3}-1)y = 1 + 8\sqrt{3}$$

### Question 12:

Find the equation of the right bisector of the line segment joining the points (3, 4) and (-1, 2).

### Answer 12:

The right bisector of a line segment bisects the line segment at  $90^\circ$ .

The end-points of the line segment are given as A (3, 4) and B (-1, 2).

Accordingly, mid-point of AB  $= \left( \frac{3-1}{2}, \frac{4+2}{2} \right) = (1, 3)$

$$\text{Slope of AB} = \frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$$

$$\therefore \text{Slope of the line perpendicular to AB} = -\frac{1}{\left(\frac{1}{2}\right)} = -2$$

The equation of the line passing through (1, 3) and having a slope of -2 is

$$(y - 3) = -2 (x - 1)$$

$$- 3 = -2x + 2$$

$$2x + y = 5$$

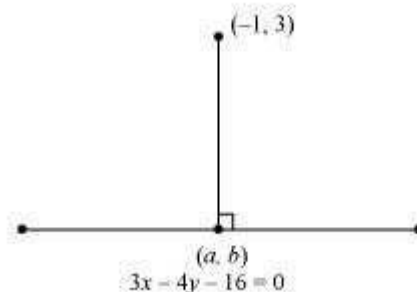
Thus, the required equation of the line is  $2x + y = 5$ .

### Question 13:

Find the coordinates of the foot of perpendicular from the point (-1, 3) to the line  $3x - 4y - 16 = 0$ .

### Answer 13:

Let (a, b) be the coordinates of the foot of the perpendicular from the point (-1, 3) to the line  $3x - 4y - 16 = 0$ .



$$\text{Slope of the line joining } (-1, 3) \text{ and } (a, b), m_1 = \frac{b-3}{a+1}$$

Slope of the line  $3x - 4y - 16 = 0$  or  $y = \frac{3}{4}x - 4$ ,  $m_2 = \frac{3}{4}$

Since these two lines are perpendicular,  $m_1 m_2 = -1$

$$\therefore \left( \frac{b-3}{a+1} \right) \times \left( \frac{3}{4} \right) = -1$$

$$\Rightarrow \frac{3b-9}{4a+4} = -1$$

$$\Rightarrow 3b-9 = -4a-4$$

$$\Rightarrow 4a+3b = 5 \quad \dots (1)$$

Point  $(a, b)$  lies on line  $3x - 4y = 16$ .

$$\therefore 3a - 4b = 16 \quad \dots (2)$$

On solving equations (1) and (2), we obtain

$$a = \frac{68}{25} \text{ and } b = -\frac{49}{25}$$

Thus, the required coordinates of the foot of the perpendicular are  $\left( \frac{68}{25}, -\frac{49}{25} \right)$ .

#### Question 14:

The perpendicular from the origin to the line  $y = mx + c$  meets it at the point  $(-1, 2)$ . Find the values of  $m$  and  $c$ .

#### Answer 14:

The given equation of line is  $y = mx + c$ .

It is given that the perpendicular from the origin meets the given line at  $(-1, 2)$ .

Therefore, the line joining the points  $(0, 0)$  and  $(-1, 2)$  is perpendicular to the given line.

$$\therefore \text{Slope of the line joining } (0, 0) \text{ and } (-1, 2) = \frac{2}{-1} = -2$$

The slope of the given line is  $m$ .

$$\therefore m \times -2 = -1 \quad [\text{The two lines are perpendicular}]$$

$$\Rightarrow m = \frac{1}{2}$$

Since point  $(-1, 2)$  lies on the given line, it satisfies the equation  $y = mx + c$ .

$$\therefore 2 = m(-1) + c$$

$$\Rightarrow 2 = \frac{1}{2}(-1) + c$$

$$\Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$$

Thus, the respective values of  $m$  and  $c$  are  $\frac{1}{2}$  and  $\frac{5}{2}$ .

### Question 15:

If  $p$  and  $q$  are the lengths of perpendiculars from the origin to the lines  $x \cos \theta - y \sin \theta = k \cos 2\theta$  and  $x \sec \theta + y \operatorname{cosec} \theta = k$ , respectively, prove that  $p^2 + 4q^2 = k^2$

### Answer 15:

The equations of given lines are

$$x \cos \theta - y \sin \theta = k \cos 2\theta \quad \dots\dots\dots (1)$$

$$x \sec \theta + y \operatorname{cosec} \theta = k \quad \dots\dots\dots (2)$$

The perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$  from a point  $(x_1, y_1)$  is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

On comparing equation (1) to the general equation of line i.e.,  $Ax + By + C = 0$ , we obtain  $A = \cos \theta$ ,  $B = -\sin \theta$ , and  $C = -k \cos 2\theta$ .

It is given that  $p$  is the length of the perpendicular from  $(0, 0)$  to line (1).

$$\therefore p = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k \cos 2\theta|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = |-k \cos 2\theta| \quad \dots(3)$$

On comparing equation (2) to the general equation of line i.e.,  $Ax + By + C = 0$ , we obtain  $A = \sec \theta$ ,  $B = \operatorname{cosec} \theta$ , and  $C = -k$ .

It is given that  $q$  is the length of the perpendicular from  $(0, 0)$  to line (2).

$$\therefore q = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \quad \dots(4)$$

From (3) and (4), we have

$$\begin{aligned}
p^2 + 4q^2 &= \left( -k \cos 2\theta \right)^2 + 4 \left( \frac{|-k|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \right)^2 \\
&= k^2 \cos^2 2\theta + \frac{4k^2}{(\sec^2 \theta + \operatorname{cosec}^2 \theta)} \\
&= k^2 \cos^2 2\theta + \frac{4k^2}{\left( \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \right)} \\
&= k^2 \cos^2 2\theta + \frac{4k^2}{\left( \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} \right)} \\
&= k^2 \cos^2 2\theta + \frac{4k^2}{\left( \frac{1}{\sin^2 \theta \cos^2 \theta} \right)} \\
&= k^2 \cos^2 2\theta + 4k^2 \sin^2 \theta \cos^2 \theta \\
&= k^2 \cos^2 2\theta + k^2 (2 \sin \theta \cos \theta)^2 \\
&= k^2 \cos^2 2\theta + k^2 \sin^2 2\theta \\
&= k^2 (\cos^2 2\theta + \sin^2 2\theta) \\
&= k^2
\end{aligned}$$

Hence, we proved that  $p^2 + 4q^2 = k^2$ .

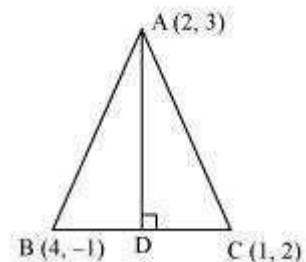
### Question 16:

In the triangle ABC with vertices A (2, 3), B (4, -1) and C (1, 2), find the equation and length of altitude from the vertex A.

### Answer 16:

Let AD be the altitude of triangle ABC from vertex A.

Accordingly,  $AD \perp BC$





The equation of the line passing through point (2, 3) and having a slope of 1 is

$$(y - 3) = 1(x - 2)$$

$$\Rightarrow x - y + 1 = 0$$

$$\Rightarrow y - x = 1$$

Therefore, equation of the altitude from vertex A =  $y - x = 1$ .

Length of AD = Length of the perpendicular from A (2, 3) to BC The equation of BC is

$$(y+1) = \frac{2+1}{1-4}(x-4)$$

$$\Rightarrow (y+1) = -1(x-4)$$

$$\Rightarrow y+1 = -x+4$$

$$\Rightarrow x+y-3=0 \quad \dots(1)$$

The perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$  from a point  $(x_1, y_1)$  is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line  $Ax + By + C = 0$ , we obtain  $A = 1$ ,  $B = 1$ , and  $C = -3$ .

$$\therefore \text{Length of AD} = \frac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}} \text{ units} = \frac{|2|}{\sqrt{2}} \text{ units} = \frac{2}{\sqrt{2}} \text{ units} = \sqrt{2} \text{ units}$$

Thus, the equation and the length of the altitude from vertex A are  $y - x = 1$  and  $\sqrt{2}$  units respectively.

### Question 17:

If  $p$  is the length of perpendicular from the origin to the line whose intercepts on the

axes are  $a$  and  $b$ , then show that:  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

### Answer 17:

It is known that the equation of a line whose intercepts on the axes are  $a$  and  $b$  is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\text{or } bx + ay = ab$$

$$\text{or } bx + ay - ab = 0 \quad \dots(1)$$

The perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$  from a point  $(x_1, y_1)$  is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

On comparing equation (1) to the general equation of line  $Ax + By + C = 0$ , we obtain  $A = b$ ,  $B = a$ , and  $C = -ab$ .

Therefore, if  $p$  is the length of the perpendicular from point  $(x_1, y_1) = (0, 0)$  to line (1), we obtain

$$p = \frac{|A(0) + B(0) - ab|}{\sqrt{b^2 + a^2}}$$

$$\Rightarrow p = \frac{|-ab|}{\sqrt{a^2 + b^2}}$$

On squaring both sides, we obtain

$$p^2 = \frac{(-ab)^2}{a^2 + b^2}$$

$$\Rightarrow p^2 (a^2 + b^2) = a^2 b^2$$

$$\Rightarrow \frac{a^2 + b^2}{a^2 b^2} = \frac{1}{p^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence, we showed that  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ .

# Mathematics

## (Chapter - 9) (Straight Lines)

(Class – XI)

### Miscellaneous Exercise on Chapter 9

#### Question 1:

Find the values of  $k$  for which the line  $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$  is

- (a) Parallel to the x-axis,
- (b) Parallel to the y-axis,
- (c) Passing through the origin.

#### Answer 1:

The given equation of line is

$$(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0 \dots (1)$$

(a) If the given line is parallel to the x-axis, then

Slope of the given line = Slope of the x-axis

The given line can be written as

$$(4-k^2)y = (k-3)x + k^2 - 7k + 6 = 0$$

$$y = \frac{(k-3)}{(4-k^2)}x + \frac{k^2 - 7k + 6}{(4-k^2)}, \text{ which is of the form } y = mx + c.$$

$$\text{Slope of the given line} = \frac{(k-3)}{(4-k^2)}$$

Slope of the x-axis = 0

$$\therefore \frac{(k-3)}{(4-k^2)} = 0$$

$$\Rightarrow k-3 = 0$$

$$\Rightarrow k = 3$$

Thus, if the given line is parallel to the x-axis, then the value of  $k$  is 3.

(b) If the given line is parallel to the y-axis, it is vertical. Hence, its slope will be undefined.

$$\text{The slope of the given line is } \frac{(k-3)}{(4-k^2)}.$$

Now,  $\frac{(k-3)}{(4-k^2)}$  is undefined at  $k^2 = 4$

$$k^2 = 4$$

$$\Rightarrow k = \pm 2$$

Thus, if the given line is parallel to the y-axis, then the value of  $k$  is  $\pm 2$ .

**(c)** If the given line is passing through the origin, then point  $(0, 0)$  satisfies the given equation of line.

$$(k-3)(0) - (4-k^2)(0) + k^2 - 7k + 6 = 0$$

$$k^2 - 7k + 6 = 0$$

$$k^2 - 6k - k + 6 = 0$$

$$(k-6)(k-1) = 0$$

$$k = 1 \text{ or } 6$$

Thus, if the given line is passing through the origin, then the value of  $k$  is either 1 or 6.

**Question 2:**

Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6, respectively.

**Answer 2:**

Let the intercepts cut by the given lines on the axes be  $a$  and  $b$ .

It is given that

$$a + b = 1 \dots\dots\dots (1)$$

$$ab = -6 \dots\dots\dots (2)$$

On solving equations (1) and (2), we obtain

$$a = 3 \text{ and } b = -2 \quad \text{or} \quad a = -2 \text{ and } b = 3$$

It is known that the equation of the line whose intercepts on the axes are  $a$  and  $b$  is

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ or } bx + ay - ab = 0$$

**Case I:**  $a = 3$  and  $b = -2$

In this case, the equation of the line is  $-2x + 3y + 6 = 0$ , i.e.,  $2x - 3y = 6$ .

**Case II:**  $a = -2$  and  $b = 3$

In this case, the equation of the line is  $3x - 2y + 6 = 0$ , i.e.,  $-3x + 2y = 6$ .

Thus, the required equation of the lines are  $2x - 3y = 6$  and  $-3x + 2y = 6$ .

**Question 3:**

What are the points on the  $y$ -axis whose distance from the line  $\frac{x}{3} + \frac{y}{4} = 1$  is 4 units.

**Answer 3:**

Let  $(0, b)$  be the point on the  $y$ -axis whose distance from line  $\frac{x}{3} + \frac{y}{4} = 1$  is 4 units.

The given line can be written as

$$4x + 3y - 12 = 0 \dots\dots\dots (1)$$

On comparing equation (1) to the general equation of line  $Ax + By + C = 0$ , we obtain

$$A = 4, B = 3, \text{ and } C = -12.$$

It is known that the perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$  from a point

$$(x_1, y_1) \text{ is given by } d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

Therefore, if  $(0, b)$  is the point on the  $y$ -axis whose distance from line  $\frac{x}{3} + \frac{y}{4} = 1$  is 4 units, then:

$$\begin{aligned} 4 &= \frac{|4(0) + 3(b) - 12|}{\sqrt{4^2 + 3^2}} \\ \Rightarrow 4 &= \frac{|3b - 12|}{5} \\ \Rightarrow 20 &= |3b - 12| \\ \Rightarrow 20 &= \pm(3b - 12) \\ \Rightarrow 20 &= (3b - 12) \text{ or } 20 = -(3b - 12) \\ \Rightarrow 3b &= 20 + 12 \text{ or } 3b = -20 + 12 \\ \Rightarrow b &= \frac{32}{3} \text{ or } b = -\frac{8}{3} \end{aligned}$$

Thus, the required points are  $\left(0, \frac{32}{3}\right)$  and  $\left(0, -\frac{8}{3}\right)$

**Question 4:**

Find perpendicular distance from the origin to the line joining the points  $(\cos \theta, \sin \theta)$  and  $(\cos \phi, \sin \phi)$ .

**Answer 4:**

The equation of the line joining the points  $(\cos \theta, \sin \theta)$  and  $(\cos \phi, \sin \phi)$  is given by

$$y - \sin \theta = \frac{\sin \phi - \sin \theta}{\cos \phi - \cos \theta} (x - \cos \theta)$$

$$y(\cos \phi - \cos \theta) - \sin \theta(\cos \phi - \cos \theta) = x(\sin \phi - \sin \theta) - \cos \theta(\sin \phi - \sin \theta)$$

$$x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \cos \theta \sin \phi - \cos \theta \sin \theta - \sin \theta \cos \phi + \sin \theta \cos \theta = 0$$

$$x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \sin(\phi - \theta) = 0$$

$$Ax + By + C = 0, \text{ where } A = \sin \theta - \sin \phi, B = \cos \phi - \cos \theta, \text{ and } C = \sin(\phi - \theta)$$

It is known that the perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$  from a point

$$(x_1, y_1) \text{ is given by } d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

Therefore, the perpendicular distance ( $d$ ) of the given line from point  $(x_1, y_1) = (0, 0)$  is

$$\begin{aligned} d &= \frac{|(\sin \theta - \sin \phi)(0) + (\cos \phi - \cos \theta)(0) + \sin(\phi - \theta)|}{\sqrt{(\sin \theta - \sin \phi)^2 + (\cos \phi - \cos \theta)^2}} \\ &= \frac{|\sin(\phi - \theta)|}{\sqrt{\sin^2 \theta + \sin^2 \phi - 2 \sin \theta \sin \phi + \cos^2 \phi + \cos^2 \theta - 2 \cos \phi \cos \theta}} \\ &= \frac{|\sin(\phi - \theta)|}{\sqrt{(\sin^2 \theta + \cos^2 \theta) + (\sin^2 \phi + \cos^2 \phi) - 2(\sin \theta \sin \phi + \cos \theta \cos \phi)}} \\ &= \frac{|\sin(\phi - \theta)|}{\sqrt{1 + 1 - 2(\cos(\phi - \theta))}} \\ &= \frac{|\sin(\phi - \theta)|}{\sqrt{2(1 - \cos(\phi - \theta))}} \\ &= \frac{|\sin(\phi - \theta)|}{\sqrt{2\left(2 \sin^2\left(\frac{\phi - \theta}{2}\right)\right)}} \\ &= \frac{|\sin(\phi - \theta)|}{2 \sin\left(\frac{\phi - \theta}{2}\right)} \end{aligned}$$

**Question 5:**

Find the equation of the line parallel to  $y$ -axis and drawn through the point of intersection of the lines  $x - 7y + 5 = 0$  and  $3x + y = 0$ .

**Answer 5:**

The equation of any line parallel to the  $y$ -axis is of the form

$$x = a \text{ ..... (1)}$$

The two given lines are

$$x - 7y + 5 = 0 \text{ .....(2)}$$

$$3x + y = 0 \text{ ..... (3)}$$

On solving equations (2) and (3), we obtain  $x = -\frac{5}{22}$  and  $y = \frac{15}{22}$  .

Therefore,  $\left(-\frac{5}{22}, \frac{15}{22}\right)$  is the point of intersection of lines (2) and (3).

Since line  $x = a$  passes through point  $\left(-\frac{5}{22}, \frac{15}{22}\right)$ . So,  $a = -\frac{5}{22}$

Thus, the required equation of the line is  $x = -\frac{5}{22}$

**Question 6:**

Find the equation of a line drawn perpendicular to the line  $\frac{x}{4} + \frac{y}{6} = 1$  through the point, where it meets the  $y$ -axis.

**Answer 6:**

The equation of the given line is  $\frac{x}{4} + \frac{y}{6} = 1$  .

This equation can also be written as  $3x + 2y - 12 = 0$

$y = \frac{-3}{2}x + 6$  , which is of the form  $y = mx + c$



∴ Slope of the  $= -\frac{3}{2}$  given line

∴ Slope of line perpendicular to the given line  $= -\frac{1}{\left(-\frac{3}{2}\right)} = \frac{2}{3}$

Let the given line intersect the  $y$ -axis at  $(0, y)$ .

On substituting  $x$  with 0 in the equation of the given line, we obtain  $\frac{y}{6} = 1 \Rightarrow y = 6$

∴ The given line intersects the  $y$ -axis at  $(0, 6)$ .

The equation of the line that has a slope of  $2/3$  and passes through point  $(0, 6)$  is

$$(y - 6) = \frac{2}{3}(x - 0)$$

$$3y - 18 = 2x$$

$$2x - 3y + 18 = 0$$

Thus, the required equation of the line is  $2x - 3y + 18 = 0$ .

### Question 7:

Find the area of the triangle formed by the lines  $y - x = 0$ ,  $x + y = 0$  and  $x - k = 0$ .

### Answer 7:

The equations of the given lines are

$$y - x = 0 \dots\dots\dots (1)$$

$$x + y = 0 \dots\dots\dots (2)$$

$$x - k = 0 \dots\dots\dots (3)$$

The point of intersection of lines (1) and (2) is given by

$$x = 0 \text{ and } y = 0$$

The point of intersection of lines (2) and (3) is given by

$$x = k \text{ and } y = -k$$

The point of intersection of lines (3) and (1) is given by

$$x = k \text{ and } y = k$$

Thus, the vertices of the triangle formed by the three given lines are  $(0, 0)$ ,  $(k, -k)$ , and  $(k, k)$ .

We know that the area of a triangle whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  is

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Therefore, area of the triangle formed by the three given lines

$$= \frac{1}{2} |0(-k - k) + k(k - 0) + k(0 + k)| \text{ square units}$$

$$= \frac{1}{2} |k^2 + k^2| \text{ square units}$$

$$= \frac{1}{2} |2k^2| \text{ square units}$$

$$= k^2 \text{ square units}$$

### Question 8:

Find the value of  $p$  so that the three lines  $3x + y - 2 = 0$ ,  $px + 2y - 3 = 0$  and  $2x - y - 3 = 0$  may intersect at one point.

### Answer 8:

The equations of the given lines are

$$3x + y - 2 = 0 \dots\dots\dots (1)$$

$$px + 2y - 3 = 0 \dots\dots\dots (2)$$

$$2x - y - 3 = 0 \dots\dots\dots (3)$$

On solving equations (1) and (3), we obtain

$$x = 1 \text{ and } y = -1$$

Since these three lines may intersect at one point, the point of intersection of lines (1) and (3) will also satisfy line (2).

$$p(1) + 2(-1) - 3 = 0$$

$$p - 2 - 3 = 0 \quad p = 5$$

Thus, the required value of  $p$  is 5.

### Question 9:

If three lines whose equations are  $y = m_1x + c_1$ ,  $y = m_2x + c_2$  and  $y = m_3x + c_3$  are

concurrent, then show that  $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$ .

**Answer 9:**

The equations of the given lines are

$$y = m_1x + c_1 \dots\dots\dots (1)$$

$$y = m_2x + c_2 \dots\dots\dots (2)$$

$$y = m_3x + c_3 \dots\dots\dots (3)$$

On subtracting equation (1) from (2), we obtain

$$0 = (m_2 - m_1)x + (c_2 - c_1)$$

$$\Rightarrow (m_1 - m_2)x = c_2 - c_1$$

$$\Rightarrow x = \frac{c_2 - c_1}{m_1 - m_2}$$

On substituting this value of x in (1), we obtain

$$y = m_1 \left( \frac{c_2 - c_1}{m_1 - m_2} \right) + c_1$$

$$y = \frac{m_1c_2 - m_1c_1}{m_1 - m_2} + c_1$$

$$y = \frac{m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1}{m_1 - m_2}$$

$$y = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

$\therefore \left( \frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2} \right)$  is the point of intersection of lines (1) and (2).

It is given that lines (1), (2), and (3) are concurrent. Hence, the point of intersection of lines (1) and (2) will also satisfy equation (3).

$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = m_3 \left( \frac{c_2 - c_1}{m_1 - m_2} \right) + c_3$$

$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = \frac{m_3c_2 - m_3c_1 + c_3m_1 - c_3m_2}{m_1 - m_2}$$

$$m_1c_2 - m_2c_1 - m_3c_2 + m_3c_1 - c_3m_1 + c_3m_2 = 0$$

$$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$

**Question 10:**

Find the equation of the lines through the point (3, 2) which make an angle of  $45^\circ$  with the line  $x - 2y = 3$ .

**Answer 10:**

Let the slope of the required line be  $m_1$ .

The given line can be written as  $y = \frac{1}{2}x - \frac{3}{2}$ , which is of the form  $y = mx + c$

$\therefore$  Slope of the given line =  $m_2 = \frac{1}{2}$

It is given that the angle between the required line and line  $x - 2y = 3$  is  $45^\circ$ .

We know that if  $\theta$  is the acute angle between lines  $l_1$  and  $l_2$  with slopes  $m_1$  and  $m_2$ , then

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

$$\therefore \tan 45^\circ = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow 1 = \left| \frac{\frac{1}{2} - m_1}{1 + \frac{m_1}{2}} \right|$$

$$\Rightarrow 1 = \left| \frac{\left( \frac{1 - 2m_1}{2} \right)}{\frac{2 + m_1}{2}} \right|$$

$$\Rightarrow 1 = \left| \frac{1 - 2m_1}{2 + m_1} \right|$$

$$\Rightarrow 1 = \pm \left( \frac{1 - 2m_1}{2 + m_1} \right)$$

$$\Rightarrow 1 = \frac{1 - 2m_1}{2 + m_1} \text{ or } 1 = - \left( \frac{1 - 2m_1}{2 + m_1} \right)$$

$$\Rightarrow 2 + m_1 = 1 - 2m_1 \text{ or } 2 + m_1 = -1 + 2m_1$$

$$\Rightarrow m_1 = -\frac{1}{3} \text{ or } m_1 = 3$$

**Case I:**  $m_1 = 3$

The equation of the line passing through (3, 2) and having a slope of 3 is:

$$y - 2 = 3(x - 3)$$

$$y - 2 = 3x - 9$$

$$3x - y = 7$$

**Case II:**  $m_1 = -\frac{1}{3}$

The equation of the line passing through (3, 2) and having a slope of  $-\frac{1}{3}$  is:

$$y - 2 = -\frac{1}{3}(x - 3)$$

$$3y - 6 = -x + 3$$

$$x + 3y = 9$$

Thus, the equations of the lines are  $3x - y = 7$  and  $x + 3y = 9$ .

**Question 11:**

Find the equation of the line passing through the point of intersection of the lines  $4x + 7y - 3 = 0$  and  $2x - 3y + 1 = 0$  that has equal intercepts on the axes.

**Answer 11:**

Let the equation of the line having equal intercepts on the axes be

$$\frac{x}{a} + \frac{y}{a} = 1$$

$$\text{Or } x + y = a \quad \dots(1)$$

On solving equations  $4x + 7y - 3 = 0$  and  $2x - 3y + 1 = 0$ , we obtain

$$x = \frac{1}{13} \text{ and } y = \frac{5}{13}$$

$\therefore \left(\frac{1}{13}, \frac{5}{13}\right)$  is the point of intersection of the two given lines.

Since equation (1) passes through  $\left(\frac{1}{13}, \frac{5}{13}\right)$  point,

$$\frac{1}{13} + \frac{5}{13} = a$$

$$\Rightarrow a = \frac{6}{13}$$

∴ Equation (1) becomes  $x + y = \frac{6}{13}$ , i.e.,  $13x + 13y = 6$

Thus, the required equation of the line is  $13x + 13y = 6$ .

### Question 12:

Show that the equation of the line passing through the origin and making an angle  $\theta$  with

the line  $y = mx + c$  is  $\frac{y}{x} = \frac{m \pm \tan \theta}{1 \mp m \tan \theta}$ .

### Answer 12:

Let the equation of the line passing through the origin be  $y = m_1x$ .

If this line makes an angle of  $\theta$  with line  $y = mx + c$ , then angle  $\theta$  is given by

$$\begin{aligned}\therefore \tan \theta &= \left| \frac{m_1 - m}{1 + m_1 m} \right| \\ \Rightarrow \tan \theta &= \left| \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right| \\ \Rightarrow \tan \theta &= \pm \left( \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right) \\ \Rightarrow \tan \theta &= \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \text{ or } \tan \theta = - \left( \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)\end{aligned}$$

Case I:

$$\begin{aligned}\tan \theta &= \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \\ \Rightarrow \tan \theta + \frac{y}{x} m \tan \theta &= \frac{y}{x} - m \\ \Rightarrow m + \tan \theta &= \frac{y}{x} (1 - m \tan \theta) \\ \Rightarrow \frac{y}{x} &= \frac{m + \tan \theta}{1 - m \tan \theta}\end{aligned}$$

**Case II:**

$$\tan \theta = - \left( \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)$$

$$\Rightarrow \tan \theta + \frac{y}{x} m \tan \theta = -\frac{y}{x} + m$$

$$\Rightarrow \frac{y}{x} (1 + m \tan \theta) = m - \tan \theta$$

$$\Rightarrow \frac{y}{x} = \frac{m - \tan \theta}{1 + m \tan \theta}$$

Therefore, the required line is given by  $\frac{y}{x} = \frac{m \pm \tan \theta}{1 \mp m \tan \theta}$  .

**Question 13:**

In what ratio, the line joining  $(-1, 1)$  and  $(5, 7)$  is divided by the line  $x + y = 4$ ?

**Answer 13:**

The equation of the line joining the points  $(-1, 1)$  and  $(5, 7)$  is given by

$$y - 1 = \frac{7 - 1}{5 - (-1)} (x + 1)$$

$$y - 1 = \frac{6}{6} (x + 1)$$

$$x - y + 2 = 0 \quad \dots(1)$$

The equation of the given line is

$$x + y - 4 = 0 \quad \dots\dots\dots (2)$$

The point of intersection of lines (1) and (2) is given by

$$x = 1 \text{ and } y = 3$$

Let point  $(1, 3)$  divide the line segment joining  $(-1, 1)$  and  $(5, 7)$  in the ratio  $1:k$ .

Accordingly, by section formula,

$$(1,3) = \left( \frac{k(-1)+1(5)}{1+k}, \frac{k(1)+1(7)}{1+k} \right)$$

$$\Rightarrow (1,3) = \left( \frac{-k+5}{1+k}, \frac{k+7}{1+k} \right)$$

$$\Rightarrow \frac{-k+5}{1+k} = 1, \frac{k+7}{1+k} = 3$$

$$\therefore \frac{-k+5}{1+k} = 1$$

$$\Rightarrow -k+5 = 1+k$$

$$\Rightarrow 2k = 4$$

$$\Rightarrow k = 2$$

Thus, the line joining the points  $(-1, 1)$  and  $(5, 7)$  is divided by line  $x + y = 4$  in the ratio 1:2.

#### Question 14:

Find the distance of the line  $4x + 7y + 5 = 0$  from the point  $(1, 2)$  along the line  $2x - y = 0$ .

#### Answer 14:

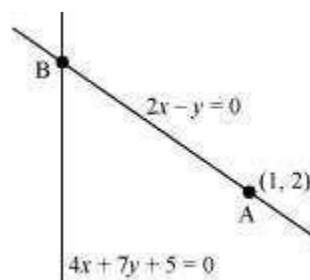
The given lines are

$$2x - y = 0 \dots\dots\dots (1)$$

$$4x + 7y + 5 = 0 \dots\dots\dots (2)$$

A  $(1, 2)$  is a point on line (1).

Let B be the point of intersection of lines (1) and (2).





On solving equations (1) and (2), we obtain  $x = \frac{-5}{18}$  and  $y = \frac{-5}{9}$

∴ Coordinates of point B are  $\left(\frac{-5}{18}, \frac{-5}{9}\right)$  .

By using distance formula, the distance between points A and B can be obtained as

$$\begin{aligned} AB &= \sqrt{\left(1 + \frac{5}{18}\right)^2 + \left(2 + \frac{5}{9}\right)^2} \text{ units} \\ &= \sqrt{\left(\frac{23}{18}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units} \\ &= \sqrt{\left(\frac{23}{2 \times 9}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units} \\ &= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{2}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units} \\ &= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{4} + 1\right)} \text{ units} \\ &= \frac{23}{9} \sqrt{\frac{5}{4}} \text{ units} \\ &= \frac{23}{9} \times \frac{\sqrt{5}}{2} \text{ units} \\ &= \frac{23\sqrt{5}}{18} \text{ units} \end{aligned}$$

Thus, the required distance is  $\frac{23\sqrt{5}}{18}$  units

### Question 15:

Find the direction in which a straight line must be drawn through the point  $(-1, 2)$  so that its point of intersection with the line  $x + y = 4$  may be at a distance of 3 units from this point.

**Answer 15:**

Let  $y = mx + c$  be the line through point  $(-1, 2)$ .

Accordingly,  $2 = m(-1) + c$ .

$$\Rightarrow 2 = -m + c$$

$$\Rightarrow c = m + 2$$

$$\therefore y = mx + m + 2 \dots\dots\dots (1)$$

The given line is

$$x + y = 4 \dots\dots\dots (2)$$

On solving equations (1) and (2), we obtain

$$x = \frac{2-m}{m+1} \text{ and } y = \frac{5m+2}{m+1}$$

$$\therefore \left( \frac{2-m}{m+1}, \frac{5m+2}{m+1} \right) \text{ is the point of intersection of lines (1) and (2).}$$

Since this point is at a distance of 3 units from point  $(-1, 2)$ , according to distance formula,

$$\begin{aligned} \sqrt{\left(\frac{2-m}{m+1} + 1\right)^2 + \left(\frac{5m+2}{m+1} - 2\right)^2} &= 3 \\ \Rightarrow \left(\frac{2-m+m+1}{m+1}\right)^2 + \left(\frac{5m+2-2m-2}{m+1}\right)^2 &= 3^2 \\ \Rightarrow \frac{9}{(m+1)^2} + \frac{9m^2}{(m+1)^2} &= 9 \\ \Rightarrow \frac{1+m^2}{(m+1)^2} &= 1 \\ \Rightarrow 1+m^2 &= m^2 + 1 + 2m \\ \Rightarrow 2m &= 0 \\ \Rightarrow m &= 0 \end{aligned}$$

Thus, the slope of the required line must be zero i.e., the line must be parallel to the  $x$ -axis.

**Question 16:**

The hypotenuse of a right angled triangle has its ends at the points (1, 3) and (-4, 1). Find an equation of the legs (perpendicular sides) of the triangle which are parallel to the axes.

**Answer:**

Let ABC be the right angles triangle, where  $\angle C = 90^\circ$

There are infinity many such lines.

Let m be the slope of AC.

$$\therefore \text{Slope of BC} = \frac{1}{m}$$

$$\text{Equation of AC : } y - 3 = m(x - 1)$$

$$\Rightarrow x - 1 = \frac{1}{m}(y - 3)$$

$$\text{Equation of BC : } y - 1 = -\frac{1}{m}(x + 4)$$

$$\Rightarrow x + 4 = -m(y - 1)$$

For a given value of m, we can get these equations

$$\text{For } m = 0, y - 3 = 0; x + 4 = 0$$

$$\text{For } m \rightarrow \infty, x - 1 = 0; y - 1 = 0$$

**Question 17:**

Find the image of the point (3, 8) with respect to the line  $x + 3y = 7$  assuming the line to be a plane mirror.

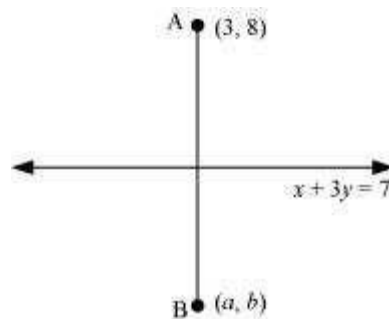
**Answer 17:**

The equation of the given line is  $x$

$$+ 3y = 7 \dots (1)$$

Let point B ( $a$ ,  $b$ ) be the image of point A (3, 8).

Accordingly, line (1) is the perpendicular bisector of AB.



$$\text{Slope of AB} = \frac{b-8}{a-3}, \text{ while the slope of line (1)} = -\frac{1}{3}$$

Since line (1) is perpendicular to AB,

$$\left(\frac{b-8}{a-3}\right) \times \left(-\frac{1}{3}\right) = -1$$

$$\Rightarrow \frac{b-8}{3a-9} = 1$$

$$\Rightarrow b-8 = 3a-9$$

$$\Rightarrow 3a-b = 1 \dots (2)$$

$$\text{Mid-point of AB} = \left(\frac{a+3}{2}, \frac{b+8}{2}\right)$$

The mid-point of line segment AB will also satisfy line (1).

Hence, from equation (1), we have

$$\left(\frac{a+3}{2}\right) + 3\left(\frac{b+8}{2}\right) = 7$$

$$\Rightarrow a+3+3b+24 = 14$$

$$\Rightarrow a+3b = -13 \dots (3)$$

On solving equations (2) and (3), we obtain  $a = -1$  and  $b = -4$ .

Thus, the image of the given point with respect to the given line is  $(-1, -4)$ .

**Question 18:**

If the lines  $y = 3x + 1$  and  $2y = x + 3$  are equally inclined to the line  $y = mx + 4$ , find the value of  $m$ .

**Answer 18:**

The equations of the given lines are

$$y = 3x + 1 \dots\dots\dots (1)$$

$$2y = x + 3 \dots\dots\dots (2)$$

$$y = mx + 4 \dots\dots\dots (3)$$

Slope of line (1),  $m_1 = 3$

Slope of line (2),  $m_2 = 1/2$

Slope of line (3),  $m_3 = m$

It is given that lines (1) and (2) are equally inclined to line (3). This means that the angle between lines (1) and (3) equals the angle between lines (2) and (3).

$$\begin{aligned}\therefore \left| \frac{m_1 - m_3}{1 + m_1 m_3} \right| &= \left| \frac{m_2 - m_3}{1 + m_2 m_3} \right| \\ \Rightarrow \left| \frac{3 - m}{1 + 3m} \right| &= \left| \frac{\frac{1}{2} - m}{1 + \frac{1}{2}m} \right| \\ \Rightarrow \left| \frac{3 - m}{1 + 3m} \right| &= \left| \frac{1 - 2m}{m + 2} \right| \\ \Rightarrow \frac{3 - m}{1 + 3m} &= \pm \left( \frac{1 - 2m}{m + 2} \right)\end{aligned}$$

$$\Rightarrow \frac{3-m}{1+3m} = \frac{1-2m}{m+2} \text{ or } \frac{3-m}{1+3m} = -\left(\frac{1-2m}{m+2}\right)$$

$$\text{If } \frac{3-m}{1+3m} = \frac{1-2m}{m+2}, \text{ then}$$

$$(3-m)(m+2) = (1-2m)(1+3m)$$

$$\Rightarrow -m^2 + m + 6 = 1 + m - 6m^2$$

$$\Rightarrow 5m^2 + 5 = 0$$

$$\Rightarrow (m^2 + 1) = 0$$

$$\Rightarrow m = \sqrt{-1}, \text{ which is not real}$$

Hence, this case is not possible.

$$\text{If } \frac{3-m}{1+3m} = -\left(\frac{1-2m}{m+2}\right), \text{ then}$$

$$\Rightarrow (3-m)(m+2) = -(1-2m)(1+3m)$$

$$\Rightarrow -m^2 + m + 6 = -(1 + m - 6m^2)$$

$$\Rightarrow 7m^2 - 2m - 7 = 0$$

$$\Rightarrow m = \frac{2 \pm \sqrt{4 - 4(7)(-7)}}{2(7)}$$

$$\Rightarrow m = \frac{2 \pm 2\sqrt{1+49}}{14}$$

$$\Rightarrow m = \frac{1 \pm 5\sqrt{2}}{7}$$

Thus, the required value of  $m$  is  $\frac{1 \pm 5\sqrt{2}}{7}$

### Question 19:

If sum of the perpendicular distances of a variable point P ( $x, y$ ) from the lines  $x + y - 5 = 0$  and  $3x - 2y + 7 = 0$  is always 10. Show that P must move on a line.

**Answer 19:**

The equations of the given lines are  $x$

$$+ y - 5 = 0 \dots (1)$$

$$3x - 2y + 7 = 0 \dots (2)$$

The perpendicular distances of P ( $x, y$ ) from lines (1) and (2) are respectively given by

$$d_1 = \frac{|x+y-5|}{\sqrt{(1)^2+(1)^2}} \text{ and } d_2 = \frac{|3x-2y+7|}{\sqrt{(3)^2+(-2)^2}}$$

$$\text{i.e., } d_1 = \frac{|x+y-5|}{\sqrt{2}} \text{ and } d_2 = \frac{|3x-2y+7|}{\sqrt{13}}$$

It is given that  $d_1 + d_2 = 10$

$$\therefore \frac{|x+y-5|}{\sqrt{2}} + \frac{|3x-2y+7|}{\sqrt{13}} = 10$$

$$\Rightarrow \sqrt{13}|x+y-5| + \sqrt{2}|3x-2y+7| - 10\sqrt{26} = 0$$

$$\Rightarrow \sqrt{13}(x+y-5) + \sqrt{2}(3x-2y+7) - 10\sqrt{26} = 0$$

[ Assuming  $(x+y-5)$  and  $(3x-2y+7)$  are positive ]

$$\Rightarrow \sqrt{13}x + \sqrt{13}y - 5\sqrt{13} + 3\sqrt{2}x - 2\sqrt{2}y + 7\sqrt{2} - 10\sqrt{26} = 0$$

$$\Rightarrow x(\sqrt{13} + 3\sqrt{2}) + y(\sqrt{13} - 2\sqrt{2}) + (7\sqrt{2} - 5\sqrt{13} - 10\sqrt{26}) = 0$$

which is the equation of a line.

Similarly, we can obtain the equation of line for any signs of  $(x+y-5)$  and  $(3x-2y+7)$

Thus, point P must move on a line.

**Question 20:**

Find equation of the line which is equidistant from parallel lines  $9x + 6y - 7 = 0$  and  $3x + 2y + 6 = 0$ .

**Answer 20:**

The equations of the given lines are

$$9x + 6y - 7 = 0 \dots (1)$$

$$3x + 2y + 6 = 0 \dots (2)$$

Let P (h, k) be the arbitrary point that is equidistant from lines (1) and (2). The perpendicular distance of P (h, k) from line (1) is given by

$$d_1 = \frac{|9h+6k-7|}{(9)^2+(6)^2} = \frac{|9h+6k-7|}{\sqrt{117}} = \frac{|9h+6k-7|}{3\sqrt{13}}$$

The perpendicular distance of P (h, k) from line (2) is given by

$$d_2 = \frac{|3h+2k+6|}{\sqrt{(3)^2+(2)^2}} = \frac{|3h+2k+6|}{\sqrt{13}}$$

Since P (h, k) is equidistant from lines (1) and (2),  $d_1 = d_2$

$$\begin{aligned} \therefore \frac{|9h+6k-7|}{3\sqrt{13}} &= \frac{|3h+2k+6|}{\sqrt{13}} \\ \Rightarrow |9h+6k-7| &= 3|3h+2k+6| \\ \Rightarrow |9h+6k-7| &= \pm 3(3h+2k+6) \\ \Rightarrow 9h+6k-7 &= 3(3h+2k+6) \text{ or } 9h+6k-7 = -3(3h+2k+6) \end{aligned}$$

The case  $9h+6k-7 = 3(3h+2k+6)$  is not possible as

$$9h+6k-7 = 3(3h+2k+6) \Rightarrow -7 = 18 \text{ (which is absurd)}$$

$$\therefore 9h+6k-7 = -3(3h+2k+6)$$

$$9h + 6k - 7 = -9h - 6k - 18$$

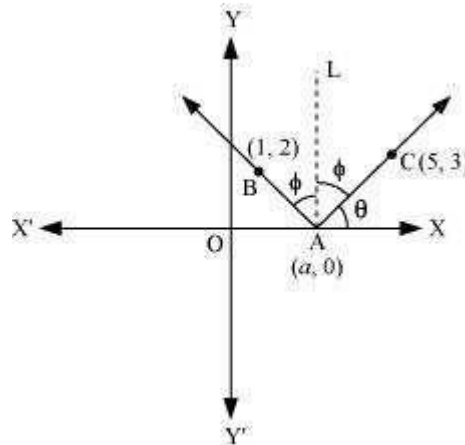
$$\Rightarrow 18h + 12k + 11 = 0$$

Thus, the required equation of the line is  $18x + 12y + 11 = 0$ .



**Question 21:**

A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

**Answer 21:**

Let the coordinates of point A be  $(a, 0)$ .

Draw a line (AL) perpendicular to the x-axis.

We know that angle of incidence is equal to angle of reflection. Hence, let

$$\angle BAL = \angle CAL = \phi$$

$$\text{Let } \angle CAX = \theta$$

$$\therefore \angle OAB = 180^\circ - (\theta + 2\phi) = 180^\circ - [\theta + 2(90^\circ - \theta)]$$

$$= 180^\circ - \theta - 180^\circ + 2\theta = \theta$$

$$\therefore \angle BAX = 180^\circ - \theta$$

$$\text{Now, slope of line AC} = \frac{3-0}{5-a}$$

$$\Rightarrow \tan \theta = \frac{3}{5-a} \quad \dots(1)$$

$$\text{Slope of line AB} = \frac{2-0}{1-a}$$

$$\Rightarrow \tan(180^\circ - \theta) = \frac{2}{1-a}$$

$$\Rightarrow -\tan \theta = \frac{2}{1-a}$$

$$\Rightarrow \tan \theta = \frac{2}{a-1} \quad \dots(2)$$

From equations (1) and (2), we obtain

$$\begin{aligned}\frac{3}{5-a} &= \frac{2}{a-1} \\ \Rightarrow 3a-3 &= 10-2a \\ \Rightarrow a &= \frac{13}{5}\end{aligned}$$

Thus, the coordinates of point A are  $\left(\frac{13}{5}, 0\right)$ .

### Question 22:

Prove that the product of the lengths of the perpendiculars drawn from the points

$$\left(\sqrt{a^2-b^2}, 0\right) \text{ and } \left(-\sqrt{a^2-b^2}, 0\right) \text{ to the line } \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \text{ is } b^2.$$

### Answer 22:

The equation of the given line is

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

$$\text{Or, } bx \cos \theta + ay \sin \theta - ab = 0 \quad \dots(1)$$

Length of the perpendicular from point  $\left(\sqrt{a^2-b^2}, 0\right)$  to line (1) is

$$p_1 = \frac{\left| b \cos \theta \left( \sqrt{a^2-b^2} \right) + a \sin \theta (0) - ab \right|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} = \frac{\left| b \cos \theta \sqrt{a^2-b^2} - ab \right|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \quad \dots(2)$$

Length of the perpendicular from point  $\left(-\sqrt{a^2-b^2}, 0\right)$  to line (2) is

$$p_2 = \frac{\left| b \cos \theta \left( -\sqrt{a^2-b^2} \right) + a \sin \theta (0) - ab \right|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} = \frac{\left| b \cos \theta \sqrt{a^2-b^2} + ab \right|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \quad \dots(3)$$

On multiplying equations (2) and (3), we obtain

$$\begin{aligned}
P_1 P_2 &= \frac{\left| b \cos \theta \sqrt{a^2 - b^2} - ab \right| \left| b \cos \theta \sqrt{a^2 - b^2} + ab \right|}{\left( \sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta} \right)^2} \\
&= \frac{\left| b \cos \theta \sqrt{a^2 - b^2} - ab \right| \left| b \cos \theta \sqrt{a^2 - b^2} + ab \right|}{\left( b^2 \cos^2 \theta + a^2 \sin^2 \theta \right)} \\
&= \frac{\left| \left( b \cos \theta \sqrt{a^2 - b^2} \right)^2 - (ab)^2 \right|}{\left( b^2 \cos^2 \theta + a^2 \sin^2 \theta \right)} \\
&= \frac{\left| b^2 \cos^2 \theta (a^2 - b^2) - a^2 b^2 \right|}{\left( b^2 \cos^2 \theta + a^2 \sin^2 \theta \right)} \\
&= \frac{\left| a^2 b^2 \cos^2 \theta - b^4 \cos^2 \theta - a^2 b^2 \right|}{b^2 \cos^2 \theta + a^2 \sin^2 \theta} \\
&= \frac{b^2 \left| a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2 \right|}{b^2 \cos^2 \theta + a^2 \sin^2 \theta} \\
&= \frac{b^2 \left| a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2 \sin^2 \theta - a^2 \cos^2 \theta \right|}{b^2 \cos^2 \theta + a^2 \sin^2 \theta} \quad \left[ \sin^2 \theta + \cos^2 \theta = 1 \right] \\
&= \frac{b^2 \left| - \left( b^2 \cos^2 \theta + a^2 \sin^2 \theta \right) \right|}{b^2 \cos^2 \theta + a^2 \sin^2 \theta} \\
&= \frac{b^2 \left( b^2 \cos^2 \theta + a^2 \sin^2 \theta \right)}{\left( b^2 \cos^2 \theta + a^2 \sin^2 \theta \right)} \\
&= b^2
\end{aligned}$$

Hence, proved.

### Question 23:

A person standing at the junction (crossing) of two straight paths represented by the equations  $2x - 3y + 4 = 0$  and  $3x + 4y - 5 = 0$  wants to reach the path whose equation is  $6x - 7y + 8 = 0$  in the least time. Find equation of the path that he should follow.

### Answer 23:

The equations of the given lines are

$$2x - 3y + 4 = 0 \dots\dots\dots (1)$$

$$3x + 4y - 5 = 0 \dots\dots\dots (2)$$

$$6x - 7y + 8 = 0 \dots\dots\dots (3)$$

The person is standing at the junction of the paths represented by lines (1) and (2).

On solving equations (1) and (2), we obtain  $x = -\frac{1}{17}$  and  $y = \frac{22}{17}$ .

Thus, the person is standing at point  $\left(-\frac{1}{17}, \frac{22}{17}\right)$

The person can reach path (3) in the least time if he walks along the perpendicular line to (3) from point  $\left(-\frac{1}{17}, \frac{22}{17}\right)$

$$\text{Slope of the line (3)} = \frac{6}{7}$$

$$\therefore \text{Slope of the line perpendicular to line (3)} = -\frac{1}{\left(\frac{6}{7}\right)} = -\frac{7}{6}$$

The equation of the line passing through  $\left(-\frac{1}{17}, \frac{22}{17}\right)$  and having a slope of  $-\frac{7}{6}$  is given by

$$\left(y - \frac{22}{17}\right) = -\frac{7}{6}\left(x + \frac{1}{17}\right)$$

$$6(17y - 22) = -7(17x + 1)$$

$$102y - 132 = -119x - 7$$

$$119x + 102y = 125$$

Hence, the path that the person should follow is  $119x + 102y = 125$  .