13

## Measure of Dispersion

### 13.01 Introduction :

In previous class, we studied about central tendency (mean, mode, median) in the measure of central tendency which is not sufficient. There may be error in the use of measures and results are not correct. A measure of central tendency gives us a rough idea where data points are centralised. But, in order to make better interpretation from the data, we should also have an idea how the data are scattered or how much they are bunched around a measure of central tendency.

For example, the marks obtained in mathematics of three groups of students is given in following table :

| | | | | | | |
|---|---|---|---|---|---|---|
| Group A : | 10 | 10 | 10 | 10 | 10 | 10 |
| Group B : | 9 | 10 | 11 | 8 | 10 | 12 |
| Group C : | 0 | 5 | 10 | 10 | 16 | 19 |

The mean of each group is 10 but there is a large difference between the marks obtained in these groups. In groups A all marks are same and there is no dispersion from mean. Hence, the mean is representing the group. In second group B the marks are different and the deviation lies between -2 to +2. Here means doesn't represents all the data of group appropriate. The deviation from marks of group C lies between -10 to +9 which is large enough hence mean doesn't represent the marks of group C. Hence, to find the detailed knowledge about the series central value is not sufficient but construction and formation of average difference series from central values of various variable is also necessary. In this chapter, we shall learn some of the important measures of dispersion and their methods of calculation for ungrouped and grouped data.

### 13.02 Dispersion :

The dispersion is the mean of terms of scattered dispension of a series. The dispersion of the series is the measure of variation of its different terms or difference of the measure.

**Measures of dispersion :** There are two types of measures -

(I) Absolute measure

(II) Relative measure

The limit of dispersion or number of quantity is absolute.

The measures of dispersion which are expressed in terms of original units of a data are termed as Absolute measures. These measures are not appropiate to compare two or more than two series. Whereas relative measures of dispersion, also known as coefficients of dispersion are obtained as ratio or percentages. These are pure numbers independent of the units of measurement and used to compare two or more sets of data values. The ratio of relative measure, absolute measure and mean which can find dispersion. These type of ratio in percentage.

There are following measures of dispersion :

1. Range
2. Quartile deviation

3. Mean deviation

4. Standard deviation and variance

## 13.03 Range :

(A) Range based on extereme values :

The difference of highest value (H) and lowest value (L) of variable in series is called as range. For example, if in a class the maximum height of a student is 72 inch and the minimum height of any one student is 58 inch, so the range of height of student of the class 72 - 58 = 14 inch.

If variables are in continuous series then the difference of highest and lowest class is called as range.

**Example 1 :** If a perticular data is divided into a class intervals as- $\mathbf{5-10,\ 10-15,\ 15-20,\ 20-25,\ 25-30,\ 30-35}$ then the range of the data $\mathbf{35-5=30}$

Range is the absolute measure of dispersion, which depends which on the extreme value of the in data but if the comparison is to be done for two different data then this method is not useful. For example if one number of terms are more than another series, then range of one is greater then other. But it may be possible that variable values of lesser range series are unequal and greater range series variables are value equally scattered.

**Example 2 :** Let us observe the two different series of data :

Series A – **2, 3, 5, 8, 11, 25, 27**

Series B – **4, 8, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56**

There are 7 observation in series A and 13 observation in series B. Range of the series are 25 and 52. We see that in series A the data is more variable still the range is less but in series B there is less variability in the data and range is more. Thus to get more accurate result we use the coefficient of range.

Coefficient of range : $(\text{C.R.}) = (H-L)/(H+L)$

For example 1 Coefficient of range :

$$= \frac{H-L}{H+L} = \frac{35-5}{35+5} = \frac{30}{40} = .75 \text{ or } 75\%$$

**Example 3 :** Calculate the range and Coefficient of range of the following frequency distribution :

| Class | **10–20** | **20–30** | **30–40** | **40–50** | **50–60** |
|---|---|---|---|---|---|
| Frequency | **6** | **13** | **10** | **5** | **3** |

**Solution :** Lower class is 10 - 20 and its lower limit is 10

Lower limit (L) = 10

Upper class is 50 - 60 and its upper limit is 60

∴ Upper limit (H) = 60

Range (R) = H–L = 60–10 =50

$$\text{coefficient of range (C.R)} = \frac{H-L}{H+L} = \frac{60-10}{60+10} = 0.714 \quad \text{or} \quad 71.4\%$$

**(B) Inter–Quartile Range :** The difference of third and first quartile of variable series is called inter quartile range. This is more effective measure than range. This is also called as range is of middle values 50 % of distribution. This can be calculated using the following formula :

Inter Quartile Range = $Q_3$ - $Q_1$

$$\text{Inter Quartile Cofficient of Range} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Where $Q_3$ and $Q_1$ are calculated as -

$$Q_1 = \ell + \frac{(N/4) - F}{f} \times h, \quad Q_3 = \ell + \frac{3(N/4) - F}{f} \times h$$

Where $\ell$ is lower limit of quartile class, h is size of the interval, $F$ is frequency of the class preceding the quartile class. $f$ is frequency of the quartile class and N sum of all frequencies.

**(C) Decile and percentile range :** If data is ordered and divided into 10 parts, then cut values are called deciles and range can be derived with the help of decile with the given formula

Decile Range = $D_9 - D_1$

where $i$ th decile $D_i = l + \frac{(i(N/10) - F)}{f} \times h, \quad i = 1, 2, 3, ..., 9$

The percentile range is based an 80 % of middle terms of the variable series and it is difference of 90th and 10th percentile. This is called percentiles range. its calculated as follows -

Percentiles range $= P_{90} - P_{10}$

where $i$ th Percentile $P_i = l + \frac{(i(N/100) - F)}{f} \times h, \quad i = 1, 2, 3, \ldots, 99$

**Example 4 :** Calculate the Inter-Quartile Range, Quartile Range, Decile, and Percentile Range of the following frequency distribution :

| Class | 0–10 | 10–20 | 20–30 | 30–40 | 40–50 |
|---|---|---|---|---|---|
| Frequency | 1 | 8 | 12 | 6 | 3 |

**Solution :** The cummulative frequency table is as follows :

| Class interval | Frequency | Cummulative frequency |
|---|---|---|
| 0–10 | 1 | 1 |
| 10–20 | 8 | 9 |
| 20–30 | 12 | 21 |
| 30–40 | 6 | 27 |
| 40–50 | 3 | 30 |

Calculation of Inter-Quartile Range : :

$$Q_3 = \ell + \frac{3(N/4) - F}{f} \times h = 30 + \frac{3 \times (30/4) - 21}{6} \times 10 = 30 + \frac{1 \cdot 5 \times 10}{6} = 30 + 2 \cdot 5 = 32 \cdot 5$$

$$Q_1 = \ell + \frac{(N/4) - F}{f} \times h = 10 + \frac{(30/4) - 1}{8} \times 10 = 10 + \frac{6 \cdot 5 \times 10}{8} = 10 + 8 \cdot 12 = 18 \cdot 12$$

Inter-Quartile Range $= Q_3 - Q_1 = 32 \cdot 5 - 18 \cdot 12 = 14 \cdot 38$

Coefficient of Inter-Quartile Range $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{32.5 - 18 \cdot 12}{32.5 + 18 \cdot 12} = \frac{14 \cdot 38}{50 \cdot 62} = 0 \cdot 284$

Decile Range :

$$\frac{9N}{10} = \frac{9\times 30}{10} = 27$$

C.F. greater than or equal to 27 is 27 whose corresponding class is 30 - 40

$\therefore \quad l = 30,\ f = 6,\ F = 21,\ h = 10$

$$\therefore \quad D_9 = 30 + \frac{9\times(30/10) - 21}{6}\times 10 = 30 + \frac{6\times 10}{6} = 30 + 10 = 40$$

$$\therefore \quad \frac{1N}{10} = \frac{1\times 30}{10} = 3$$

C.F. greater than or equal to 3 is 9 whose coorresponding class is 10 - 20

$\therefore \quad l = 10,\ f = 8,\ F = 1,\ h = 10$

$$\therefore \quad D_1 = 10 + \frac{1\times(30/10) - 1}{8}\times 10 = 10 + \frac{2\times 10}{8} = 10 + 2\cdot 5 = 12\cdot 5$$

$\therefore$ Decile Range $= D_9 - D_1 = 40 - 12\cdot 5 = 27\cdot 5$

Percentile Range :

$$\frac{90N}{100} = \frac{90\times 30}{100} = 27$$

C.F. greater than or equal to 27 is 27 whose coorresponding class is 30 - 40

$\therefore \quad l = 30,\ f = 6,\ F = 21,\ h = 10$

$$\therefore \quad P_{90} = 30 + \frac{90\times(30/100) - 21}{6}\times 10 = 30 + \frac{6\times 10}{6} = 30 + 10 = 40$$

$$\frac{10N}{100} = \frac{10\times 30}{100} = 3$$

C.F. greater than or equal to 3 is 9 whose coorresponding class is 10 - 20

$\therefore \quad l = 10, f = 8, F = 1, h = 10$

$$\therefore \quad P_{10} = 10 + \frac{10\times(30/100) - 1}{8}\times 10 = 10 + \frac{2\times 10}{8} = 10 + 2\cdot 5 = 12\cdot 5$$

$\therefore$ Percentile Range $= P_{90} - P_{10} = 40 - 12\cdot 5 = 27\cdot 5$

## 13.04 Quartile deviation :

The half of difference between 3rd quartile $Q_3$ and 1st quartile $Q_1$ is called Quartile deviation or Semi–inter quartile range. This can be done by th following formula :

Quartile deviation $(Q.D.) = \dfrac{Q_3 - Q_1}{2}$

**Coefficient of quartile deviation :** Quartile deviation is an absolute measure of dispersion. To compare two or more variable series the coefficient of Quartile deviation is used for calculation. Its formula is given below :

Coefficient of quartile deviation $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

**Example 5 :** The heights in inches of 31 students are given below : **55, 56, 57, 57, 58, 58, 59, 59, 60, 60, 60, 61, 62, 62, 62, 63, 63, 63, 64, 64, 65, 65, 65, 66, 66, 66, 67, 68, 68, 69, 70** find the quartile deviation.

**Solution :** Total observations are 31

$$Q_1 = \frac{31+1}{4} = 8, \text{ value of 8th observation} = 59$$

and $$Q_3 = \frac{3(31+1)}{4} = 24, \therefore \text{ value of 8th observation} = 66$$

$$\therefore \text{ quartile deviation } = \frac{Q_3 - Q_1}{2} = \frac{66-59}{2} = \frac{7}{2} = 3.5$$

To find the quartile deviation in a discontinuous frequency distribution an example is given below.

**Example 6 :** Calculate the quartile deviation and coefficient of quartile deviation of the following frequency distribution table :

| Weight (kg) | 32 | 35 | 38 | 43 | 50 | 56 | 60 |
|---|---|---|---|---|---|---|---|
| frequency | 2 | 4 | 8 | 9 | 4 | 3 | 1 |

**Solution :** The following table is formed

| Weight(kg) | frequency | C.F. |
|---|---|---|
| 32 | 2 | 2 |
| 35 | 4 | 6 |
| 38 | 8 | 14 |
| 43 | 9 | 23 |
| 50 | 4 | 27 |
| 56 | 3 | 30 |
| 60 | 1 | 31 |

$$Q_1 = \frac{(31+1)}{4} = 8, \text{ 8th term is} = 38 \text{ kg}$$

$$Q_3 = \frac{3(31+1)}{4} = 24, \text{ 24th term is} = 50 \text{ kg}$$

$$\text{quartile deviation } = \frac{Q_3 - Q_1}{2} = \frac{50-38}{2} = \frac{12}{2} = 6$$

$$\text{Coefficient of quartile deviation } = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{50-38}{50+38} = \frac{12}{88} = 0 \cdot 136 \text{ या } 13 \cdot 6\%$$

The method to find the quartile deviation in continuous series is same that as of general series $Q_1$ and $Q_3$ are can be found as :

$$Q_1 = \ell + \frac{(N/4) - F}{f} \times h \quad \text{and} \quad Q_3 = \ell + \frac{(3N/4) - F}{f} \times h$$

where $\ell$ is lower limit of quartile class, $h$ is the size of the class interval, , $F$ is frequency of the class preceding the quartile class, $f$ is frequency of the quartile class and $N$ is sum of all frequencies.

**Example 7 :** Calculate the Quatile deviation and Coefficient of quartile deviation of the following frequency distribution table : :

| wages (Rs) | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 | 80–90 | 90–100 |
|---|---|---|---|---|---|---|---|
| frequency | 11 | 26 | 63 | 81 | 35 | 21 | 13 |

**Solution :** This can be shown through the following C.F. table :

| wages (Rs) | frequency | C.F. |
|---|---|---|
| 30–40 | 11 | 11 |
| 40–50 | 26 | 37 |
| 50–60 | 63 | 100 |
| 60–70 | 81 | 181 |
| 70–80 | 35 | 216 |
| 80–90 | 21 | 237 |
| 90–100 | 13 | 250 |

$$\frac{N+1}{4} = \frac{(250+1)}{4} = \frac{251}{4} = 62\cdot75 \text{ value}$$

$$Q_1 = 50 + \frac{62.75-37}{63} \times 10 = 50 + \frac{25\cdot75}{63} \times 10 = 54\cdot09$$

$$\frac{3(N+1)}{4} = \frac{3(250+1)}{4} = \frac{753}{4} = 188\cdot25 \text{ value}$$

$$Q_3 = 70 + \frac{188\cdot25-181}{35} \times 10 = 70 + \frac{7\cdot25}{35} \times 10 = 72\cdot07$$

Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{72\cdot07 - 54\cdot09}{2} = 8\cdot99$

Coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{72\cdot07 - 54\cdot09}{72\cdot07 + 54\cdot09} = \frac{17\cdot98}{126\cdot16} = \cdot14$

## Exercise 13.1

1. Write the formula for Coefficient of quartile deviation.
2. For any data if $Q_1 = 61$ and $Q_3 = 121$ then calculate its Quartile deviation.
3. Calculate its Quartile deviation and Coefficient of quartile deviation for the below given data :
   3, 8, 11, 13, 17, 19, 20, 22, 23, 27, 31
4. Calculate the range and coeffiecient of range for the following distribution table :

| $x$ | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 | 11.5 |
|---|---|---|---|---|---|---|---|---|
| $f$ | 4 | 5 | 6 | 3 | 2 | 1 | 3 | 5 |

5. Calculate the inter quartile range and coeffiecient of range for the following distribution table :

| $x$ | 1 | 3 | 5 | 7 |
|---|---|---|---|---|
| $f$ | 10 | 15 | 3 | 2 |

6. Calculate the coeffiecient of range for the following distribution table :

| size | 10 – 15 | 15 – 20 | 20 – 25 | 25 – 30 |
|---|---|---|---|---|
| frequency | 2 | 4 | 6 | 8 |

7. Calculate the decile range and percentile range for the following distribution table :

| $x$ | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 |
|---|---|---|---|---|---|---|---|---|
| $f$ | 3 | 9 | 8 | 5 | 7 | 5 | 7 | 6 |

8. Calculate the decile range and percentile range for the following distribution table :

| Marks $x$ | 0 – 10 | 10–20 | 20 –30 | 30–40 | 40–50 |
|---|---|---|---|---|---|
| Frequency $f$ | 5 | 8 | 20 | 14 | 3 |

## 13.05 Mean deviation :

It is the average of the absolute values of the deviation from the standard mean (mean, median or mode).

**(i) For Raw (ungrouped) data :** If the values of any variable are $x_1, x_2, x_3, ....., x_n$ and their numerical mean is A then,

$$\text{mean deviation} = \frac{1}{n}\sum_{i=1}^{n} |x_i - A|, \quad (1)$$

here $|x_i - A|$, $x_i$ $|x_i - A|$ is always positive

we write $\sum_{i=1}^{n}$ as $\sum$

**(ii) For Discrete frequency distribution :** For the data $x_1, x_2, x_3, ...., x_n$ if the frequency is $f_1, f_2, f_3, ...., x_n$ and let A be the statical mean

$$\text{Mean deviation of } A = \frac{1}{N}\Sigma f_i |x_i - A|, \quad \text{where } N = \Sigma f_i . \quad (2)$$

**(iii) For Discrete frequency distribution :**

The above given formula (2) is used, here $x_1, x_2, x_3, ...., x_n$ denotes the class mark of the given intervals.

**(iv) Coefficient of mean deviation :**

$$= \frac{\text{Mean Deviation}}{A}$$

**Note :**

(1) Mean deviation taken from dedian is always less then deviation takenfrom other measures.

(2) For convenience we write Mean deviation from average as Mean deviation only.

(3) Coefficient of mean deviation is a relative measure of Dispersion.

(4) Mean deviation is easy to calculate but since mean deviation has less mathematical value, it is rarely applied for biological statisgical analysis. It is also not meaning because negative sign of deviations is ignored.

(A) **Mean deviation from mean :**

(I) For Raw (ungrouped) data :

(i) Calculate the absolute measure of deviation from $\overline{x}$ .

Here $\bar{x} = \frac{\sum x_i}{n}$

(ii) Find the sum of all deviation.

(iii) Find the mean of the absolute values of the deviations.

i.e. mean deviation $= \frac{\Sigma\ |x_i - \bar{x}|}{n}$

**Example 8 :** Calculate the mean deviation of weight of 8 students of a class : **42, 47, 52, 47, 37, 60, 55, 38**

**Solution :** The following table is shown on the given data :

| $x_i$ | $\|x_i - \bar{x}\|$ |
|---|---|
| 42 | 5·25 |
| 47 | 0·25 |
| 52 | 4·75 |
| 47 | 0·25 |
| 37 | 10·25 |
| 60 | 12·75 |
| 55 | 7·75 |
| 38 | 9·25 |
| $\Sigma x_i$ $= 378$ | $\Sigma\|x_i - \bar{x}\|$ $= 50.50$ |

Mean $\bar{x} = \frac{\Sigma x_i}{n} = \frac{378}{8} = 47 \cdot 25$

Mean Deviation $= \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{50 \cdot 50}{8} = 6 \cdot 31$

**(II) For Discrete frequency distribution :**

(i) Find $\bar{x} = \frac{\Sigma f_i x_i}{N}, N = \Sigma f_i$ where $\sum f_i x_i$ denotes the sum of the products of observations $x_i$ with their respective frequencies *fi* and $N = \sum f_i$

(ii) Then, we find the deviations of observations $x_i$ from the mean $\bar{x}$ and take their absolute values.

(iii) After this, find the mean of the absolute values of the deviation, which is the required mean deviation about the mean. Thus

Mean Deviation $= \frac{\Sigma f_i |x_i - \bar{x}|}{N}$ , where $N = \Sigma f_i$

**Example 9 :** Calculate the Mean Deviation of the following distribution :

| $x$ | 3 | 9 | 17 | 23 | 27 |
|---|---|---|---|---|---|
| $f$ | 8 | 10 | 12 | 9 | 5 |

**Solution :** The following table is formed :

| $x$ | $f$ | $fx$ | $\|x-15\|$ | $f\|x-15\|$ |
|---|---|---|---|---|
| 3 | 8 | 24 | 12 | 96 |
| 9 | 10 | 90 | 6 | 60 |
| 17 | 12 | 204 | 2 | 24 |
| 23 | 9 | 207 | 8 | 72 |
| 27 | 5 | 135 | 12 | 60 |
| | $N=\Sigma f_i$ $=44$ | $\Sigma f_i x_i$ $=660$ | | $\Sigma f_i\|x_i-15\|$ $=312$ |

Mean $\quad \bar{x}=\frac{\Sigma f_i x_i}{N}=\frac{660}{44}=15$

Mean Deviation $=\frac{\Sigma f_i|x_i-\bar{x}|}{N}=\frac{312}{44}=7\cdot 09$

(III) Continuous frequency distribution

The same formula is applied to find the Mean Deviation. here mid value of each class will be taken at the place of values of variables.

**Example 10 :** Calculate the Mean Deviation of the following distribution :

| Class | 0–10 | 10–20 | 20–30 | 30–40 | 40–50 |
|---|---|---|---|---|---|
| frequency | 3 | 28 | 42 | 20 | 7 |

**Solution :** The following table is formed :

| Class | frequency $f$ | Class marks $x$ | $fx$ | $\|x-25\|$ | $fx\| -25\|$ |
|---|---|---|---|---|---|
| 0–10 | 3 | 5 | 15 | 20 | 60 |
| 10–20 | 28 | 15 | 420 | 10 | 280 |
| 20–30 | 42 | 25 | 1050 | 0 | 0 |
| 30–40 | 20 | 35 | 700 | 10 | 200 |
| 40–50 | 7 | 45 | 315 | 20 | 140 |
| | $N=\Sigma f_i$ $=100$ | | $\Sigma f_i x_i$ $=2500$ | | $\sum f_i\|x_i-25\|$ $=680$ |

$\therefore \quad \bar{x}=\frac{\Sigma f_i x_i}{N}=\frac{2500}{100}=25$

Mean Deviation $\frac{\Sigma f_i|x_i-\bar{x}|}{N}=\frac{680}{100}=6\cdot 80$

**Example 11 :** Find the Mean Deviation taken from the mean 20 for the following distribution table :

| $x$ | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
|---|---|---|---|---|---|---|---|---|
| $f$ | 5 | 8 | 21 | 24 | 18 | 15 | 7 | 2 |

**Solution :** The following table shows the calculation :

| $x$ | $f$ | $\lvert x_i - 20 \rvert$ | $f_i \lvert x_i - 20 \rvert$ |
|---|---|---|---|
| 10 | 5 | 10 | 50 |
| 12 | 8 | 8 | 64 |
| 14 | 21 | 6 | 126 |
| 16 | 24 | 4 | 96 |
| 18 | 18 | 2 | 36 |
| 20 | 15 | 0 | 0 |
| 22 | 7 | 2 | 14 |
| 24 | 2 | 4 | 8 |
| | $\Sigma f_i = 100$ | | $\Sigma f_i \lvert x_i - 20 \rvert = 394$ |

Mean Deviation taken from mean 20 $= \frac{1}{N} \Sigma f_i |x_i - 20|$

$$= \frac{394}{100} = 3 \cdot 94$$

**Example 12 :** Find the Mean Deviation and coefficient of Mean Deviation taken from origin point and scale transformation.

| Class | $0-3$ | $3-6$ | $6-9$ | $9-12$ | $12-15$ | $15-18$ | $18-21$ | |
|---|---|---|---|---|---|---|---|---|
| Frequency | 2 | 7 | 10 | 12 | 9 | 6 | 4 | |

**Solution :**

Here $a = 10 \cdot 5$, $h = 3$, where '$a$" is the assumed mean of class 9-12

| Class | frequency $f_i$ | mean $x_i$ | $u_i = \frac{x_i - 10.5}{3}$ | $f_i u_i$ | $\lvert x_i - \bar{x} \rvert$ | $f_i \lvert x_i - \bar{x} \rvert$ |
|---|---|---|---|---|---|---|
| 0–3 | 2 | 1·5 | –3 | –6 | 9·18 | 18·36 |
| 3–6 | 7 | 4·5 | –2 | –14 | 6·18 | 43·26 |
| 6–9 | 10 | 7·5 | –1 | –10 | 3·18 | 31·80 |
| 9–12 | 12 | 10·5 | 0 | 0 | 0·18 | 2·16 |
| 12–15 | 9 | 13·5 | 1 | 9 | 2·82 | 25·38 |
| 15–18 | 6 | 16·5 | 2 | 12 | 5·82 | 34·92 |
| 18–21 | 4 | 19·5 | 3 | 12 | 8·82 | 35·28 |
| | $N = \Sigma f_i = 50$ | | | $\Sigma f_i u_i = 3$ | | $\Sigma f_i \lvert x_i - \bar{x} \rvert = 19116$ |

$$\bar{x} = a + h \frac{\Sigma f_i u_i}{N} = 10 \cdot 5 + 3 \times \frac{3}{50} = 10 \cdot 68$$

Mean Deviation $= \frac{\Sigma f_i |x_i - \bar{x}|}{N} = \frac{191 \cdot 16}{50} = 3 \cdot 8232$

Coefficient of Mean Deviation $= \frac{\text{Mean Deviation}}{\bar{x}} = \frac{3 \cdot 8232}{10 \cdot 68} = 0 \cdot 358$

**(B) Mean deviation from median :**

(I) For Raw (ungrouped) data :

(i) To find the median, arrange the given values of $n$ data in ascending or descending order, then

(a) When $n$ is odd median will be $M=(n+1)/2$ th term.

(b) When $n$ is even than median will be the mean of $n/2$ and $(n/2)+1$ th term.

(ii) Find the value of deviations of $M$ from the given data (discard the sign)

(iii) Add the deviations.

(iv) Now divide the sum by the total number of terms in (iii).

Mean Deviation (Median) $(\delta_m)=\dfrac{\Sigma|x_i-M|}{n}$

**Example 13 :** Calculate the mean deviation from median using the following observation :

**38, 70, 48, 34, 42, 55, 63, 46, 54, 44**

**Solution :** Arranging the data in ascending order

34, 38, 42, 44, 46, 48, 54, 55, 63, 70 here $n=10$

$\therefore$ Median = Mean of 5th and 6th terms

$$M=\frac{46+48}{2}=47$$

Now we make a table using the given data :

| $x$ | $\lvert x-M\rvert$ |
|---|---|
| 38 | 9 |
| 70 | 23 |
| 48 | 1 |
| 34 | 13 |
| 42 | 5 |
| 55 | 8 |
| 63 | 16 |
| 46 | 1 |
| 54 | 7 |
| 44 | 3 |
| $n=10$ | $\dfrac{\Sigma\lvert x_i-M\rvert}{n}=86$ |

Mean deviation from median $(\delta_m)=\dfrac{\Sigma|x_i-47|}{10}=\dfrac{86}{10}=8\cdot 6$

**(II) For Continuous frequency distribution :**

(i) The process of finding the mean deviation about median for a continuous frequency distribution is similar as we did for mean deviation about the mean. The only difference lies in the replacement of the mean by median in distribution. The data is first arranged in asending order. Then the median of continuous frequency distribution is obtained by first identifying the class in which median lies (median class).

(ii) Find the unique value of deviations (ignoring the sign) of values of variable from median $M$.

(iii) Add the deviations.

(iv) Now find mean deviation by dividing sum obtained in (iii) to number of terms.

Mean deviation from median $(\delta_m) = \frac{\Sigma f_i |x_i - M|}{N}$

**Example 14 :** Calculate the mean deviation about median for the following data :

| Variable (x) | 4 | 6 | 8 | 10 | 12 | 14 | 16 | |
|---|---|---|---|---|---|---|---|---|
| Frequency (f) | 2 | 4 | 5 | 3 | 2 | 1 | 4 | |

**Solution :**

| Variable $x$ | frequency $f$ | commulative frequency $cf$ | $\|x - M\|$ | $f\|x - M\|$ |
|---|---|---|---|---|
| 4 | 2 | 2 | 4 | 8 |
| 6 | 4 | 6 | 2 | 8 |
| 8 | 5 | 11 | 0 | 0 |
| 10 | 3 | 14 | 2 | 6 |
| 12 | 2 | 16 | 4 | 8 |
| 14 | 1 | 17 | 6 | 6 |
| 16 | 4 | 21 | 8 | 32 |
| | $N = \Sigma f_i = 21$ | | | $\Sigma f_i \|x_i - M\| = 68$ |

Here $\frac{N}{2} = \frac{21}{2} = 10.5$ lies in the value of cummulative frequency 11, hence median M = 8

$\therefore$ Mean deviation from median $(\delta_m) = \frac{1}{N}\Sigma f_i |x_i - M| = \frac{68}{21} = 3 \cdot 238$

**(III) For grouped frequency distribution :**

(i) Same as (II) by finding the just greater cummulative frequency class, from mid term of commulative frequency also called as median class. Now median can be calculated using following formula :

$$\text{Median } (M) = \ell + \left[\frac{(N/2) - F}{f}\right] \times h$$

Wherer $\ell$ = lower limit of median class $N$ = sum of frequencies, $F$ = commulative frequency of class preceding the median class, $f$ = frequencyof median class, $h$ = class interval.

**Method :**

(ii) Find the value of deviations of values of variable of variable from median M (ignoring sign).

(iii) Sum up the deviations.

(iv) Now divide sum obtained in (iii) by number of terms and find mean deviation.

Mean deviation from median $(\delta_m) = \frac{\Sigma f_i |x_i - M|}{N}$

**Example 15 :** Calculate the median and mean deviation from median of marks obtained by 100 students :

| Marks | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
|---|---|---|---|---|---|
| No. of students | 8 | 30 | 40 | 12 | 10 |

**Solution :** Form the following table from the given data :

| Class interval | mean $x$ | No. of student $f$ | Commulative frequency $cf$ | $\|x-M\|$ | $f\|x-M\|$ |
|---|---|---|---|---|---|
| 0–10 | 5 | 8 | 8 | 18 | 144 |
| 10–20 | 15 | 30 | 38 | 8 | 240 |
| 20–30 | 25 | 40 | 78 | 2 | 80 |
| 30–40 | 35 | 12 | 90 | 12 | 144 |
| 40–50 | 45 | 10 | 100 | 22 | 220 |
| | | $N=\Sigma f_i = 100$ | | | $\Sigma f_I\|x_i - M\| = 828$ |

Here $\frac{N}{2}=\frac{100}{2}=50$ lies in the C.F. class 78 thus median class is 20-30

Median $M=\ell+\frac{(N/2)-F}{f}\times h=20+\frac{50-38}{40}\times 10=20+\frac{12}{4}=23$

Mean deviation from median $(\delta_m)=\frac{1}{N}\Sigma f_i\left|x_i-M\right|=\frac{828}{100}=8\cdot 28$

**(C) Mean deviation from mode** $(\delta_z)$ :

(I) For Raw (ungrouped) data :

(i) Find the value (variable) that occurs maximum times in data, which is called as mode ($z$).

(ii) $|x-z|$ is determined by substracting mode from the given observations.

(iii) Then the sum $\Sigma|x-z|$ is calculated

(iv) Using the formula $(\delta_z)=\frac{1}{n}\Sigma|x-z|$ mean deviation is obtained.

**Example 16 :** Calculate the Modal value and the mean deviation from that of the following data :

**8, 10, 15, 10, 12, 15, 8, 10, 8, 10, 12, 13.**

**Solution :** The maximum frequency of 10 is 4 thus $Z=10$

| Variable $x$ | $\|x-Z\|$ |
|---|---|
| 8 | 2 |
| 10 | 0 |
| 15 | 5 |
| 10 | 0 |
| 12 | 2 |
| 15 | 5 |
| 8 | 2 |
| 10 | 0 |
| 8 | 2 |
| 10 | 0 |
| 12 | 2 |
| 13 | 3 |
| | $\Sigma\|x_i - Z\| = 23$ |

Mean Deviation from mode $(\delta_z) = \frac{1}{n}\Sigma|x_i - Z| = \frac{23}{12} = 1.916$

**(II) method for ungrouped distribution :**

In ungroup distribution the variable whose frequency is maximum, the value of that variable is called as mode (z). Now remaining process of finding mean from mode is same as case (1).

**(III) Grouped frequency distribution :**

(i) In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. here, we can only locate a class with the maximum frequency, called the modal class. The mode is a value less in the modal class, and is given by the formula :

(ii) Formula mode $z = l + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$

where $f_0$ = frequency of the prodeding the modal class, $f_1$=frequency of the modal class, $f_2$ = frequency of the class succeeding the modal class, $l$ = lower limit of the modal class, $h$= size of the class interval (assuming all class sizes to be equal).

(iii) Find the absolute value by substracting mode from and mid value of class interval.

(iv) Multiplying absulute value with freqyencies and getting the sum, after than find mean deviation by divides the sum by sum of friquencies.

**Example 17 :** Calculare the model value and the mean deviation from mode of the following

| Class | 140 – 150 | 150 – – 160 | 160 – 170 | 170 – 180 | 180 – 190 | 190 – 200 | | |
|---|---|---|---|---|---|---|---|---|
| Frequencies | 4 | 5 | 10 | 12 | 9 | 3 | | |

**Solution :** Here taking Z = 174

| Class | mean $x$ | frequency $f$ | $\lvert x - z\rvert$ | $f\lvert x - z\rvert$ |
|---|---|---|---|---|
| 140–150 | 145 | 4 | 29 | 116 |
| 150–160 | 155 | 6 | 19 | 114 |
| 160–170 | 165 | 10 | 9 | 90 |
| 170–180 | 175 | 12 | 1 | 12 |
| 180–190 | 185 | 9 | 11 | 99 |
| 190–200 | 195 | 3 | 21 | 63 |
| | | $N = 44$ | | $\Sigma f_i\lvert x_i - z\rvert = 494$ |

here maximum frequency is 12, hence modal class is 170–180

$$\therefore \text{ Mode } z = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

$$= 170 + \frac{12 - 10}{2 \times 12 - 10 - 9} \times 10 = 170 + \frac{2 \times 10}{5} = 174$$

Mean Deviation (Mode) $= \frac{1}{N}\Sigma f_i\,|x_i - z| = \frac{494}{44} = 11 \cdot 22$

## Exercise 13.2

In Q.1 and Q.2 calculate the mean deviation about Mean.

1. 4, 7, 8, 9, 10, 12, 13, 17
2. 28, 60, 38, 30, 32, 45, 53, 36, 44, 34

In Q.3 and Q.4 calculate the mean deviation about Median.

3. 13, 10, 12, 13, 15, 18, 17, 11, 14, 16, 12
4. 26, 32, 35, 39, 41, 62, 36, 50, 43

In Q.5 and Q.6 calculate the mean deviation about Mode.

5. 2, 4, 6, 4, 8, 6, 4, 10, 4, 8
6. 2.2, 2.5, 2.1, 2.5, 2.9, 2.8, 2.5, 2.3

In Q.7 and Q.8 calculate the mean deviation about Mean .

7.

| $x_i$ | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| $f_i$ | 7 | 4 | 6 | 3 | 5 |

8.

| $x_i$ | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|
| $f_i$ | 2 | 12 | 14 | 8 | 4 |

In Q.9 and Q.10 calculate the mean deviation about Median.

9.

| $x_i$ | 5 | 7 | 9 | 10 | 12 | 15 |
|---|---|---|---|---|---|---|
| $f_i$ | 8 | 6 | 2 | 2 | 2 | 6 |

10.

| $x_i$ | 10 | 16 | 22 | 25 | 30 |
|---|---|---|---|---|---|
| $f_i$ | 3 | 5 | 6 | 7 | 8 |

In Q.11 and Q.12 calculate the mean deviation about Mode.

11.

| $x_i$ | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| $f_i$ | 2 | 4 | 6 | 3 | 2 | 1 |

12.

| $x_i$ | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| $f_i$ | 2 | 8 | 16 | 26 | 20 | 16 | 7 | 5 |

In Q.13 and Q.14 calculate the mean deviation about Mean.

13.

| Salary | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 |
|---|---|---|---|---|---|---|---|---|
| frequencies | 4 | 8 | 9 | 10 | 7 | 5 | 4 | 3 |

14.

| Height | 95 – 105 | 105 – 115 | 115 – 125 | 125 – 135 | 135 – 145 | 145 – 155 |
|---|---|---|---|---|---|---|
| Number | 9 | 13 | 26 | 30 | 12 | 10 |

In Q.15 and Q.16 calculate the mean deviation about Median.

15.

| Marks | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 |
|---|---|---|---|---|---|---|
| Number | 3 | 4 | 7 | 8 | 2 | 1 |

16.

| Age | 16 – 20 | 21 – 25 | 26 – 30 | 31 – 35 | 36 – 40 | 41 – 45 | 46 – 50 | 51 – 55 |
|---|---|---|---|---|---|---|---|---|
| Number | 5 | 6 | 12 | 14 | 26 | 12 | 16 | 9 |

In Q.17 and Q.18 calculate the mean deviation about Mode.

17.

| Marks | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 |
|---|---|---|---|---|---|
| Number | 8 | 24 | 42 | 20 | 6 |

18.

| Height (inch) | 52 – 55 | 55 – 58 | 58 – 61 | 61 – 64 |
|---|---|---|---|---|
| Number | 10 | 20 | 35 | 10 |

## 13.06 Standard deviation :

The mean deviation is the mean of total deviation ignoring of positive (+) and negative (-) signs. The sum of the deviations from the mean (minus signs ignored) is more than the sum of the deviations from median. Therefore, the mean deciation about the mean is not very scientific. Thus, in many cases, mean deviation may give unsatisfactory results. Also mean deviation is calcvulated on the basis of absolute values of the deviations and therefore, cannot be subjected to further algebraic treatment. This implies that we must have some other measure of dispersion. Standard deviation is such a measure dispersion.

**Definitions :**

**Standard deviation :** Taking square root of mean of squares of deviations is considered as Standard Deviation.

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$$

**Coefficient of standard deviation :** Standard deviation is a relative measure. To compare the two quantities we make use Standard deviation which is known as coefficient of standard deviation. The formula used is :

$$\text{Coefficient of standard deviation} = \frac{\sigma}{\bar{x}} \text{ or } \frac{\text{Standard Deviation}}{\text{Mean}}$$

Variance : The square of standard deviation is known as Variance.

$$\text{Variance } (\sigma^2) = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

**Coefficient of variation :** We want to compare the variability of two series or more than two series with same mean, which are measured in different units, we do not merely calculate the measures of dispersion but we require that measure which is independent of the unit. The measure of variability which is independent of unit is called coefficient of variation (denoted as C.V.). The coefficient fo variation is defined as

$$\text{Coefficient of Variation } = \frac{\sigma}{\bar{x}} \times 100$$

We shall now learn to calculate standard deviation for different data.

**(I) For Raw (Ungrouped) data :** let there be $n$ number of observation. $x_1, x_2, x_3, \ldots, x_n$ and their mean be $\bar{x}$ then.

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$$

another simplified formula can be used :

$$\sigma = \frac{1}{n}\sqrt{n\Sigma x_i^2 - (\Sigma x_i)^2}$$

**Example 18 :** Calculate the Standard deviation of marks obtained in Maths by 5 students in a class. 23, 46, 16, 25 and 20.

**Solution :** Arithmetic Mann $\overline{x} = \frac{23+46+16+25+20}{5} = \frac{130}{5} = 26$

| $x$ | $x-\overline{x}$ | $(x-\overline{x})^2$ |
|---|---|---|
| 23 | −3 | 9 |
| 46 | 20 | 400 |
| 16 | −10 | 100 |
| 25 | −1 | 1 |
| 20 | −6 | 36 |
| | | $\sum(x_i-\overline{x})^2 = 546$ |

Standard deviation $\sigma = \sqrt{\frac{\sum(x_i-\overline{x})^2}{n}} = \sqrt{\frac{546}{5}} = \sqrt{109 \cdot 2} = 10 \cdot 45$

**(II) For Ungrouped Frequency Distribution** : If for $n$ number of observations $x_1, x_2, x_3, ..., x_n$ with frequencies be $f_1, f_2, f_3, ..., f_n$ and their mean be $\overline{x}$ then

Standard deviation $\overline{x} = \sqrt{\frac{\sum f\left(x_i-\overline{x}\right)^2}{N}}$, where $\overline{x} = \frac{\sum f_i x_i}{N}$, $N = \sum f_i$

**Example 19 :** Calculate the Standard deviation for the following distribution

| **x** | **10** | **12** | **14** | **16** | **18** | **20** | **22** | **24** |
|---|---|---|---|---|---|---|---|---|
| **f** | **5** | **8** | **21** | **24** | **18** | **15** | **7** | **2** |

**Solution :**

| $x$ | $f$ | $fx$ | $(x-\overline{x})$ | $(x-\overline{x})^2$ | $f(x-\overline{x})^2$ |
|---|---|---|---|---|---|
| 10 | 5 | 50 | −6·5 | 42·25 | 211·25 |
| 12 | 8 | 96 | −4·5 | 20·25 | 162·00 |
| 14 | 21 | 294 | −2·5 | 6·25 | 131·25 |
| 16 | 24 | 384 | −0·5 | 0·25 | 6·00 |
| 18 | 18 | 324 | 1·5 | 2·25 | 40·50 |
| 20 | 15 | 300 | 3·5 | 12·25 | 183·75 |
| 22 | 7 | 154 | 5·5 | 30·25 | 211·75 |
| 24 | 2 | 48 | 7·5 | 56·25 | 112·50 |
| | $N=100$ | $\sum f_i x_i = 1650$ | | | $\sum f_i\left(x_i-\overline{x}\right)^2 = 1059 \cdot 00$ |

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{1650}{100} = 16 \cdot 50$$

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}} = \sqrt{\frac{1059}{100}} = 3 \cdot 25$$

**Short–cut methods for calculation of standard deviation** :

(i) As per the definition

$$\sigma^2{}_x = \frac{1}{N} \Sigma f_i (x_i - \bar{x})^2$$

$$= \frac{1}{N} \Sigma f_i (x_i^2 + \bar{x}^2 - 2x_i \bar{x})$$

$$= \frac{1}{N} \left( \Sigma f_i x_i{}^2 + \bar{x}^2 \Sigma f_i - 2\bar{x} \Sigma f_i x_i \right)$$

$$= \frac{1}{N} \left( \Sigma f_i x_i{}^2 + N\,\bar{x}^2 - 2N\,\bar{x}^2 \right) \qquad \left[ \because N = \Sigma f_i, \bar{x} = \frac{\Sigma f_i x_i}{N} \right]$$

$$= \frac{1}{N} \left( \Sigma f_i x_i{}^2 - N\,\bar{x}^2 \right)$$

$$= \frac{1}{N} \Sigma f_i x_i{}^2 - \left( \frac{1}{N} \Sigma f_i x_i \right)^2$$

Hence $\sigma_x = \sqrt{\frac{1}{N} \Sigma f_i x_i{}^2 - \left( \frac{1}{N} \Sigma f_i x_i \right)^2}$

(ii) Let Assumed Mean $= a$ and $x_i - a = d_i$ , then

$$\sigma_x^2 = \frac{1}{N} \Sigma f_i (x_i - \bar{x})^2$$

$$= \frac{1}{N} \Sigma f_i (x_i - a + a - \bar{x})^2$$

$$= \frac{1}{N} \Sigma f_i (d_i - d)^2 , \qquad \text{Where } d_i = x_i - a$$

$$= \frac{1}{N} \Sigma f_i d_i{}^2 - \left( \frac{1}{N} \Sigma f_i d_i \right)^2 = \sigma_d^2$$

Hence $\sigma_x = \sqrt{\frac{1}{N} \Sigma f_i d_i^2 - \left( \frac{1}{N} \Sigma f_i d_i \right)^2} = \sigma_d$, , (simplified)

(iii) **Step deviation method** :

If class intervals are equal in classified frequency series. By using step deviation method, it is possible to simplify the procedure. The formula is used as follows :

$$\sigma_x = h \times \sqrt{\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2} = h\sigma_u ,$$

Where $u_i = \frac{x_i - a}{h} = \frac{d_i}{h}$ and $d_i = hu_i$

In this method, Mean $\bar{x} = a + h\frac{\Sigma f_i u_i}{N}$

**Note :** Symbolically standard deviation $\sigma = \sigma_x = \sigma_d = h\sigma_u$

**Example 20 :** Calculate the Standard Deviation, Coefficient of Standard Deviation and Coefficient of variation of the following distribution :

| Class | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 |
|---|---|---|---|---|---|
| Frequency | 2 | 5 | 15 | 7 | 1 |

**Solution :**

| Class | Mid point $x$ | Frequency $f$ | $fx$ | $fx^2$ |
|---|---|---|---|---|
| 0–2 | 1 | 2 | 2 | 2 |
| 2–4 | 3 | 5 | 15 | 45 |
| 4–6 | 5 | 15 | 75 | 375 |
| 6–8 | 7 | 7 | 49 | 343 |
| 8–10 | 9 | 1 | 9 | 81 |
| | | $N = \Sigma f_i = 30$ | $\Sigma f_i x_i = 150$ | $\Sigma f_i x_i^2 = 846$ |

Standard Deviation $(\sigma) = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$

$$= \sqrt{\frac{846}{30} - \left(\frac{150}{30}\right)^2}$$

$$= \sqrt{28.2 - 25} = 1 \cdot 79$$

Coefficient of Standard Deviation $(\text{C.S.D.}) = \frac{\sigma}{\bar{x}} = \frac{1 \cdot 79}{5} = 0 \cdot 36$

Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100 = 0 \cdot 36 \times 100 = 36$

**Example 21 :** Calculate theCoefficient of Standard Deviation and Coefficient of variation from the following distribution :

| $x$ | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
|---|---|---|---|---|---|---|---|---|
| $f$ | 20 | 60 | 150 | 250 | 200 | 120 | 50 | 40 |

**Solution :**

Let the Assumed mean $a = 18$

| $x$ | $f$ | $d=x-18$ | $d^2$ | $fd$ | $fd^2$ |
|---|---|---|---|---|---|
| 9 | 20 | −9 | 81 | −180 | 1620 |
| 12 | 60 | −6 | 36 | −360 | 2160 |
| 15 | 150 | −3 | 9 | −450 | 1350 |
| 18 | 250 | 0 | 0 | 0 | 0 |
| 21 | 200 | 3 | 9 | 600 | 1800 |
| 24 | 120 | 6 | 36 | 720 | 4320 |
| 27 | 50 | 9 | 81 | 450 | 4050 |
| 30 | 40 | 12 | 144 | 480 | 5760 |
| | $N=$ 890 | | | $\Sigma f_i d_i$ =1260 | $\Sigma f_i d_i^2$ =21060 |

Standard Deviation $\sigma=\sqrt{\frac{1}{N}\Sigma f_i d_i^2-\left(\frac{1}{N}\Sigma f_i d_i\right)^2}=\sqrt{\frac{21060}{890}-\left(\frac{1260}{890}\right)^2}=\sqrt{23\cdot 66-2\cdot 004}=\sqrt{21\cdot 656}=4\cdot 65$

Mean $\bar{x}=a+\frac{1}{N}\Sigma f_i d_i=18+\frac{1260}{890}=19\cdot 41$

Cofficient of Standard Deviation $=\frac{\sigma}{\bar{x}}=\frac{4\cdot 65}{19\cdot 14}=0\cdot 242$

Cofficient of variation $=\frac{\sigma}{\bar{x}}\times 100 \quad =0\cdot 242\times 100=24\cdot 2$

**Example 22 :** Calculate the mean and standard deviation of sollowing distribution

| Class | 0 − 10 | 10 − 20 | 20 − 30 | 30 − 40 | 40 − 50 |
|---|---|---|---|---|---|
| No.of student | 5 | 8 | 15 | 16 | 6 |

**Solution :** We will find the solution using step deviation method. Let assumed mean $a=25$ which is mid value of class 20-30

| Class | Mean $x$ | No. of student $f$ | $u_i=\frac{x-25}{10}$ | $u_i^2$ | $f_i u$ | $f_i u^2$ |
|---|---|---|---|---|---|---|
| 0–10 | 5 | 5 | −2 | 4 | −10 | 20 |
| 10–20 | 15 | 8 | −1 | 1 | −8 | 8 |
| 20–30 | 25 | 15 | 0 | 0 | 0 | 0 |
| 30–40 | 35 | 16 | 1 | 1 | 16 | 16 |
| 40–50 | 45 | 6 | 2 | 4 | 12 | 24 |
| | | $N=50$ | | 10 | $\Sigma f_i u_i$ = 10 | $\Sigma f_i u_i^2$ = 68 |

Mean $\bar{x}=a+h\times\frac{\Sigma f_i u_i}{N}=25+\frac{10\times 10}{50}=27$

Standard Deviation $\sigma = h \times \sqrt{\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2}$

$$= 10 \times \sqrt{\frac{68}{50} - \left(\frac{10}{50}\right)^2} = 10 \times \sqrt{1 \cdot 32} = 10 \times 1 \cdot 1489 = 11 \cdot 489$$

For grouped frequency distribution : For grouped frequency distribution having equal class internal, find the Standard Deviation using the step deviation method discussed in short cut method.

## Exercise 13.3

**In Q.1 and Q.2 calculate the mean and variance.**

1.

| $x_i$ | 6 | 10 | 14 | 18 | 24 | 28 | 30 |
|---|---|---|---|---|---|---|---|
| $f_i$ | 2 | 4 | 7 | 12 | 8 | 4 | 3 |

2.

| $x_i$ | 82 | 83 | 87 | 88 | 92 | 94 | 99 |
|---|---|---|---|---|---|---|---|
| $f_i$ | 3 | 2 | 3 | 2 | 6 | 3 | 3 |

3. **Using short cut method calculate the mean and standard deviation :**

| $x_i$ | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 |
|---|---|---|---|---|---|---|---|---|---|
| $f_i$ | 2 | 1 | 12 | 29 | 25 | 12 | 10 | 4 | 5 |

**In Q.4 and Q.5 calculate the mean and variance.**

4.

| Class | 0 – 30 | 30 – 60 | 60 – 90 | 90 – 120 | 120 – 150 | 150 – 180 | 180 – 210 |
|---|---|---|---|---|---|---|---|
| Frequencies | 2 | 3 | 5 | 10 | 3 | 5 | 2 |

5.

| Class | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
|---|---|---|---|---|---|
| Number | 5 | 8 | 15 | 16 | 6 |

6. **Using short cut method find the the mean, variance and standard deviation of the following distribution :**

| Height (cm) | 70 – 75 | 75 – 80 | 80 – 85 | 85 – 90 | 90 – 95 | 95 – 100 | 100 – 105 | 105 – 110 | 110 – 115 |
|---|---|---|---|---|---|---|---|---|---|
| Frequency | 3 | 4 | 7 | 7 | 15 | 9 | 6 | 6 | 3 |

7. **Calculate the standard deviation of the diameters of circles :**

| Diam. (mm) | 43 – 46 | 47 – 50 | 51 – 54 | 55 – 58 | 59 – 62 |
|---|---|---|---|---|---|
| Circles | 15 | 17 | 21 | 22 | 25 |

8. **Calculate the standard deviation, coefficient of standard deviation and coefficient of variance of the following distribution :**

| Class | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 |
|---|---|---|---|---|---|
| Frequency | 2 | 5 | 15 | 7 | 1 |

**9.** **Calculate the standard deviation taking the assumed mean 35 of the following frequency distribution :**

35, 25, 33, 50, 37, 35, 33, 37, 30

**10.** **Calculate the standard deviation and coefficient of standard deviation by mean, median and mode :**

| **Monthly wages (Rs)** | **Number of Tenants** |
|---|---|
| less than 10 | 3 |
| less than 20 | 8 |
| less than 30 | 16 |
| less than 40 | 26 |
| less than 50 | 37 |
| less than 60 | 50 |
| less than 70 | 56 |
| less than 80 | 60 |

## Miscellaneous Exercise-13

**1.** The range of marks obtained by 5 students 20, 25, 15, 35 and 30 is -

(A) 15 (B) 20 (C) 25 (D) 30

**2.** The formula for inter Quartile range is -

(A) $Q_3 + Q_1$ (B) $Q_3 - Q_1$ (C) $Q_3 - Q_2$ (D) $Q_3 - Q_4$

**3.** Coefficient of range when the maximum value of a product is Rs 500 and minimum is Rs 75, is -

(A) 0.739 (B) 0.937 (C) 7.39 (D) 73.9

**4.** Coefficient of range for tha data 10, 20, 30, 40, 50, 60 is -

(A) 3 / 2 (B) 5 / 6 (C) 7 / 5 (D) 5 / 7

**5.** Mean deviation is least from :

(A) Mean (B) Median (C) Mode (D) Origin

**6.** Mean deviation of the marks of 4 students 25, 35, 45 and 55 is -

(A) 10 (B) 1 (C) 0 (D) 40

**7.** Mean deviation about median for the distribution 25, 35, 45 व 55 is -

(A) 13 / 7 (B) 1 / 2 (C) 11 / 7 (D) 2

**8.** If the mean for the distribution is $\overline{x} = 773$ and its mean deviation is 64.4 then its coefficient of mean deviation is -

(A) 0.065 (B) 12.003 (C) 0.083 (D) 0.073

**9.** Standard deviation of 6, 10, 4, 7, 4, 5

(A) $\sqrt{13/3}$ (B) 13 / 3 (C) $\sqrt{26}$ (D) $\sqrt{26}/6$

**10.** If the standard deviation is 1.4 then variance will be -

(A) 1.2 (B) 0.38 (C) 1.96 (D) 1.4

**11.** If the variance is $\sigma = \sqrt{\frac{\sum fd^2}{k} - \left(\frac{\sum fd}{30}\right)^2}$ then the value of $k$ is -

(A) 10 (B) 20 (C) 30 (D) 60

**12.** If in a distribution, coefficient of deviation is 30% and its standard deviation is 15 then its mean is -

(A) 0.5 (B) 5 (C) 2 (D) 50

**13.** Find the standard deviation of $\sum x^2 = 100,\ n = 5$ and $\sum x = 20$ -

(A) 16 (B) 2 (C) 4 (D) 8

**14.** The temperature in Centigrade of a city is $18, 12, 6, -7, -12, 5, -4$, the range will be -

(A) 6 (B) 30 (C) 22 (D) 14

**15.** If $N = 10,\ \sum x = 120$ and $\sigma_x = 60$ then coifficient of variation is -

(A) 5 (B) 50 (C) 500 (D) 0.5

**16.** The sum of deviation taken from mean is -

(A) Negative (B) Positive (C) Different everytime (D) Zero

**17.** If $\overline{x} = 6,\ \sum x = 60$ and $\sum x^2 = 1000$ then the value of $\sigma_x$

(A) 6 (B) 8 (C) 64 (D) 10

**18.** Coefficient of range is defined as -

(A) $\frac{H-L}{2}$ (B) $\frac{H+L}{2}$ (C) $\frac{H-L}{H+L}$ (D) $\frac{H+L}{H-L}$

**19.** If the terms in the data are all equal then find the value of dispersion.

**20.** Write the formula to find standard deviation.

**21.** If the standard deviation is 20.5 and the Arithmetic mean is 60 then find its coefficient of standard deviation.

**22.** Calculate the inter quartile range, coefficient of Inter quartile range, Quartile deviation and coefficient of quartile deviation for the following distribution.

| Class | 0 | 15 | 30 | 45 | 60 | 75 | 90 | 105 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 150 | 140 | 100 | 80 | 70 | 30 | 14 | 0 |

**23.** using the step deviation method find the mean and standard deviation for the distribution -

| Class | 140–150 | 150––160 | 160–170 | 170–180 | 180–190 | 190–200 | |
|---|---|---|---|---|---|---|---|
| Frequencies | 4 | 5 | 10 | 12 | 9 | 3 | |

**24.** Calculate the mean deviation from mode and its coefficient -

| $x$ | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|
| $f$ | 3 | 6 | 9 | 13 | 8 | 5 | 4 |

**25.** Calculate the variance of the following frequency distribution -

| Class | 32 – 38 | 38 – 44 | 44 – 50 | 50 – 56 | 56 – 62 | 62 – 68 |
|---|---|---|---|---|---|---|
| Frequency | 3 | 6 | 9 | 13 | 8 | 5 |

## Important Points

1. measures of dispersion range, quartile deviation , mean deviation, variance, standard deciation are measures of dispersion.
   Range = Maximum value - Minimum value
2. Range : In series, the difference of highest ($H$) and lowest ($L$) value of variable is called range :
   Coefficient of range $(C.R.) = \frac{H-L}{H+L}$
3. Inter quartile range $(I.Q.R.) = Q_3 - Q_1$

4. Quartile deviation $(Q.D.) = \frac{Q_3 - Q_1}{2}$ = Semi Inter quartile range

5. Coefficient of Quartile deviation $(C.Q.D.) = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

6. Mean deviation : In any series, the arithmetics means of various values of variable and the value of deviation from their numerical mean (mean, mode or median) is called its mean deviation.

   (i) When data grouped

   Mean deviation from $A = \frac{\Sigma f_i |x_i - A|}{N}$, where $N$ is numerical mean

   (ii) For ungrouped frequency distribution :

   Mean deviation from $A = \frac{1}{N} \Sigma f_i |x_i - A|$, where $N = \Sigma f_i$

   (iii) When data in grouped from then the formula in step (ii) is used. here $x_i$ is the mid value of corresponding class.

7. Coefficient of mean deviation $= \frac{\text{Mean Deviation}}{A}$ where A is the mean from which mean deviation has been taken.

8. Variance and standard deviation : The mean of squares of deviations, from arithmetic mean of variable values of series is called variance. The positive square root of variance is called as standard deviation.

   (i) When data is ungrouped :

   Standard deviation $(\sigma) = \sqrt{\frac{\Sigma (x_i - \overline{x})^2}{n}} = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

   (ii) When data is in the form of ungrouped or grouped frequency distribution

   Standard deviation $\sigma = \sqrt{\frac{\Sigma f_i (x_i - \overline{x})^2}{N}} = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$

9. Changing the origin : If assumed mean is $a$,

   (i) then for ungrouped data.

   $$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}, \text{ where } d_i = x_i - a$$

   (ii) For ungrouped data and grouped frequency distribution

   $$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

   (iii) For step deviation : If assumed mean is $a$ and class internal of each class in grouped frequency is $h$ then taking $u_i = \frac{x_i - a}{h}$

   Standard deviation $\sigma = h\sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2}$

   Coefficient of Standard deviation $= \frac{\sigma}{\overline{x}}$

Coefficient of variation $= \frac{\sigma}{\overline{x}} \times 100$

# Answers

## Exercise 13.1

**1.** $\frac{Q_3 - Q_1}{Q_3 - Q_1}$ **2.** 30 **3.** 6, 0.352 **4.** 7, 0.44 **5.** 2, 0.5

**6.** 0.375 **7.** 59.44, 59.44 **8.** 34.375, 34.375

## Exercise 13.2

**1.** 3 **2.** 8.4 **3.** 2 **4.** 7 **5.** 2 **6.** 0.2 **7.** 6.32

**8.** 16 **9.** 3.33 **10.** 5.1 **11.** 1 **12.** 13.1 **13.** 15.792 **14.** 11.28

**15.** 10.34 **16.** 7.35 **17.** 7.38 **18.** 2.075

## Exercise 13.3

**1.** 19, 43.4 **2.** 90, 29.09 **3.** 74, 1.69 **4.** 107, 2276 **5.** 27, 132

**6.** 93, 105.52, 10.27 **7.** 5.55 **8.** 17.9, 0.358, 35.8 **9.** 6.376

**10.** 15.18, 15.36, 15.22, 0.348, 0.363, 0.351

## Miscellaneous Exercise 13

**1.** (B) **2.** (B) **3.** (A) **4.** (D) **5.** (B) **6.** (A) **7.** (D)

**8.** (C) **9.** (A) **10.** (C) **11.** (C) **12.** (D) **13.** (B) **14.** (B)

**15.** (C) **16.** (D) **17.** (B) **18.** (C) **19.** 0 **20.** $\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

**21.** 0.34 **22.** 46.875; 0.48; 23.4375; 0.48 **23.** $\overline{x} = 17$, $\sigma_x = 6.44$

**24.** $\delta_z = 1.265$, $\delta_z = 0.141$ **25.** 113.4